Case Studies in Thermal Engineering 26 (2021) 101185

Contents lists available at ScienceDirect

Case Studies in Thermal Engineering

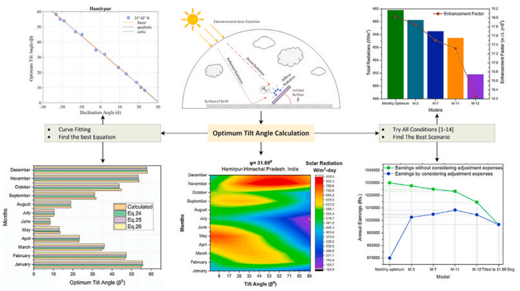
journal homepage: www.elsevier.com/locate/csite

Correlation formulation for optimum tilt angle for maximizing the solar radiation on solar collector in the Western Himalayan region

Ashutosh Sharma [a], Mehmet Ali Kallioğlu [b], Anchal Awasthi [c], Ranchan Chauhan [a], Gusztáv Fekete [d], Tej Singh [d,*]

[a] *Department of Mechanical Engineering, Dr. B. R. Ambedkar NIT, Jalandhar, 144011, India*
[b] *Faculty of Technology, Batman University, Batman, 72100, Turkey*
[c] *Department of Electrical Engineering, A. P. Goyal Shimla University, Shimla, 171009, India*
[d] *Savaria Institute of Technology, Faculty of Informatics, ELTE Eötvös Loránd University, Szombathely, 9700, Hungary*

GRAPHICAL ABSTRACT



ARTICLE INFO

Keywords:
Solar collector
Tilt angle
Utilizable models
Cost-benefit analysis
Correlation
Statistical analysis

ABSTRACT

Solar tracking devices are efficacious in maximizing solar irradiation collection. However, higher price makes these systems less frequently used. As an alternative, optimum tilt angle estimation has the advantage that it does not involve tracking cost expenditure. In this study, optimum tilt angles for different months have been computed for Hamirpur, Himachal Pradesh, India (L 31° 42′ N). Several mathematical models and statistical tools have been employed to forecast the monthly optimum tilt angles for the selected site. Different scenarios have been presented by considering five, four, three, and two annual adjustment models to increase the practical usage of the estimated optimum tilt angles. Additionally, cost-benefit analysis has also be performed on the PV panels. Based on the results, it has been concluded that model M − 11, with three annual adjustments, stipulates maximum benefits over the other models. This study can serve as fundamental guidance for setting up solar energy plants in this specified region with the highest efficiency.

* Corresponding author.
E-mail address: sht@inf.elte.hu (T. Singh).

https://doi.org/10.1016/j.csite.2021.101185
Received 25 July 2020; Received in revised form 20 May 2021; Accepted 22 June 2021
Available online 25 June 2021

Nomenclature

φ	Latitude
n	Julian day
δ	Declination angle
θ	Incidence angle
ω	Hour angle
θ_Z	Zenith angle
γ	Azimuth angle
γ_S	Solar azimuth angle
ω_S	Sunset angle
$\beta_{opt.,monthly}$	Monthly optimum tilt angle
$\beta_{opt.,yearly}$	Yearly optimum tilt angle
H_B	Beam radiations on the tilted surface
H_n	Maximum direct beam radiations
H	Beam radiations normal to the horizontal surface
R_B	Beam radiation view factor
H_R	Reflected radiations
H_o	Daily extraterrestrial radiations
k_T	Day's clearness ratio
G_{SC}	Global solar constant
H_d	Diffuse radiations on the horizontal surface
H_D	Diffuse radiations on the tilted surface
$H_{g,t}$	Global irradiance
ρ_g	Reflection factor
GI	Global Irradiance
SC	Solar collector
PV	Photo Voltaic

1. Introduction

Energy is an essential requisite for every living being to sustain life and development. The resources of energy are categorized as conventional and non-conventional resources. Conventional energy resources are conventionally utilized for complying the energy requirements. However, these resources are nearly exhausted, and their utilization is non-ecofriendly [1,2]. Non-conventional energy resources are abundant while also more sustainable than conventional resources [3,4]. Among numerous energy resources available, solar energy distinguishes itself as one of the cleanest ones. The solar energy received on a particular location on earth changes throughout the day and year because of the rotary motion along its axis and revolutionary movement along with the sun, respectively [5–8]. The most feasible method to collect maximum solar irradiations is by establishing the solar tracking system. However, solar energy entrepreneurs dodge the solar tracking systems due to their high cost [9]. Alternative expedient means to harvest an extensive amount of solar irradiations is by exercising the tilt angle adjustment of the solar collector (SC). Attaining considerable amount of solar energy by the means of SC system is a convoluted process. It comprises examining the climatology, positioning, and tilt adjustment with the wide-open surface horizon [10]. Therefore, climatology and optimum tilt angle assessment is desirable prior to the onset installation of an SC system.

Recently, various studies have been reported respecting to SC tilt angle optimization and correlation assessment across the world. Wessley and co-workers [11] designed a model for optimum SC tilt angles by considering different Indian cities. Similarly, Benghanem [12] calculated the monthly optimum tilt angles ($\beta_{opt.,monthly}$) for Madinah, Saudi Arabia. Siraki and Pillay [13] studied the effect of tilt angle at different latitudes and the consideration of obstacles in an urban area. It was found that the optimum tilt angle is not only influenced by obstruction at an urban place but also by the azimuth angle. In an another study, Soulayman and Sabbagh [14] intended to work on assessing the tilt angle at tropical regions and found a percentage increment of 11 to 18% from a conventional system. Several authors have reported that latitude angle is a significant parameter for determining the variation in monthly and yearly optimum tilt angles [15–18].

As seen in various studies, optimum tilt angle correlation development for a particular latitude range can bring forth substantial application possibilities. Most of the correlations are developed based on latitude or the declination angle. Morad et al. [19] calculated the monthly and $\beta_{opt.,monthly}$ and $\beta_{opt.,yearly}$ for Diyala and Tikrit of Baghdad. The $\beta_{opt.,yearly}$ assessment and correlation development for 26 Indian cities were carried out by Yadav and Chandel [20]. Al-Sayyab et al. [21] conducted an experimental and theoretical study for assessing tilt angle variation on photovoltaic cells' output in Basrah city of Iraq. The authors also checked the accuracy of their developed correlations using different statistical tools. Similarly, Stanciu and Stanciu [22] developed a correlation for $\beta_{opt.,monthly}$ for various locations in the northern hemisphere in terms of declination angle.

Theoretically, $\beta_{opt.,monthly}$ assessment can be conducted effortlessly. However, it is tedious work to change the tilt angle of the SC each month practically. Thereby, various annual adjustment models have been devised and suggested in the literature. Skeiker [23], Sultan et al. [24], Ismail et al. [25], and Khorasanizadeh et al. [26] considered the winter starting from December to February, and some other authors like Talebizadeh et al. [27] suggested winter from November to January. Idowu et al. [28] evaluated the optimum tilt angle for each month at low latitude tropical regions. It was found that tilt angles$\varphi + 25^\circ$for November, December, and January,$\varphi + 15^\circ$for February, September, and October; $\varphi - 15^\circ$for August; $\varphi - 25^\circ$for May, June, and July; and $\varphi$for March and April provided the most relevant results.

The literature suggests that the solar irradiation collection can be significantly improved by tilt angle optimization. In this article, the optimum tilt angle assessment has been conducted for a location (Hamirpur (L31°42$'N$)) in the western Himalayan region. The correlations have been developed to generalize the obtained results. The accuracy of these correlations has been checked by statistical tools such as coefficient of determination (R^2), the sum of the square of the relative error (SSRE), relative standard error (RSE), and root mean square error (RMSE). Different scenarios have been devised by considering five, four, three, and two annual adjustment models for increasing the practical usage of these tilt angles. Adjustments of the tilt angle also improved the operational cost of the system. A cost-benefit analysis has also been conducted to estimate the actual effectiveness of these models [29]. This research would provide a window to the process of making most of the solar energy and developing an alternative to the solar tracking system.

