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Corrigendum

Corrigendum to "Nucleon-nucleon potential from skyrmion dipole interactions" [Nucl. Phys. B 967 (2021) 115430]

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The authors regret that there were two errors in their calculation. The first is in eq. (3.5) for the charge distribution of a dipole, which is missing one term and should read:

$$T := \left(\left(\boldsymbol{c} - \tfrac{1}{2}|\dot{\boldsymbol{X}}|^2\boldsymbol{c} - \tfrac{1}{2}(\dot{\boldsymbol{X}}\cdot\boldsymbol{c})\dot{\boldsymbol{X}}\right)\cdot\nabla + \dot{\boldsymbol{X}}\cdot\boldsymbol{c}\times\boldsymbol{\omega} + \ddot{\boldsymbol{X}}\cdot\boldsymbol{c}\right). \tag{1}$$

This in turn means that eqs. (3.17)–(3.20) should read:

$$A_{ab;ij} = \varepsilon_{ajc}(-\delta_{ib}\nabla_c e^{-mr}/r - \tfrac{1}{2}\nabla_{ibc}e^{-mr}/m) \tag{2}$$

$$B_{ab;ij} = -\varepsilon_{aic}\varepsilon_{bjd}\nabla_{cd}e^{-mr}/m \tag{3}$$

$$\begin{aligned} C_{ab;ij} &= \tfrac{1}{2}\delta_{ij}\nabla_{ab}e^{-mr}/r - \tfrac{1}{4}\nabla_{abij}e^{-mr}/m \\ &\quad - \tfrac{3}{8}(\delta_{jb}\nabla_{ia} + \delta_{ja}\nabla_{ib} + \delta_{ib}\nabla_{ja} + \delta_{ia}\nabla_{jb})e^{-mr}/r \end{aligned} \tag{4}$$

$$D_{ab} = \nabla_{ab}e^{-mr}/r. \tag{5}$$

The second error is in eq. (4.3), which is missing two terms and should read:

$$\begin{aligned} H = &-\frac{\hbar^2}{2}\triangle_g + V + \frac{\hbar^2}{2}E_\kappa g^{\kappa\lambda}\delta g_{\lambda\mu}g^{\mu\nu}E_\nu - \frac{\hbar^2}{2}E_\kappa g^{\kappa\lambda}\delta g_{\lambda\mu}g^{\mu\nu}\delta g_{\nu\rho}g^{\rho\sigma}E_\sigma \\ &+ \frac{\hbar^2}{32}g^{\mu\nu}[E_\mu, g^{\kappa\lambda}\delta g_{\lambda\kappa}][E_\nu, g^{\rho\sigma}\delta g_{\sigma\rho}] + \frac{\hbar^2}{8}[E_\mu, g^{\mu\nu}[E_\nu, g^{\kappa\lambda}\delta g_{\kappa\lambda} - \frac{1}{2}g^{\kappa\lambda}\delta g_{\lambda\rho}g^{\rho\sigma}\delta g_{\sigma\lambda}]] \\ &- \frac{\hbar^2}{8}[E_\mu, g^{\mu\nu}\delta g_{\nu\lambda}g^{\lambda\kappa}[E_\kappa, g^{\rho\sigma}\delta g_{\rho\sigma}]] + O(\delta g^3). \end{aligned} \tag{6}$$

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These errors mean that the formulae given for the nucleon-nucleon potentials in eqs. (5.39)–(5.47) and (E.1)–(E.8) are incorrect. The corrected versions of (E.1)–(E.8) are appended, and the corrected versions of (5.39)–(5.47) are obtained from these by setting $s = 0$. We have recalibrated our model using the same method as in our original paper but with the corrected potentials. The revised calibration is

$$F_\pi = 201 \text{ MeV}, \quad e = 3.30 \quad \text{and} \quad m_\pi = 199 \text{ MeV}. \tag{7}$$

This is not too different from the calibration originally proposed in (5.50). It leads to $M = 2389$ MeV, $\Lambda = 321$ MeV fm^2 and $\rho = 143$ MeV fm^3 in place of (5.51). The revised version of Fig. 1 of the paper is plotted in Fig. 1 below.

As can be seen from Fig. 1, the main consequence of these changes is that the isoscalar spin-orbit potential is no longer attractive in the range of interest. This is mainly due to the correction to eq. (3.5); the correction to eq. (4.3) makes little difference to the potentials. Apart from the two spin-orbit potentials, the remaining 6 potentials are in good agreement with the Paris potential. In a forthcoming paper, we show that correct signs are obtained for the spin-orbit potentials in a more sophisticated approximation based on instanton holonomies [1].

