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Noise Bubbles

Mario Forni, Luca Gambetti, Marco Lippi and Luca Sala

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Mario Forni*
Università di Modena e Reggio Emilia
CEPR and RECent

Luca Gambetti†
Universitat Autonoma de Barcelona
and Barcelona GSE

Marco Lippi
Università di Roma La Sapienza and EIEF

Luca Sala
Università Bocconi and IGIER

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Abstract

We introduce noisy information into a standard present value stock price model. Agents receive a noisy signal about the structural shock driving future dividend variations. The resulting equilibrium stock price includes a transitory component —the "noise bubble"— which can be responsible for boom and bust episodes unrelated to economic fundamentals. We propose a non-standard VAR procedure to estimate the structural shock and the "noise" shock, their impulse response functions and the bubble component of stock prices. We apply such procedure to US data and find that noise explains a large fraction of stock price volatility. In particular the dot-com bubble is entirely explained by noise. On the contrary the stock price boom peaking in 2007 is not a bubble, whereas the following stock market crisis is largely due to negative noise shocks.

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*Contact: Dipartimento di Economia Politica, via Berengario 51, 41100, Modena, Italy. Tel. +39 0592056851; e-mail: mario.forni@unimore.it. The contribution of the MIUR for the Prin project 2010J3LZEN_003 is gratefully acknowledged.

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1 Introduction

Stock markets react to news about events whose actual consequences on economic fundamentals are often highly uncertain. An international crisis may be resolved peacefully or escalate into war; inventions may take a lot of time, or even fail, to produce important technological improvements; a sovereign debt crisis may be solved by sound policy measures or end up with a ruinous default. On the one hand, there are news which anticipate major changes of future dividends; on the other hand, there are news whose potential effects never materialize. Typically, when a piece of news arrives, investors do not know which of the two types the news belongs to, but they have to take a decision immediately. Since such decisions affect prices, part of stock prices fluctuations can be driven by news unrelated to economic fundamentals.

In this paper we introduce noisy information into a standard present value stock price model.[1] Dividends are driven by a structural economic shock, let us say the "dividend" shock. The effects of such shock are delayed, so that traders cannot see it by looking at current dividends. Agents have some information about the current shock, in that they see a signal, given by the sum of the dividend shock and a "noise" shock, not affecting fundamentals.[2] On impact, investors react to both the dividend shock and the noise shock in just the same way, being unable to distinguish between them. As time goes on, however, agents learn about the true nature of past dividend shocks by looking at realized dividends, and adjust their initial response. Thus a noise shock announcing good news leads to a kind of "rational exuberance": dividends are expected to rise and stock prices go up. But in the end agents realize that the shock was in fact noise and the bubble bursts.

The noise shock can generate transitory boom and bust episodes, the "noise bubbles", unrelated to the intrinsic value of equities. The key difference with the standard theory of rational bubbles is that here bubbles are a component of what is usually referred to as the "fundamental" value of securities, i.e. the present value of expected dividends. Hence, unlike standard bubbles,[3] noise bubbles have nothing to do with multiple equilibria and self-fulfilling expectations and are not ruled out by the transversality condition (see, e.g. Santos and Woodford, 1997). Our theory is different from that presented in Adam, Marcet and Nicolini (2007) where agents are assumed to have limited information about model's parameters and form their expectations through a learning mechanism. Here on the contrary agents know the model and information is limited only in that the shocks

[1] Campbell and Shiller, 1988.

[2] We follow here suggestions coming from the recent news-noise business cycle literature (Beaudry and Portier, 2004, 2006, Christiano *et al.*, 2008, Lorenzoni, 2009, Angeletos and La' O, 2010, Forni Gambetti and Sala, 2013).

[3] See e.g. Samuelson (1985), Tirole (1985), and more recently Martin and Ventura, (2012).

are not observable.

The ideas presented here are not necessarily in contrast with other explanations of bubbles. In particular, Abreu and Brunnermaier, 2003, show that, once a bubble has started, it can survive despite the presence of rational arbitrageurs, who are aware that the market will eventually collapse, but are willing to ride the bubble until it can generate profits. This theory of bubble persistence does not explain why a bubble arises in the first place. The authors ascribe the responsibility to irrational traders.

While not denying the role of irrational behavior, which is well documented in the literature,[4] we show here that rational bubbles may arise from imperfect information about the structural shocks affecting fundamentals.

Noisy information has dramatic implications for empirical analysis: if agents do not see the structural shocks, standard structural VAR methods fail. This is because economic data reflect agents' behavior, which in turn depends on their information. If agents observe current shocks, the econometrician can in principle infer them from existing data; but if agents do not distinguish the shocks, present and past values of observable variables cannot embed the relevant information (Blanchard, Lorenzoni and L'Huillier, 2010).

Despite this, in our theoretical setting, structural VAR methods can still be used successfully, provided that identification is generalized to include dynamic transformations of the VAR residuals. The reason is that, as times goes by, realized dividends reveal whether past signals were true dividend shocks or noise. Hence, current dividend and noise shocks, while not being combinations of current VAR residuals, are combinations of *future* values of such residuals.[5] A general treatment of dynamic structural VAR identification is found in Lippi and Reichlin, 1994. Here we propose a specific identification scheme to recover the structural shocks along with the related impulse response functions within a noisy information framework.[6]

In the empirical section we apply our structural VAR identification technique to US stock market and dividend data. We find that noise shocks, while not affecting fundamentals, explain a large fraction of stock price volatility at short and medium-run horizons. On the other hand, the dividend shock has a limited impact in the short run, but have permanent effects and explain a good deal of stock market fluctuations in the long run. The component of log stock prices driven by noise measures the percentage deviation of prices form the intrinsic value of stocks. Hence the historical decomposition estimated with the VAR enables us to identify the duration and the size of past relevant

[4]See e.g. Shiller, 2000.

[5]This feature is not shared by the business cycle models of Blanchard, Lorenzoni and L'Huillier, 2010 and Barski and Sims, 2012, where agents never learn completely the true nature of past structural shocks.

[6]See also the companion paper Forni, Gambetti, Lippi and Sala, 2013, where a similar news-noise setting is applied to business cycle issues.

bubble episodes. The largest noise bubble of the last half century was the dot-com episode starting in 1997:Q3 and ending in 2002:Q1; the peak was reached in the second quarter of 2000, when prices deviated from the intrinsic value by 56%. The boom peaking in 2007 was not a bubble, whereas the stock market crisis of 2008 was partially due to negative noise shocks.

The reminder of the paper is organized as follows. In Section 2 we present the model. Section 3 discusses the econometric implications and presents our dynamic, structural VAR identification scheme. Section 4 presents our empirical results. Section 5 concludes.

2 Economics

The idea that stock prices are affected by news about economic and political happenings is largely accepted. Figure 1 depicts the growth rate of the S&P 500 index as well as vertical lines in coincidence with news about major economic and political events. In many of these episodes, the index displays large drops and peaks. For instance the index dropped by about 20% in coincidence of the Franklin National Bank collapse and the Worldcom bankruptcy and increased by around 10% the quarter before the official end of the Vietnam war.

From a rational expectations perspective, the interpretation of the above findings is that stock prices change because agents expect future dividends to change in consequence of the event agents have become aware of. Figure 2 plots the quarterly series of log-dividends and log-prices after four major episodes: the Watergate scandal and the Franklin National Bank collapse, the end of the Vietnam War, the Worldcom bankruptcy, and the Lehman Brothers bankruptcy. The series are normalized to zero in period 0 which is the period before the event occurs. The vertical line coincides with the event. The drop in stock prices following the Lehman Brothers bankruptcy clearly anticipates a decline of future dividends. This happens, but to a much lesser extent and with a longer delay, also in other two episodes, namely the Vietnam War (with a reversed sign) and the Watergate scandal. The fall in prices associated to the Worldcom bankruptcy, however, is associated with a mild increase in future dividends. This last fact can be reconciled with the rational expectation paradigm provided that one admits the possibility that, at the time the news arrives, agents are unable to predict its effects on future dividends. The rationale of this could be that agents are simply uncertain about the nature of the shock behind the news. In other words, agents could be uncertain about whether the shock leading Worldcom to bankruptcy is a bad financial shock with disastrous consequences on the financial system or simply a temporary and isolated episode with no further consequences on the economy. Below we develop formally a model based on this idea.

2.1 Noisy news in the present value model

We adopt here the log-linear version of the present value model proposed by Campbell and Shiller, 1988. The log of price is determined by the expected discounted sum of future log dividends. For the sake of simplicity we assume that the rate of return on equities is fixed over time, so that the "discount rate" component of prices is constant. Formally, the log of prices, p_t, is given by

$$p_t = \frac{k}{1-\rho} + \frac{1-\rho}{\rho} \sum_{j=1}^{\infty} \rho^j E_t d_{t+j}, \tag{1}$$

where d_t are dividends, expressed in logs, E_t denotes expected value, conditional to information available at time t, $\rho = 1/(1+e^{\mu})$, where $\mu = E(d_t - p_t)$, and $k = -\log(1+r) - \log\rho + (1-\rho)\log(1/\rho - 1)$, r being the constant rate of return on equities. Observe that, in the above equation, speculative bubbles, as defined in standard textbook models, are ruled out and stock prices are simply given by what is usually referred to as the "fundamental" value.[7]

We assume that dividends are driven by a structural shock whose effects are slow and delayed. This is essential to the model, since otherwise agents could infer the shock by looking at current and past dividends. Precisely, we assume that d_t follows the stochastic equation

$$d_t = d_{t-1} + \sum_{j=1}^{\infty} c_j a_{t-j}, \tag{2}$$

where a_t is the structural dividend shock, a gaussian white noise with variance σ_a^2. The corresponding impulse response function is the absolutely-summable sequence c_j, $j = 1, \ldots, \infty$. Notice that a_t does not affect d_t on impact.

The basic novelty of our model is that agents have incomplete information. Precisely, agents do not see the current dividend shock, but observe only the noisy signal

$$s_t = a_t + e_t, \tag{3}$$

where e_t (the "noise") is a gaussian white noise orthogonal to a_t at all leads and lags. The signal sometimes conveys relevant information about the future (when e_t is small), sometimes is essentially misleading (when e_t is large).

Finally, we assume that economic agents observe d_t at time t, so that agents' information set at time t, say Ω_t, is given by the linear space spanned by present and past values of dividends and the signal s_t. Below we shall compare results for Ω_t with what obtained with complete information, i.e. the information set Φ_t, spanned by present and past values of a_t and e_t.

[7] Equation (1) is derived in the Appendix.

2.2 An illustrative example: The "noise bubble"

Let us begin by studying a simplified version of the model, which will be sufficient to illustrate most of the economic implications of the general model, i.e.

$$d_t = d_{t-1} + a_{t-1}. \tag{4}$$

Hence

$$E_t d_{t+1} = E_t d_t + E_t a_t = d_t + E_t a_t.$$

Moreover, $E_t d_{t+2} = E_t d_{t+1} + E_t a_{t+1}$. Since a_{t+1} is unpredictable, we have $E_t d_{t+2} = E_t d_{t+1}$. Proceeding recursively we get

$$E_t d_{t+j} = E_t d_{t+1} = d_t + E_t a_t \quad \text{for } j \geq 1.$$

Applying equation (1) we get

$$p_t = \frac{k}{1-\rho} + d_t + E_t a_t. \tag{5}$$

Now let us consider the expectation of a_t. For the sake of comparison, we begin by deriving the stock price equation under the assumption that a_t is observable, i.e. the information set is Φ_t. Denoting by E_t^{Φ} the expectation conditional to Φ_t, we have $E_t^{\Phi} a_t = a_t$. Using (4) and (5) we get

$$p_t^{\Phi} = p_{t-1}^{\Phi} + a_t. \tag{6}$$

When a positive shock arrives, the market reacts immediately by rising prices by the amount a_t.

