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Brief paper

The lower convergence tendency of imitators compared to best responders☆

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ABSTRACT

Imitation is widely observed in nature and often used to model populations of decision-making agents, but it is *not* yet known under what conditions a network of imitators will reach a state where they are satisfied with their decisions. We show that every network in which agents imitate the best performing strategy in their neighborhood will reach an equilibrium in finite time, provided that all agents are *opponent coordinating*, i.e., earn a higher payoff if their opponent plays the same strategy as they do. It follows that any non-convergence observed in imitative networks is not necessarily a result of population heterogeneity nor special network topology, but rather must be caused by other factors such as the presence of non-opponent-coordinating agents. To strengthen this result, we show that large classes of imitative networks containing non-opponent-coordinating agents never equilibrate even when the population is homogeneous. Comparing to best-response dynamics where equilibration is guaranteed for every network of homogeneous agents playing 2×2 matrix games, our results imply that networks of imitators have a lower equilibration tendency.

1. Introduction

In social, economic, biological, technological, and other types of networks, the dynamics of interconnected agents may give rise to complex and seemingly unpredictable behaviors. While some networks may converge to a state of equilibrium, others may perpetually cycle or experience chaotic fluctuations (Govaert, Cenedese, Grammatico, & Cao, 2019; Meloni, Arenas, & Moreno, 2009; Skyrms & Pemantle, 2000; Strogatz, 2001). Unfortunately, localized analysis may reveal little about the underlying causes of these emergent behaviors, in part because the major factors driving the dynamics may lie not in the individual agents but in their complex interconnections. However, studying the system from a broader perspective, at the cost of simplifying the agent-level dynamics, allows studying critical problems such as finding the conditions on the agents under which a network is likely to converge or not (Riehl, Ramazi, & Cao, 2018; Sandholm, 2010). Indeed we have seen a substantial transition from local to network-based analysis across various disciplines in the physical and social sciences and engineering. This network-oriented approach (captured by graphical games (Kearns, Littman, & Singh, 2013)) has led to many influential discoveries of collective behaviors, even when the individual agents are autonomous and highly complex (De Domenico, Granell, Porter, & Arenas, 2016; Kitsak et al., 2010).

Two common update rules describing individuals' strategy revisions in collective behaviors are *best-response* (Abouheaf, Lewis, Vamvoudakis, Haesaert, & Babuska, 2014; Cortés & Martínez, 2014; Mäs & Nax, 2016; Ramazi & Cao, 2017; Swenson, Murray, & Kar, 2018), that is, to play the strategy maximizing your payoff against your neighbors, and *imitation* (Barreiro-Gomez & Tembine, 2018; Chang, Piraveenan, & Prokopenko, 2019; Cheng, Qi, He, Xu, & Dong, 2014; Hu, Zhang, & Tian, 2019; Zhu, Xia, & Wu, 2016), that is, to play the strategy of the maximum-earning individual in your neighborhood. A best-responder needs to know the strategy distribution among her neighbors and how her payoff depends on her own and her neighbors' strategies, whereas an imitator needs to know her neighbors' payoffs and the strategy of the maximum earner. Therefore, although leading to possibly lower cooperation levels in the network (van den Berg, Molleman, & Weissing, 2015), imitation is preferred in situations where little, if not none, is known about the available strategies or their consequences (Ackermann, Berenbrink, Fischer, & Hoefer, 2016). A fundamental yet less-studied problem for networks governed

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by these two update rules is whether each individual will settle on a particular action, or equivalently, whether the network will equilibrate. We have recently shown that any network of individuals, either all playing coordination or all playing anticoordination games, and updating according to the best-response rule, will reach an equilibrium in finite time (Ramazi, Riehl, & Cao, 2016). Therefore, none of network topology, individual heterogeneity and the sequence in which agents revise their strategies can cause non-convergence behavior in these cases. For imitation, however, it is not yet known under what conditions, and particularly for which types of games, the network will settle at an equilibrium. Previous studies have investigated this problem for well-mixed populations (Fu & Ramazi, 2019) and when individuals are homogeneous, i.e., share the same utility function (Griffin, Rajtmajer, Squicciarini, & Belmonte, 2019), where the evolution of mixed, rather than pure, strategies is studied (He & Tadmor, 2019). By means of numerical simulations, it was shown in Cimini, Castellano, and Sánchez (2015) and Govaert, Ramazi, and Cao (2017) that even for homogeneous agents updating synchronously, imitation does not generally result in equilibrium convergence, although it appears likely for coordination games. Moreover, networks of homogeneous individuals playing congestion games are shown to converge to an equilibrium in Ackermann et al. (2016) and similar convergence results hold for potential games (Zino, Como, & Fagnani, 2017). It remains to be seen under what conditions arbitrary networks of heterogeneous imitators, updating synchronously or asynchronously, and choosing from an arbitrary number of available strategies, can be expected to equilibrate, and that is the primary goal of this paper.