2. Solar irradiation on the tilted surface

Solar irradiation intensity over the SC changes continuously due to the change in the sun's position with respect to the earth (Fig. 1). The slope of the SC is defined by the tilt angle of the collector (β) about the horizontal surface. The azimuth angle (γ) (westward positive and eastward negative) determines the SC's orientation concerning the south. The other associated angles which are used for tracking the sun w.r.t. any latitude on the earth are declination angle (δ), incidence angle (θ), hour angle (ω), zenith angle (θ_Z), and sunset angle (ω_S) [30].

Global irradiance (GI) is the total solar power received by the surface of SC. GI is the sum of direct, diffuse, and reflection irradiation from the nearby surfaces (Fig. 2).

$$H_{g,t} = H_{Beam} + H_{Diffuse} + H_{Reflected} \tag{1}$$

$$H_{g,t} = H \cdot R_B + H_d \cdot \left(\frac{(1+\cos\beta)}{2}\right) + \left(\left(H + H_d\right) \cdot \rho_g \cdot \left(\frac{(1-\cos\beta)}{2}\right)\right) \tag{2}$$

R_B is the beam radiation view factor, H is the direct beam irradiation normal to the horizontal surface, H_d is the monthly average diffuse irradiation, and ρ_g is the reflection factor. Correlations for various associated entities are readily available in the literature [9,17,24,30].

For estimating the diffuse radiation component on a tilted surface, two types of models, viz., isotropic and anisotropic models, are implemented. Yadav and Chandel [33] performed a comparative analysis of different diffuse radiation models for a location in the northern hemisphere. They uncovered that the results obtained from the isotropic model (Liu and Jordan [31]) were maximally effective. Therefore, the same model has been used in this study.

3. Tilt angle assessment

3.1. Data collection

Computations for tilt angle optimization have been conducted for Hamirpur, Himachal Pradesh (L31°42$'N$), located in the Western

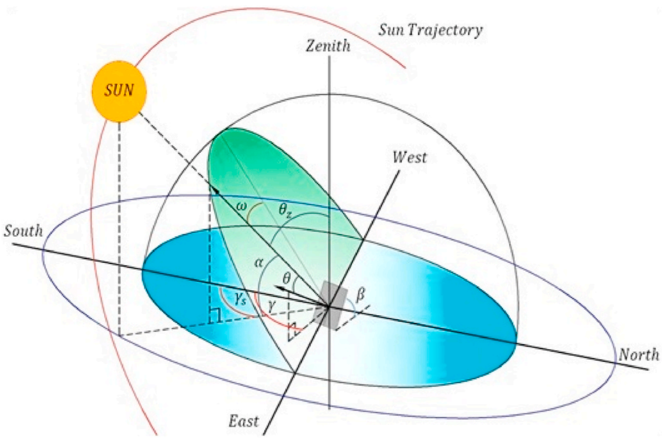


Fig. 1. The schematic is representing different solar angles and orientations for the flat-SC.

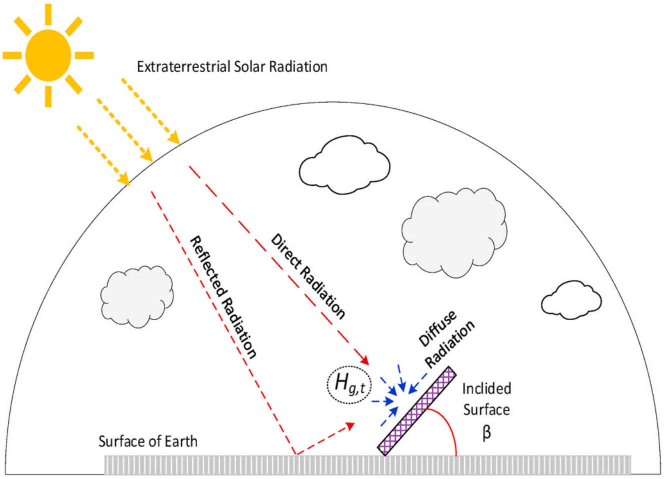


Fig. 2. The schematic is representing the sky and different constituents of solar irradiation over a tilted collector. Mathematically, GI is given as [31,32].

Himalaya region. Data for diffuse and global irradiance from 2012 to 2017 have been taken from the Center for Energy and Environmental Engineering situated at the National Institute of Technology, Hamirpur. Solar irradiance at the selected location varies throughout the year (Fig. 3). Two peaks of solar irradiance and sunshine hours have been observed in April–May and August–September.

3.2. Monthly optimum tilt angle ($\beta_{opt.,monthly}$)

Solar irradiance on the tilted surface has been computed using Eq. (2). Since the location is in the northern hemisphere, the SC is oriented toward the true south (i.e., γ=0°). The tilt angles have been varied from 0° to 90° for the calculations. The tilt angle corresponding to the maximum solar irradiance has been selected as the optimum tilt angle.

3.3. Correlations for optimum tilt angles

The methodology for developing optimum tilt angle correlations has been discussed in this section. Three mathematical models (Linear, second-degree polynomial, and third-degree polynomial equations) have been developed to estimate the optimum tilt angle based on the declination angle. Declination angles have been evaluated

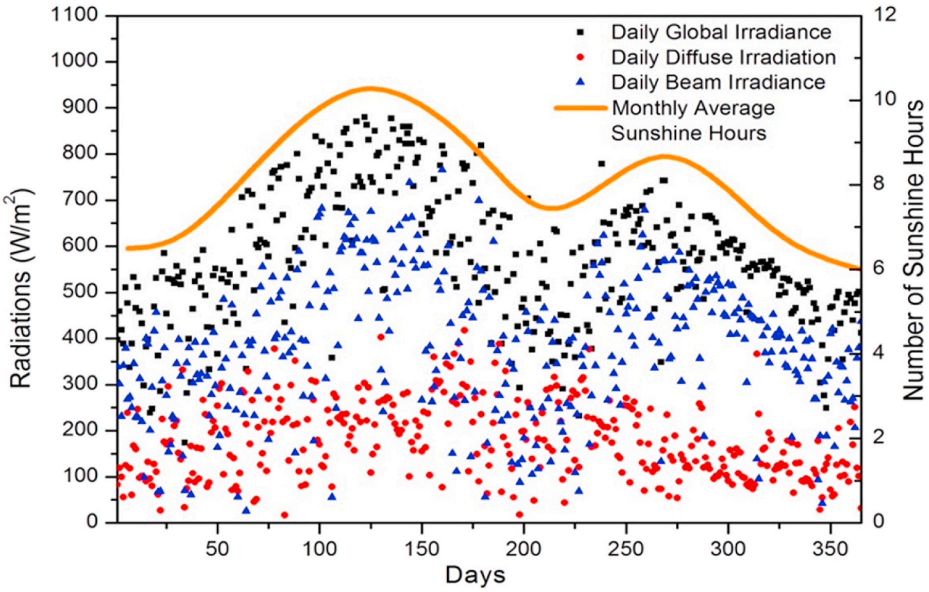


Fig. 3. Daily contribution of different components of the GI on the horizontal surface and the number of sunshine hours.

corresponding to the Julian day at the center of each month. These values are depicted in Table 1.

3.4. Statistical analysis

The developed correlations have beenstatistically analyzed with four statistical methods [35–37]. The selected methods are coefficient of determination (R2), sum square relative error (SSRE), relative standard error (RSE), and root mean bias error (RMSE).

The coefficient of determination (R2) shows how much one variable depends on another. This statistical tool provides a linear relationship between the measured and calculated data. The value of R2 ranges from 0 to 1, and its best value is close to 1. It is evaluated by Eq. (4). Here, *ca* and *ma* are the average of the calculated and measured quantities, respectively.

$$R^2 = \frac{\sum_{i=1}^{n}(ci - ca) \times (mi - ma)}{\sqrt{\left[\sum_{i=1}^{n}(ci - ca)^2\right] \times \left[\sum_{i=1}^{n}(mi - ma)^2\right]}} \tag{3}$$

The sum of the square of the relative error (SSRE) shows the total square error and is normalized by dividing it by the absolute error of correlation. The outcome of this statistical tool is a positive integer, and the ideal value is zero.

$$SSRE = \sum_{i=1}^{n}\left(\frac{mi - ci}{ci - \bar{c}}\right)^2 \ where, \ \bar{c} = \frac{1}{n}\sum_{i=1}^{n} ci \tag{4}$$

The relative standard error (RSE) determines the accuracy degree of the correlation and the actual results. The RSE is calculated using,

$$RSE = \sqrt{\frac{\sum_{i=1}^{n}\left(\frac{mi-ci}{mi}\right)^2}{n}} \tag{5}$$

Root mean square error (RMSE) is another significant parameter to compare short-term measured and predicted model performance. Its outcome is a positive integer, and the ideal value is zero. It is calculated as follows:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(ci - mi)^2} \tag{6}$$

These equations determine certain margins of error in predicted and measured results.