Coming to the present setting, a_t is not observed. The information set of the agents is given by Ω_t. By equation (4), d_t reveals the past of a_t, but is completely uninformative about the present. Similarly, past values of s_t do not tell anything about a_t. Hence, $E_t^{\Omega} a_t$ is simply the projection of a_t on s_t, i.e. $(\sigma_a^2/\sigma_s^2)s_t = (\sigma_a^2/\sigma_s^2)a_t + (\sigma_a^2/\sigma_s^2)e_t$. Replacing in (5), taking the first difference and rearranging terms gives

$$\Delta p_t = \frac{\sigma_a^2}{\sigma_s^2}\left(a_t + \frac{\sigma_e^2}{\sigma_a^2} a_{t-1}\right) + \frac{\sigma_a^2}{\sigma_s^2}\left(e_t - e_{t-1}\right). \tag{7}$$

To interpret the above equation, let us assume that, at time t, the signal is perfectly correct, i.e. the noise is zero and $s_t = a_t$. Agents do not see this, so that they are too much cautious and under-react on impact, the coefficient being σ_a^2/σ_s^2, which is less than the perfect information response of one. After one period, however, by observing d_t, agents realize that the signal was indeed correct and adjust their behavior to get a cumulated response equal to $\sigma_a^2/\sigma_s^2 + \sigma_e^2/\sigma_s^2 = 1$.

At the opposite extreme, when the signal is completely false, i.e. the structural economic shock is zero in t and $s_t = e_t$, agents are too much optimistic, if e_t is positive, or pessimistic, if e_t is negative, and over-react on impact, with coefficient σ_a^2/σ_s^2, which of course is greater that the "correct" response zero. Notice that the impact response to e_t is the same of a_t, since people cannot distinguish false news from true news on impact. Again, after one period, this kind of "rational exuberance" disappears and prices go back to the previous level.

According to (7), the noise shock affects stock prices, though dividends are noise free. Price changes are driven by two components, let us say the "structural component" or the "intrinsic value" and the "noise component". The latter is similar to a bubble, in that it is not related to fundamentals. Noticeably, the effect of false news is transitory, the cumulated response being zero (whereas the effect of true news is permanent, the cumulated response being one). Hence, the noise bubble is fated to burst, like traditional bubbles.

On the other hand, there are important differences. In the present value model, traditional bubbles are related to multiple equilibria; they arise because the economy is shifting to an unstable equilibrium, for reasons which are not specified by the theory. Sooner or later they burst, but the theory has nothing to say about when this will happen. By contrast, the noise bubble (i) is part of the stable equilibrium; (ii) arise when the market exaggerate the implications of current signals about future economic fundamentals; (iii) lasts until agents learn that the signal was in fact noise.

Two interesting limit cases are $\sigma_e^2 = 0$, i.e. there is no noise at any t, and $\sigma_e^2 \to \infty$, i.e. false news are largely predominant. When $\sigma_e^2 = 0$, the signal s_t is equal to a_t, so that agents can see the true economic shock. Obviously in this case the noise bubble is not there and equation (7) reduces to (6).

Somewhat surprisingly, the noise bubble disappears even in the opposite case, when σ_e^2 goes to infinity. For, the variance of the noise component is $2\sigma_a^4\sigma_e^2/\sigma_s^4$, which vanishes for $\sigma_e^2 \to \infty$. The economic intuition is that, when e_t is very large, the signal is not reliable, so that the stock market does not react to it. Equation (7) reduces to $p_t = p_{t-1} + a_{t-1}$, reflecting the fact that, s_t being not informative, agents see only a_{t-1} and therefore respond to the structural shock with delay.

The noise bubble is large when dividend and noise shocks have approximately the same size. To see this, let us compute the ratio of the variance of the noise component to the variance of Δp_t. The structural component in equation (7) has variance $\sigma_a^2(\sigma_a^4 + \sigma_e^4)/\sigma_s^4$, whereas the variance of the noise component is $2\sigma_a^4\sigma_e^2/\sigma_s^4$. Summing the two variances gives the variance of Δp_t, i. e. σ_a^2. The ratio of the variance of the noise component to the total variance is then $2\sigma_e^2\sigma_a^2/\sigma_s^4$. Such ratio is zero for both $\sigma_e^2 = 0$ and $\sigma_e^2 \to \infty$, as observed above, and reaches its maximum 1/2 for $\sigma_e^2 = \sigma_a^2$.

2.3 Information sets and learning

Now let us go back to the general model. Since a basic feature of our model is that the information set of the agents does not coincide with the information set spanned by the structural shocks —the dividend shock and the noise shock— we start by studying the relation between these information sets.

In the lag operator notation, equation (2) becomes

$$\Delta d_t = c(L)a_t, \tag{8}$$

where $c(0) = 0$. The relation between the information set of the agents, Ω_t, and the complete information set, Φ_t, is characterized by the relation linking the vectors $(\Delta d_t \; s_t)'$ and $(a_t \; e_t)'$. We have

$$\begin{pmatrix} \Delta d_t \\ s_t \end{pmatrix} = \begin{pmatrix} c(L) & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} a_t \\ e_t \end{pmatrix}, \tag{9}$$

This relation is not invertible, since the determinant of the MA matrix is $c(L)$, which by assumption vanishes for $L = 0$, which is less than 1 in modulus. Non-invertibility implies that we do not have a VAR representation for Δd_t and s_t in the structural shocks, and that present and past values of the observed variables Δd_t and s_t contain strictly less information than present and past values of a_t and e_t. [8]

Representation (9) is not the only MA representation of Δd_t and s_t. In particular, there is a "fundamental" representation, i.e. an MA representation in the innovations of Ω_t.[9] Let r_j, $j = 1, \dots, n$, be the roots of $c(L)$ which are smaller than one in modulus and

$$b(L) = \prod_{j=1}^{n} \frac{L - r_j}{1 - \bar{r}_j L}, \tag{10}$$

where $\bar{r}_j$ is the complex conjugate of r_j. Then let us consider the representation:

$$\begin{pmatrix} \Delta d_t \\ s_t \end{pmatrix} = \begin{pmatrix} c(L)/b(L)\sigma_s^2 & c(L)\sigma_a^2/\sigma_s^2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u_t \\ s_t \end{pmatrix}, \tag{11}$$

where

$$\begin{pmatrix} u_t \\ s_t \end{pmatrix} = \begin{pmatrix} b(L)\sigma_e^2 & -b(L)\sigma_a^2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} a_t \\ e_t \end{pmatrix}. \tag{12}$$

[8] Notice that, if the representation were invertible, such a VAR would exist, so that the structural shocks could be written as a linear combination of present and past values of observable variables, and the information sets Φ_t and Ω_t would be equal, contrary to the assumption that the dividend shock does not belong to the information set of the agents.

[9] "Fundamental" in the present context is a term of time series theory, which has nothing to do with the "fundamental" value of a security or economic "fundamentals".

It is easily verified that (11) and (12) imply (9). Moreover, u_t and s_t are jointly white noise and orthogonal.[10] Finally, the determinant of the matrix in (11), i.e. $c(L)/b(L)\sigma_s^2$, vanishes only for $|L| \geq 1$ because of the definition of $b(L)$. It follows that u_t and s_t are orthogonal innovations for Ω_t, i.e. $\Omega_t = \text{span}(u_{t-k}, s_{t-k}, j = 1, \ldots, m, k \geq 0)$.

The shock u_t, let us call it the *learning shock*, must be interpreted as agents' new information resulting from observation of Δd_t. The contemporaneous value of Δd_t conveys information concerning the past of dividend and noise shocks (there is no information about the present, since, by the definition of $b(L)$, the condition $c(0) = 0$ implies that $b(0) = 0$).

In the long run, observation of economic fundamentals completely unveils whether past signals were true or not. To make this point clear, consider that the roots of the determinant of the matrix in (12), $b(L)\sigma_s^2$, are smaller than one in modulus by the definition. Hence representation (12), though not invertible toward the past, can be inverted toward the future. Considering that $1/b(L) = b(F)$, where $F = L^{-1}$ is the forward operator, we get

$$\begin{pmatrix} a_t \\ e_t \end{pmatrix} = \frac{1}{\sigma_s^2} \begin{pmatrix} b(F) & \sigma_a^2 \\ -b(F) & \sigma_e^2 \end{pmatrix} \begin{pmatrix} u_t \\ s_t \end{pmatrix}. \tag{13}$$

The above equation shows that the structural shock and the noise shock are linear combinations of *future* values of the learning shock u_t and the present of s_t. This point is crucial for the identification of the econometric model, as we shall see in Section 3.

2.4 Deriving stock prices

In the Appendix we derive the following formula for a generic process $\Delta y_t = \alpha(L)\epsilon_t$, where ϵ_t is observed:

$$\frac{1-\rho}{\rho} \sum_{j=1}^{\infty} \rho^j E_t y_{t+j} = y_t + \tilde{\alpha}(L)\epsilon_t, \tag{14}$$

where $\tilde{\alpha}(L) = [\alpha(L) - \alpha(\rho)]/(L - \rho)$. Applying the above formula to $\Delta d_t = c(L)a_t$ and taking differences gives prices for the complete information case, i.e.

$$\Delta p_t^{\Phi} = c^*(L)a_t. \tag{15}$$

where $c^*(L) = c(L) + (1 - L)\tilde{c}(L) = [c(L)(1 - \rho) - c(\rho)(1 - L)]/(L - \rho)$.

[10] Let us first observe that $u_t = b(L)(\sigma_e^2 a_t - \sigma_a^2 e_t)$ is a white noise process. To see this, consider that $\sigma_e^2 a_t - \sigma_a^2 e_t$ is a white noise (being the sum of two white noise processes, orthogonal at all leads and lags) and $b(L)$ is a so called "Blaschke" factor, such that $b(L)b(L^{-1}) = 1$. Hence the covariance generating function is $\sigma_u^2 b(L)b(L^{-1}) = \sigma_u^2$, so that all lagged covariances are zero. Obviously, $s_t = a_t + e_t$ is a white noise as well. In addition, u_t is orthogonal to s_t at all leads and lags, since $\sigma_e^2 a_t - \sigma_a^2 e_t$ is orthogonal to $a_t + e_t$.

When studying the incomplete information case, it is seen seen from (11) and (2) that dividends can be represented as

$$\Delta d_t = \frac{\gamma(L)}{\sigma_s^2} u_t + \frac{c(L)\sigma_a^2}{\sigma_s^2} s_t,$$

where $\gamma(L) = c(L)/b(L)$. Applying formula (14) we get

$$\Delta p_t = \frac{1}{\sigma_s^2} \gamma^*(L) u_t + \frac{\sigma_a^2}{\sigma_s^2} c^*(L) s_t. \tag{16}$$

Finally, using (12) and taking into account that $\gamma(L)b(L) = c(L)$ we get

$$\Delta p_t = c^*(L) a_t - \frac{\sigma_e^2}{\sigma_s^2}(1-L)\gamma(\rho)\tilde{b}(L) a_t + \frac{\sigma_a^2}{\sigma_s^2}(1-L)\gamma(\rho)\tilde{b}(L) e_t. \tag{17}$$

The third term on the right hand side of equation (17)is the noise bubble. When a false signal arrives, agents over-react by an amount proportional to σ_a^2/σ_s^2, which measures the confidence of agents in the signal s_t. The first term is the reaction to a_t of the complete information equation (15). The second term reduces the reaction of agents to the dividend shock with respect to the complete information response. When true news arrive, the market prudentially under-react, and the reduction is larger, the larger σ_e^2/σ_s^2, i.e the noise to signal variance ratio.