The key equilibration criterion turns out to be whether agents earn a higher payoff when their opponents play the same strategy as them. We call such agents *opponent-coordinating*, and we show that every network consisting entirely of opponent-coordinating agents will equilibrate under asynchronous updates. Although not equal, the set of payoff matrices corresponding to such agents has a significant intersection with that of coordination games, which explains why they are reported to be more likely to drive the network to equilibration. We then show that if the agents satisfy the more-constrained condition of being *strongly opponent-coordinating*, then equilibration is guaranteed even under fully or partially synchronous updates. In addition to illuminating causes of perpetual fluctuations, knowing when a network is expected to converge opens up avenues for further research into control. As the second contribution, we provide several never-equilibrating examples on heterogeneous networks, including non-opponent-coordinating agents, which can be extended to substantial classes of networks of homogeneous agents. The comparison of these results with those of the best-response update rule (Kreindler & Young, 2014; Montanari & Saberi, 2010; Young, 2011), known to govern any network of homogeneous agents to an equilibrium (Ramazi et al., 2016), postulates convergence under imitation as a relatively rare phenomenon, and also helps to explain why networks in which imitation is prevalent may exhibit cyclic or chaotic behavior more often (Fu & Ramazi, 2020).

2. Asynchronous imitation updates

Consider an undirected network $\mathbb{G} = (\mathcal{V}, \mathcal{E})$ where the nodes $\mathcal{V} = \{1, \dots, n\}$ correspond to agents who over time $k \in \{0, 1, \dots\}$, play 2-player games with their neighbors indicated by the edges $\mathcal{E}$. The agents start with one of the strategies $1, 2, \dots, m$ at $k = 0$. At each time step, every agent earns an accumulated payoff against her neighbors, and then one random agent activates to mimic the strategy of the most successful agent in her neighborhood. More specifically, the possible payoffs of an agent $i \in \mathcal{V}$ against another agent j are summarized in the *payoff matrix* $\pi^i \in \mathbb{R}^{m\times m}$ whose pqth entry corresponds to the strategy pair p-against-q where $p, q \in \{1, \dots, m\}$. Let $x_i \in \{1, \dots, m\}$ denote the strategy of agent i. Then the (accumulated) payoff or utility of agent i against her neighbors is calculated by

$$u_i = \sum_{j\in\mathcal{N}_i} \pi^i_{x_i,x_j},$$

where $\mathcal{N}_i$ denotes the set of neighbors of agent i.

Consequently, agents with more neighbors may earn more regardless of their strategies. However, this heterogeneous setup allows the payoff matrices to be normalized such that the maximum possible utility for every agent becomes the same. The imitation update rule for agent i, active at time k, dictates that agent i updates her strategy at $k+1$ to the strategy of the agent earning the highest payoff at k in her neighborhood $\mathcal{N}_i \cup \{i\}$. In case several agents with different strategies earn the highest payoff, we assume agent i sticks to her current strategy if she is also earning the highest payoff. Otherwise, we assume a preference on the strategies such that agent i chooses the smallest strategy in magnitude, namely

$$x_i(k+1) = \begin{cases} x_i(k) & x_i(k) \in \mathcal{S}^M_i(k) \\ \min \mathcal{S}^M_i(k) & x_i(k) \notin \mathcal{S}^M_i(k) \end{cases} \tag{1}$$

where $\mathcal{S}^M_i(k)$ is the set of strategies resulting in the maximum payoff at time k in the neighborhood of agent i, that is

$$\mathcal{S}^M_i(k) \triangleq \left\{ x_j(k) \,\middle|\, j = \arg\max_{r\in\mathcal{N}_i\cup\{i\}} u_r(k) \right\}.$$

Without such a deterministic tie breaking rule, situations could arise where agents switch between strategies of their equally high-earning neighbors. This could unrealistically reduce the number of equilibrium states and generally make the network dynamics less likely to converge. We study the evolution of the strategy vector $x(k) = (x_1(k), \dots, x_n(k))^\top$, under update rule (1) and the activation sequence of the agents, defining the asynchronous imitation dynamics. We assume that the activation sequence is *persistent*; that is, each agent activates infinitely many times.

We provide three motivating examples. The agents may be seen as researchers who decide to work on the same scientific topic as their highly-reputed fellows do. Their utilities may be their satisfaction on the quality and quantity of their published work (Fu & Ramazi, 2020). A similar setup applies to people deciding whether to take the vaccination of a seasonal disease, based on the anecdotal evidence (Fu, Rosenbloom, Wang, & Nowak, 2011). The agents can also represent autonomous robots deciding to push or not push an obstacle they encounter during a foregoing task. Each robot's utility indicates changes in its energy level and whether the group task, i.e., removing the obstacle, has been fulfilled (Wang, Chen, Xie, & Cao, 2017). Each robot can be programmed to occasionally imitate those with the highest utility at the previous encounter.

An equilibrium of the dynamics is a state x^* at which none of the agents will change strategies if active, implying that if $x(k) = x^*$ for some $k \geq 0$, then $x(k+1) = x^*$, regardless of which agent is active at k. This is different from a *Nash equilibrium* where no agent earns more by switching strategies. For example, a state where all agents play the same strategy, say *to cooperate*, is an equilibrium of the imitation dynamics, yet it is not a Nash equilibrium if the payoff matrices correspond to that of the prisoner's dilemma (Riehl et al., 2018). Because of the implicit stochasticity caused by the activation sequence and the nonlinearity of the imitation dynamics, convergence of $x(k)$ to an equilibrium is not guaranteed. Indeed, we provide examples where $x(k)$ fluctuates in a set of several states in the long run and never converges to a single state. However, there are types of payoff matrices for which the network always reaches an equilibrium, as we show in the next section.