3.5. Utilizable scenarios and models

Four scenarios, including a total of fourteen models, have been developed in this study. These models have been framed based on seasons, average monthly solar irradiance, and range of $\beta_{opt.,monthly}$ for the current site. Further explanation of the model formulation methodology has been discussed in section 4.5.

3.6. Cost-benefit analysis

Solar irradiation collection through SCs is a function of the incident GI. However, the economy of this process depends on other variables as well. The professional technicians adjust the tilt angle and charge a considerable amount. Consequently, an increase in adjustments can also affect the overall economy of the solar energy collection. The SC tilted at $\beta_{opt.,monthly}$ can collect maximum solar energy. However, the number of adjustments associated with this model is also high; therefore, it cannot be considered the best model. A market-based cost-benefit analysis [29] has been executed to discover the most practicable model. Cost-benefit analysis is an economic tool for estimating the overall earning by evaluating different benefits and costs and subtracting the service's prices to ultimately acknowledge the overall earnings [38]. The primary considerations for the cost-benefit analysis are as follows:

(a) PV panels have been considered as SCs to calculate solar energy collection economics. PV panels are supposed to spread over a trivial area of 1500 m^2.
(b) PV panel's efficiency is the ratio of electricity generated to the received solar irradiation [39]. At present, the solar cells can work with maximum efficiency of 30% [40]. For the current estimation, the PV efficiency is assumed as 20%.

Table 1
Suggested values of Julian day and declination angle for different months [34].

Months	Jan.	Feb.	Mar.	Apr.	May.	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
Day of the year	17	47	75	105	135	162	198	228	258	288	318	344
Date	17/1	16/2	16/3	15/4	15/5	11/6	17/7	16/8	15/9	15/10	14/11	10/12
Declination (δ)	−20.92	−13.29	−2.42	9.41	18.79	23.09	21.18	13.45	2.22	−9.60	−18.91	−23.05

(c) The thermal, transmission, and other miscellaneous losses reduce the generated electricity significantly at the application end. It is well-known that the temperature of the surroundings strongly influences the output of the PV panel [41]. For every degree rise in temperature from the standard (i.e., 25°C), the PV panel's efficiency decreases by 0.4-0.65% [42]. So, the percentage of energy loss by an increase in temperature is assumed to be 5% in this study. Conversion losses of DC to AC, with transmission losses, are considered to be 5%. Other miscellaneous losses due to dust and shadow have been assumed to be about 10% of total energy conversion.

(d) The price of electricity has been calculated by kWh of energy consumption. Thus kWh/day energy is estimated by multiplying the output energy of PV panels by the total number of sunshine hours per day. Average sunshine hours are 9.66 h/day for this location. The total sunshine hours have been considered 7 h/day for this study by bearing in mind extreme environmental conditions.

(e) The price of electricity in India is different in every state. For Himachal Pradesh, the per-unit charge slab starts from Rs. 1 to a maximum of Rs. 4.35 for domestic subsidized supply. The least price of solar electricity is 2.44 Rupees. In this study, the electricity price is assumed to be 2.5 Rs./kWh.

Earnings have been evaluated by considering various losses from the most feasible models of each scenario. Aggregate earnings or benefits from PV panels are a function of tilt angle adjustment cost. Final earnings have been evaluated considering this factor, and results have been discussed.

4. Results and discussion

4.1. Optimum tilt angle assessment

Daily solar irradiance on a tilted SC at different angles has been computed using Eq. (2). The results are plotted in Fig. 4(a). Optimum tilt angles corresponding to maximum irradiance have been shown in Fig. 4(b).

The graph shows that the optimum tilt angles reaches to maximal values between November and January. However, its smallest values are observed from May to July. GI for $\beta_{opt.,monthly}$ is compared with the cases when the SC is fitted at a $\beta=0°$ and $\beta=\varphi$. Month-wise comparison data for solar irradiance over differently tilted surfaces are presented in Table 2. The data depict that the enhancement in solar irradiance from $\beta=0°$ increases gradually from June to December, and afterward, it decreases from January to June. The enhancement from $\beta=31.69°$ has two peaks: one between October and February and another between April and August. The minimum value has been observed in March and September. The monthly maximum solar irradiance enhancements for $\beta_{opt.,monthly}$ are 58.7% and 8.8% from $\beta=0°$ and $\beta=31.69°$ respectively. These data show that the SC oriented at $\beta_{opt.,monthly}$ could significantly collect more irradiations.

4.2. The correlations for optimum tilt angles

The correlations have been formulated for the *βopt.,monthly*. Eq. (12), Eq. (13), and Eq. (14) represent the linear, second-degree polynomial, and third-degree polynomial mathematical models, respectively (Fig. 5). By using these mathematical models, the *βopt.,monthly* can be assessed. The following section discusses the statistical analysis of these developed models.

$$\beta_{OPT.} = -1.0728(\delta) + 33.285 \tag{12}$$

$$\beta_{OPT.} = -0.0009(\delta)^2 - 1.0727(\delta) + 33.515 \tag{13}$$

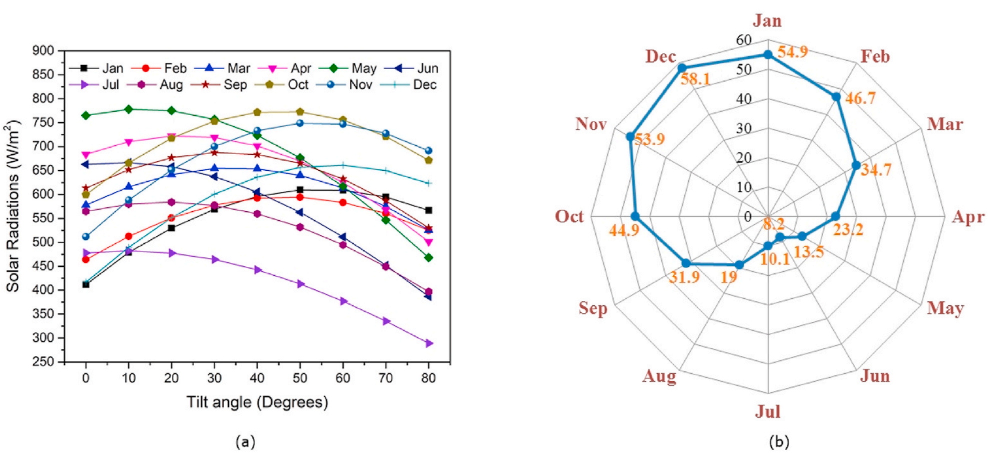


Fig. 4. (a) Variation of solar insolation with tilt position for different months, (b) Monthly optimum tilt angle (Degrees).

Table 2
Comparison of average GI (W/m^2) over the surface at different values of tilt angle.

