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Evaluation of the uncertainty due to dynamic effects in linear measuring devices – Preliminary results

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ABSTRACT

Systematic dynamic effects in linear measuring devices include possible amplification or attenuation, and phase shift of the spectral components of the indicated signal in respect to the original phenomenon. When this effect is non negligible, dynamic compensation should be applied. Yet in any case uncertainty on the modulus and the phase of the frequency response of the device will cause uncertainty on the final measurement results. Therefore, a simple formula for the evaluation of such uncertainty is presented, for periodic or harmonic dynamic phenomena.

1. Introduction

Dynamic measurement, that is measurement where the measurand value varies along time, is the object of increasing attention today, due to its application importance and to the scientific and technological challenges it still poses [1]. Dynamic measurement can be classified as either direct, where the property to be measured is the time history of the quantity of interest [2], or indirect, where some other characteristic of the quantity is searched such as, most frequently, the spectral distribution of the energy of the phenomenon [3]. Here only the case of direct dynamic measurement will be considered.

In this regard, the scientific and technical debate has developed along four main lines, strictly related to each other, but with a focus on.

- generic modelling [4,5],
- dynamic calibration [6,7],
- dynamic compensation [8–10],
- uncertainty reduction end evaluation [11–13],

where the list above includes just a few examples, among many others.

Concerning generic modelling, dynamic measurement can be considered a part of a generic framework for measurement, which has been a key topic of measurement sciences, over the years. A main concern has been the possibility of developing a common approach between physical and social sciences [4,14–16,5]. The specific aspects of dynamic measurement have also been discussed [15,17–19], and the possibility of a probabilistic framework common to static and dynamic measurement has been addressed [2].

In this context, systematic dynamic effects in the measurement system constitute an important point for improving the quality of the measurement process and for evaluating and declaring its uncertainty. In linear measuring devices such effects include a possible amplification or attenuation, and phase shift of the spectral components of the indicated signal, in respect to the original phenomenon. When this effect is non negligible, typically when operating outside the recommended band for the instrument, dynamic compensation should be applied, as discussed elsewhere [16,6]. When operating within the recommended band, uncertainty due to non-ideal behaviour the measuring device should be anyway evaluated and included in the uncertainty budget. Here a simple practical formula for doing that will be derived and proposed, for periodic or harmonic dynamic phenomena. Such evaluation should also be done even when dynamic compensation is applied, to account for residual uncertainty remaining after such compensation. Yet in any case uncertainty on the modulus and the phase of the frequency response of the device will cause uncertainty on the final measurement results.

2. Modelling dynamic effects in measuring devices: introductory example

The dynamic behaviour of a linear measuring device can be modelled through its frequency response:

$$H(f) = k\alpha(f)exp(j\phi(f)) \tag{1}$$

where f is the frequency, $H(f)$ the (complex) frequency response (FR), k is the sensitivity, $\alpha(f)$ is the (dimensionless) modulus of the FR, $\phi(f)$ is its phase, and j denotes the imaginary unit. For example, in the case of a simple contact thermometer, the modulus is:

$$\alpha(f) = \left(1 + (2\pi fT)^2\right)^{-1/2} \tag{2}$$

and the phase is [9]:

$$\phi(f) = \tan^{-1}(2\pi fT) \tag{3}$$

where T is the time constant of the thermometer.

Suppose now that the measurand is a simple cosinusoidal process:

$$x(t) = x_0 \cos(2\pi f_0 t + \phi_0), \tag{4}$$

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where $f_0 = T_p^{-1}$, and T_p is the period.

Considering, by now, systematic effects only, i.e., neglecting the noise, the instrument indication will then be

$$y(t) = k\alpha(f_0)x_0 \cos(2\pi f_0 t + \phi_0 + \phi(f_0)) \tag{5}$$

The measured signal, with no dynamic compensation [14], will then be:

$$\widehat{x}(t) = k^{-1}y(t) = \alpha(f_0)x_0 \cos(2\pi f_0 t + \phi_0 + \phi(f_0)) \tag{6}$$

Then, the dynamic effect can be expressed by the *error*:

$$e(t) = \widehat{x}(t) - x(t). \tag{7}$$

If the (ideal) *non-distortion conditions* hold true, i.e., if $\alpha(f) = 1$ and $\phi(f) = 0$ for all the frequencies of interest, $\widehat{x}(t) = x(t)$ and no systematic deviation occurs. Therefore, for discussing the actual behaviour of the system, it is convenient to assume $\alpha(f) = 1 + \delta\alpha(f)$ and $\phi(f) = 0 + \delta\phi(f)$. Yet, in a typical practical case the exact values of $\delta\alpha(f)$ and $\delta\phi(f)$ would be unknown, it makes sense to model them as probabilistic variables. Lastly, since we do not know the "functions" $\delta\alpha(f)$ and $\delta\phi(f)$ but only some global figures about them, such as their standard deviations (i.e., σ_α and σ_ϕ), or their ranges (i.e. $\pm\Delta_\alpha$ and $\pm\Delta_\phi$), we will neglect their dependence upon frequency, thus definitely setting:

$$\alpha(f) = 1 + \delta\alpha$$

$$\phi(f) = 0 + \delta\phi \tag{8}$$

It thus results, for the measured signal:

$$\widehat{x}(t) = x_0(1 + \delta\alpha)\cos(2\pi f_0 t + \phi_0 + \delta\phi)$$

$$= x_0(1 + \delta\alpha)[\cos(2\pi f_0 t + \phi_0)\cos \delta\phi - \sin(2\pi f_0 t + \phi_0)\sin \delta\phi]$$

Since $\delta\phi$ is usually small, let us assume $\cos \delta\phi \cong 1$ and $\sin \delta\phi \cong \delta\phi$. Then, after neglecting second order terms, we finally obtain for the error:

$$e(t) = \delta\alpha x_0 \cos(2\pi f_0 t + \phi_0)$$

$$-\delta\phi x_0 \sin(2\pi f_0 t + \phi_0) \tag{9}$$

Therefore, for any given dynamic process $x(t)$, the error is a stochastic process, depending upon the two random parameters $\delta\alpha$ and $\delta\phi$, that can be modelled as probabilistic variables. Since such variables can be assumed as zero-mean, the error will be also zero-mean.

Up to now, a deterministic model of the process has been assumed. In fact the process has been modelled as a member of the set of cosinusoidal functions, each identified by its amplitude, frequency and phase.

For each possible such function, let us calculate the variance of the error, assuming that the two variables, $\delta\alpha$ and $\delta\phi$, are zero-mean and uncorrelated. We obtain:

$$\begin{aligned}\sigma_e^2(t) &= \mathrm{E}\left(e^2(t)\right)\\ &= \mathrm{E}\left[\delta\alpha^2 x_0^2 \cos^2(2\pi f_0 t + \phi_0) + \delta\phi^2 x_0^2 \sin^2(2\pi f_0 t + \phi_0)\right.\\ &\quad \left. - 2\delta a \delta\phi x_0^2 \cos(2\pi f_0 t + \phi_0)\sin(2\pi f_0 t + \phi_0)\right]\\ &= \sigma_\alpha^2 {x_0}^2 \cos^2(2\pi f_0 t + \phi_0) + \sigma_\phi^2 {x_0}^2 \sin^2(2\pi f_0 t + \phi_0),\end{aligned} \tag{10}$$

where $\mathrm{E}(\cdot)$ denotes the expectation operator.

The variance is thus time dependent, and such is also the standard deviation due to dynamic effects. Yet this is not practical, and a constant global value is rather of interest. To obtain that, time averaging over one period may be considered, yielding:

$$\begin{aligned}\overline{\sigma}_e^2 &= \frac{1}{T_p}\int_{-T_p/2}^{+T_p/2} \sigma_e^2(t)\mathrm{d}t\\ &= \frac{1}{T_p}\int_{-T_p/2}^{+T_p/2} \left[\sigma_\alpha^2 {x_0}^2 \cos^2(2\pi f_0 t + \phi_0) + +\sigma_\phi^2 {x_0}^2 \sin^2(2\pi f_0 t + \phi_0)\right]\mathrm{d}t\\ &= \frac{{x_0}^2}{2}\left(\sigma_\alpha^2 + \sigma_\phi^2\right)\end{aligned} \tag{11}$$

Let us introduce now the "power" of the signal, i.e., its mean square value. For any positive integer n , and for T being a generic time duration, we obtain:

$$P_x = \frac{1}{T_p}\int_{-T_p/2}^{+T_p/2} x^2(t)\mathrm{d}t = \frac{1}{nT_p}\int_{-nT_p/2}^{+nT_p/2} x^2(t)\mathrm{d}t$$

$$= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{+T/2} x^2(t)\mathrm{d}t = \frac{{x_0}^2}{2} \tag{12}$$

Lastly, considering the usual notation for standard uncertainty, and denoting by $u_d$the standard uncertainty due to *dynamical* effects in the measuring device, we obtain, noteworthy:

$$\frac{u_d}{x_{rms}} = \sqrt{\sigma_\alpha^2 + \sigma_\phi^2} = \sqrt{u_\alpha^2 + u_\phi^2} \tag{13}$$

where u_α, u_ϕdenote the uncertainty contribution due to the modulus and phase. Equation (13) establishes a simple, elegant, and practical relation between the relative standard uncertainty due to dynamical effects and the uncertainty on the modulus and the phase of the frequency response of the measuring device.