3. Convergence under asynchronous updates

In a 2-player matrix game, the best response strategy is the one corresponding to the maximum entry in the column of the payoff matrix defined by the opponent's strategy. It is, therefore, intuitive that convergence relies on the ordering of the payoff values within each column (which determines whether the agents are coordinating or anti-coordinating Ramazi et al., 2016). However, under imitation, an agent never compares the payoffs she may earn upon playing different strategies. In what follows, we show that the ordering of payoff values within each row appears to be the key. We say a payoff matrix π is *opponent-coordination* if each diagonal entry is greater than all off-diagonal entries in the same row, that is

$$\pi_{p,p} > \pi_{p,q} \qquad \forall p, q \in \{1, \dots, m\}, p \neq q. \tag{2}$$

Intuitively, if a neighbor of agent i switches her strategy to that of agent i, then agent i's payoff increases. The payoff matrix of a coordination game satisfies a similar condition but over the columns, i.e., each diagonal entry is the greatest in its column. If each diagonal entry is additionally also greatest in its row, then the coordination payoff matrix becomes opponent-coordination. Hence, payoff matrices of coordination games may or may not satisfy the condition, however, those of snowdrift and prisoners' dilemma games never do (Riehl et al., 2018). We call agents with opponent-coordination payoff matrices, *opponent coordinating*.

Theorem 1. *Every network of opponent-coordinating agents reaches an equilibrium under the asynchronous imitation update rule.*

Back to our examples, the theorem implies that if each scholar experiences higher quality and quantity research, and thus, higher satisfaction when more researchers work on her topic, then we expect the network of researchers to settle on their topics of research eventually. While the same holds for autonomous robots, it may not be the case for the vaccination example as more individuals taking the vaccination yields a more immune community, lowering the infection risk and motivation for vaccination.

For the proof, we make use of the energy-like function $u_{\mathcal{M}}$, the payoff of the agent(s) with maximum payoff (a similar idea has been discussed in Blanchini, Casagrande, Giordano, & Viaro, 2019). Define $\mathcal{M}(k)$ as the set of agents in the network that have the maximum payoff at time k, i.e.,

$$\mathcal{M}(k) = \left\{ i \,\middle|\, u_i(k) = \max_{j \in \mathcal{V}} u_j(k) \right\}.$$

Clearly, $\mathcal{M}(k)$ is nonempty for all $k \geq 0$. At time k, every member of $\mathcal{M}(k)$ earns the same payoff that we denote by $u_{\mathcal{M}(k)}$, i.e., $u_{\mathcal{M}(k)} = u_i(k) \forall i \in \mathcal{M}(k)$. Since $u_{\mathcal{M}}$ is the sum of a finite set of fixed payoff values, it is trivially upper bounded. The following lemma guarantees that $u_{\mathcal{M}}$ is always non-decreasing, and increases when any of the neighbors of any of the agents with the maximum payoff switches strategies. To simplify the statements of the lemmas, we make the following assumption.

Assumption 1. The payoff matrix of every agent in the network is opponent-coordination.

Lemma 1. *The following holds under Assumption 1:*

$$u_{\mathcal{M}(k+1)} \geq u_{\mathcal{M}(k)} \qquad \forall k \geq 0. \tag{3}$$

Moreover, given $k \geq 0$, if a neighbor of an agent $i \in \mathcal{M}(k)$ switches strategies at $k+1$, then

$$u_{\mathcal{M}(k+1)} > u_{\mathcal{M}(k)}. \tag{4}$$

Proof. Consider an agent $i \in \mathcal{M}(k)$. This agent does not change her strategy at $k+1$ since she is already earning the maximum payoff in the network. In general, one of the following two cases takes place.

Case 1: None of the neighbors of agent i switch strategies at $k+1$. Then the payoff of agent i does not change, i.e.,

$$u_i(k+1) = u_i(k). \tag{5}$$

Now one of the following two cases can happen at $k+1$.

Case 1-1: $i \in \mathcal{M}(k+1)$. Then according to (5), $u_{\mathcal{M}(k+1)} = u_{\mathcal{M}(k)}$.
Case 1-2: $i \notin \mathcal{M}(k+1)$. Then there exists an agent $j \in \mathcal{M}(k+1)$ who earns more than agent i at $k+1$. This in view of (5) implies $u_j(k+1) > u_i(k)$, resulting in $u_{\mathcal{M}(k+1)} > u_{\mathcal{M}(k)}$. By summarizing the two cases, we arrive at (3).

Case 2: A neighbor r of agent i, switches strategies at $k+1$. According to update rule (1), agent r changes her strategy to that of one of her neighbors, say j, that has the highest payoff among the rest (j may equal i). Hence, since agent i is already earning the highest payoff in the network at k, agent j must also do so, i.e., $j \in \mathcal{M}(k)$. Therefore, agent j does not switch strategies at $k+1$. Moreover, because of the asynchronous updating, no other neighbor of j except for agent r switches strategies at $k+1$. So, the payoff of agent j at k equals

$$u_j(k) = \pi^j_{x_j(k),x_r(k)} + \sum_{s \in \mathcal{N}_j - \{r\}} \pi^j_{x_j(k),x_s(k)}$$

and at $k+1$ equals

$$u_j(k+1) = \pi^j_{x_j(k),x_j(k)} + \sum_{s \in \mathcal{N}_j - \{r\}} \pi^j_{x_j(k),x_s(k)}$$

Hence, according to (2),

$$u_j(k+1) - u_j(k) = \pi^j_{x_j(k),x_j(k)} - \pi^j_{x_j(k),x_r(k)} > 0. \tag{6}$$

Now one of the following two cases can happen at $k+1$.