Month	$(H_{g,t})_{\beta=0^\circ}$	$(H_{g,t})_{\beta=\varphi}$	$(H_{g,t})_{\beta=monthly\ opt.}$	Solar irradiation collection enhancement for $\beta_{opt.,monthly}$ (%)	
				From β=0°	From β=31.69°
Jan	411	551	583	41.7	5.8
Feb	464	581	595	28.3	2.4
Mar	578	655	656	13.5	0.1
Apr	684	717	723	5.7	0.8
May	765	752	779	1.8	3.5
Jun	663	633	667	0.6	5.4
Jul	478	461	482	0.9	4.5
Aug	565	575	584	3.4	1.6
Sep	613	688	688	12.2	0.0
Oct	599	758	775	29.2	2.2
Nov	512	707	750	46.6	6.1
Dec	417	607	662	58.7	8.8

$$\beta_{OPT.} = -0.00002(\delta)^3 - 0.0009(\delta)^2 - 1.0647(\delta) + 33.515 \quad (14)$$

4.3. Statistical analysis

The R2, RSE, SSRE, and RMSE have been used to check the accuracy of developed mathematical models. The outcomes of these statistical tools are presented in Table 3. The ideal product of R2 is 1. Eq. (13) and Eq. (14) (0.9986 for each) provide the most relevant result of R2 as compared to Eq. (12) (0.9985). For RSE, SSRE, and RMSE, the most suitable outcome is near to zero. Eq. (14) provides comparatively more relevant results for RSE (0.0206), SSRE (0.005), and RMSE (0.6633). Fig. 6 shows the calculated and predicted *βopt.,monthly*. No significant deviations observed when *βopt.,monthly* are estimated by Eqs. (12)–(14). The stable results have been obtained from all the developed equations for monthly percentage error (Table 4). The highest value is in July for all the equations, while the lowest value is in December. Average absolute error values for Eq. (12)-13-14are 2.07, 1.63, and 1.60, respectively. Here, the equation that best represents all values is Eq.(14).

4.4. Comparison

The comparison of the current procured results with the published literature has been carried out. For this comparison, the $\beta_{opt.,yearly}$ has been calculated by averaging the $\beta_{opt.,monthly}$. Afterward, the comparison of the $\beta_{opt.,yearly}$ has been conducted with the correlation provided in the literature.

Table 5 presents the comparison of previous studies in the literature at Hamirpur and the values of the current study. These reference equations are developed particularly for the northern hemisphere. The table shows that the $\beta_{opt.,yearly}$ obtained from the correlations significantly deviate from the current study [30,43–46]. However, the variation has been observed least from the correlations developed for a particular region. The Darhmaoui and Lahjouji [47] estimated the $\beta_{opt.,yearly}$ for Mediterranean regions and

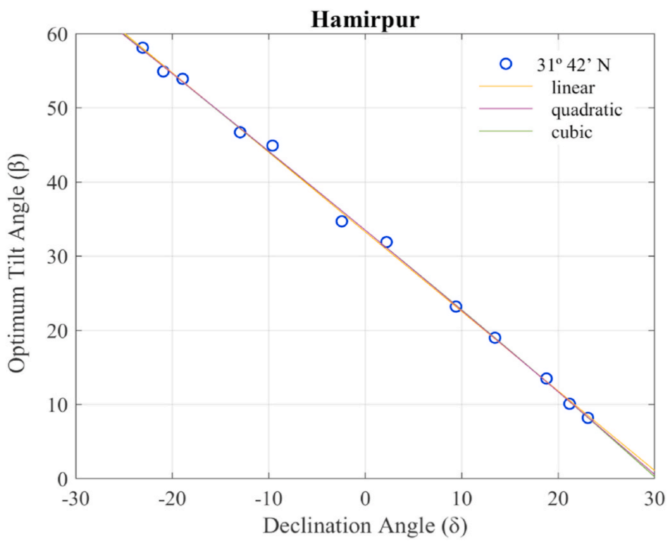


Fig. 5. Correlation providing monthly optimum tilt angles for Hamirpur (Eq. (12) to Eq. (14)).

Table 3
Statistical analysis of thedevelopedmathematical models.

Statistical analysis	Eq. 12	Eq. 13	Eq. 14
R^2	0.9985	**0.9986**	**0.9986**
RSE	0.0254	0.0211	**0.0206**
SSRE	0.0072	0.0051	**0.0050**
RMSE	0.6831	0.6690	**0.6633**

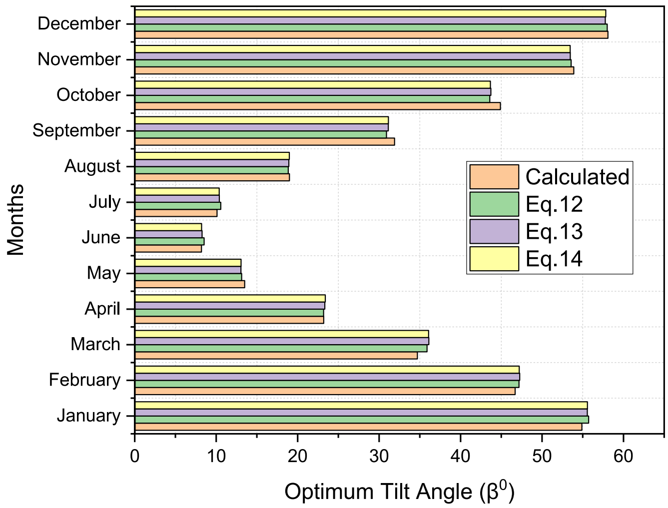


Fig. 6. Calculated and predicted values comparison for Hamirpur.

Table 4
Statistical error (%) of thedeveloped equations for Hamirpur.

Months	Eq.12	Eq.13	Eq.14
January	−1.5	−1.1	−1.1
February	−0.9	−1.1	−1.1
March	−3.2	−4.0	−3.7
April	0.4	−0.4	−0.4
May	3.0	3.7	3.7
June	−3.7	0	0
July	−4.0	−2.0	−2.0
August	1.1	0.5	0.5
September	3.1	2.5	2.5
October	3.1	2.7	2.9
November	0.7	0.9	0.9
December	0.2	0.7	0.5

Table 5
Optimum tilt angle by current and literature studies for Hamirpur.

Reference	Model	$\beta_{opt., yearly}$ (Degrees)
Wessley et al. [11]	$\beta_{opt., yearly} = 0.823\varnothing + 8.8274$	34.90
Duffie and Beckman [30]	$\beta_{opt., yearly} = (\varnothing + 15) \pm 15$	39.19
Jamil et al. [43]	$\beta_{opt., yearly} = \varnothing$	31.69
Ajao et al. [44]	$\beta_{opt., yearly} = \varnothing + 13.5$	45.19
Kern and Herris [45]	$\beta_{opt., yearly} = \varnothing + 10$	41.69
Yellot [46]	$\beta_{opt., yearly} = \varnothing + 20$	51.69
Darhmaoui and Lahjouji [47]	$\beta_{opt.yearly} = 1.25351\ \varnothing - 0.00728944\ \varnothing^2$	32.64
Present study		33.26

developed a correlation in terms of latitude angle. A trivial difference of 0.64° has been observed between Darhmaoui and Lahjouji [47] and the current study. Similarly, Wessley et al. [11] evaluated the optimum tilt angle for Indian cities and developed a correlation for $\beta_{opt.,yearly}$. A minor deviation of 1.64° has been observed between Wessley et al. [11] and the current study. This comparison can also be detected as a validation for this study.

4.5. Utilization models

The models under different scenarios have been developed to increase the practical applicability of the assessed optimum tilt angles. For scenario I, three models have been designed with five annual adjustments. The averaging of the $\beta_{opt.,monthly}$ is an effective method to devise the different seasons. Thereby, the M − 1 and M − 3 have been developed based on the range of optimum tilt angle. Fig. 4(b) demonstrates that the maximum $\beta_{opt.,monthly}$ values have been observed from November to February. Whereas approximately similar values have been observed for March and September, April and August, May and July. For M − 1, the November to February has kept in one adjustment group while the other adjustment groups have been defined relatively.

For designing the M − 2, the variation in solar irradiance for the consecutive month has been chosen as a basis. Four models have been designed for scenario II, which requires four adjustments per year. Models M − 4 and M − 6 have been designed as per the suggestion of [27] and M − 5, according to Ref. [24]. For the selected site, winter, summer, monsoon, and autumn last from December to March, April–May, June–August, and September–November, respectively (Fig. 7 and [48]). Based on this data, M − 7 has been designed. The models under scenarios III and IV encompass three and two adjustments per year, respectively. The M − 10 and M − 14 have been formulated as per the convenience of dividing the months of a year.