We should point out that the incorrect eq. (4.3) was taken from eq. (6.15) of [2], which we believe should read

$$\rho_1(t, \boldsymbol{x}) = -[(\dot{\boldsymbol{u}} \cdot \boldsymbol{p}) + (\boldsymbol{u} \cdot \boldsymbol{p} \times \boldsymbol{\alpha}) + (1 - \tfrac{1}{2}|\boldsymbol{u}|^2 \boldsymbol{p} \cdot \nabla - \tfrac{1}{2}(\boldsymbol{u}) \cdot \boldsymbol{p})(\boldsymbol{u} \cdot \nabla)]\delta^3(\boldsymbol{x} - \boldsymbol{X}). \tag{8}$$

In [2] the term $(\dot{\boldsymbol{u}} \cdot \boldsymbol{p})$ was neglected. If we follow through the calculation of [2], and eliminate terms involving accelerations by subtracting total time derivatives, we obtain a lagrangian

$$\begin{aligned}\tilde{L}_{\text{int}} = 2\kappa\big[&- \nabla \cdot \mathcal{O}\nabla + \boldsymbol{u} \cdot \mathcal{O}(\nabla \times \boldsymbol{\beta}) - (\nabla \times \boldsymbol{\alpha}) \cdot \mathcal{O}\boldsymbol{v} + \tfrac{1}{2}(|\boldsymbol{u}|^2 + |\boldsymbol{v}|^2)\nabla \cdot \mathcal{O}\nabla \\ &+ (\boldsymbol{u} \cdot \mathcal{O}\nabla)(\boldsymbol{v} \cdot \nabla) + (\nabla \cdot \mathcal{O}\boldsymbol{v})(\boldsymbol{u} \cdot \nabla) - \tfrac{1}{2}(\boldsymbol{u} \cdot \nabla)(\boldsymbol{u} \cdot \mathcal{O}\nabla) - \tfrac{1}{2}(\boldsymbol{v} \cdot \nabla)(\nabla \cdot \mathcal{O}\boldsymbol{v})\big]\frac{1}{R} \\ &+ \kappa\big[(\nabla \times \boldsymbol{\alpha}) \cdot \mathcal{O}(\nabla \times \boldsymbol{\beta}) - (\nabla \cdot \mathcal{O}\nabla)(\boldsymbol{u} \cdot \nabla)(\boldsymbol{v} \cdot \nabla) \\ &+ \boldsymbol{v} \cdot \nabla(\nabla \times \boldsymbol{\alpha} \cdot \mathcal{O}\nabla) - (\boldsymbol{u} \cdot \nabla)(\nabla \cdot \mathcal{O}(\nabla \times \boldsymbol{\beta}))\big]R.\end{aligned} \tag{9}$$

This has similar structure to eq. (6.22) of [2], but differs in a few places.

One of the results of [2] was that the lagrangians calculated using the dipole approximation and the product approximation agree. In eqs. (7.25)–(7.29) of the calculation based on the product approximation, total derivatives are added and terms involving acceleration are discarded. In general, discarding an acceleration from a lagrangian can lead to incorrect results: for example, subtracting the derivative of $\frac{1}{2}m\boldsymbol{x} \cdot \dot{\boldsymbol{x}}$ from the standard lagrangian $\frac{1}{2}m|\dot{\boldsymbol{x}}|^2 - V(\boldsymbol{x})$ and discarding the acceleration eliminates the kinetic term.

If we repeat the calculation in [2] and omit this step, we obtain a lagrangian that agrees exactly with (9). So the important result that the dipole and product approximations lead to the same lagrangian for well-separated skyrmions does appear to be correct. We have also checked by numerical calculation that the product approximation agrees with (9) at large separations; in fact this numerical observation was what prompted us to discover the mistake in eq. (3.5) of our paper.

We would like to apologise for any inconvenience caused.

1. Corrected formulae for potentials

The following correct eqs. (E.1)–(E.8) in the original manuscript. As in those equations, $s := m_\pi r/\hbar$.