Case 2-1: $j \in \mathcal{M}(k+1)$. Then according to (6), $u_{\mathcal{M}(k+1)} > u_{\mathcal{M}(k)}$.

Case 2-2: $j \notin \mathcal{M}(k+1)$. Then there exists an agent $q \in \mathcal{M}(k+1)$ who earns more than agent j at $k+1$. This in view of (6) implies $u_q(k+1) > u_j(k)$, resulting in $u_{\mathcal{M}(k+1)} > u_{\mathcal{M}(k)}$. By summarizing the two cases, we arrive at (4), which completes the proof. □

Using Lemma 1, we can show the convergence of the strategies of the agents with maximum payoffs.

Lemma 2. *Under Assumption 1, there exists some time $k_1 \geq 0$ after which the strategies of all agents in $\mathcal{M}(k_1)$ and their neighbors remain unchanged, i.e.,*

$$x_j(k) = x_j(k_1) \; \forall j \in \mathcal{N}_i \cup \{i\} \; \forall i \in \mathcal{M}(k_1) \forall k \geq k_1. \tag{7}$$

Proof. Since $u_{\mathcal{M}(k)}$ is upper-bounded, Lemma 1 implies the existence of some time $k_{\mathcal{M}} \geq 0$, such that

$$u_{\mathcal{M}(k)} = u_{\mathcal{M}(k_{\mathcal{M}})} \qquad \forall k \geq k_{\mathcal{M}}, \tag{8}$$

and that for $k \geq k_{\mathcal{M}}$, none of the neighbors of any agent $i \in \mathcal{M}(k)$ switch strategies at $k+1$ as otherwise (4) would be violated:

$$x_j(k) = x_j(k+1) \qquad \forall j \in \mathcal{N}_i \; \forall i \in \mathcal{M}(k), \; \forall k \geq k_{\mathcal{M}}. \tag{9}$$

Now, (8) implies that if an agent earns the maximum payoff at some time $k \geq k_{\mathcal{M}}$, she will keep doing so in future time steps, yielding $\mathcal{M}(k) \subseteq \mathcal{M}(k+1)$ for all $k \geq k_{\mathcal{M}}$. Therefore, since $\mathcal{M}$ is upper-bounded by $\mathcal{V}$, it will become fixed after some time $k_1 \geq k_{\mathcal{M}}$, i.e.,

$$\mathcal{M}(k) = \mathcal{M}(k_1) \forall k \geq k_1.$$

Hence, in view of (9),

$$x_j(k) = x_j(k+1) \qquad \forall j \in \mathcal{N}_i \, \forall i \in \mathcal{M}(k_1) \forall k \geq k_1,$$

resulting in

$$x_j(k) = x_j(k_1) \qquad \forall j \in \mathcal{N}_i \, \forall i \in \mathcal{M}(k_1) \forall k \geq k_1.$$

Now since the strategies of all neighbors of each agent $i \in \mathcal{M}(k_1)$ are fixed, agent i's strategy also becomes fixed in view of update rule (1), which results in (7). □

Now consider the time k_1 in Lemma 2, and define the set $\mathcal{V}_2 = \mathcal{V} - \mathcal{M}(k_1)$. If $\mathcal{V}_2$ is empty, the network has reached a state which has to be an equilibrium due to the persistent assumption on the activation sequence. Otherwise, define $\mathcal{M}_2$ similar to how $\mathcal{M}$ is defined, i.e.,

$$\mathcal{M}_2(k) = \left\{ i \,\middle|\, u_i(k) = \max_{j \in \mathcal{V}_2} u_j(k) \right\}.$$

Clearly, $\mathcal{M}_2(k)$ is nonempty for all $k \geq k_1$. Denote by $u_{\mathcal{M}_2(k)}$ the payoff of any member in $\mathcal{M}_2(k)$ at time k, i.e.,

$$u_{\mathcal{M}_2(k)} = u_i(k) \forall i \in \mathcal{M}_2(k).$$

One can prove a similar result to that of Lemma 1, for $u_{\mathcal{M}_2(k)}$, as stated in the following lemma. The proof is omitted due to limited space.

Lemma 3. *Consider the time k_1 in Lemma 2. Under Assumption 1, it holds that*

$$u_{\mathcal{M}_2(k+1)} \geq u_{\mathcal{M}_2(k)} \forall k \geq k_1.$$

Moreover, given $k \geq k_1$, if a neighbor of an agent $i \in \mathcal{M}_2(k)$ switches strategies at $k+1$, then

$$u_{\mathcal{M}_2(k+1)} > u_{\mathcal{M}_2(k)}.$$

The following lemma can also be proven similar to Lemma 2, which guarantees the convergence of the strategies of agents with the second-maximum payoffs.

Lemma 4. *Consider the time k_1 in Lemma 2. Under Assumption 1, there exists some time $k_2 \geq k_1$ after which the strategies of all agents in $\mathcal{M}_2(k_2)$ and their neighbors remain unchanged, i.e.,*

$$x_j(k) = x_j(k_1) \qquad \forall j \in \mathcal{N}_i \cup \{i\} \, \forall i \in \mathcal{M}_2(k_2) \forall k \geq k_2.$$

Now consider the time k_2 in Lemma 4, and define the set $\mathcal{V}_3 = \mathcal{V}_2 - \mathcal{M}_2(k_2)$. If $\mathcal{V}_3$ is empty, the network has reached an equilibrium. Otherwise, we again define $\mathcal{M}_3$ similar to how $\mathcal{M}_2$ is defined, and show that the same result as in Lemma 4 holds for $\mathcal{M}_3$. We continue this procedure and define $\mathcal{V}_3, \mathcal{V}_4, \ldots$ until we reach an empty set $\mathcal{V}_l$, $l > 0$. This will certainly happen since $|\mathcal{V}_i| < |\mathcal{V}_{i-1}|$ for all $i \geq 1$ such that $\mathcal{V}_i \neq \emptyset$. Then the network has reached an equilibrium, which completes the proof of Theorem 1.