The M − 8 and M − 9 have been chosen, as suggested by Refs. [24,27], respectively. The M − 11, M − 12, and M − 13 have been designed based on the range of $\beta_{opt.,monthly}$ for different months. Fig. 7 additionally clarifies the concept of model formulation. Yearly average solar irradiance on the tilted surface from each model has been estimated and compared with each other. The range of months for different models and their respective tilt angles have been presented in Table 6 and Table 7, respectively.

4.5.1. Scenario I

In this scenario, it has been assumed that five adjustments per year are sufficient for irradiation collection. The annual average solar irradiance for M − 1, M − 2, and M − 3 has been computed and presented in Fig. 8(a). M − 3 provides maximum average irradiance compared to M − 1 and M − 2 in Scenario I.

4.5.2. Scenario II

This scenario assumes that only four adjustments per year are sufficient for collecting solar irradiations. Four models have been developed, and four seasons are considered for each model. The annual average of incident irradiance has been computed for each model and compared with each other. Approximately the same solar irradiance has been observed for all the models under this scenario (see Fig. 8 (b)). The descending order of models based on annual average incident irradiances is M-7>M-5>M-4>M-6.

4.5.3. Scenario III

For this scenario, four models have been devised in which each model requires three annual adjustments. As the numbers of adjustments carried throughout the year are comparatively low, so are the incident irradiance on the tilted SC. The annual average

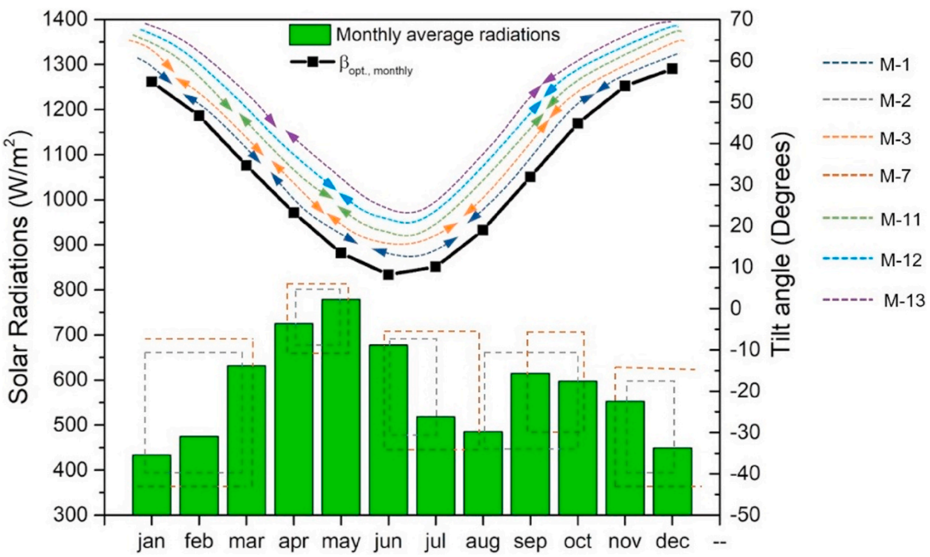


Fig. 7. The concept for the model formulation.

Table 6
Scenarios and models for the tilt angle.

S. No.	Scenario I			Scenario II				Scenario III				Scenario IV		
	M-1	M-2	M-3	M-4	M-5	M-6	M-7	M-8	M-9	M-10	M-11	M-12	M-13	M-14
1	Nov.–Jan.	Jan.–Mar.	Oct.–Jan.	Nov.–Jan.	Dec.–Feb.	Jan.–Mar.	Dec.–Mar.	Nov.–Feb.	Dec.–Mar.	Jan.–Apr.	Oct.–Feb.	Oct.–May	Oct.–Mar.	Jan.–Jun.
2	Feb.–Mar.	Apr.–May	Feb.–Mar.	Feb.–Apr.	Mar.–May	Apr.–Jun.	Apr.–May	Mar.–Jun.	Apr.–Jul.	May–Aug.	Mar.–Apr.	Jun.–Sep.	Apr.–Sep.	Jul.–Dec.
3	Apr.–May	Jun.–Jul.	Apr.	May–Jul.	Jun.–Aug.	Jul.–Sep.	Jun.–Aug.	Jul.–Oct.	Aug.–Nov.	Sep.–Dec.	May–Sep.			
4	Jun.–Jul.	Aug.–Oct.	May -Jul.	Aug.–Oct.	Sep.–Nov.	Oct.–Dec.	Sep.–Nov.							
5	Aug.–Oct.	Nov.–Dec.	Aug.–Sep.											

Table 7
Tilt angle (degrees) for different scenarios and models.

Month	Scenario I			Scenario II				Scenario III				Scenario IV		
	M-1	M-2	M-3	M-4	M-5	M-6	M-7	M-8	M-9	M-10	M-11	M-12	M-13	M-14
Jan	55.5	45.4	52.9	55.6	53.2	45.4	48.6	53.4	48.6	40	51.7	41.2	48.9	30.2
Feb	40.7	45.4	40.7	34.9	53.2	45.4	48.6	53.4	48.6	40	51.7	41.2	48.9	30.2
Mar	40.7	45.4	40.7	34.9	23.6	45.4	48.6	20	48.6	40	29	41.2	48.9	30.2
Apr	18.3	18.3	23.1	34.9	23.6	14.9	18.4	20	13.8	40	29	41.2	17.7	30.2
May	18.3	18.3	10.6	10.6	23.6	14.9	18.4	20	13.8	12.7	16.6	41.2	17.7	30.2
Jun	9.13	9.13	10.6	10.6	12.4	14.9	12.4	20	13.8	12.7	16.6	17	17.7	30.2
Jul	9.13	9.13	10.6	10.6	12.4	20.4	12.4	26.5	13.8	12.7	16.6	17	17.7	36.2
Aug	31.9	31.9	25.5	31.9	12.4	20.4	12.4	26.5	37.5	12.7	16.6	17	17.7	36.2
Sep	31.9	31.9	25.5	31.9	43.6	20.4	43.6	26.5	37.5	47.2	16.6	17	17.7	36.2
Oct	31.9	31.9	52.9	31.9	43.6	52.3	43.6	26.5	37.5	47.2	51.7	41.2	48.9	36.2
Nov	55.5	55.9	52.9	55.6	43.6	52.3	43.6	53.4	37.5	47.2	51.7	41.2	48.9	36.2
Dec	55.5	55.9	52.9	55.6	53.2	52.3	48.6	53.4	48.6	47.2	51.7	41.2	48.9	36.2

