Perbandingan Performa Pelacakan antara Repetitive Controller dan PI Controller

On Comparison Between Repetitive Controller and PI Controller Tracking Performance

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Abstract

Tracking periodic signals are common task in many control problems. One of the examples is movement control of pick and place robot in industry. The requirement of high tracking accuracy becomes very important in many applications. Therefore, a sophisticated control algorithm that manages to achieve high accuracy tracking of periodic command is required. Repetitive Control (RC) based on internal model principle is one of control schemes that can be employed to achieve perfect tracking of periodic signal. On the other hand, Proportional Integral (PI) controller can also be used for tracking. This paper compares the tracking performance of PI controller, RC, and PI with RC, where PI with RC here is integration between PI controller and RC. Step by step design to obtain the parameters of PI, RC and PI with RC are given. A simulation on servo motor system is carried out to assess the performance of RC, PI, and PI with RC respectively. From the simulation results, the transient response and tracking accuracy are thoroughly discussed.

Keywords: *Repetitive Control, PI controller, tracking, periodic signal*

Abstrak

Pelacakan sinyal periodik adalah kegiatan umum dalam banyak permasalahan kontrol. Salah satu contoh adalah kontrol gerakan *pick and place* robot di industri . Kebutuhan akurasi yang tinggi menjadi sangat penting dalam banyak aplikasi kontrol. Oleh karena itu, algoritma kontrol untuk mencapai pelacakan akurasi yang tinggi khususnya untuk sinyal periodik sangat diperlukan . *Repetitive Controller* (RC) berdasarkan prinsip *internal model* adalah salah satu dari banyak algoritma kontrol yang dapat digunakan untuk pelacakan sempurna sinyal periodik. Di sisi lain, *Proportional Integral* (PI) *controller* juga dapat digunakan untuk pelacakan. Makalah ini membandingkan kinerja pelacakan PI *controller*, RC , dan PI dengan RC, di mana PI dengan RC di sini merupakan integrasi antara PI *controller* dan RC . Langkah-langkah untuk mendapatkan parameter PI, RC, dan PI dengan RC di berikan dalam makalah ini. Simulasi untuk sistem servo motor dilakukan untuk menguji kinerja RC , PI , dan PI dengan RC . Dari hasil simulasi, *Transient response* dan akurasi pelacakan dibahas secara mendalam.

Kata kunci: Repetitive Control, PI controller, pelacakan, sinyal periodik

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1. Introduction

Tracking control can be found in many industrial applications such as contour tracking in machining processes [1], trajectory tracking of robot manipulator [2], and etc. Proportional Integral (PI) controller becomes most common feedback control , and considered as first basic solution in the control of industrial system [3]–[5]. Proportional Integral is part of Proportional Integral Derivative (PID) controller, which is used more than 90% of control loop in today use [4]. Most loops are PI because derivative action is not used very often.

In some repetitive processes, using PI controller may not be satisfactory, and a more sophisticated controller is needed to achieve control objectives. There is another tracking controller named as Repetitive Controller (RC), that can be used for tracking, especially for periodic signal. Tracking periodic signal is also common problem found in many industrial applications. As listed in [6]-[7], RC has been succesfully used for robot control, accurate

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position control of piezoelectric actuators, and etc. Recently, RC has been used for tracking control in underwater applications [8], tracking control of engine valve system [9][10], and tracking of contouring tasks in an industrial biaxial precision gantry [11].

RC is based on the idea of internal model principle by [12], that uses the model of reference signal in the controller. The internal model has a capability to learn from the previous cycle error, then generate control signal that can refine the tracking output to be as close as possible to the reference signal. Besides tracking periodic reference, RC can also be employed for rejecting period disturbance. However, the control objective discussed in this paper is only for tracking reference signal.

This paper investigates the tracking performance between PI controller , RC, and PI with RC. The pros and cons of each control algorithm will be reviewed.