3.1. Convergence time

Define

$$L \triangleq nm \sum_{i \in \mathcal{V}} \binom{\deg_i + m - 1}{m-1}.$$

The following result provides an upper bound for the equilibrium convergence time in terms of the number of agents' switches of strategies.

Proposition 1. *Every network of agents with opponent-coordination payoff matrices equilibrates after at most $(L+1)^{n-1}$ number of strategy switches.*

Proof. Consider a network of opponent-coordinators $\mathbb{G}$. Starting from an arbitrary initial state and under any activation sequence, let $\bar{L}$ denote the maximum number of changes in the maximum utility $u_{\mathcal{M}}$, and r denote the maximum number of switches between two consecutive changes of $u_{\mathcal{M}}$. According to Lemma 1, $u_{\mathcal{M}}$ is a monotonically increasing function. Thus, by at most $(r+1)\bar{L}$ switches, $u_{\mathcal{M}}$ reaches its maximum. Again, starting from an arbitrary initial state and under any activation sequence, let $M(\mathbb{G})$ denote the maximum number of switches that it takes for the network to equilibrate, and $M_i(\mathbb{G})$ be the maximum number of switches that all non-neighbors of agent i, i.e., $\mathcal{V} - \mathcal{N}_i - \{i\}$, can make, given that agent i and her neighbors remain fixed in their strategies. Let p_1 be an agent with the maximum $M_i(\mathbb{G})$, i.e., $p_1 \in \arg\max_{i \in \mathcal{V}} M_i(\mathbb{G})$. Once $u_{\mathcal{M}}$ reaches its maximum, at most $M_{p_1}(\mathbb{G})$ more switches can happen until the network equilibrates. Hence,

$$M(\mathbb{G}) \leq (r+1)\bar{L} + M_{p_1}(\mathbb{G}). \tag{10}$$

Now, we obtain $\bar{L}$. First, we find the number of different utilities u_i that an agent i may earn. The utility of agent i can be written as $u_i = \mathbf{1}_{x_i}^\top \pi^i s$, where $\mathbf{1}_j$ is the jth column of the $m \times m$ identity matrix and $s \in \{0, 1, \ldots, \deg_i\}^m$ is the stacking of s_j, the number of agent i's neighbors who play strategy j, for $j = 1, \ldots, m$. Hence, the number of distinct values of u_i is at most equal to the number of distinct combinations of $\mathbf{1}_{x_i}$ and s. The vector $\mathbf{1}_{x_i}$ takes m different forms. The number of possible vectors s equals the number of distinct solutions to the equation

$$s_1 + s_2 + \cdots + s_m = \deg_i, \quad s_j \in \mathbb{Z}_{\geq 0} \forall j \in \{1, \ldots, m\},$$

which is the number of m-compositions of $\deg_i$ (Heubach & Mansour, 2004):

$$\binom{\deg_i + m - 1}{m-1}.$$

Thus, the number of distinct u_i's for an agent i is at most m times the above term, and over all agents i's is at most nm times the above term. Consequently, $u_{\mathcal{M}}$ takes at most L different values, yielding $\bar{L} = L$.

Now, we upper-bound r. Starting from an arbitrary state, for the value of $u_{\mathcal{M}}$ to change, either a neighbor of a highest earner must switch strategies, which happens after at most $M_{p_1}(\mathbb{G})$ switches, or an agent that is not a highest earner must switch strategies and become the new highest earner, which takes at most $M_{p_1}(\mathbb{G})$ switches. Hence, $r \leq M_i(\mathbb{G})$. Therefor, (10) becomes

$$M(\mathbb{G}) \leq M_{p_1}(\mathbb{G})(L+1) + L.$$

A similar bound is obtained for $M_{p_1}(\mathbb{G})$ by considering $u_{\mathcal{M}_2}$ defined as the maximum utility of the agents in $\mathcal{V}_2 = \mathcal{V} - \{p_1\}$:

$$M_{p_1}(\mathbb{G}) \leq M_{p_1,p_2}(\mathbb{G})(L+1) + L,$$

where $M_{p_1,v}(\mathbb{G})$ is the maximum number of switches that the agents in $\mathcal{V} - \mathcal{N}_{p_1} - \mathcal{N}_v - \{p_1, v\}$ can make, given that the strategies of agents p_1 and v and their neighbors are fixed over time, and $p_2 \in \max_{v \in \mathcal{V}_2} M_{p_1,v}(\mathbb{G})$. This recursive process can be repeated for at most n steps, because at each step, the strategy of at least one agent becomes fixed. Hence, we obtain the following general recursive inequality for $i = 2, \ldots, n$:

$$M_{p_1,\ldots,p_{i-1}}(\mathbb{G}) \leq M_{p_1,\ldots,p_i}(\mathbb{G})(L+1) + L.$$