3. Periodic and harmonic phenomena

Let us now generalize the above ideas by proposing their application to two of the most important classes of (models of) dynamic phenomena, namely the (zero-mean) periodic and the harmonic ones. In the case of periodic phenomena, a deterministic model can be based on its limited Fourier series expansion [10]:

$$x(t) = \sum_{i=1}^{n} c_i \cos(i2\pi f_0 t + \phi_i), \tag{14}$$

where $x(t)$ is (the time history of) the measurand and $f_0 = T_p^{-1}$is its fundamental frequency.

Harmonic processes can be instead modelled as [20]:

$$x(t) = \sum_{i=1}^{n} c_i \cos(2\pi f_i t + \phi_i) \tag{15}$$

Since eq. (15) is a generalization of eq. (14), which can be obtained from (15) by putting

$$f_i = if_0 \tag{16}$$

it is sufficient to discuss the latter.

Firstly, let us calculate the "power" (mean square value) of a harmonic process:

$$P_x = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{+T/2} x^2(t)\mathrm{d}t = \sum_{i=1}^{n}\frac{{c_i}^2}{2}. \tag{17}$$

Then, following the approach outlined in section 2, we obtain for the instrument indication:

$$y(t) = k\sum_{i=1}^{n} \alpha(f_i)c_i \cos(2\pi f_i t + \phi_i + \phi(f_i)), \quad (18)$$

where k is the sensitivity of the measurement device. Thus, the measured signal can be derived as:

$$\hat{x}(t) = k^{-1}y(t) = \sum_{i=1}^{n} \alpha(f_i)c_i \cos(2\pi f_i t + \phi_i + \phi(f_i)), \quad (19)$$

and for the error due to dynamic effects, still accounting for assumptions (8), we obtain:

$$e(t) = \delta\alpha \sum_{i=1}^{n} c_i \cos(2\pi f_i t + \phi_i)$$

$$-\delta\phi \sum_{i=1}^{n} c_i \sin(2\pi f_i t + \phi_i) \quad (20)$$

The variance of the error is:

$$\sigma_e^2(t) = \sigma_\alpha^2 \sum_{i=1}^{n} {c_i}^2 \cos^2(2\pi f_i t + \phi_i) + \sigma_\phi^2 \sum_{i=1}^{n} {c_i}^2 \sin^2(2\pi f_i t + \phi_i). \quad (21)$$

Accounting for eq. (17), the average error variance is now:

$$\overline{\sigma}_e^2 = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{+T/2} \sigma_e^2(t)dt = P_x\left(\sigma_\alpha^2 + \sigma_\phi^2\right) \quad (22)$$

and, lastly, we still obtain eq. (13), here recalled for clarity:

$$\frac{u_d}{x_{rms}} = \sqrt{u_\alpha^2 + u_\phi^2} \quad (23)$$

Therefore, *this result, originally obtained for mono-tone processes results to be applicable also to the important classes of periodic and harmonic processes.*

4. Hints for practical uncertainty evaluation

Let us now discuss the application of the above method to typical dynamic measurements, such as vibration measurement. Typical transducers for such measurements are either piezo-electric accelerometers, for absolute motion monitoring, or eddy-currents proximity probes, for relative motion. Let us then assume a more general model of instrument indication, capable to account for other typical uncertainty sources, that is:

$$y(t) = (k + \delta k)\sum_{i=1}^{n} \alpha(f_i)c_i \cos(2\pi f_i t + \phi_i + \phi(f_i)) + (h + \delta h) + v(t), \quad (24)$$

where h is an additive term which accounts for a possible no-zero output, in correspondence to a zero input, which is typically the case with eddy-current proximity probes, δk and δh are multiplicative and additive deviations, and $v(t)$ is additive noise that includes the effect of noise in the process, due to a non-perfectly harmonic phenomenon, and electric measurement noise.

The corresponding expression for the error will then be:

$$e(t) = \left(\frac{\delta k}{k} + \delta\alpha\right)\sum_{i=1}^{n} c_i \cos(2\pi f_i t + \phi_i)$$

$$-\delta\phi \sum_{i=1}^{n} c_i \sin(2\pi f_i t + \phi_i) + k^{-1}(h + v(t)) \quad (25)$$

It should be noted that the multiplicative systematic effect due to sensitivity (normalised) deviation $\delta k/k$ behaves in a way much similar to $\delta\alpha$;therefore it is again convenient to average over time. Consequently, the final expression for relative standard uncertainty evaluation, accounting for all the above considered uncertainty sources, is:

$$\frac{u}{x_{rms}} = \sqrt{u_\alpha^2 + u_\phi^2 + k^{-2}\left(u_k^2 + x_{rms}^{-2}\left(u_h^2 + v_{rms}^2\right)\right)} \quad (26)$$

The relationship between relative standard uncertainty and signal-to-noise ratio (SNR), a common feature in dynamic measurement, can be also noted. Indeed:

$$\mathrm{SNR} = 10\log_{10}\left(\frac{x_{rms}}{u}\right)^2 = 20\log_{10}\left(\frac{x_{rms}}{u}\right) \quad (27)$$

Let us then briefly discuss its practical application, leaving apart, by now, the evaluation of noise, to be treated at the end of this section.