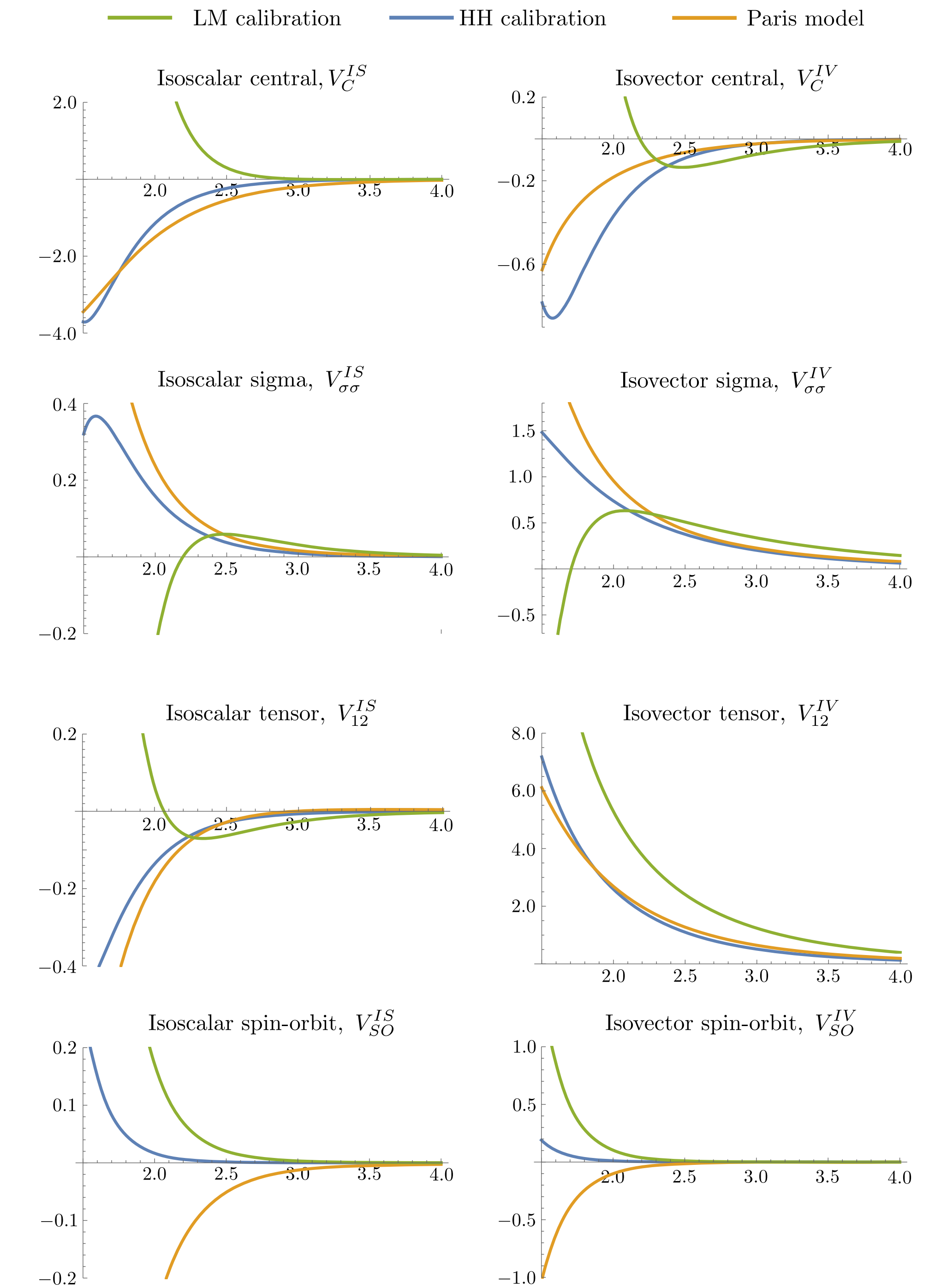

Fig. 1. A comparison between the potentials generated from our calculation and the phenomenological Paris potential. All are plots of the potential (MeV) against separation r (fm). This corrects Fig. 1 from the published paper.