At step $n-1$, we end up at $M_{p_1,p_2,\ldots,p_{n-1}}(\mathbb{G})$ that is the maximum number of switches that the agents in $\mathcal{V} - \sum_{i=1}^{n-1} \mathcal{N}_{p_i} \cup \{p_i\}$ can make, given that the remaining agents have fixed their strategies. Since $\mathcal{V}-\sum_{i=1}^{n-1}\mathcal{N}_{p_i}\cup\{p_i\}$ includes at most one agent, which is isolated, it holds that $M_{p_1,p_2,\ldots,p_{n-1}}(\mathbb{G})=0$. Thus, the corresponding recurrence inequality yields

$$M(\mathbb{G}) \le M_{p_1,p_2,\ldots,p_{n-1}}(\mathbb{G})(L+1)^{n-1} + L\sum_{i=0}^{n-2}(L+1)^i = (L+1)^{n-1}-1 \le (L+1)^{n-1}.$$

3.2. Rational imitation

One can show the same convergence result for the case when instead of imitation, some agents decide based on the *rational imitation* update rule (Govaert, Ramazi, & Cao, 2021), implying that they imitate the highest-earner only if doing so earns them a higher payoff:

$$x_i(k+1) = \begin{cases} y_i(k) & u_i(y_i(k),k) > u_i(k) \\ x_i(k) & \text{otherwise,} \end{cases}$$

where $y_i(k) = \min \mathcal{S}_i^M(k)$, and $u_i(s,k)$ is the payoff agent i earns if she plays strategy s and the remaining agents play their strategies at time k.

Corollary 1. *A network of opponent-coordinating agents reaches an equilibrium if every agent updates asynchronously according to either the imitation or the rational imitation update rule.*

The proof can be done similarly to that of Theorem 1. In particular, all previous lemmas remain valid. The difference is that in the same situation, a rational imitator switches less often compared to an imitator, yet still if she switches, the maximum payoff does not decrease.

4. Convergence under arbitrary number of simultaneous updates

The result of Theorem 1 can be extended to when multiple agents can update simultaneously at any time step. Then the activation sequence becomes $\{\mathcal{A}_k\}_{k=0}^{\infty}$ where $A_k \subseteq \mathcal{V}$ consists of agents that are active at k, and where $|\mathcal{A}_i|$ and $|\mathcal{A}_j|$ are not necessarily equal for $i \neq j$. However, in order to guarantee convergence, the agents must satisfy a stronger condition than being opponent-coordinating; namely, their payoff matrices must satisfy

$$\pi^i_{p,p} + (\deg_i - 1)\pi^i_{p,p_{\min}} > \deg_i \pi^i_{p,p_{\max}} \tag{11}$$

where $\deg_i$ denotes the degree of agent i, $p_{\min}$ denotes the column of the minimum off-diagonal entry of the pth row in π^i and $p_{\max}$ denotes the column of the maximum off-diagonal entry of the pth row in π^i. We call agents satisfying the above condition *strongly-opponent-coordinating*. Intuitively, each diagonal entry of such agents' payoff matrix is sufficiently greater than the off-diagonal entries in the same row. As with asynchronous updates, here we assume the activation sequence is persistent. The proof of the following results is similar to that of Theorem 1.

Corollary 2. *Every network of strongly-opponent-coordinating agents reaches an equilibrium under the imitation update rule, regardless of how many agents update simultaneously at any time step.*

For the special case of $m = 2$, an opponent-coordinating agent turns out to be also strongly row-coordinating, yielding the following result.

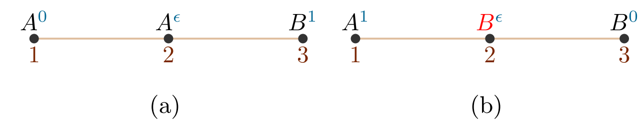

Fig. 1. Three anti-coordinating agents in a line. The payoff matrices of agents 1, 2 and 3 are π_0, $\epsilon\pi_0$ and π_0, respectively. The superscripts in blue indicate the payoffs. (a) Agents' initial strategies. (b) Agents' strategies at the first time step an agent switches strategies. The agents' strategies at the next time step when an agent switches will be the same as those in (a), resulting in an oscillating behavior.

Corollary 3. *Every network of opponent coordinating agents with only two available strategies, i.e., $m = 2$, reaches an equilibrium under the imitation update rule, regardless of how many agents update at the same time.*

Remark 1. Theorem 1 and Corollary 2 hold even when the persistence assumption does not hold; however, then after some finite time, the network reaches and remains at a final state that may not be an equilibrium.

5. Non-convergence behavior

We provide counterexamples to demonstrate cases in which networks containing non-opponent-coordinating agents never reach an equilibrium.

5.1. Three anti-coordinating agents in a line

In a network containing only two agents, any asynchronous imitation will result in an equilibrium. Therefore, the smallest network in which asynchronous imitation may lead to non-convergence is one consisting of three agents. Such a network can be constructed out of three anti-coordinating agents connected in a path, *i.e.* one edge connects agents 1 and 2 and the other edge connects agents 2 and 3 (Fig. 1). The minimum number of strategies required for non-convergence is two, i.e., $m = 2$. We refer to strategies 1 and 2 as A and B, respectively. Define the payoff matrix $\pi_0 = ((0\ 1)^\top, (1\ 0)^\top)$, and let $\pi^1 = \pi^3 := \pi_0$, and $\pi^2 = \epsilon\pi_0$, where $\epsilon < 1$. The network undergoes a cycle of length 2 and will never reach equilibrium. This configuration can also appear embedded in much larger networks and demonstrates that imitative anti-coordinating agents who receive less payoff than their neighbors for the same types of interactions can quite easily be made to waver between the strategies of more steadfast neighbors.