This paper is structured as follows; Section 2 and 3 presents the overview of PI and RC respectively. Numerical example that covers the simulations resuts and discussion is given in Section 4. Section 5 concludes the paper.

2. Overview: PI Controller

The PID controller has the following transfer function:

$$C(s) = K_p + K_i \frac{1}{s} + K_d s \qquad (1)$$

where K_p is proportional gain, K_i is integral gain, K_d is derivative gain, $\frac{1}{s}$ is integrator, and s is differentiator, and it is also a Laplace operator .

If we omit derivative part as derivative action is not used very often [4], then we have the following PI transfer function.

$$C(s) = K_p + K_i \frac{1}{s} \qquad (2)$$

Proportional, Integral control is based on the present (P) and the past (I) control error, where error is obtained from the difference between the actual and desired output. This can be seen from the control law as follows:

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)\, d\tau \qquad (3)$$

We can see that Proportional part uses the current control error, while the Integral part accumulates the previous control error values.

Tuning PI gains becomes a very crucial part in the design of PI controller, and PI tuning method has been a large research area.There are many aspects that should be taken into account when designing the PI controller. Desirable features of a design procedure are [4]:

(a) It should give a controller that meets the design specifications.
(b) It should be based on the available process knowledge
(c) It should meet limitations on computation power and resources available.

There are several methods for tuning PI gains such as *Manual tuning*, *Ziegler–Nichols*, *Software tools*, *Cohen–Coon*, and etc. The most effective ways usually involve the knowledge of process/plant model, then choosing P, I, gains based on the plant model parameters. Effects of increasing PI gains independently is shown in Table 1 [13].

Table 1. Effects of increasing K_p and K_i independently

Gain	Rise time	Over-shoot	Settling time	Steady-state error
K_p	Decrease	Increase	Small Change	Decrease
K_i	Decrease	Increase	Increase	Eliminate

From the Table 1, it shows that increasing K_i eliminates the steady state error which is required for tracking accuracy. However, it comes with the trade-off such as increasing overshoot and rise time,which represents poor transient response. Therefore, tuning PI gains is an effort to obtain optimal parameters that meets the design specifications, or not to obtain best parameters that satisfies all aspects.

PI controller shown in (2) is in contionous-time form, in which it needs to be digitized in order to be implemented in digital computer. The digital/discrete-time PI is basically an approximation of the contionous-time form. Using forward difference approximation, the discrete-time PI is formulated as follows:

$$C(z) = K_p + K_i T \frac{1}{z-1} \qquad (4)$$

where T is sampling time, and z is a discrete frequency domain operator.

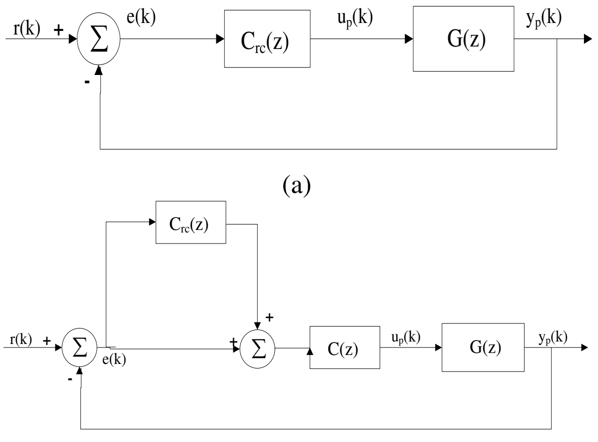
3. Overview: Repetitive Control

Since the introduction of the digital computer, the use of digital control has greatly expanded for several reasons, such as being cheaper, smaller, and more flexible than analogue hardware. The first digital RC was introduced in [14], where the digital RC has the following transfer function:

$$C_{rc}(z) = F(z)\frac{z^{-N}}{1 - z^{N}} \tag{5}$$

where $N = {}^{T_r}/_{T}$ is the number of samples per reference period and it has to be integer, T_r is reference period, T is sampling time, and $F(z)$ is RC compensator.