A few remarks are in order. Firstly, the factor $(1-L)$ appearing in the third term on the right-hand side confirms that the noise shock has transitory effects even in the general model. By contrast, the long-run effect of a_t on stock prices is $c^*(1) = c(1) = \sum_{j=1}^{\infty} c_j$. Second, we have $c^*(0) = c(\rho)/\rho$, $\tilde{b}(0) = b(\rho)/\rho$ and $\gamma(\rho)b(\rho) = c(\rho)$, so that the impact effects of both the structural and the noise shock is $\sigma_a^2 c(\rho)/(\sigma_s^2\rho)$. The impact effects are equal because agents cannot distinguish false and true news on impact.[11] Third, when $\sigma_e^2 = 0$, the noise bubble disappears as well as the "caution" term, so that (17) reduces to (15). On the other hand, when σ_e^2 goes to infinity, the signal becomes unreliable. As we have already seen for the illustrative example, the variance of the term driven by the noise shock goes to zero, being $O(\sigma_a^4\sigma_e^2/\sigma_s^4)$, $\sigma_e^2/\sigma_s^2 \to 1$ and

$$\Delta p_t \to [c^*(L) - (1-L)\gamma(\rho)\tilde{b}(L)] a_t.$$

Since $c^*(0) - \gamma(\rho)\tilde{b}(0) = c^*(\rho)/\rho - \gamma(\rho)b(\rho)/\rho = 0$, p_t reacts to a_t with delay, reflecting the fact that the signal is uninformative and agents do not learn anything about a_t at time t. The maximal effect of the noise shock is obtained somewhere in the open interval $(0 < \sigma_e^2/\sigma_s^2 < 1)$, but in the general model neither the maximum point nor the maximum itself are easily found.

[11] In the empirical Section, where, following the VAR tradition, the shocks are normalized to have unit variance, the impact responses are proportional to the standard deviations of the shocks.

3 Econometrics

In Section 2.3 we have analyzed the joint MA representation of Δd_t and s_t. Such representation however is not suitable for estimation, since s_t is not directly observable by the econometrician. In the present section we focus on the joint representation of Δd_t and Δp_t, which are both observable.

From (9) and (17) it is seen that the structural representation of Δd_t and Δp_t can be written as

$$\begin{pmatrix} \Delta d_t \\ \Delta p_t \end{pmatrix} = \begin{pmatrix} \sigma_a c(L) & 0 \\ d(L) & e(L) \end{pmatrix} \begin{pmatrix} a_t/\sigma_a \\ e_t/\sigma_e \end{pmatrix}, \tag{18}$$

where $d(L) = \sigma_a[c^*(L) - \frac{\sigma_e^2}{\sigma_s^2}\gamma(\rho)(1-L)\tilde{b}(L)]$ and $e(L) = \frac{\sigma_a^2\sigma_e}{\sigma_s^2}\gamma(\rho)(1-L)\tilde{b}(L)$ and $\tilde{b}(L) = [b(L) - b(\rho)]/(L-\rho)$. The shocks are normalized to have unit variance, as usual in structural VAR analysis. Our target is to estimate the shocks and the impulse response functions of the above representation.

Just like in representation (9), however, the determinant of the MA matrix vanishes for $L = 0$, since $c(0) = 0$. It follows that the representation is not invertible. This has dramatic consequences for empirical analysis.

3.1 Non-invertibility in models with noisy shocks

The problem of non-invertibility, or "non-fundamentalness" is a debated issue in the structural VAR literature. Early references are Hansen and Sargent, 1991 and Lippi and Reichlin, 1993, 1994; more recent contributions include Giannone and Reichlin, 2006, Fernandez-Villaverde *et al.*, 2007, Chari *et al.*, 2008, Forni and Gambetti, 2011. In essence, the problem is that standard SVAR methods assume that the structural shocks are linear combinations of the residuals obtained by estimating a VAR. If the structural MA representation of the variables included in the VAR is non-fundamental, the structural shocks are not linear combinations of such residuals, so that the method fails.[12]

In most of the economic literature, the structural shocks are elements of agents' information set and non-fundamentalness may arise if the econometrician uses less information than the agents. In this case, non-fundamentalness can in principle be solved by

[12] An MA representation is fundamental if and only if its associated matrix is non-singular for all L with modulus less than one (see Rozanov, 1967, Ch. 2). This condition is slightly different from invertibility, since invertibility requires non-singularity also when L is unit modulus. Hence non-fundamentalness implies non-invertibility, whereas the converse is not true. When the variables are cointegrated, for instance, the MA representation of the first differences is not invertible, but nonetheless can be fundamental. In such a case, non-invertibility can be easily circumvented by resorting to structural ECM or level VAR estimation. Non-fundamentalness is a kind of non-invertibility which cannot be solved in this way.

enlarging the information set used by the econometrician (Forni, Giannone, Lippi and Reichlin, 2009, Forni and Gambetti, 2011). But in the present setting non-fundamentalness stems from agents' ignorance and cannot be solved by adding variables to the VAR.[13] The economic intuition is that agents' behavior cannot reveal information that agents do not have. Stock prices or other variables which are the outcome of agents' decisions do not add anything to the information already contained in d_t and s_t. More generally, in models assuming that agents cannot see the structural shocks, the structural representation is non fundamental for whatever set of observable variables. For, if it were, agents could infer the shocks from the variables themselves, contrary to the assumption (unless we assume that there are variables that are observable for the econometrician but not for the agents).

In our theoretical framework, if identification is generalized to include dynamic unitary transformations (i.e. Blasckhe matrices), structural VAR estimation may still be successful. Dynamic unitary transformations are rotations which may involve, besides current values, past and future values of the VAR residuals. In fact, we have already seen in Section 2.3, equation (13), that the structural shocks, dividend and noise, can be written as linear combinations of the current signal and future values of the learning shock, which in principle can be found with a standard VAR procedure.

A general treatment of dynamic identification in structural VARs can be found in Lippi and Reichlin, 1994. When considering the more general class of dynamic rotations, identification is more demanding than in the standard, contemporaneous rotation setting, because it requires stronger theoretical restrictions. A contribution of the present paper (and the companion paper Forni, Gambetti, Lippi and Sala, 2013) is to show that in models with noisy signals the restrictions arising naturally from the theory are sufficient to identify the structural shocks. Below we explain in detail how to find the structural and the noise shock, as well as the corresponding impulse response functions.[14]

[13]See also Blanchard *et al.*, 2010.

[14]Blanchard *et al.*, 2010, and Barski and Sims, 2012, present news-noise models where agents never learn the true nature of past shocks. The basic difference with respect to our model in this respect is that in both papers there are three structural shocks, whereas agents see just two dynamically independent sources of information. Since the dynamic dimension of the structural shocks is larger than the dynamic dimension of agents' information space, there is no way for the agents to see such shocks, even when assuming known the future values of the observable series. For the same reason, the econometrician cannot recover the shocks and the impulse response functions by means of a structural VAR, even by resorting to dynamic transformations of the VAR residuals.

3.2 Dynamic identification of the bivariate VAR

Let us come to our identification and estimation strategy. From (11) and (16) it is obtained[15]

$$\begin{pmatrix} \Delta d_t \\ \Delta p_t \end{pmatrix} = \begin{pmatrix} a_{11}(L) & a_{12}(L) \\ a_{21}(L) & a_{22}(L) \end{pmatrix} \begin{pmatrix} u_t/\sigma_u \\ s_t/\sigma_s \end{pmatrix} = \frac{\sigma_a}{\sigma_s} \begin{pmatrix} \sigma_e \gamma(L) & \sigma_a c(L) \\ \sigma_e \gamma^*(L) & \sigma_a c^*(L) \end{pmatrix} \begin{pmatrix} u_t/\sigma_u \\ s_t/\sigma_s \end{pmatrix}, \quad (19)$$

where $\gamma^*(L)$ and $c^*(L)$ are defined as in (15). Estimation of (19) is our first step to identify and estimate the structural representation.

Unfortunately, there is no convincing motivation to assume that (19) is fundamental. However, in this case, whether (19) is fundamental or not can be checked by a simple test, see Section 3.3. We proceed in two steps. Firstly, we assume that representation (19) is fundamental, so that it can be estimated by means of standard structural VAR procedures. Secondly, in Section 3.3, we discuss the testing procedure, which is then employed in Section 4.

Assuming that (19) is fundamental, an estimate of $[a_{ij}(L)]_{i=1,2;j=1,2}$ is obtained by estimating and inverting an unrestricted VAR. Identification is obtained by imposing $\hat{a}_{12}(0) = 0$, which corresponds to the condition $c(0) = 0$, which is derived by the theory. The theory imposes further restrictions on the entries of the MA matrix appearing in (19). We do not use such restrictions for estimation, since we want to use them for testing purposes (see below).

Now, let us re-write equation (12) as

$$\begin{pmatrix} u_t/\sigma_u \\ s_t/\sigma_s \end{pmatrix} = \frac{1}{\sigma_s} \begin{pmatrix} b(L)\sigma_e & -b(L)\sigma_a \\ \sigma_a & \sigma_e \end{pmatrix} \begin{pmatrix} a_t/\sigma_a \\ e_t/\sigma_e \end{pmatrix}. \quad (20)$$

where the shocks are normalized to have unit variance. Since (18) is obtained from (19) and (20), our target is to get an estimate of the latter representation.

First, we need an estimate of $b(L)$, which is given by the roots of $c(L)$ that are smaller than 1 in modulus (see equation (10)). Such roots are revealed by our estimate of $\hat{a}_{12}(L)$, which is proportional to $c(L)$ (of course, one out of these roots will be zero because of the identification constraint $\hat{a}_{12}(0) = 0$). This is the crucial step of our procedure. The proportionality of the reaction of dividends to the dividend shock, on the one hand, and the signal shock, on the other hand, is due to the assumption that noise shocks do not affect dividends at any lag —an assumption which is essential, from a theoretical point of view, to distinguish the dividend shock from the noise shock.

Next, we need an estimate of σ_a/σ_s and σ_e/σ_s. Since $\gamma(L) = c(L)/b(L)$ and $b(1) = 1$, we have $\gamma(1) = c(1)$ so that an estimate of σ_a/σ_e can be obtained as

$$\widehat{\sigma_a/\sigma_e} = \frac{\hat{a}_{12}(1)}{\hat{a}_{11}(1)}.$$

[15] Let us remind that $\sigma_u = \sigma_e \sigma_a \sigma_s$.

Considering that $\sigma_a^2/\sigma_s^2+\sigma_e^2/\sigma_s^2 = 1$, it is seen that σ_a/σ_s and σ_e/σ_s are the sine and the cosine, respectively, of the angle whose tangent is σ_a/σ_e. Hence $\widehat{\sigma_a/\sigma_s}$ and $\widehat{\sigma_e/\sigma_s}$ can be obtained as $\sin(\arctan(\widehat{\sigma_a/\sigma_e}))$ and $\cos(\arctan(\widehat{\sigma_a/\sigma_e}))$, respectively.

