

N -soft sets: OWA aggregation operators and multi-agent decisions — Slides in 22nd IPMC 2022 (1/3)

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Abstract

The 22nd International Pure Mathematics Conference 2022 (**22nd IPMC 2022**) on Algebra, Analysis and Geometry, was held in Islamabad (Pakistan) from August 21–23, 2022. It provides a stimulating opportunity to interact with experts from various countries in a variety of branches of pure mathematics. The conference is organized in hybrid mode, with a first day face-to-face and the other two days online.

The emeritus professor Qaiser Mushtaq, Department of Mathematics, Quaid-i-Azam University, Islamabad and the Organizing Committee has been organizing the International Pure Mathematics Conference (IPMC) annually in Islamabad since 2000.

Here are the slides of the lecture given by the author.

***N*-soft sets: OWA aggregation operators and multi-agent decisions**

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August 25, 2022 at Islamabad, Pakistan

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Goals

- ▶ Understand the semantics of N -soft sets (Fatimah, Rosadi, Hakim and Alcantud, 2018).

This is necessary for example, for a correct evaluation of the alternatives, or for interpreting aggregation.

- ▶ Two different approaches to the aggregation of N -soft sets.

Aggregation is interesting, e.g., for multi-agent decision making.

Why semantical analyses?

From Dubois and Prade "The three semantics of fuzzy sets", Fuzzy Sets and Systems (1997):

(...) there is no uniformity in the interpretation of what a membership grade means. (...) Most negative statements expressed in the literature turn around the question of interpreting and eliciting membership grades. Our claim in this position paper is that, far from being a weakness, the existence of several understandings of what a membership grade may mean proves the potential richness of the concept of fuzzy set (...)

Three main semantics for membership functions seem to exist in the literature: similarity, preference and uncertainty.

Recommended bibliography

- ▷ J.C.R. Alcantud: “**The semantics of N -soft sets, their applications, and a coda about three-way decision**”, Information Sciences **606** (2022), 837–852. *Open Access*.

Largely based on J. Yang, Y. Yao: “Semantics of soft sets and three-way decision with soft sets”, Knowledge-Based Systems **194** (2020), 105538.

- ▷ J.C.R. Alcantud, G. Santos-García, M. Akram: “**OWA aggregation operators and multi-agent decisions with N -soft sets**”, Expert Systems with Applications **203** (2022), 117430. *Open Access*.

Table of contents

1. N -soft sets
2. Semantics of N -soft sets
3. Aggregation of N -soft sets
4. Conclusions

***N*-soft sets**

Soft sets and N -soft sets

Conceptual definition (finite setting). Consider $T = \{t_1, \dots, t_q\}$, a set of ‘attributes’.

An N -soft set on a set $O = \{o_1, \dots, o_p\}$ is defined by Table 1.

Table 1: Representation of an N -soft set (Fatimah, Rosadi, Hakim, Alcantud, 2018).

(F, T, N)	t_1	t_q
o_1	r_{11}	r_{1q}
\vdots	\vdots		\vdots
o_p	r_{p1}	r_{pq}

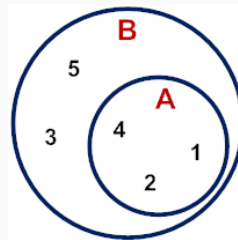
Each r_{ij} (a ‘grade’) is in $G = \{0, 1, \dots, N-1\}$ \leftarrow a convenient default

When $N = 2$ we have a soft set (Molodtsov, 1999).

Crisp sets vs. soft sets: An example

Let $B = \{1, 2, 3, 4, 5\}$.

(a) $A = \{1, 2, 4\}$ is a **crisp** subset of B .



Identified by its characteristic function $\chi_A: B \rightarrow \{0, 1\}$ with

$$\chi_A(1) = \chi_A(2) = \chi_A(4) = 1, \quad \chi_A(3) = \chi_A(5) = 0.$$

Or a vector with 5 components and binary values: $(1, 1, 0, 1, 0)$.

(b) A **soft set** over B is identified by several vectors with 5 components (one column vector for each relevant attribute) and binary values.

7

Crisp sets vs. N -soft sets: An example

(c) An N -soft set over B is identified by several vectors with 5 components (one column vector for each relevant attribute) and values from $G = \{0, 1, \dots, N - 1\}$.

Example with 3 characterizing attributes:

$(F, T, 4)$	t_1	t_2	t_3
1	1	1	2
2	3	2	0
3	0	1	2
4	2	3	2
5	1	0	3

What can we capture with this table? semantics of attributes and values.

Important: Real-life examples are given in various references.

8

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