$$
\begin{aligned}
V_C^{IS} = & \Big(-\frac{32\Lambda}{9\hbar^2 r^6} - \frac{64\Lambda s}{9\hbar^2 r^6} - \frac{160\Lambda\, s^2}{27\hbar^2 r^6} - \frac{64\Lambda\, s^3}{27\hbar^2 r^6} - \frac{16\Lambda\, s^4}{27\hbar^2 r^6} \\
& - \frac{8}{27\Lambda\, r^4} - \frac{16s}{27\Lambda\, r^4} - \frac{8s^2}{27\Lambda\, r^4} - \frac{4s^3}{27\Lambda\, r^4} + \frac{17\hbar^2}{54\Lambda^3 r^2} + \frac{17\hbar^2 s^2}{108\Lambda^3 r^2} \Big) \frac{\rho^2}{e^{2s}} \\
& + \Big(\frac{800\Lambda^2}{27\hbar^2 r^8} + \frac{1600\Lambda^2 s}{27\hbar^2 r^8} + \frac{160\Lambda^2 s^2}{3\hbar^2 r^8} + \frac{2240\Lambda^2 s^3}{81\hbar^2 r^8} + \frac{80\Lambda^2 s^4}{9\hbar^2 r^8} + \frac{160\Lambda^2 s^5}{81\hbar^2 r^8} \\
& + \frac{80\Lambda^2 s^6}{243\hbar^2 r^8} + \frac{472}{27r^6} + \frac{944s}{27r^6} + \frac{2504s^2}{81r^6} + \frac{1232s^3}{81r^6} + \frac{380s^4}{81r^6} + \frac{260s^5}{243r^6} \\
& + \frac{10\hbar^2}{81\Lambda^2 r^4} + \frac{20\hbar^2 s}{81\Lambda^2 r^4} - \frac{71\hbar^2 s^2}{81\Lambda^2 r^4} + \frac{5\hbar^2 s^3}{3\Lambda^2 r^4} - \frac{563\hbar^2 s^4}{486\Lambda^2 r^4} \Big) \frac{\rho^2}{Me^{2s}}
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
V_{\sigma\sigma}^{IS} = & \Big(\frac{8\Lambda}{27\hbar^2 r^6} + \frac{16\Lambda s}{27\hbar^2 r^6} + \frac{40\Lambda\, s^2}{81\hbar^2 r^6} + \frac{16\Lambda\, s^3}{81\hbar^2 r^6} + \frac{14}{81\Lambda\, r^4} + \frac{28s}{81\Lambda\, r^4} + \frac{28s^2}{81\Lambda\, r^4} \\
& - \frac{13\hbar^2}{648\Lambda^3 r^2} + \frac{13\hbar^2 s}{324\Lambda^3 r^2} \Big) \frac{\rho^2}{e^{2s}} \\
& + \Big(-\frac{280\Lambda^2}{81\hbar^2 r^8} - \frac{560\Lambda^2 s}{81\hbar^2 r^8} - \frac{56\Lambda^2 s^2}{9\hbar^2 r^8} - \frac{784\Lambda^2 s^3}{243\hbar^2 r^8} - \frac{728\Lambda^2 s^4}{729\hbar^2 r^8} - \frac{112\Lambda^2 s^5}{729\hbar^2 r^8} \\
& - \frac{134}{81r^6} - \frac{268s}{81r^6} - \frac{2156s^2}{729r^6} - \frac{1096s^3}{729r^6} - \frac{292s^4}{729r^6} \\
& - \frac{265\hbar^2}{5832\Lambda^2 r^4} - \frac{265\hbar^2 s}{2916\Lambda^2 r^4} - \frac{445\hbar^2 s^2}{1944\Lambda^2 r^4} - \frac{661\hbar^2 s^3}{2916\Lambda^2 r^4} \Big) \frac{\rho^2}{Me^{2s}}
\end{aligned} \tag{11}
$$