Finally, the (normalized) structural shocks a_t/σ_a and e_t/σ_e can be estimated by inverting equation (20):

$$\begin{pmatrix} a_t/\sigma_a \\ e_t/\sigma_e \end{pmatrix} = \frac{1}{\sigma_s}\begin{pmatrix} b(F)\sigma_e & \sigma_a \\ -b(F)\sigma_a & \sigma_e \end{pmatrix}\begin{pmatrix} u_t/\sigma_u \\ s_t/\sigma_s \end{pmatrix}. \tag{21}$$

The above relation involves future values of u_t and s_t, so that the structural shocks cannot be estimated consistently at the end of the sample. This is perfectly in line with the assumption that neither the agents, nor the econometrician can see the current values of the structural shocks. However, in the middle of the sample future is known and relation (21) can in principle provide reliable estimates of a_t/σ_a and e_t/σ_e.

Summing up, our estimation strategy is the following.

1. We estimate an (unrestricted) reduced form VAR for d_t and p_t[16] and identify by imposing $\hat{a}_{12}(0) = 0$, i.e. that s_t does not affect d_t on impact. In such a way we get an estimate of the matrix $[a_{ij}(L)]_{i=1,2;j=1,2}$ appearing in (19) as well as the normalized learning and signal shocks.

2. We estimate $b(L)$ by computing the roots of $\hat{a}_{12}(L)$, selecting those which are smaller than one in modulus and using (10).

3. We estimate σ_a/σ_e as the ratio $\hat{a}_{12}(1)/\hat{a}_{11}(1)$.[17] Then we get $\widehat{\sigma_a/\sigma_s}$ and $\widehat{\sigma_e/\sigma_s}$ as $\sin(\arctan(\widehat{\sigma_a/\sigma_e}))$ and $\cos(\arctan(\widehat{\sigma_a/\sigma_e}))$, respectively. Steps 2 and 3 provide an estimate of (20).

4. Finally, we estimate the structural impulse response functions in (18) by using (19) and (20). Moreover, we estimate the structural shocks by using relation (21).

3.3 Testing and additional estimation issues

As already noticed, the restrictions appearing in representation (19) which are not used for identification can be used for testing. In particular, we can test the theoretical implication that e_t has temporary effects on prices. Moreover, $\hat{a}_{12}(L)\hat{b}(L)\widehat{\sigma_a/\sigma_e}$ should be equal to $\hat{a}_{12}(L)$.[18] Such condition implies that in the structural representation (18)

[16] We estimate the VAR in levels for reasons which will be clarified below.

[17] In practice we compute the cumulated long-run effects as the effects at forty quarters.

[18] Our identification conditions imply that such relation is satisfied both on impact and in the long run. At intermediate lags the relation can be violated.

the upper-right response function is zero, which can be tested by verifying whether the confidence bands include the x-axis for all lags.

Let us now go back to the first step of our estimation procedure, i.e. estimation of (19). The determinant of the MA matrix appearing in (19) is proportional to

$$\gamma(L)c^*(L) - \gamma^*(L)c(L) = \gamma(\rho)(1-L)\gamma(L)\frac{b(L)-b(\rho)}{L-\rho}, \tag{22}$$

which vanishes for $L = 1$, so that d_t and p_t are cointegrated. This problem is solved, as is well known, by estimating an ECM or a VAR in the levels of the variables, rather than the first differences. Our choice, see Section 4, is a VAR in levels.

Moreover, as observed in Section 3.2, representation (19) might be non-fundamental, i.e. the determinant (22) may have roots within the unit circle. As $\gamma(L)$ can vanish only outside the unit circle by definition, the problem arises with the factor $[b(L)-b(\rho)]/(L-\rho)$. Clearly, when $b(L)$ has just one zero, so does $b(L)-b(\rho)$, its root being ρ, and the ratio reduces to a constant. Indeed, in such case $b(L) = L$, by the definition of $b(L)$ along with the assumption that $c(0) = 0$, and the ratio is equal to 1. However, when $b(L)$ has two or more zeros, roots smaller than one in modulus may occur.

To see this, assume for instance that $c(L)$ has the "wrong" root r, besides the root zero. In this case, $b(L) = L(L-r)/(1-rL)$ and the ratio $[b(L)-b(\rho)]/(L-\rho)$ reduces to $(\rho - r + L)/(1-\rho L)$, which vanishes for $L = \rho - r$. Since $|r| < 1$, the root is larger than one in modulus when r lies in the open interval $(-1, \rho - 1)$. Notice that a second zero root for $c(L)$ would violate the condition, since $\rho < 1$.

As a further example, consider the case $b(L) = L^k$. All of the roots of $L^k - \rho^k$ lie on the circumference centered in the origin and with radius ρ in the complex plane. When dividing by $L - \rho$ one of these roots disappears, but the others do not. Hence for $k > 1$ we do not have a fundamental representation.

Here non-fundamentalness, when it is there, does not arise from the fact that agents do not have enough information, but from the fact that the econometrician uses less information than the agents, i.e. $\text{span}(\Delta d_{t-k}, \Delta p_{t-k}, k \geq 0) \subset \Omega_t = \text{span}(u_{t-k}, s_{t-k}, k \geq 0)$, because present and past values of d_t and p_t do not reveal completely u_t and s_t.

A simple check for fundamentalness, which can be used in this case, has been proposed by Forni and Gambetti, 2011. The test consists in verifying whether the estimated shocks are orthogonal to past values of the principal components of a large data set of macroeconomic series. In the empirical application below we replace the principal components with a set of selected control variables. If orthogonality is rejected, the shocks cannot be innovations with respect to available information, and the VAR should be amended by adding variables reflecting agents' information. A multivariate specification may help solving the problem, by closing the gap between the information used by agents and the one used by the econometrician.

3.4 Higher-dimensional specifications

Let Δy_t be the $n-2$-dimensional vector of additional variables. In order to have a square system, it is convenient to assume that there are also $n-2$ additional shocks, potentially affecting d_t and p_t. The innovation representation becomes

$$\begin{pmatrix} \Delta y_t \\ \Delta d_t \\ \Delta p_t \end{pmatrix} = \begin{pmatrix} Q(L) & f(L) & g(L) \\ h(L) & \sigma_a \sigma_e \gamma(L)/\sigma_s & \sigma_a^2 c(L)/\sigma_s \\ p(L) & \sigma_a \sigma_e \gamma^*(L)/\sigma_s & \sigma_a^2 c^*(L)/\sigma_s \end{pmatrix} \begin{pmatrix} v_t \\ u_t/\sigma_u \\ s_t/\sigma_s \end{pmatrix}, \tag{23}$$

where $f(L)$, $g(L)$, $h(L)$, $p(L)$ and the entries of the $n-2 \times n-2$ matrix $Q(L)$ are absolutely summable.

Within the multivariate framework, the condition that the dividend shock does not affect d_t on impact is no longer sufficient, by itself, to identify the model. In the empirical section we impose a Cholesky triangularization with y_t ordered first, d_t ordered second, and p_t ordered third, i.e. $f(0) = g(0) = 0$ and $Q(0)$ lower triangular. The reason for this ordering is that we want to allow for a contemporary effect of v_t on dividends and stock prices. Stock prices, in particular, can in principle react on impact to shocks affecting dividends and interest rates.

The corresponding structural representation is obtained by postmultiplying the above matrix by

$$\begin{pmatrix} I_{n-2} & 0 & 0 \\ 0' & b(L)\sigma_e/\sigma_s & -b(L)\sigma_a/\sigma_s \\ 0' & \sigma_a/\sigma_s & \sigma_e/\sigma_s \end{pmatrix},$$

where 0 denotes the $(n-2)$-dimensional null column vector.

4 Empirics

In this section we present our empirical analysis. Our benchmark specification is a four-variable VAR with dividends, stock prices, and two interest rates. We find that, in line with the theory, noise shocks do not affect dividends and have transitory effects on stock prices. Despite this, noise explains a large fraction of stock market fluctuations at short and medium run horizons and is responsible for large deviations of stock prices from the intrinsic value of equities. Noise explain most of the information technology bubble, as well as other boom-bust episodes, including a sizable fraction of the stock market crash of 2008-2009.

4.1 The data

We use US quarterly series covering the period 1960:Q1— 2010:Q4. The stock price series is the monthly average of the Standard & Poor's Index of 500 Common Stocks reported by Datastream (code US500STK). We converted the series in quarterly figures by taking simple averages and dividing the resulting series by the GDP implicit price deflator in order to express it in real terms. Dividends are NIPA Net Corporate Dividends, divided by the GDP implicit price deflator and population aged 16 years or more (the BLS Civilian Non-institutional Population, converted to quarterly frequency by taking monthly averages). Both dividends and stock prices are taken in log-levels rather than differences to avoid estimation problems related to cointegration. The interest rates included in our baseline specification are the 3-Month Treasury Bill, Secondary Market Rate, and the Moody's Seasoned Aaa Corporate Bond Yield. We take the monthly averages of business days (original source: Board of Governors of the Federal Reserve System) and converted the monthly series in quarterly figures by taking simple averages. Interest rates are taken in levels.

To test for fundamentalness, we use an additional interest rate, the 10-Year Treasury Constant Maturity Rate, the inflation rate and two leading indexes. The interest rate is treated as the interest rates described above. The inflation rate is the NIPA GDP Implicit Price Deflator, taken in first differences of the logs. The leading indexes are the Conference Board Leading Economic Indicators Index (Datastram code USCYLEAD) and the Michigan University Survey of Consumers Expected Index.

Stock prices and the Conference Board leading index are taken from Datastream, the consumer confidence index is taken from the website of the Michigan University, whereas all other series are downloaded from the FRED data base.

4.2 The effects of dividend and noise shocks

As a first exercise, we estimate the two-variable VAR with dividends and stock prices. We include 4 lags, according to the AIC criterion and identify the signal, learning, dividend and noise shocks as explained in Section 3.2. Then we test for fundamentalness as explained in Section 3.3, by regressing the estimated shocks onto 2 and 4 lags of the 3-Months Treasury Bill, the Aaa Corporate Bond Yield, and the four control variables described above, one at a time. Dividend and noise shocks are truncated at time $T-4$ since the filter obtained by inverting (20) involves the leads of the signal and the learning shocks, producing an end-of-sample bias.[19] The p-values of the F-statistic of these regressions are reported in Table 1. The null hypothesis that the signal is orthogonal to the past of the regressors is rejected at the 5% level for all interest rates and the inflation rate. A similar result holds for the noise shock. We conclude that dividends and stock prices do not contain enough information to represent adequately agents' information set.

[19] The truncation size is chosen on the basis of the estimated filter.

Hence we amend the VAR by adding two control variables, i.e the 3-Month Treasury Bill, Secondary Market Rate and the Moody's Seasoned Aaa Corporate Bond Yield. We estimate the four-variable VAR with 4 lags according to the AIC criterion and identify by imposing a Cholesky scheme with the interest rates ordered first as explained in Section 3.4. Again, we perform the orthogonality test. As shown in Table 2, orthogonality cannot be rejected, even at the 10% level, for all regressions. Hence we use the four-variable VAR as our baseline specification.

Figures 3a and 3b show the impulse response functions of dividends and stock prices to signal, learning and interest rate shocks. The dark gray and the light gray areas show the 68% and the 90% confidence bands, respectively, obtained by performing 500 bootstrap replications. A positive signal, anticipating future dividend growth, has large and significant contemporaneous effects on stock prices. On the other hand, the stock market reacts more cautiously and gradually to a positive learning shock, which has large contemporaneous and permanent effects on dividends. Positive interest rates shocks have little effects on dividends, but, as expected, have significant negative impact on stock prices in the short run.