In the case of high quality piezo-electric accelerometers, explicit statements on the uncertainties of the modulus and of the phase, for a selected frequency range, in the form:

$$\delta\alpha = \pm\Delta\alpha,$$

$$\delta\phi = \pm\Delta\phi. \quad (28)$$

The uncertainty on k is typically expressed as a percentage/relative value, i.e., in the form $\pm\Delta k/k$ and the uncertainty on h are often not mentioned, which may be interpreted as they are considered negligeable as compared to the dynamic effects. Let us also assume that uncertainty related to environmental conditions is negligeable as well. Then, apart from noise, relative standard uncertainty can be evaluated by:

$$\frac{u}{x_{rms}} = \sqrt{\frac{\Delta\alpha^2 + \Delta\phi^2 + (\Delta k/k)^2}{3}}. \quad (29)$$

where uniform distributions have been assumed for the variable involved. For example, if, in the frequency range of the device, the uncertainty on the sensitivity is rated within $\pm 10\%$, that on the module also within $\pm 10\%$, and that on the phase within $\pm 1^\circ$, we obtain:

$$\frac{u}{x_{rms}} = \sqrt{\frac{(0.1)^2 + (0.0175)^2 + (0.1)^2}{3}} = 0.082, \quad (30)$$

which corresponds to SNR $= 22$ dB. It may be noted that the phase effect, in this case, is negligeable as compared to the modulus effect.

In the case of eddy-current proximity probes, the dynamic behaviour may be documented by presenting typical frequency responses curves for both modulus and phase. Such curves usually have a low-pass character, with deviation from the ideal behaviour asymmetrical in respect to the zero, typically in a range $(-\Delta\alpha, 0)$ and $(-\Delta\phi, 0)$, respectively. Yet an asymmetrical distribution would imply some correction of the result, which, in this case, would mean to perform dynamic compensation. Yet this is usually avoided, in practical application. Then, symmetric ranges can be assumed, i.e., $(-\Delta\alpha, +\Delta\alpha)$ and $(-\Delta\phi, +\Delta\phi)$. Concerning the other uncertainty sources, here both $\delta k/k$ ad δh are present, usually denoted as (uncertainty on the) *incremental scale factor* (ISL) and *deviation from best fit straight line* (DSL). Therefore, apart from noise, eq. (24) can be used. If, for example, in a given frequency range, viz. up to 1 kHz, the maximum deviation of the modulus is -0.25 dB, of the phase -10°, the ISL is rate within $\pm 5\%$, the DSL is ± 0.025 mm, for $x_{rms} = 1.0$ mm, we obtain:

$$\frac{u}{x_{rms}} = \sqrt{\frac{(0.03)^2 + (0.175)^2 + (0.05)^2 + (0.025/1.0)^2}{3}} = 0.11, \quad (31)$$

with $SNR = 19$. Here the uncertainty on the phase is the most important effect.

Lastly, let us briefly discuss the evaluation of the rms value of the noise that directly affects the result as an additional uncertainty source. In practice, this can be hardly obtained from the data sheets of the devices, since it is strongly related to the experimental and environmental conditions.

One possibility, when applicable, is to record the output of measuring system, for a zero measurand input, and to compute the corresponding rms value. But zeroing the input is often impossible, especially in the field.

Then, if the maximum frequency of interest for the phenomenon, call it f_{max}, is significantly smaller then $f_s/2$, where f_s is the sampling frequency, the noise in the band $(f_{max}, f_s/2)$ can be estimated as the difference between the original signal e the signal low-pass filtered up to f_{max}. If v'_{rms} is its rms value, the rms value of the noise can be estimated as:

$$v_{rms} = v'_{rms}\sqrt{\frac{f_s/2}{f_s/2 - f_{max}}}. \quad (32)$$

Lastly, if neither that is possible, after estimating the spectrum of the signal, the noise can be estimated as the difference between the original signal and the signal reconstructed through (14) or (15), where only the significant spectral components are included.

5. Conclusions

Systematic dynamic effects in linear measuring devices have been considered and a simple formula has been derived, for evaluating the relative standard uncertainty associated to such effects, in the case of periodic or harmonic phenomena. Its practical application, in case of absolute or relative vibration measurement has been discussed. The relation with the signal to noise ratio has also been outlined. As a future development, the application of this approach to other classes of dynamic phenomena is envisaged.

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