5.2. Extension to a homogeneous network

We extend the previous example to a network of homogeneous agents with the payoff matrix

$$\pi^i = \begin{pmatrix} R & S \\ T & P \end{pmatrix},\ P > 0,\ 0 < R < T,\ S > P + R. \tag{12}$$

For this, we need to add α_1 initially A-playing neighbors to agent 1, and α_2 initially B-playing neighbors to agent 3 (Fig. 2) where $\alpha_1, \alpha_2 \ge 1$ are positive integers satisfying

$$R\alpha_1 \ge S, \quad P\alpha_2 \ge T, \tag{13}$$

$$\lfloor \frac{R}{T}(\alpha_1+1) - \frac{P}{T}\alpha_2 \rfloor = 0. \tag{14}$$

Such α_1 and α_2 exist since one can first choose them to be large enough to satisfy (13), then increase α_2 so that $R(\alpha_1+1) - P\alpha_2 < 0$, and then start increasing α_1 until the first time $R(\alpha_1+1) - P\alpha_2$

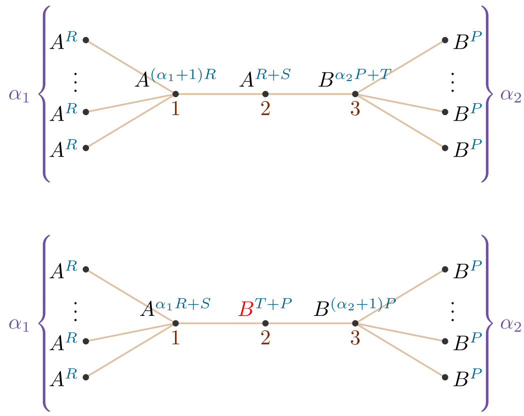

Fig. 2. Extension of the example in Section 5.1 to a homogeneous network. All agents' payoff matrices equal that in (12), implying a homogeneous network. The superscripts in blue indicate the payoffs. (a) Agents' initial strategies. (b) Agents' strategies at the first time step when an agent switches strategies. The agents' strategies at the next time step when an agent switches will be the same as those in (a), resulting in an oscillating behavior.

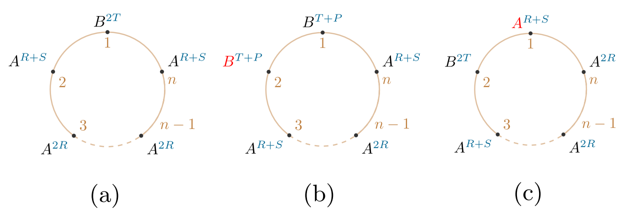

Fig. 3. Ring of asynchronous homogeneous non-opponent-coordinating agents. All agents' payoff matrices equal that in (15). The superscripts in blue indicate the payoffs. (a) Agents' initial strategies. It holds that $x(0) \in \hat{\mathcal{X}}$. (b) Agents' strategies at $k = 1$ when agent 2 was active at $k = 0$. It holds that $x(1) \in \bar{\mathcal{X}}$. (c) Agents' strategies at $k = 2$ when agent 1 was active at $k = 1$. It holds that $x(1) \in \hat{\mathcal{X}}$, meaning that the strategy configuration has reached the same structure as it had at $k = 0$. A similar process takes place for any other activation sequence. Therefore, the network never reaches an equilibrium.

becomes positive. Since $R < T$, it then follows that $0 < R(\alpha_1 + 1) - P\alpha_2 < T$, satisfying (14), proving the existence of α_1 and α_2.

As in the previous example, we assume that the initial strategies of agents 1, 2 and 3 are (A, A, B). It then follows that the network never equilibrates and the same can be shown when the last three inequalities in (12) are replaced with $R > 0$, $0 < P < S$ and $T > P + R$. Both cases correspond to significant classes of payoff matrices, implying that even networks of homogeneous agents may often not converge. This is further supported by the next example.

5.3. Ring of asynchronous, homogeneous, non-opponent-coordinating agents

Consider a ring network $\mathbb{G} = (\mathcal{V}, \mathcal{E})$ where $\mathcal{E} = \{(1, 2), (2, 3), \dots, (n-1, n), (n, 1)\}$. The network is homogeneous in that the agents' payoff matrices π^i, $i \in \mathcal{V}$, are the same and equal to π^i in (12), but when

$$R < T, \;\; T + P < R + S < 2T. \tag{15}$$

The agents' initial strategies are as follows: $x_1(0) = B$ and $x_i(0) = A$ for all $i \in \mathcal{V} - \{1\}$ (Fig. 3(a)). One can show that the network fluctuates between two sets of states: first, $\hat{\mathcal{X}} = \{\hat{x}^i \mid i \in \mathcal{V}\}$, where $\hat{x}^i \in \{A, B\}^n$ and is defined by $\hat{x}^i_j = B$ if $j = i$ and $\hat{x}^i_j = A$ otherwise; and second, $\bar{\mathcal{X}} = \{\bar{x}^i \mid i \in \mathcal{V}\}$ where $\bar{x}^i \in \{A, B\}^n$ and is defined by $\bar{x}^i_j = B$ if $j = i$ or $j = i + 1$ and $\bar{x}^i_j = A$ otherwise, where $i + 1$ is counted modulo n. One can further construct an activation sequence that results in a cycle.