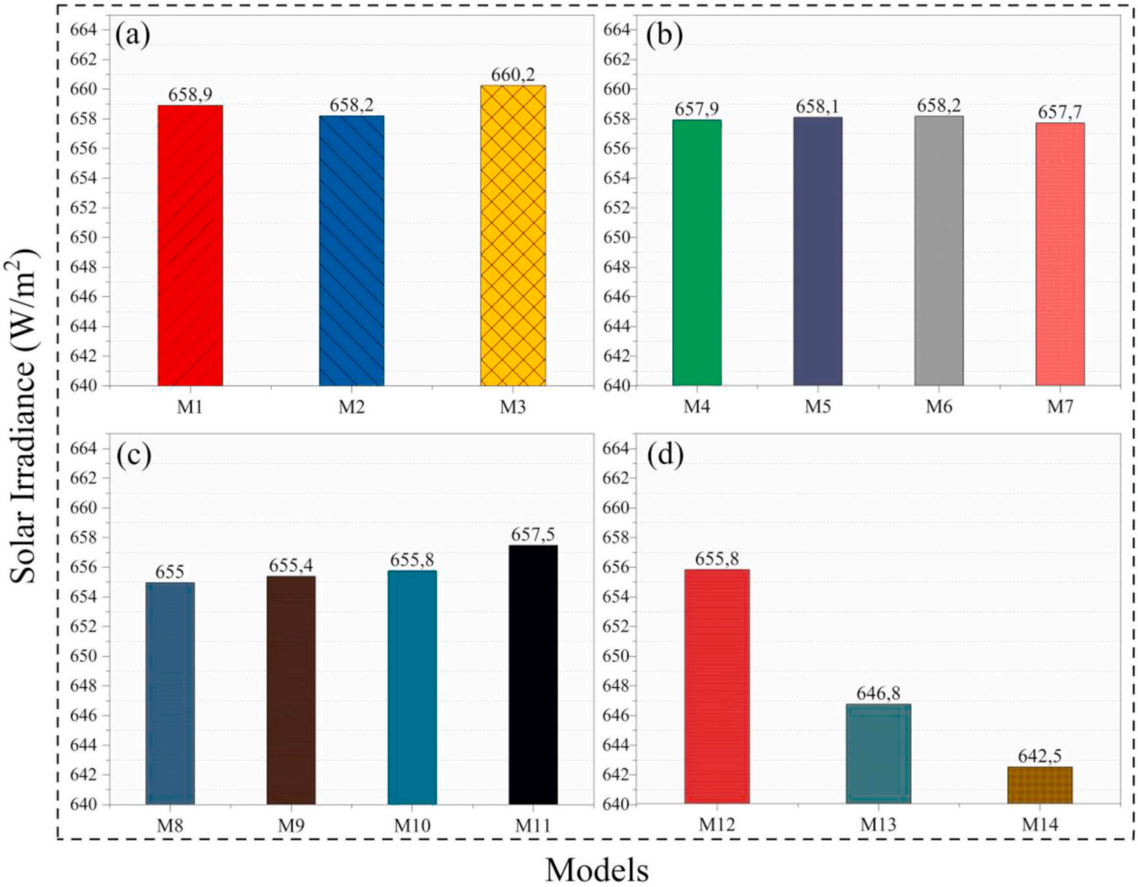


Fig. 8. Average GI for (a) scenario I, (b) scenario II, (c) scenario III and (d) scenario IV.

incident irradiance on the SC surface for models under Scenario III (i.e., M − 8, M − 9, M − 10, and M − 11) are shown in Fig. 8(c). The figure supports the idea that M − 11 can collect more irradiation than any other model in this scenario.

4.5.4. Scenario IV

Two annual adjustment models have been considered under this scenario. Incident irradiations on the tilted surface are low for this scenario. The annual average irradiance for models M − 12, M − 13, and M − 14 are presented in Fig. 8(d). Based on annual average irradiance, the descending order of models is M-12>M-13>M-14.

4.5.5. Inter-scenario comparison

An inter-scenario comparison has been conducted to find the most feasible model. Table 8 shows the monthly average irradiance for M − 3, M − 7, M − 11, and M-12. Average GI is a function of the number of adjustments conducted on the collector. As the number of adjustments performed in the monthly optimum model is the most and the least for M − 12, these models collect maximum and minimum irradiance, respectively. Table 8 also represents the enhancement for each model concerning the horizontally placed SC. The table proposes that the enhancement is highest for $\beta_{opt.,monthly}$ followed by models from the scenario I, II, III, and IV, respectively. A cost-benefit analysis on PV panels has been carried out and presented in the next section to find the most feasible model.

4.6. Most practicable model estimation using cost-benefit analysis

A speculative cost-benefit analysis has been carried outto find the most practicable model. Table 9 shows the number of units generated and the amount of money earned (in Rs.) from the selected models. It can be seen that the model where PV panels are tilted at a $\beta_{opt.,monthly}$ provides maximum earnings.

This amount earned from PV panels is also subjected to maintenance and tilt angle adjustment charges. The maintenance charges implied are the same for each model. Thereby, maintenance charges are neglected in this study. Since a trivial area of 1500 m^2 has been considered in the study, that can be adjusted by two technicians in one day only. Local PV panel technician charges an amount of 2500 Rs./day. Table 10 depicts the calculation for tilt angle adjustment charges. Table 11 shows the actual amount earned by considering adjustment charges for different models. Annual earnings from different models have been plotted in Fig. 9. Total earnings without considering the adjustment expenses are maximum for $\beta_{opt.,monthly}$. However, the same does not remain valid when the effect of adjustment expenses was considered. For the $\beta_{opt.,monthly}$ model, the maximum budget is spent on adjustment expenses, due to which it is the most undesirable model. M − 11 gives the most earnings when the effect of adjustment charges are considered. The descending order of models based on their earnings is: M-11>M-7>M-12>M-3>β=31.69°>Monthly Optimum.

5. Conclusions

Analytical computations have been conducted for the optimum tilt angle assessment at a location (Hamirpur L31°42′) of the Western Himalayan Region. Mathematical models have been developed and statistically analyzed. For practical implementations, some useable models have been formulated and compared with each other. The following results can be drawn from this study.

a) SC tilted at $\beta_{opt.,monthly}$ provides 18.03% and 4.0% enhancement from conventional SC fitted horizontally and tilted equal to latitude angle, respectively.
b) Three mathematical models for linear, second-degree polynomial, and third-degree polynomial have been developed (Eq. (12), Eq. (13), and Eq. (14)) for $\beta_{opt.,monthly}$ based on the declination angle. It is proposed to use the linear model due to its simplicity and slightest deviation.
c) Different statistical tools have been used for checking the accuracy of the developed mathematical models. The mathematical model of the third-degree polynomial (Eq. (14)) provides the most relevant results with R^2 (0.9986), RSE (0.0206), SSRE (0.0050), and RMSE (0.6633).
d) For practical implementation of optimum tilt angles, fourteen models with different annual adjustments have been formulated. The model involving five annual adjustments (M − 3) provides the most enhancements, while the model with two annual adjustments (M − 14) collects the least solar irradiations.
e) A speculative cost-benefit analysis has been carried out to estimate the effect of adjustment expenses. The implementation of M − 11 (three annual adjustment model) achieves maximum earnings. The descending order of the models, based on their earnings is M − 11 > M − 7 > M − 12 > M − 3 > β=31.69° > Monthly Optimum.

Table 8
Monthly comparison of GI (W/m^2) for most feasible models of different scenarios, monthly optimum, horizontal surface, and tilted surface to latitude.

Irradiance	β=0°	β=31.69°	Monthly optimum	M-3	M-7	M-11	M-12
Jan	411.4	550.7	582.8	582.6	580.6	581.3	575.3
Feb	463.9	581.1	595.0	592.9	594.8	592.3	589.2
Mar	577.7	655.1	655.7	653.2	642.8	652.5	636.7
Apr	683.9	717.1	722.6	722.6	721.0	719.0	715.0
May	765.0	752.2	778.6	778.1	776.5	776.7	771.6
Jun	662.9	632.6	666.8	666.4	665.6	661.2	655.5
Jul	477.7	461.1	482.0	482.0	481.8	479.0	473.8
Aug	564.7	574.9	584.0	581.5	581.7	582.7	578.4
Sep	613.2	687.7	687.8	684.4	678.4	668.8	666.7
Oct	599.5	757.6	774.4	769.4	774.1	770.0	767.9
Nov	511.8	707.0	750.2	750.1	740.7	748.7	742.4
Dec	416.9	607.8	661.5	659.5	654.6	657.4	649.5
Average	562.4	640.4	661.8	660.2	658.5	657.5	651.8
Enhancement $\left(\frac{(H_{g,t})_{\beta=Optimum} - (H_{g,t})_{\beta=0^\circ}}{(H_{g,t})_{\beta=0^\circ}} \times 100\right)$			**18.03**	**17.85**	**17.5**	**17.32**	**16.25**

Table 9
Annual money earned through various models by considering losses.

S. No.	Model	Irradiance (W/m^2)	Irradiance collected over 1500 m^2 (W)	Energy collected per day (kWh/day)	Ideal energy converted by PV panels (kWh/day)	Energy converted by considering different losses (kWh/day)	Amount earned per day (Rs. /day)	Money earned annually (Rs. /day)
1	Monthly optimum	661.8	992670	6948.69	1389.738	1128.815	2822	1030043
2	M-3	660.2	990360	6932.52	1386.504	1126.188	2815	1027646
3	M-7	658.5	987780	6914.46	1382.892	1123.254	2808	1024969
4	M-11	657.5	986205	6903.435	1380.687	1121.463	2803	1023335
5	M-12	651.8	977745	6844.215	1368.843	1111.843	2779	1014556
6	β=31.69°	640.4	960600	6724.2	1344.84	1092.346	2730	996766

Table 10
Expenses on adjustments for different models.