$$
\begin{aligned}
V_{12}^{IS} = & \Big(-\frac{8\Lambda}{27\hbar^2 r^6} - \frac{16\Lambda s}{27\hbar^2 r^6} - \frac{32\Lambda\, s^2}{81\hbar^2 r^6} - \frac{8\Lambda\, s^3}{81\hbar^2 r^6} \\
& - \frac{28}{81\Lambda\, r^4} - \frac{35s}{81\Lambda\, r^4} - \frac{14s^2}{81\Lambda\, r^4} - \frac{13\hbar^2}{648\Lambda^3 r^2} - \frac{13\hbar^2 s}{648\Lambda^3 r^2} \Big) \frac{\rho^2}{e^{2s}} \\
& + \Big(\frac{224\Lambda^2}{81\hbar^2 r^8} + \frac{448\Lambda^2 s}{81\hbar^2 r^8} + \frac{392\Lambda^2 s^2}{81\hbar^2 r^8} + \frac{560\Lambda^2 s^3}{243\hbar^2 r^8} + \frac{448\Lambda^2 s^4}{729\hbar^2 r^8} + \frac{56\Lambda^2 s^5}{729\hbar^2 r^8} \\
& + \frac{134}{81r^6} + \frac{268s}{81r^6} + \frac{1900s^2}{729r^6} + \frac{767s^3}{729r^6} + \frac{146s^4}{729r^6} \\
& + \frac{751\hbar^2}{2916\Lambda^2 r^4} + \frac{281\hbar^2 s}{729\Lambda^2 r^4} + \frac{553\hbar^2 s^2}{1944\Lambda^2 r^4} + \frac{661\hbar^2 s^3}{5832\Lambda^2 r^4} \Big) \frac{\rho^2}{Me^{2s}}
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
V_{LS}^{IS} = & \Big(\frac{16\Lambda^2}{27\hbar^2 r^8} + \frac{32\Lambda^2 s}{27\hbar^2 r^8} + \frac{80\Lambda^2 s^2}{81\hbar^2 r^8} + \frac{32\Lambda^2 s^3}{81\hbar^2 r^8} + \frac{16\Lambda^2 s^4}{243\hbar^2 r^8} + \frac{16}{81r^6} + \frac{32s}{81r^6} \\
& - \frac{8s^2}{243r^6} - \frac{56s^3}{243r^6} - \frac{101\hbar^2}{972\Lambda^2 r^4} + \frac{7\hbar^2 s}{486\Lambda^2 r^4} + \frac{115\hbar^2 s^2}{972\Lambda^2 r^4} \Big) \frac{\rho^2}{Me^{2s}}
\end{aligned} \tag{13}
$$

$$
\begin{aligned}
V_C^{IV} = & \Big(-\frac{16\Lambda}{27\hbar^2 r^6} - \frac{32\Lambda s}{27\hbar^2 r^6} - \frac{80\Lambda\, s^2}{81\hbar^2 r^6} - \frac{32\Lambda\, s^3}{81\hbar^2 r^6} - \frac{8\Lambda\, s^4}{81\hbar^2 r^6} \\
& - \frac{28}{81\Lambda\, r^4} - \frac{56s}{81\Lambda\, r^4} - \frac{28s^2}{81\Lambda\, r^4} - \frac{14s^3}{81\Lambda\, r^4} - \frac{13\hbar^2}{324\Lambda^3 r^2} - \frac{13\hbar^2 s^2}{648\Lambda^3 r^2} \Big) \frac{\rho^2}{e^{2s}}
\end{aligned}
$$

$$+\left(\frac{560\Lambda^2}{81\hbar^2r^8}+\frac{1120\Lambda^2s}{81\hbar^2r^8}+\frac{112\Lambda^2s^2}{9\hbar^2r^8}+\frac{1568\Lambda^2s^3}{243\hbar^2r^8}+\frac{56\Lambda^2s^4}{27\hbar^2r^8}+\frac{112\Lambda^2s^5}{243\hbar^2r^8}\right.$$
$$+\frac{56\Lambda^2s^6}{729\hbar^2r^8}+\frac{268}{81r^6}+\frac{536s}{81r^6}+\frac{1388s^2}{243r^6}+\frac{632s^3}{243r^6}+\frac{182s^4}{243r^6}+\frac{146s^5}{729r^6}$$
$$\left.+\frac{373\hbar^2}{972\Lambda^2r^4}+\frac{373\hbar^2s}{486\Lambda^2r^4}+\frac{505\hbar^2s^2}{972\Lambda^2r^4}+\frac{11\hbar^2s^3}{81\Lambda^2r^4}+\frac{661\hbar^2s^4}{5832\Lambda^2r^4}\right)\frac{\rho^2}{Me^{2s}} \quad (14)$$