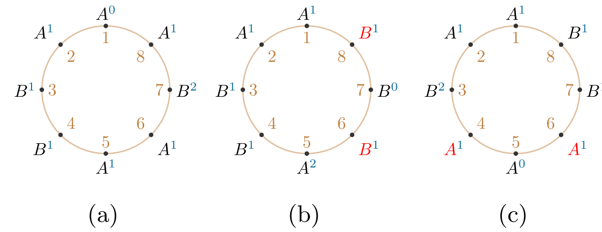

Fig. 4. Ring of synchronous homogeneous non-opponent-coordinating agents. All agents' payoff matrices equal π_0, implying a homogeneous network. The superscripts in blue indicate the payoffs. (a) Agents' initial strategies. (b) Agents' strategies at $k = 1$. (c) Agents' strategies at $k = 2$. The resulting strategy configuration is the same as that at $k = 0$ but shifted by 4 agents. Therefore, the network game undergoes a cycle of length 4 and never converges to an equilibrium.

Such behavior never shows up for a network of opponent coordinating agents in view of Theorem 1. Indeed for an opponent-coordinating agent, we have that

$$P > T \Rightarrow T + P > 2T,$$

violating the inequality in (15). Other types, however, may satisfy the inequality, including most well-known types of agents. For example, the payoff matrices $\pi^{SD_1} = ((1\ 4)^\top, (3\ 1)^\top)$ and $\pi^{SD_2} = ((2\ 3)^\top, (3\ 1)^\top)$ corresponding to two snowdrift games, both satisfy (15). So any homogeneous network of agents with the payoff matrix π^{SD_1} or homogeneous network of agents with the payoff matrix π^{SD_2} never converges to a single state.

The results can also be extended to heterogeneous networks by modifying the condition in (15) as follows. Assume that each agent $i \in \mathcal{V}$ has a possibly unique payoff matrix as in (15), but when the payoffs are replaced with the personalized values $R_i, S_i, T_i, P_i \in \mathbb{R}$, and that for any (not necessarily distinct) $i, j \in \mathcal{V}$, it holds that $(R_i < T_j)$, $(T_i + P_i < R_j + S_j)$, and $(R_j + S_j < 2T_i)$. Then it can be verified that again starting from $\hat{x}^1$, the network does not converge to a single state. As a result, any network of agents, some of which having π^{SD_1} and others π^{SD_2}, never converges.

5.4. Long cycles in synchronous networks

When networks of anti-coordinating agents update in full synchrony under imitation dynamics, it is possible for relatively long cycles to emerge. This is in sharp contrast with synchronous best-response dynamics, in which it has been shown that cycles of length at most 2 can occur (Adam, Dahleh, Ozdaglar, et al., 2012). Following is an example of how cycles of length $\frac{n}{2}$ can appear in rings of synchronous anti-coordinating agents. Consider a ring of $4p$ agents, where $p \in \{2, 3, \dots\}$, and let the payoffs of each agent be given by the anti-coordinating matrix π_0. The network will then persist in cycles of length $2p$ if it starts with the initial condition (Fig. 4(a)) $x_i(0) = A$ if $(i-1) \bmod 4 \leq 1$ or $i = n$ and $x_i(0) = B$ otherwise. For example, in a ring of 8 agents, the initial strategies are $(A^0, A^1, B^1, B^1, A^1, A^1, B^2, A^1)$, where the superscripts indicate the initial payoffs (Fig. 4).

6. Concluding remarks

The opponent-coordination setup applies to collective networks where agents' payoffs are highest when their opponents also play the same strategy, e.g., financial investments (Decker & Günther, 2016), the spread of social norms (Montanari & Saberi, 2010), technological innovations (Young, 2011) and voting opinions (Moreno & del Pino Ramos-Sosa, 2017). In all of these cases, our results imply that if the agents simply imitate the most successful in their neighborhood, then regardless of how the agents are linked (network topology) and how differently they perceive the interaction outcomes (heterogeneity of the payoff matrices), every agent will eventually fix her strategy. Consequently, non-convergent behavior in such situations may imply the presence of individuals who perceive the social context extremely differently from the conventional understanding, e.g., anti-coordinators who earn more when they play the opposite strategy of the majority of their neighbors.

The second contribution of this paper has been to show that compared to best-response, convergence under imitation seems to be a relatively rare phenomenon. In the case of two available strategies, best response guarantees equilibration for every network of homogeneous agents and every network of heterogeneous agents that consists of either all coordinating or all anticoordinating (Ramazi et al., 2016). Imitation, however, may drive a network of all anticoordinating agents to perpetual fluctuations, and the same holds for networks of homogeneous agents. We have provided significant classes of payoff matrices that can result into this non-equilibration. Even a well-mixed population of all anticoordinating agents may not equilibrate if a few agents imitate rather than best-respond (Le & Ramazi, 2021). The comparison suggests that frequency-based learners (best-responders) are more influential in leading an entire network to satisfactory decisions than success-based learners (imitators), at least in anti-coordination social contexts, where agents benefit from playing opposite strategies. The reason may be that imitators are less independent (or less self-confident) and ignore their own options. Exceptions to this general claim may correspond to special situations, e.g., every agent follows the same most-successful fellow who does not change over time.

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