S. No.	Model	Adjustments required annually	Annual expenses on adjustments (Rs. /year)
1	Monthly optimum	12	60000
2	M-3	5	25000
3	M-7	4	20000
4	M-11	3	15000
5	M-12	2	10000
6	β=31.69°	0	None

Table 11
The actual amount earned by different models considering adjustment expenses.

S. No.	Model	Annual expenses on adjustments (Rs. /Year)	Money earned annually (Rs. /Year)	The actual amount earned (Rs. /Year)
1	Monthly optimum	60000	1030043	970043
2	M-3	25000	1027646	1002646
3	M-7	20000	1024969	1004969
4	M-11	15000	1023335	1008335
5	M-12	10000	1014556	1004556
6	β=31.69°	0	996766	996766

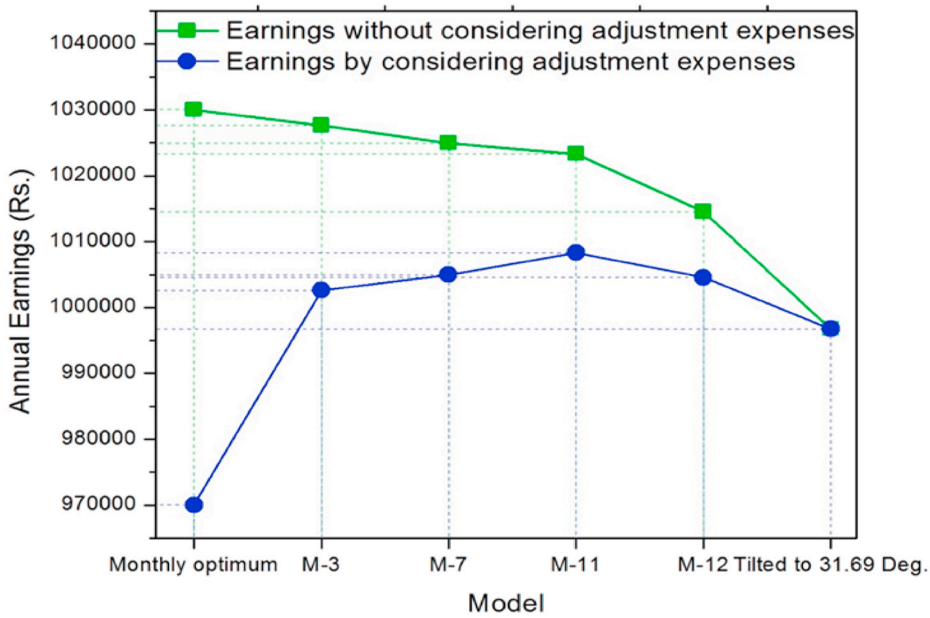


Fig. 9. Annual earnings from PV panels with and without considering adjustment expenses.

This study suggests that the output of the SC significantly improves with tilt angle optimization. The obtained results can be highly beneficial in the installation of SCs at the specified region.

CRediT authorship contribution statement

Ashutosh Sharma: Conceptualization, Methodology, Formal analysis, Writing – original draft. **Mehmet Ali Kallioğlu:** Conceptualization, Methodology, Formal analysis. **Anchal Awasthi:** Conceptualization, Methodology, Formal analysis. **Ranchan Chauhan:** Supervision, Conceptualization, Methodology, Writing – review & editing. **Gusztáv Fekete:** Formal analysis, Writing – review & editing. **Tej Singh:** Supervision, Conceptualization, Formal analysis, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgment

Data for solar irradiance for the current location are provided by the National Institute of Technology, Hamirpur, Himachal Pradesh, India, for which their support is highly appreciated.

References

[1] F.C. Eboh, P. Ahlström, T. Richards, Evaluating improvements in a waste-to-energy combined heat and power plant, Case Stud. Therm. Eng. 14 (2019) 100476, https://doi.org/10.1016/j.csite.2019.100476.
[2] A. Sharma, R. Chauhan, T. Singh, A. Kumar, R. Kumar, A. Kumar, M. Sethi, Optimizing discrete V obstacle parameters using a novel Entropy-VIKOR approach in a solar air flow channel, Renew. Energy 106 (2017) 310–320, https://doi.org/10.1016/j.renene.2017.01.010.
[3] A.N. Vilarinho, J.B.L.M. Campos, C. Pinho, Energy and exergy Analysis of an aromatics plant, Case Stud. Therm. Eng. 8 (2016) 115–127, https://doi.org/10.1016/j.csite.2016.06.003.
[4] T. Srinivas, B.V. Reddy, Hybrid solar–biomass power plant without energy storage, Case Stud. Therm. Eng. 2 (2014) 75–81, https://doi.org/10.1016/j.csite.2013.12.004.
[5] A. Inayat, H.H. Ang, M. Raza, B.A.A. Yousef, C. Ghenai, M. Ayoub, S.I.U.H. Gilani, Integration and simulation of solar energy with hot flue gas system for the district cooling application, Case Stud. Therm. Eng. 19 (2020) 100620, https://doi.org/10.1016/j.csite.2020.100620.
[6] F.M. Hoffmann, R.F. Molz, J.V. Kothe, E.O.B. Nara, L.P.C. Tedesco, Monthly profile analysis based on a two-axis solar tracker proposal for photovoltaic panels, Renew. Energy 115 (2018) 750–759, https://doi.org/10.1016/j.renene.2017.08.079.
[7] H. Gnaifaid, H. Ozcan, Multi-objective optimization of a concentrated solar energy driven trigeneration plant with thermal energy storage: a case study for Turkey, Case Stud. Therm. Eng. 20 (2020) 100642, https://doi.org/10.1016/j.csite.2020.100642.
[8] K.-M. Chung, C.-C. Chen, K.-C. Chang, Effect of diffuse solar radiation on the thermal performance of solar collectors, Case Stud. Therm. Eng. 12 (2018) 759–764, https://doi.org/10.1016/j.csite.2018.10.006.
[9] M. Despotovic, V. Nedic, Comparison of optimum tilt angles of solar collectors determined at yearly, seasonal and monthly levels, Energy Convers. Manag. 97 (2015) 121–131, https://doi.org/10.1016/j.enconman.2015.03.054.
[10] M. Shehadi, Optimizing solar cooling systems, Case Stud. Therm. Eng. 21 (2020) 100663, https://doi.org/10.1016/j.csite.2020.100663.
[11] G.J.J. Wessley, R.N. Starbell, S. Sandhya, Modelling of optimal tilt angle for solar collectors across eight Indian cities, Int. J. Renew. Energy Resour. 7 (2017) 353–358.
[12] M. Benghanem, Optimization of tilt angle for solar panel: case study for Madinah, Saudi Arabia, Appl. Energy 88 (2011) 1427–1433, https://doi.org/10.1016/j.apenergy.2010.10.001.
[13] A.G. Siraki, P. Pillay, Study of optimum tilt angles for solar panels in different latitudes for urban applications, Sol. Energy 86 (2012) 1920–1928, https://doi.org/10.1016/j.solener.2012.02.030.
[14] S. Soulayman, W. Sabbagh, Optimum tilt angle at tropical region, Int. J. Renew. Energy Dev. 4 (2015) 48–54, https://doi.org/10.14710/ijred.4.1.48-54.
[15] E. Calabrò, An algorithm to determine the optimum tilt angle of a solar panel from global horizontal solar radiation, J. Renew. Energy. 2013 (2013), https://doi.org/10.1155/2013/307547.
[16] H.K. Elminir, A.E. Ghitas, F. El-Hussainy, R. Hamid, M.M. Beheary, K.M. Abdel-Moneim, Optimum solar flat-plate collector slope: case study for Helwan, Egypt, Energy Convers. Manag. 47 (2006) 624–637, https://doi.org/10.1016/j.enconman.2005.05.015.
[17] H.R. Ghosh, N.C. Bhowmik, M. Hussain, Determining seasonal optimum tilt angles, solar radiations on variously oriented, single and double axis tracking surfaces at Dhaka, Renew. Energy 35 (2010) 1292–1297, https://doi.org/10.1016/j.renene.2009.11.041.
[18] E.D. Mehleri, P.L. Zervas, H. Sarimveis, J.A. Palyvos, N.C. Markatos, Determination of the optimal tilt angle and orientation for solar photovoltaic arrays, Renew. Energy 35 (2010) 2468–2475, https://doi.org/10.1016/j.renene.2010.03.006.
[19] A.M. Ali Morad, A.K. Shaker Al-Sayyab, M.A. Abdulwahid, Optimisation of tilted angles of a photovoltaic cell to determine the maximum generated electric power: a case study of some Iraqi cities, Case Stud. Therm. Eng. 12 (2018) 484–488, https://doi.org/10.1016/j.csite.2018.07.001.
[20] A.K. Yadav, S.S. Chandel, Formulation of new correlations in terms of extraterrestrial radiation by optimization of tilt angle for installation of solar photovoltaic systems for maximum power generation: case study of 26 cities in India, Sādhanā 43 (2018) 81, https://doi.org/10.1007/s12046-018-0858-2.
[21] A.K. Shaker Al-Sayyab, Z.Y. Al Tmari, M.K. Taher, Theoretical and experimental investigation of photovoltaic cell performance, with optimum tilted angle: basra city case study, Case Stud. Therm. Eng. 14 (2019) 100421, https://doi.org/10.1016/j.csite.2019.100421.
[22] C. Stanciu, D. Stanciu, Optimum tilt angle for flat plate collectors all over the World – a declination dependence formula and comparisons of three solar radiation models, Energy Convers. Manag. 81 (2014) 133–143, https://doi.org/10.1016/j.enconman.2014.02.016.
[23] K. Skeiker, Optimum tilt angle and orientation for solar collectors in Syria, Energy Convers. Manag. 50 (2009) 2439–2448, https://doi.org/10.1016/j.enconman.2009.05.031.
[24] DrF. Sultan, F.A. Ali, Tariq Khalid A. Razaq, Tilt angle optimization of solar collectors for maximum radiation in three Iraqi cities, Int. J. Eng. Ind. 3 (2012) 99–107. https://doi.org/10.4156/ijei.vol3.issue4.11.
[25] M.S. Ismail, M. Moghavvemi, T.M.I. Mahlia, Analysis and evaluation of various aspects of solar radiation in the Palestinian territories, Energy Convers. Manag. 73 (2013) 57–68, https://doi.org/10.1016/j.enconman.2013.04.026.
[26] H. Khorasanizadeh, K. Mohammadi, A. Mostafaeipour, Establishing a diffuse solar radiation model for determining the optimum tilt angle of solar surfaces in Tabass, Iran, Energy Convers. Manag. 78 (2014) 805–814, https://doi.org/10.1016/j.enconman.2013.11.048.