RC involves two main designs; Internal model and compensator design. The internal model is a generator of periodic signal which has a capability to generate periodic signal so the perfect tracking of periodic reference can be achieved. The internal model is shown as the term $\left({z^{-N}}/{1 - z^{N}}\right)$ in (5). RC compensator $F(z)$ is required to stabilize the closed-loop system. RC can be designed either in standard or plug-in manner as shown in Figure 1(a) and 1(b) respectively.



(a)

(b)

Figure 1. Block diagram of Repetitive Control System (a) Standard (b) Plug-in.

The plug-in RC has been introduced by Cosner et al [15], where RC is added in plug-in manner as shown in Figure1(b). $C(z)$ is a nominal controller used (e.g P, PI, PID, Lead, Lag). A compensator design based on Zero Phase Tracking Error Controller (ZPTEC) proposed in [16], [17] can be used to obtain a compensator $F(z)$. For standard RC system shown in Figure 1(a), $F(z)$ is designed to compensate the dynamics of plant $G(z)$. If $G(z)$ is stable minimum phase plant, $F(z)$ can be designed as the inverse of $G(z)$

$$F(z) = \frac{1}{G(z)} \tag{6}$$

While for plug-in RC shown in Figure 2(b), $F(z)$ is designed to compensate the dynamics of closed-loop model as follows:

$$G_c(z) = \frac{C(z)G(z)}{1 + C(z)G(z)} \tag{7}$$

$$F_p(z) = \frac{1}{G_c(z)} \tag{8}$$

where $F_p(z)$ is a compensator of the plug-in RC system.

The standard RC system is stable if the following two conditions are satisfied [15][17]:

1. $G(z)$ is a stable transfer function
2. $\|1 - G(z)F(z)\|_{\infty} < 1$

where $\|.\|_{\infty}$ denotes the norm infinity of the transfer function.

For plug-in RC system to be stable, the term $G(z)$ in stability conditions (1) and (2) is replaced with $G_c(z)$.

4. Numerical Example

Numerical examples are given to investigate the performance of PI, RC, and PI with RC (Plug-in RC). The following continous plant model is used in the simulation

$$G(s) = \frac{649.3}{s^2 + 37.31\,s + 649.3} \tag{9}$$

which is a transfer function of stabilized servo motor used in [18].

4.1 Standard RC Design

Let the sampling time be $T = 0.005s$, and the reference signal r(k) to be tracked has a period 1.25 s. This gives the number of samples N as 250. The discrete model of (9) with the chosen sampling period is given as follows:

$$G(z) = \frac{0.007623\,z + 0.007164}{z^2 - 1.815\,z + 0.8298} \tag{10}$$

which is a minimum phase stable plant as its zero and poles located inside the unit circle.

The first stability condition of RC system is satisfied due to the plant model is stable. The choice of compensator $F(z)$ as the inverse of $G(z)$ makes the second stability condition is fulfilled. Thus, the digital RC can be formulated as follows:

$$C_{rc}(z) = 131.23\frac{z^2 - 1.815\,z + 0.8298}{z^{251} + 0.9402\,z^{250} - z - 0.9402} \tag{11}$$

4.2 PI Design

Let the design in continous time, then PI controller will be digitized with the sampling period $\mathrm{T} = 0.005\mathrm{s}$. The function pidtune in MATLAB can used to tuned gain of PI. The obtained gain gives the following PI Controller in continous time.