Let us now consider the structural representation. We begin the analysis by examining the series of the estimated shocks. First of all notice that the point estimate of σ_a/σ_s is 0.44 (standard error 0.3), which entails a large noise, i.e. $\sigma_e/\sigma_s = 0.90$ (standard error 0.15). Figure 4 plots the two shocks. The vertical lines report events, most of them exogenous, coinciding with peaks and troughs in the estimated series of the signal. All of the events coincide with peaks or troughs of the noise shock. For instance the largest negative noise shock is observed in 1987:Q4 and corresponds to the Black Monday (October 1987). Other negative shocks are registered in coincidence with the collapse of the Franklin National Bank, the Gulf War I and the bankruptcy of the Lehman Brothers. Positive noise shocks are found in coincidence of the Bush re-election and the 2009 fiscal stimulus. In coincidence of some of these events the dividend shock has the same sign as the noise shock although is somehow smaller in terms of magnitudes. A notable difference between the dividend shock and the noise shock is observed in 2004:Q4, in coincidence with the Bush re-election, where the two shocks have opposite sign. According to our estimates, the Bush re-election is an episode with large negative effects on economic fundamentals, accompanied by a large positive noise shock responsible for the under-reaction of the stock market.

Figure 5a shows the impulse response functions of dividend and noise shocks on dividends and stock prices. Positive dividend shocks are followed by an increase in dividends, which reach their new long-run level after three quarters. Stock prices react immediately by a similar percentage amount and then remain approximately stable at the new level. In line with the theory, the effect of the noise shock on dividends is small and not significant at all horizons, even considering the tighter confidence region. By contrast, the effect of noise on the stock market, large and strongly significant on impact, declines sharply after a few quarters and approaches zero in the long run, confirming the temporary effect predicted by the model.

Table 3 reports the estimated decomposition of the forecast error variance at different horizons. The signal explains about 20% of dividend variation at medium and long-run horizons (two years or more), while the bulk of dividend volatility is captured by the learning shock. As for stock prices, the role of the signal and the learning shocks are inverted: the signal explains the bulk of stock price volatility, whereas learning has a sizable effect only in the long run (about 20%). Learning and signal explain together about 90% of stock price variation on impact and about 70% at longer horizons, the remaining 30% being explained by interest rates shocks. The dividend shock explains about 20% of stock price variation on impact and almost one half at the ten year horizon. Noise is very important, in that it explains the bulk of stock price variance on impact (about 70%) and in the short-medium run (about 50% and 40% at the 2-year and the 4-year horizons, respectively).

Figure 5b shows the impulse response functions of the two interest rates to dividend and noise shocks. Both shocks induce a monetary policy tightening; the T-Bill increases significantly for a few quarters according to the narrower bands. Interestingly enough, after about two years the response of the T-Bill rate to the noise shock becomes negative and significant at the 68% confidence level. Given that the noise shock has negligible real effects, the dividends are largely unaffected, the result seems to support the idea that monetary policy, to some extent, responds to fluctuations in stock prices. Nonetheless the response turns out to be relatively small. In fact the T-Bill increases up to 0.2% in front of an increase of about 6% of stock prices. Moreover only a small fraction of the interest rate, about 5%, is explained by noise.

4.3 Measuring historical boom-bust episodes

We have seen that stock prices, expressed in logs, are equal to the sum of the noise component —the "noise bubble"— plus the structural component, which, being the only one anticipating future dividends, can be interpreted as the intrinsic value. Hence the noise component has a noticeable interpretation: it measures the percentage deviation of current prices from the "true" value of equities. A positive (negative) value means that prices are overvalued (undervalued) with respect to the intrinsic value by a certain percentage. Such a measure is obtained by filtering the estimated noise shock with the corresponding impulse response function.[20]

[20] Formally the bubble is

$$\widehat{b}_t = \hat{a}(L)\hat{e}_t$$

Figure 6 shows the bubble component (solid line), estimated with the four-variable and the five-variable VAR, respectively. For the sake of comparison, the figures also report the stock price series (dashed line) and the structural component, i.e. the intrinsic value (dotted line). The estimates show several episodes of prolonged and sustained deviations from the fundamental price. Here we limit our attention only to those episodes in which deviations are 20% or higher.[21] We find eight of such episodes: four positive and four negative bubbles. The positive bubbles episodes are:

1. First half of the 70s (span: 1972:Q3-1973:Q1, max: 22.1% in 1972:Q3)
2. Second half of the 80s (span: 1987:Q3; max: 22.5% in 1987:Q3)
3. Dot-com (span: 1997:Q3-2002:Q1; max: 56.4% in 2000:Q2)
4. Mid 2000 (span: 2005:Q1; max: 21.1% in 2005:Q1)

The negative bubbles are

5. 1974 stock market crash (span: 1974:Q4; min: -23.7% in 1974:Q4)
6. Second half of the 70s (span: 1977:Q4-1979.Q4; min: -29.8% in 1978:Q1)
7. First half of the 80s (span: 1982:Q3-1983.Q2; min: -27.1% in 1983:Q1)
8. Great Recession (span: 2008:Q1-2009:Q4; min: -41.7% in 2009:Q1)

The dot-com bubble represents, by far, the episode with the largest and longest-lasting deviations. Between 1997 and 2002 prices have been over-evaluated on average by 40% with a peak of 56% in 2000:Q2. From the figures it emerges clearly that the bulk of fluctuations in prices around these years is attributable to news having no effect on future fundamentals, which were largely interpreted as genuine good news.

Notice that while our estimates point to a relatively large noise component in 2005, they show that the peak in 2007:Q2 was not a bubble. On the contrary, stock prices were undervaluated by about 10% compared to their fundamental value in that quarter. Finally, the noise component accounts, to a large extent, for the large drop in prices in 2008.

We conclude this section with a historical digression on the conduct of monetary policy in response to the noise bubble. Figure 7 plots the 3M T-Bill rate (dashed line), the noise component (solid line) and the difference between the variable and the noise

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where $\hat{a}(L) = \hat{a}_0 + \hat{a}_1 L + \hat{a}_2 L^2 + ...$ is the estimated impulse response function of the log of stock prices to the noise shock and $\hat{e}_t$ is the estimated noise shock.

[21] Adalid and Detken (2006) identify boom-bust episodes for a number of industrial countries. For the US they find two episodes, the 1986-87 and the dot-com bubble, which appear also in our list.

component (dotted line). Figure 8 plots the noise component of the 3M T-Bill rate and the stock prices together. In general, fluctuations in the interest rate driven by noise have become more volatile since late 90s. More specifically, at the onset of the dot-com bubble monetary policy responded to the increase in prices by increasing the interest rate by about 1%. However, around 1997, while prices kept rapidly growing, the interest rate stalled. The stock prices bust is followed by a huge drop in the interest rate, by around 3%. Actually during the second half of the 2000, absent the bubble, the interest rate would have been much higher, around 4%, than the observed value of 1.5%. Given that the real effects of the noise shock are relatively limited, the huge fall of the interest rate supports the idea that monetary policy has reacted quite strongly to the burst of the bubble and that the low levels of the interest rate observed until 2005 were driven by factors disconnected from economic fundamentals.

5 Conclusions

In this paper we have studied a simple present value stock price model where rational traders receive noisy signals about future economic fundamentals. We have shown that the resulting stock price equilibrium includes a transitory component which can be responsible for boom and bust episodes unrelated to fluctuations of economic fundamentals —the "noise bubbles". Noise bubbles are a component of what is usually referred to as the "fundamental" value of securities, i.e. the present value of expected dividends, so that they have nothing to do with multiple equilibria and self-fulfilling expectations.

We have shown that, in our theoretical framework, the structural shocks —"dividend" and "noise" shocks— can be estimated by using a non-standard structural VAR procedure, where identification is obtained by imposing a "dynamic" rotation of the VAR residuals, involving their future values.

In the empirical section we have applied our procedure to US data. We have found that, consistently with the theory, the noise shock has transitory effects on stock prices, whereas the dividend shock has permanent effects. Moreover, noise is very important, in that it explains the bulk of stock price fluctuations at short and medium-run horizons. Finally, the historical decomposition shows that the component of stock prices driven by the noise shock is responsible for the information technology bubble; the boom peaking in 2007 was entirely driven by genuine news, whereas the following stock market crisis is largely accountable to a negative noise bubble.

Appendix

The log-linear present value model

To obtain formula (1), let us start from the accounting identity

$$P_t = \frac{1}{1+r_{t+1}}(P_{t+1} + D_{t+1}),$$

where P_t is the price of equities, D_t is dividends and r_t is the rate of return on equities. Setting $r_t = r$ and taking logs we get

$$p_t = -\log(1+r) + \log\left(e^{p_{t+1}} + e^{d_{t+1}}\right) = -\log(1+r) + p_{t+1} + \log\left(1 + e^{d_{t+1}-p_{t+1}}\right), \quad (24)$$

where $p_t = \log P_t$ and $d_t = \log D_t$. Now let us set $w_t = d_t - p_t$ and linearize $\log\left(1 + e^{w_{t+1}}\right)$ with respect to w_{t+1} around $\mu = Ew_t$. We obtain

$$\log\left(1 + e^{w_{t+1}}\right) \approx \log\left(1 + e^{\mu}\right) + \frac{e^{\mu}}{1+e^{\mu}}(w_{t+1} - \mu) = -\log\rho - \mu(1-\rho) + (1-\rho)(d_{t+1} - p_{t+1}),$$

where $\rho = (1 + e^{\mu})^{-1}$. Replacing in (24) we get the approximate accounting identity

$$p_t = k + \rho p_{t+1} + (1-\rho)d_{t+1}, \quad (25)$$

where $k = -\log(1+r) - \log\rho - \mu(1-\rho) = -\log(1+r) - \log\rho + (1-\rho)\log(1/\rho - 1)$. Equation (1) is obtained by solving forward and taking expectations at time t on both sides.

Derivation of formula (14)

Let $S_t = \frac{1-\rho}{\rho}\sum_{j=1}^{\infty}\rho^j E_t y_{t+j}$ and $\Delta y_t = \alpha(L)\epsilon_t$, ϵ_t being the innovation of the relevant information set. We have

$$\begin{aligned}
\rho E_t y_{t+1} &= \rho y_t + \rho E_t \Delta y_{t+1} \\
\rho^2 E_t y_{t+2} &= \rho^2 y_t + \rho^2 E_t \Delta y_{t+1} + \rho^2 E_t \Delta y_{t+2} \\
\cdots &= \cdots
\end{aligned}$$

Summing terms we get

$$\sum_{j=1}^{\infty}\rho^j E_t y_{t+j} = \frac{\rho}{1-\rho}y_t + \frac{\rho}{1-\rho}E_t\Delta y_{t+1} + \frac{\rho^2}{1-\rho}E_t\Delta y_{t+2} + \cdots$$

Considering that $E_{t-1}\Delta y_t = (\alpha(L) - \alpha_0)\epsilon_t$ and $E_t\Delta y_{t+j} - E_{t-1}\Delta y_{t+j} = \alpha_j\epsilon_t$, we have

$$\begin{aligned}
LS_t &= y_{t-1} + E_{t-1}\Delta y_t + \rho E_{t-1}\Delta y_{t+1} + \rho^2 E_{t-1}\Delta y_{t+2} + \cdots \\
\rho S_t &= \rho y_t + \rho E_t \Delta y_{t+1} + \rho^2 E_t \Delta y_{t+2} + \cdots \\
(L-\rho)S_t &= (L-\rho)y_t + \alpha(L)\epsilon_t - \alpha_0\epsilon_t - \rho\alpha_1\epsilon_t - \rho^2\alpha_2\epsilon_t - \cdots
\end{aligned}$$

Hence

$$S_t = \frac{1-\rho}{\rho}\sum_{j=1}^{\infty}\rho^j E_t y_{t+j} = y_t + \frac{\alpha(L)-\alpha(\rho)}{L-\rho}\epsilon_t.$$

About stock price volatility

Given the volatility of dividends, stock price volatility, as predicted by the present value model, is much smaller than volatility observed in real data (Shiller, 1981, Le Roy and Porter, 1981, Campbell and Shiller, 1988, West, 1988). Intuition suggest that noise shocks, by adding uncertainty, may enlarge stock price volatility, and therefore may help explaining the puzzle.