[27] P. Talebizadeh, M.A. Mehrabian, M. Abdolzadeh, Prediction of the optimum slope and surface azimuth angles using the Genetic Algorithm, Energy Build. 43 (2011) 2998–3005, https://doi.org/10.1016/j.enbuild.2011.07.013.
[28] O.S. Idowu, O.M. Olarenwaju, O.I. Ifedayo, Determination of optimum tilt angles for solar collectors in low-latitude tropical region, Int. J. Energy Environ. Eng. 4 (2013) 1–10, https://doi.org/10.1186/2251-6832-4-29.
[29] A.R. Prest, R. Turvey, Cost-benefit analysis: a survey, econ, J 75 (1965) 683–735, https://doi.org/10.2307/2229670.
[30] J.A. Duffie, W.A. Beckman, N. Blair, Solar Engineering of Thermal Processes, Photovoltaics and Wind, fifth ed., John Wiley & Sons, 2020.
[31] B. Liu, R. Jordan, Daily insolation on surfaces tilted towards equator, ASHRAE J U. S. 10 (1961). https://www.osti.gov/biblio/5047843-daily-insolation-surfaces-tilted-towards-equator. (Accessed 21 June 2020).
[32] M. Kacira, M. Simsek, Y. Babur, S. Demirkol, Determining optimum tilt angles and orientations of photovoltaic panels in Sanliurfa, Turkey, Renew. Energy 29 (2004) 1265–1275, https://doi.org/10.1016/j.renene.2003.12.014.
[33] P. Yadav, S.S. Chandel, Comparative Analysis of diffused solar radiation models for optimum tilt angle determination for Indian locations, Appl. Sol. Energy 50 (2014) 53–59, https://doi.org/10.3103/S0003701X14010137.
[34] M.A. Kallioğlu, A. Durmuş, H. Karakaya, A. Yılmaz, Empirical calculation of the optimal tilt angle for solar collectors in northern hemisphere, Energy Sources Part Recovery Util, Environ. Eff. 42 (2020) 1335–1358, https://doi.org/10.1080/15567036.2019.1663315.
[35] M. Despotovic, V. Nedic, D. Despotovic, S. Cvetanovic, Review and statistical analysis of different global solar radiation sunshine models, Renew. Sustain. Energy Rev. 52 (2015) 1869–1880, https://doi.org/10.1016/j.rser.2015.08.035.
[36] C.J. Willmott, K. Matsuura, Advantages of the mean absolute error (MAE) over the root mean square error (RMSE) in assessing average model performance, Clim. Res. 30 (2005) 79–82, https://doi.org/10.3354/cr030079.
[37] C.A. Gueymard, A review of validation methodologies and statistical performance indicators for modeled solar radiation data: towards a better bankability of solar projects, Renew. Sustain. Energy Rev. 39 (2014) 1024–1034, https://doi.org/10.1016/j.rser.2014.07.117.
[38] E.J. Mishan, Cost-benefit analysis. https://doi.org/10.2307/2230232, 1971.
[39] W. Shockley, H.J. Queisser, Detailed balance limit of efficiency of p-n junction solar cells, J. Appl. Phys. 32 (1961) 510–519, https://doi.org/10.1063/1.1736034.
[40] A. Blakers, N. Zin, K.R. McIntosh, K. Fong, High efficiency silicon solar cells, Energy Procedia 33 (2013) 1–10, https://doi.org/10.1016/j.egypro.2013.05.033.
[41] V. Poulek, T. Matuška, M. Libra, E. Kachalouski, J. Sedláček, Influence of increased temperature on energy production of roof integrated PV panels, Energy Build. 166 (2018) 418–425, https://doi.org/10.1016/j.enbuild.2018.01.063.
[42] A. Hasan, S.J. McCormack, M.J. Huang, B. Norton, Evaluation of phase change materials for thermal regulation enhancement of building integrated photovoltaics, Sol. Energy 84 (2010) 1601–1612, https://doi.org/10.1016/j.solener.2010.06.010.
[43] M.J. Ahmad, G.N. Tiwari, Optimization of tilt angle for solar collector to receive maximum radiation, Open Renew. Energy J. 2 (2009) 19–24.
[44] K.R. Ajao, R.M. Ambali, M.O. Mahmoud, Determination of the optimal tilt angle for solar photovoltaic panel in Ilorin, Nigeria, J. Eng. Sci. Technol. Rev. 6 (2013) 87–90, https://doi.org/10.25103/jestr.061.17.
[45] J. Kern, I. Harris, On the optimum tilt of a solar collector, Sol. Energy 17 (1975) 97–102, https://doi.org/10.1016/0038-092X(75)90064-X.
[46] J. Yellott, Utilization of sun and sky radiation for heating and cooling of buildings, ASHRAE J U. S. (1973). https://www.osti.gov/biblio/5043741. (Accessed 14 March 2021).
[47] H. Darhmaoui, D. Lahjouji, Latitude based model for tilt angle optimization for solar collectors in the mediterranean region, Energy Procedia 42 (2013) 426–435, https://doi.org/10.1016/j.egypro.2013.11.043.
[48] About District | District Hamirpur, Government of Himachal Pradesh - INDIA | India, (n.d.). https://hphamirpur.nic.in/about-district/(accessed December 16, 2020).