$$C(s) = 1.100 + 18.900\frac{1}{s} \tag{12}$$

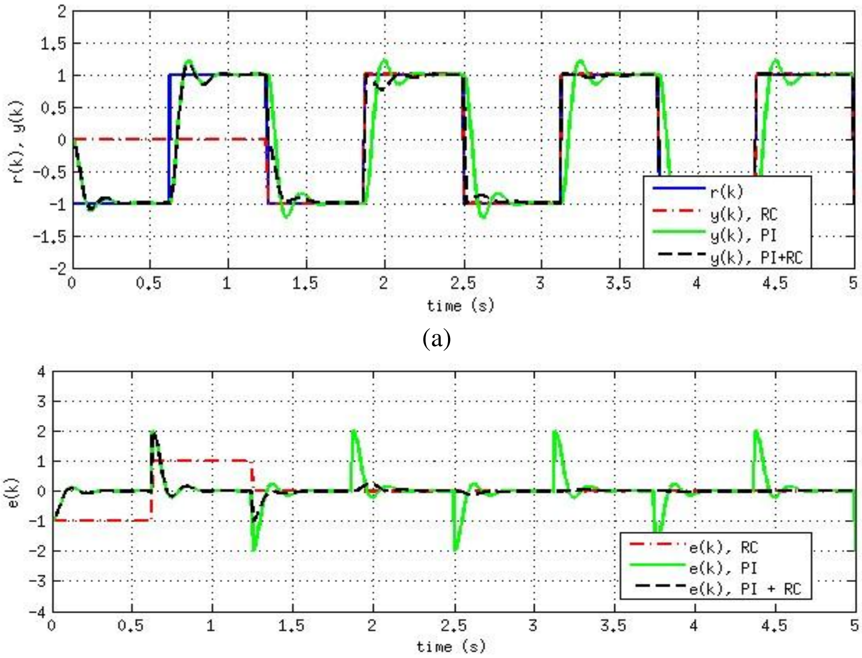
The digital PI is given as follows:

$$C(z) = 1.100 + 18.900T\frac{1}{z-1} \tag{13}$$

, where T is sampling period.

4.3 Plug-in RC (PI with RC) Design

Let the PI controller given in (13) is the nominal controller used in the plug-in RC system as shown Figure 1(b). The closed-loop model as formulated in (7) is :



(a)

(b)

Figure 2. (a) Tracking Output of Square Signal, (b) Tracking Error.

$$G_c(z) = \frac{8.38\times 10^{-3}(z^2 + 0.025z - 0.859)}{z^3 - 2.807z^2 + 2.645z - 0.837} \tag{14}$$

Then, the digital plug-in RC is :

$$C_{rc}(z) = \frac{119.24[z^3 - 2.807z^2}{z^{252} + 0.026z^{251}} \frac{+2.645z - 0.837]}{+0.859z^{250} - z^2 - 0.025z + 0.858} \tag{15}$$

4.4 Results and Discussion

Two tracking scenarios are carried out in the simulation. SIMULINK is used to simulate the tracking controls of RC, PI, and PI with RC. The block diagram of the SIMULINK model is given in Appendix.The first scenario is that the plant output is required to track square reference signal. The tracking outputs and errors of RC, PI, and PI with RC are shown in Figure 2(a) and (b) respectively. From Figure 2(a), tracking output of RC starts to perfectly follow the reference signal at second cycle (1.25 – 2.5 s), but the output is zero in the first cycle (0 – 1.25 s). We can also observe from Figure 2(b) that the tracking error of RC in the first cycle is equal to the reference signal, while the tracking error in the next cycle is equal to zero. This is due to RC uses the first cycle as learning period, and no control signal is generated in this interval. This also means that standard RC gives zero response in the first cycle, but a good response in the second cycle and ahead.