Comparing the variances of Δp_t for the cases of complete and incomplete information is extremely complicated for the general case. Therefore we shall limit ourselves to the interesting special case $c(L) = L^k$.

In this case, it is easily seen that $\gamma(L) = \gamma^*(L) = 1$. Moreover,

$$c^*(L) = \frac{L^k(1-\rho) - \rho^k(1-L)}{L-\rho} = (1-\rho)\sum_{j=1}^{k-1}\rho^{j-i}L^{k-j} + \rho^{k-1}.$$

The sum of squared coefficients is then $\lambda = (1-\rho)^2(1+\rho^2+\cdots+\rho^{2k-4}) + \rho^{2k-2} = (1-\rho+\rho^{2k-1})/(1+\rho)$. Using (15), it is seen that the variance of Δp_t^{Φ} is

$$\text{var}(\Delta p_t^{\Phi}) = \sigma_a^2\lambda.$$

Since $0 < \rho < 1$, λ is smaller than 1 for $k > 1$. Moreover, it decreases monotonically with k, approaching $(1-\rho)/(1+\rho)$ as $k \to \infty$. Volatility is smaller when k is large because uncertainty is smaller, since agents can predict perfectly what will happen to dividends far in the future.

As for incomplete information, from (16) we get $\sigma_u^2/\sigma_s^4 + \sigma_a^4\lambda/\sigma_s^2$. From (12) we see that $\sigma_u^2 = \sigma_a^2\sigma_e^2\sigma_s^2$, so that the variance of prices is

$$\text{var}(\Delta p_t) = \frac{\sigma_a^2\sigma_e^2}{\sigma_s^2} + \frac{\sigma_a^4\lambda}{\sigma_s^2}.$$

The second term is the variance of the signal component. It is decreasing in k, like the complete information case, since when k gets larger, forecasts improve. Moreover, it is decreasing in σ_e^2, because, as we have already seen, when the noise is large, the signal is not reliable and agents do not react to it. On the contrary, the first term, i.e. the variance of the learning component, is increasing in σ_e^2. This is because the noise worsen forecasts and therefore the learning shock gets larger. In other words, prices change little following the signal, since the market does not trust in it, but change a lot when agents can see the large forecast errors produced by the signal and adjust their response.

On balance, provided that $k > 1$ and therefore $\lambda < 1$, total variance increases monotonically with σ_e^2, starting from $\sigma_a^2 \lambda$, with $\sigma_e^2 = 0$, and approaching σ_a^2 as $\sigma_e^2 \to \infty$. The effect is sizable when k is large and therefore λ is small. Hence, the intuition that noise enlarges uncertainty and therefore stock price volatility is correct. When k is small, forecast errors are already large, so that the noise cannot change things that much. But when forecast errors are small, the noise has large volatility effects.

Finally, let us consider the variance ratio. Dividing the variance of Δp_t by the variance of Δp_t^Φ we obtain

$$\frac{\operatorname{var}(\Delta p_t)}{\operatorname{var}(\Delta p_t^\Phi)} = \frac{\lambda^{-1}\sigma_e^2}{\sigma_s^2} + \frac{\sigma_a^2}{\sigma_s^2} = 1 + (\lambda^{-1} - 1)\frac{\sigma_e^2}{\sigma_s^2}.$$

Therefore the variance ratio is greater than 1 for $k > 1$, grows monotonically with k, and is increasing in σ_e^2 when $k > 1$.

However, for reasonable values of the parameters the variance effect is not large. For instance, with $k = 4$, $\rho = 0.92$ and $\sigma_e^2/\sigma_s^2 = 0.8$, the variance of Δp_t under incomplete information is about 50% larger than the variance of the complete information case. More generally, Shiller, 1981, and West, 1988, argue convincingly that information cannot explain completely excess volatiliy in the context of the present value model. In our empirical setting, excess volatilitily can be detected by looking at the reaction of prices to both learning and signal shocks, which is too large with respect to what is predicted by the theory and reported in equation (11). Adding uncertainty about interest rates goes in the right direction; but we think that excess volatility calls for some additional explanation, which cannot be found in the present value framework.

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Tables

Shock	Lags	Regressors					
		(1)	(2)	(3)	(4)	(5)	(6)
Learning	2	0.66	0.82	0.98	0.92	0.66	0.93
	4	0.07	0.09	0.42	0.64	0.41	0.88
Signal	2	0.03	0.00	0.00	0.02	0.81	0.05
	4	0.10	0.01	0.01	0.05	0.91	0.05
Dividend	2	0.21	0.39	0.73	0.86	0.51	0.21
	4	0.39	0.48	0.78	0.97	0.62	0.41
Noise	2	0.01	0.00	0.00	0.02	0.63	0.03
	4	0.03	0.00	0.01	0.05	0.86	0.05

Table 1. Results of the fundamentalness test in the bivariate VAR. The table reports the p-values of the F-test in the regressions of the estimated shocks on 2 and 4 lags of the regressors (1)-(6). Dividend and noise shocks are truncated at time $T-4$ since end-of-sample estimates are inaccurate. Regressors: (1) 3-Month Treasury Bill: Secondary Market Rate; (2) 10-Year Treasury Constant Maturity Rate; (3) Moody's Seasoned Aaa Corporate Bond Yield; (4) GDP Implicit Price Deflator; (5) The Conference Board Leading Economic Indicators Index; (6) Michigan University Consumer Confidence Expected Index.

Shock	Lags	Regressors					
		(1)	(2)	(3)	(4)	(5)	(6)
Learning	2	1.00	0.91	1.00	0.79	0.82	0.60
	4	1.00	0.95	1.00	0.84	0.98	0.70
Signal	2	1.00	0.98	1.00	0.43	0.95	0.53
	4	1.00	0.97	1.00	0.59	0.98	0.27
Dividend	2	0.91	0.90	0.98	0.80	0.89	0.22
	4	0.96	0.72	0.97	0.91	0.98	0.51
Noise	2	0.95	0.96	0.93	0.31	0.73	0.20
	4	0.99	0.99	0.99	0.58	0.93	0.14

Table 2. Results of the fundamentalness test in the 4-variable VAR. The table reports the p-values of the F-test in the regressions of the estimated shocks on 2 and 4 lags of the regressors (1)-(6). Dividend and noise shocks are truncated at time $T-4$ since end-of-sample estimates are inaccurate. Regressors: (1) 3-Month Treasury Bill: Secondary Market Rate; (2) 10-Year Treasury Constant Maturity Rate; (3) Moody's Seasoned Aaa Corporate Bond Yield; (4) GDP Implicit Price Deflator; (5) The Conference Board Leading Economic Indicators Index; (6) Michigan University Consumer Confidence Expected Index.

Variable	Horizon				
	Impact	1-Year	2-Year	4-Years	10-Years
	Learning				
3-M T. Bill Rate	0.0 (0.0)	0.3 (1.5)	0.5 (2.7)	0.6 (3.6)	1.2 (5.1)
AAA C. Bond Yield	0.0 (0.0)	0.2 (1.3)	0.1 (2.0)	0.5 (3.7)	0.8 (5.9)
Dividends	99.7 (10.7)	81.7 (9.0)	75.2 (11.4)	73.8 (12.5)	74.0 (15.9)
Stock Prices	2.5 (2.1)	1.3 (2.7)	1.9 (4.7)	4.6 (8.9)	18.4 (14.6)
	Signal				
3-M T. Bill Rate	0.0 (0.0)	5.4 (4.2)	7.3 (6.1)	8.0 (6.4)	17.0 (8.8)
AAA C. Bond Yield	0.0 (0.0)	3.0 (3.4)	3.8 (4.9)	2.7 (5.6)	11.1 (9.2)
Dividends	0.0 (0.0)	14.7 (7.0)	20.7 (10.1)	17.3 (11.6)	17.8 (15.4)
Stock Prices	87.5 (5.0)	72.1 (9.5)	67.7 (11.5)	63.1 (13.7)	56.8 (15.8)
	Dividend shock				
3-M T. Bill Rate	0.0 (0.0)	1.4 (2.8)	2.8 (4.6)	2.6 (4.8)	5.9 (8.1)
AAA C. Bond Yield	0.0 (0.0)	0.3 (1.7)	0.4 (2.7)	0.7 (3.6)	1.7 (7.5)
Dividends	0.0 (0.0)	94.2 (11.5)	94.4 (10.8)	90.0 (13.4)	91.3 (13.7)
Stock Prices	17.3 (21.3)	21.4 (18.8)	21.5 (18.1)	26.1 (18.3)	45.0 (19.7)
	Noise				
3-M T. Bill Rate	0.0 (0.0)	4.2 (3.6)	4.9 (4.8)	6.0 (6.2)	12.2 (8.0)
AAA C. Bond Yield	0.0 (0.0)	2.8 (2.9)	3.6 (4.3)	2.5 (5.3)	10.2 (8.8)
Dividends	0.0 (0.0)	1.3 (9.6)	1.1 (7.7)	0.8 (6.3)	0.4 (4.3)
Stock Prices	72.4 (21.5)	52.0 (19.1)	47.9 (18.1)	41.4 (16.7)	30.0 (14.1)

Table 3. Variance decomposition in the 4-variable VAR. The entries are the percentages of forecast error variance explained by the shocks at the specified horizons. Standard errors in brackets.

Figures

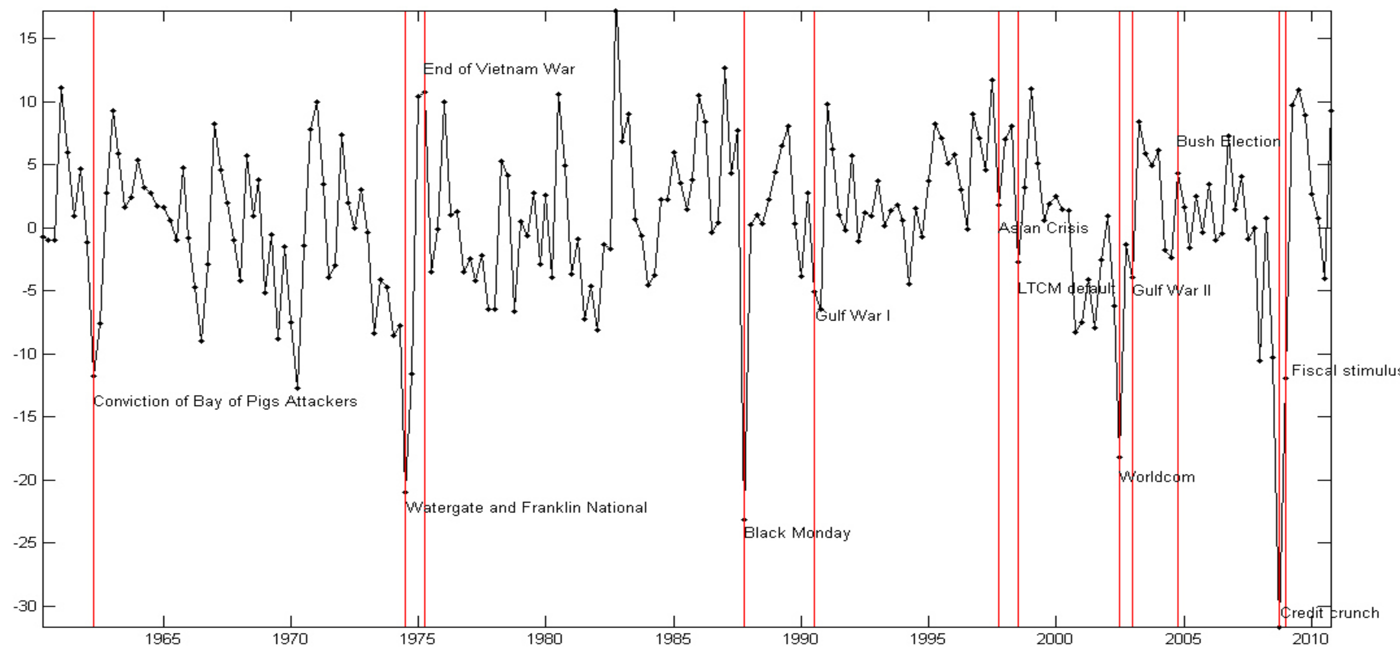

Figure 1. Log of the deflated S&P 500 index with vertical lines corresponding to economic and political events.