$$V^{IV}_{\sigma\sigma}=\left(\frac{2s^2}{27r^3}-\frac{2\hbar^2}{27\Lambda^2r}+\frac{\hbar^2s}{27\Lambda^2r}\right)\frac{\rho}{e^s}$$
$$+\left(-\frac{\hbar^2s^3}{54\Lambda r^3}\right)\frac{\rho}{Me^s}$$
$$+\left(\frac{4\Lambda}{27\hbar^2r^6}+\frac{8\Lambda s}{27\hbar^2r^6}+\frac{20\Lambda s^2}{81\hbar^2r^6}+\frac{8\Lambda s^3}{81\hbar^2r^6}\right.$$
$$\left.+\frac{1}{27\Lambda r^4}+\frac{2s}{27\Lambda r^4}+\frac{2s^2}{27\Lambda r^4}+\frac{43\hbar^2}{1296\Lambda^3r^2}-\frac{43\hbar^2s}{648\Lambda^3r^2}\right)\frac{\rho^2}{e^{2s}}$$
$$+\left(-\frac{340\Lambda^2}{243\hbar^2r^8}-\frac{680\Lambda^2s}{243\hbar^2r^8}-\frac{68\Lambda^2s^2}{27\hbar^2r^8}-\frac{952\Lambda^2s^3}{729\hbar^2r^8}-\frac{884\Lambda^2s^4}{2187\hbar^2r^8}-\frac{136\Lambda^2s^5}{2187\hbar^2r^8}\right.$$
$$-\frac{185}{243r^6}-\frac{370s}{243r^6}-\frac{2978s^2}{2187r^6}-\frac{1516s^3}{2187r^6}-\frac{406s^4}{2187r^6}$$
$$\left.-\frac{2621\hbar^2}{34992\Lambda^2r^4}-\frac{2621\hbar^2s}{17496\Lambda^2r^4}-\frac{1973\hbar^2s^2}{11664\Lambda^2r^4}-\frac{29\hbar^2s^3}{17496\Lambda^2r^4}\right)\frac{\rho^2}{Me^{2s}} \quad (15)$$

$$V^{IV}_{12}=\left(\frac{2}{9r^3}+\frac{2s}{9r^3}+\frac{2s^2}{27r^3}+\frac{\hbar^2}{27\Lambda^2r}+\frac{\hbar^2s}{27\Lambda^2r}\right)\frac{\rho}{e^s}$$
$$+\left(-\frac{\hbar^2}{9\Lambda r^3}-\frac{\hbar^2s}{9\Lambda r^3}-\frac{\hbar^2s^2}{18\Lambda r^3}-\frac{\hbar^2s^3}{54\Lambda r^3}\right)\frac{\rho}{Me^s}$$
$$+\left(-\frac{4\Lambda}{27\hbar^2r^6}-\frac{8\Lambda s}{27\hbar^2r^6}-\frac{16\Lambda s^2}{81\hbar^2r^6}-\frac{4\Lambda s^3}{81\hbar^2r^6}\right.$$
$$\left.-\frac{2}{27\Lambda r^4}-\frac{5s}{54\Lambda r^4}-\frac{s^2}{27\Lambda r^4}+\frac{43\hbar^2}{1296\Lambda^3r^2}+\frac{43\hbar^2s}{1296\Lambda^3r^2}\right)\frac{\rho^2}{e^{2s}}$$
$$+\left(\frac{272\Lambda^2}{243\hbar^2r^8}+\frac{544\Lambda^2s}{243\hbar^2r^8}+\frac{476\Lambda^2s^2}{243\hbar^2r^8}+\frac{680\Lambda^2s^3}{729\hbar^2r^8}+\frac{544\Lambda^2s^4}{2187\hbar^2r^8}+\frac{68\Lambda^2s^5}{2187\hbar^2r^8}\right.$$
$$+\frac{185}{243r^6}+\frac{370s}{243r^6}+\frac{2626s^2}{2187r^6}+\frac{2125s^3}{4374r^6}+\frac{203s^4}{2187r^6}$$
$$\left.+\frac{2135\hbar^2}{17496\Lambda^2r^4}+\frac{1649\hbar^2s}{8748\Lambda^2r^4}+\frac{1001\hbar^2s^2}{11664\Lambda^2r^4}+\frac{29\hbar^2s^3}{34992\Lambda^2r^4}\right)\frac{\rho^2}{Me^{2s}} \quad (16)$$

$$V^{IV}_{LS}=\left(\frac{40\Lambda^2}{81\hbar^2r^8}+\frac{80\Lambda^2s}{81\hbar^2r^8}+\frac{200\Lambda^2s^2}{243\hbar^2r^8}+\frac{80\Lambda^2s^3}{243\hbar^2r^8}+\frac{40\Lambda^2s^4}{729\hbar^2r^8}\right.$$
$$-\frac{14}{243r^6}-\frac{28s}{243r^6}-\frac{128s^2}{729r^6}-\frac{86s^3}{729r^6}$$

$$+\frac{197\hbar^2}{5832\Lambda^2 r^4}+\frac{89\hbar^2 s}{2916\Lambda^2 r^4}-\frac{19\hbar^2 s^2}{5832\Lambda^2 r^4}\bigg)\frac{\rho^2}{Me^{2s}} \tag{17}$$

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