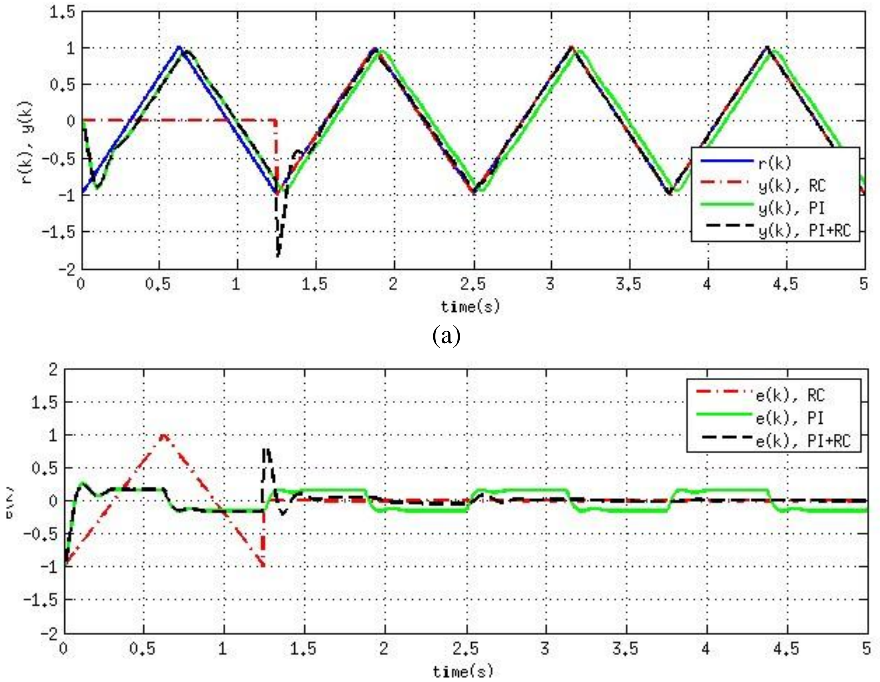
The tracking output of PI is shown in green line. It gives fast response in the first cycle, but overshoot and oscillation still remain in every pulse change. After overshoot and oscillation occur, the tracking output moves and settles to the reference signal value. However, this steady state condition does not last longer due to pulse change. This implies that no matter how many cycles of reference signal have been tracked, zero tracking error can not be achieved by using PI controller.

The tracking output of PI with RC (Plug-in RC) is indicated by black dash line. In the first cycle, the tracking output shows better response compared to the RC's, and also shows similar response to the PI's. This is due to the RC control signal is still inactive, while PI control signal is already active and do the tracking. On the second cycle, small oveshoot stilll appear, but it is smaller than PI's. Starting at the third cycle, zero tracking error is achieved as shown in Figure 2(b).

The first scenario uses periodic signal with step type as a reference. The second scenario is that the plant output is required to track triangle reference signal. Triangle signal is ramp type signal that may effect to the tracking performance of PI.

For RC, tracking performance is similar to the performance as shown in the first scenario. An error equal to the reference signal appears in the first cycle, and zero tracking error is started at the second cycle.

For PI, the tracking output shows small overshoot for every peak change. After the overshoot occurs, the tracking output never settle to the reference value. This performance differs with the performance shown in the first scenario.



(a)

(b)

Figure 3. (a) Tracking Output of Triangle Signal, (b) Tracking Error.

This error is typical response when Type-1 system is fed by ramp input. When the plant model (9) is cascaded with PI controller, results in an open-loop model with a single integrator, which is a Type-1 system. Therefore, steady-state error of Type-1 system fed by ramp input is always non-zero, while steady-state error of Type-1 system fed by step input is zero. This can also be proven from final value theorem.

The tracking output of PI with RC shows similar performance compared to the performance shown in the first scenario.

In summary, PI with RC provides better tracking performance compared to both PI and RC. PI with RC combines the fast response feature of PI and learning feature of RC. Therefore, both fast response and zero-tracking error can be achieved by employing this controller. However, PI with RC gives higher order controller compared to both PI and RC. We can see from (11) (13), and (15) that the order of PI with RC is 252, while the order of RC and PI is 250 and 2 respectively.

5. Conclusion

This paper presents the tracking performance comparison of PI controller , RC, and PI with RC. The transient response and tracking accuracy are also discussed. Overall, PI with RC provides better tracking performance compared to both PI and RC. This is due to PI with RC integrates the fast response feature of PI and learning feature of RC at once. In exchange, PI with RC gives higher order controller compared to both PI and RC.