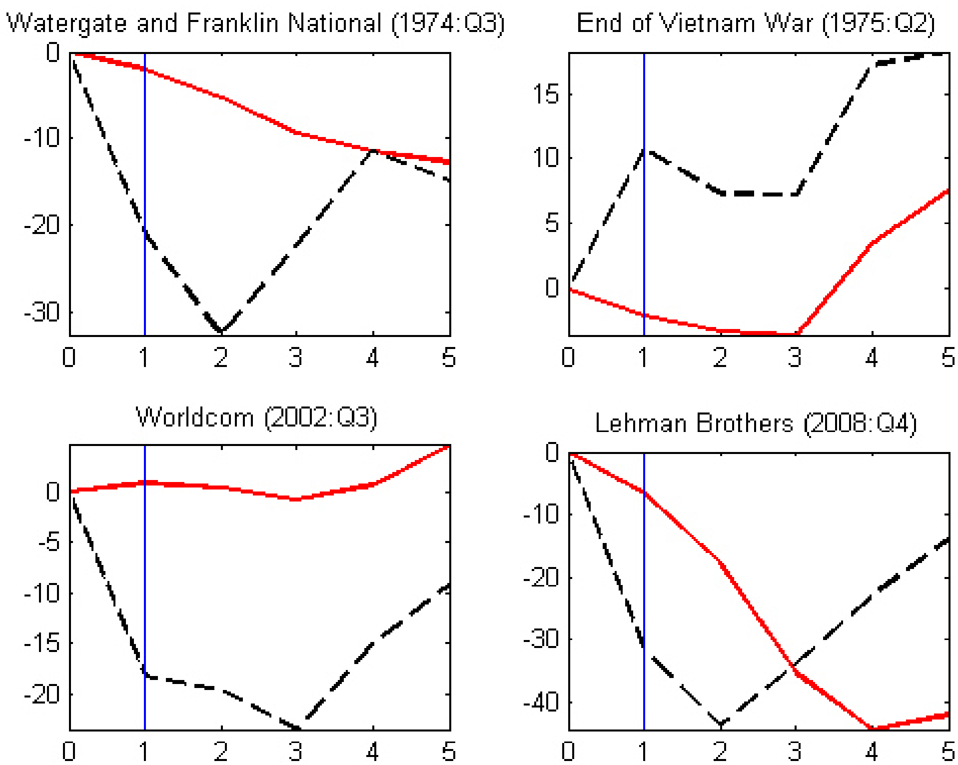

Figure 2. S&P 500 (dotted line) and Net Corporate Dividends (solid line) both in logs. On the x-axis there are the quarters. The events occurs in quarter 1.

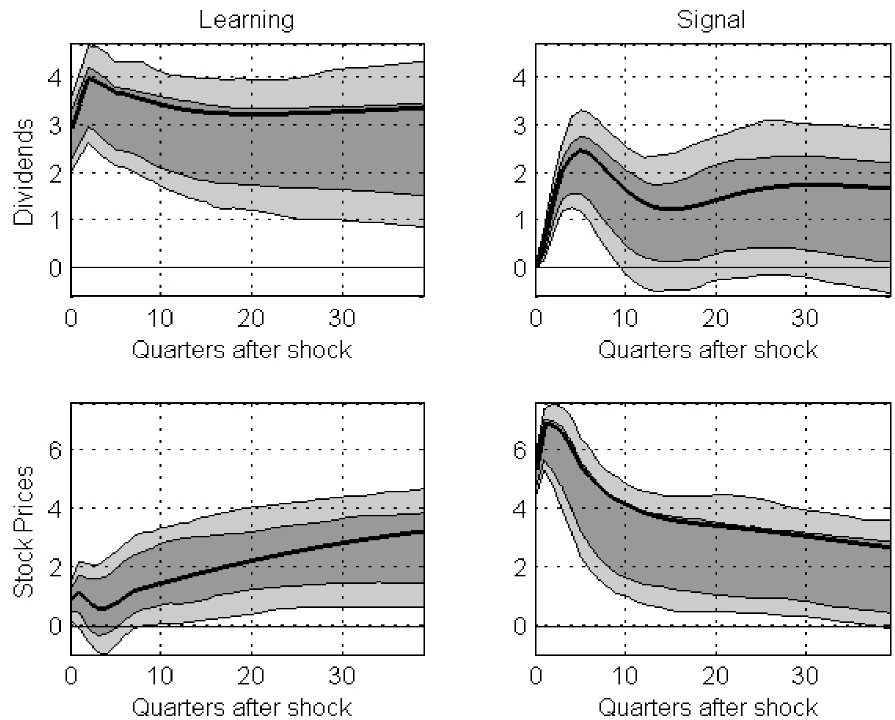

Figure 3a. Impulse response functions of dividends and stock prices to signal and learning shocks in the 4-variable VAR. Solid line: point estimates. Dark gray area: 68% confidence bands. Light gray area: 90% confidence bands.

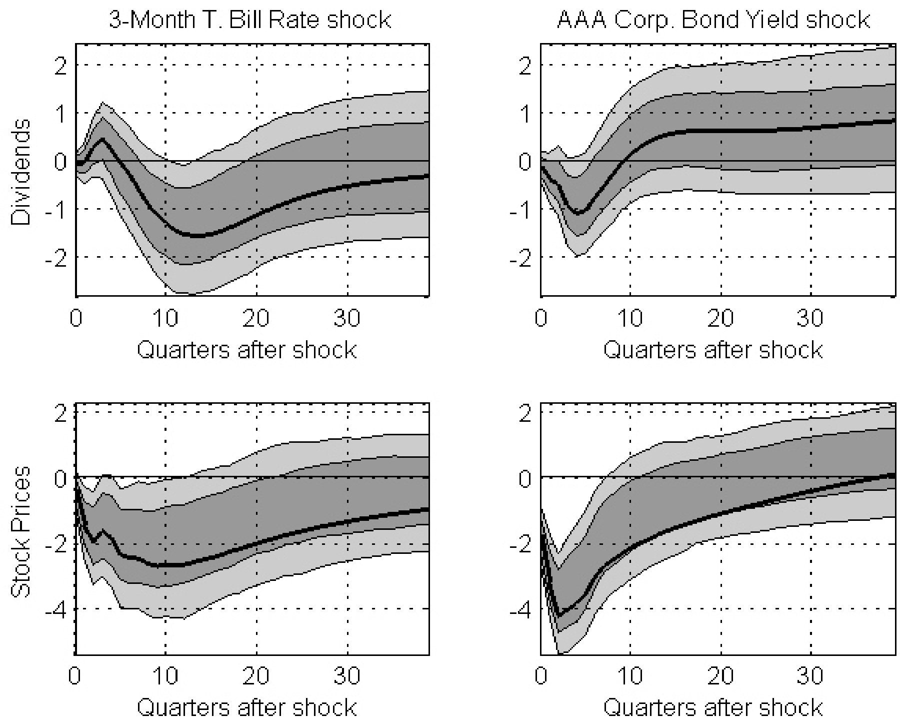

Figure 3b. Impulse response functions of dividends and stock prices to the 3-month Treasury bill, secondary market interest rate shock and the AAA corporate bond yield shock in the 4-variable VAR. Solid line: point estimates. Dark gray area: 68% confidence bands. Light gray area: 90% confidence bands.

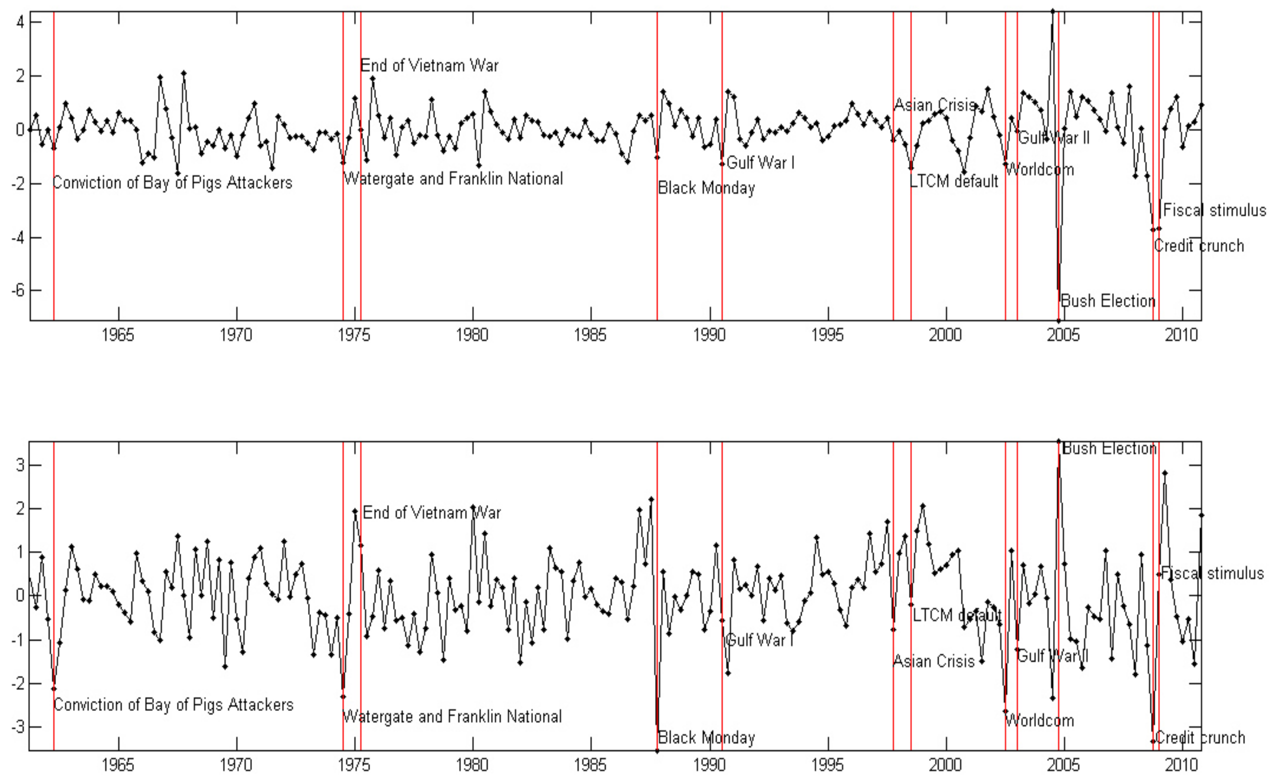

Figure 4. The estimated dividend shock (upper panel) and noise shock (lower panel) with vertical lines corresponding to economic and political events.

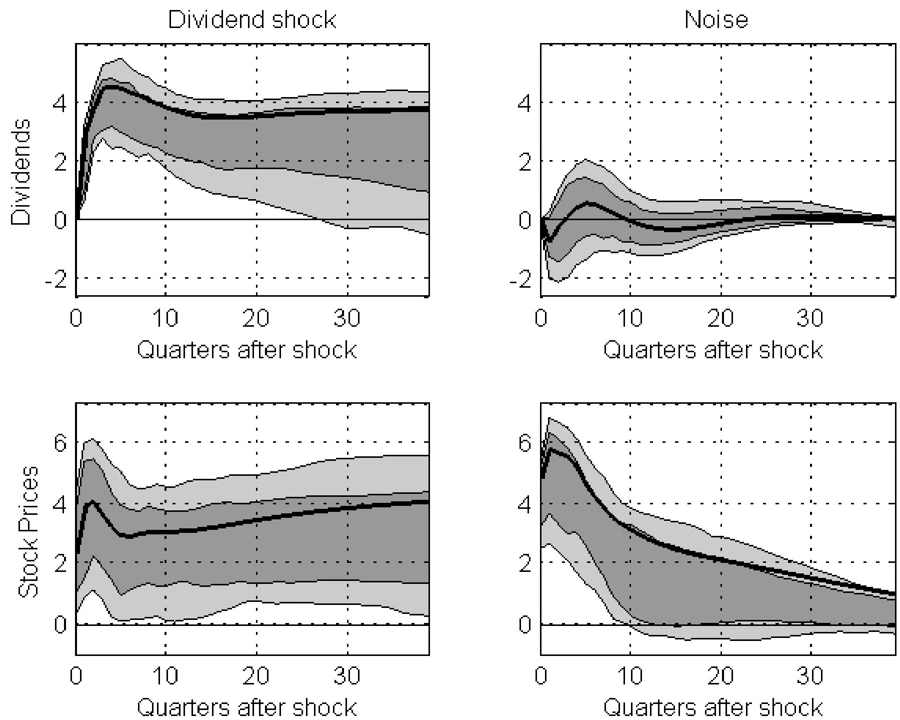

Figure 5a. Impulse response functions of dividends and stock prices to dividend and noise shocks in the 4-variable VAR. Solid line: point estimates. Dark gray area: 68% confidence bands. Light gray area: 90% confidence bands.

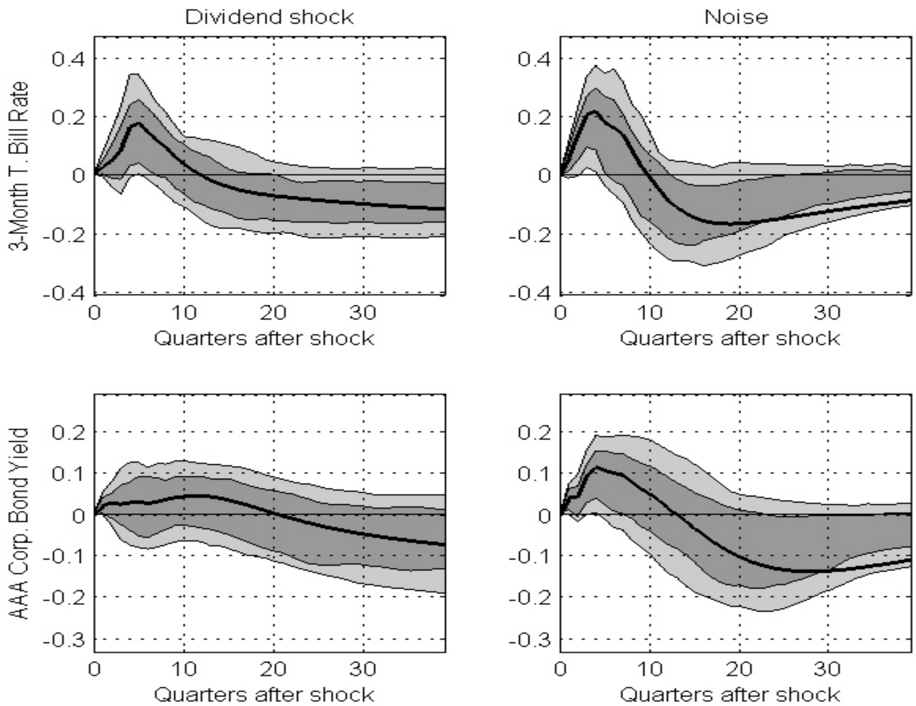

Figure 5b. Impulse response functions of 3M T-Bill and AAA Bond yield to dividend and noise shocks in the 4-variable VAR. Solid line: point estimates. Dark gray area: 68% confidence bands. Light gray area: 90% confidence bands.

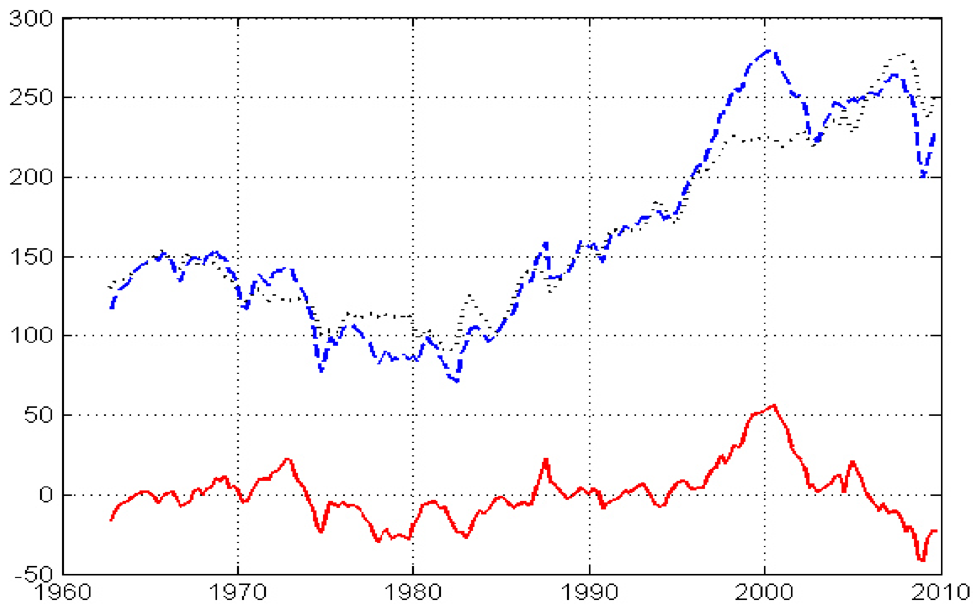

Figure 6. Historical decomposition in the 4-variable VAR. Dashed line: log of the S&P 500 stock price index divided by the GDP deflator. Solid line: noise component of the stock price index. Dotted line: difference between the stock price index and the noise component. The decomposition is truncated at time $T-4$ since end-of-sample estimates are inaccurate.

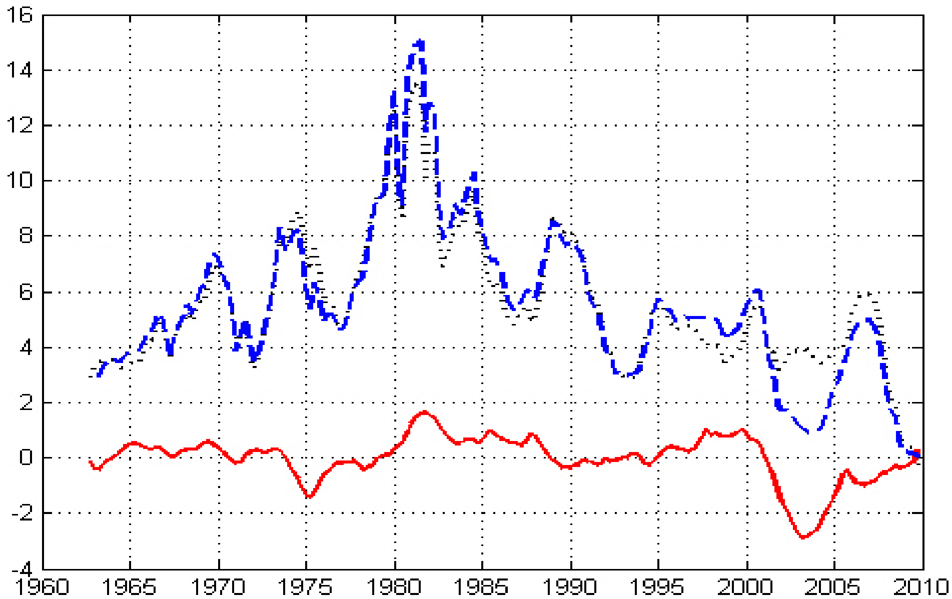

Figure 7. Historical decomposition in the 4-variable VAR. Dashed line: 3M T-Bill. Solid line: noise component of the 3M T-Bill. Dotted line: difference between the 3M T-Bill and the noise component. The decomposition is truncated at time $T-4$ since end-of-sample estimates are inaccurate.

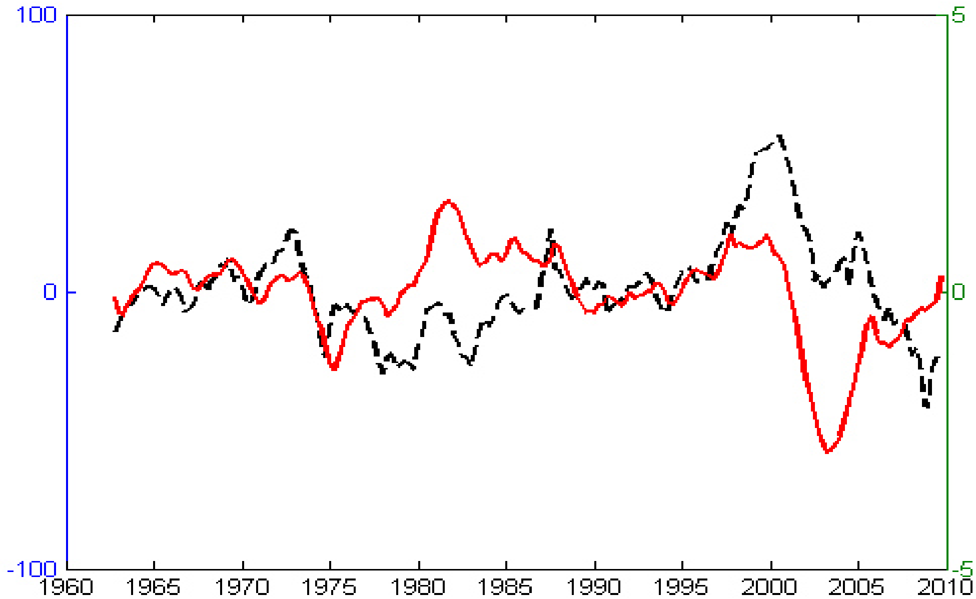

Figure 8. Historical decomposition in the 4-variable VAR. Dashed line: noise component of the log of the (real) S&P 500 stock price index. Solid line: noise component of the 3M T-Bill. The decomposition is truncated at time $T-4$ since end-of-sample estimates are inaccurate.

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