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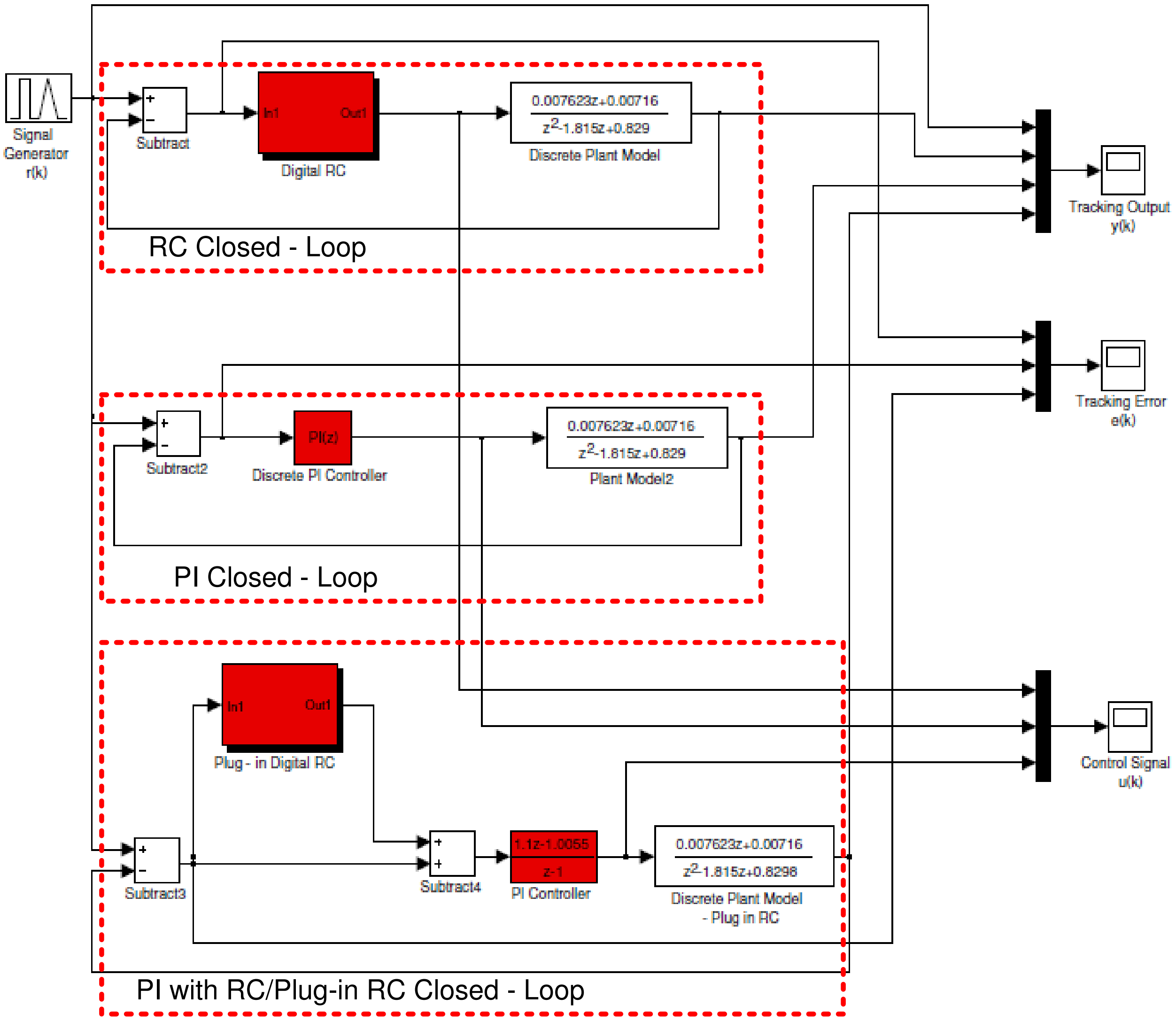
Appendix



Block Diagram of SIMULINK model