Assessing higher levels of learning through real-life problems in engineering mathematics

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***Abstract*— This paper discusses the design of engineering mathematics assessment that encourages learning beyond algorithmic recall. Our approach is based on the MATH (mathematical assessment task hierarchy) taxonomy. We propose using mathematics as a tool to analyse relatable problems and produce clear engineering deliverables to ensure that academic knowledge is translated to real-life situations. Creating engineering scenarios was instrumental in fostering active engagement, enquiry, creativity and reducing opportunities for academic misconduct in our first-year engineering mathematics assessments. An example of an exam question covering the topics of calculus, linear algebra, and dimensional analysis is given to illustrate the concepts discussed. Qualitative feedback from 203 students on an assessment paper containing the question shown herein is also included. Our data shows that most students had little to no previous exposure to real-world questions and that most of their time engaging with the assessment was spent critically analysing the problems. In addition, student feedback showed that contextual assessment is perceived as more challenging and exciting than pure mathematics problems and that students believe contextual assessment adds value to their education.**

Keywords*— *Engineering mathematics, contextual learning, assessment, deep learning, higher level of learning

I. INTRODUCTION

Assessment is the primary tool used by universities in expressing and assuring undergraduate academic standards. Through assessment, teachers can communicate the knowledge, skills, and attitudes valuable to students throughout their degree and professional life. Assessment practice, however, is often dissonant to the requirements of a fast-changing education landscape. Institutions have been under an increasing amount of pressure to meet the complex needs of society, employers, and a fee-paying student body. This tension is particularly notable in engineering courses, where a conservative approach to mathematics teaching often emphasises reciting formulas and replicating procedural knowledge at the expense of synthesis and critical analysis [1]. This deprives students of opportunities to engage with conceptual understanding [2], and students can frequently achieve high marks in mathematics exams by the mere recall of protocols that they have previously observed [3].

The obsoleteness of traditional mathematics teaching practice was evident when teaching and assessment moved online due to the COVID-19 pandemic. Social distancing implied challenges in coordinating and invigilating formal examinations, which were widely applied in an online, open-book format. This conjecture generated opportunities and temptation for students to collaborate or solve problems using tools such as Wolfram Alpha and MATLAB. Although the issue of academic misconduct is not the focus of this paper, it leads to the questioning of common assumptions about essential academic standards when a significant part of these standards consists of replication of procedures available in textbooks or on the internet. This picture reveals students that have outgrown an obsolete syllabus and challenges the perceived value of traditional mathematics training in engineering courses.

Significant work has been done on designing mathematics examinations and curricula that assess a range of learning levels beyond simple recall and procedural knowledge [3, 4, 5]. However, adopting and implementing inclusive, progressive mathematics curricula and assessments is not free of challenges. This change involves collaborative institutional action and allocating resources and expert pedagogical support for teaching staff [6]. In the aftermath, it is still imperative that foundational mathematics modules in engineering courses equip students with technical skills that will allow them to engage with expert subjects later in the course. Moreover, diversity in students' abilities, previous instruction and aspirations in engineering need to be considered. Finally, there is strong evidence that the structure of STEM courses can exacerbate inequalities and reduce student retention [7, 8].

The possibility of recurring social distancing restrictions warrants interest in developing non-invigilated examinations that assess higher levels of learning in engineering mathematics. Not only as a precautionary measure but as an effort to produce meaningful assessment practices that promote deep learning, are not detrimental to student mental health and well-being, and align with the complex skillset required from graduate professionals in modern society [1, 2]. This paper exemplifies how the MATH taxonomy can be applied in designing real-life questions for introductory undergraduate engineering mathematics questions on calculus and linear algebra.

II. ASSESSMENT DESIGN

A. *Overview of the MATH Taxonomy*

The assessment format is known to play a significant role in the way students revise and engage with educational material. On the one hand, this was exemplified in studies showing that closed-book examinations favour a surface approach to learning, primarily based on recall of formulae and procedures. On the other hand, open-book exams encouraged students to produce more notes and engage with deep learning of the material [1, 2].

The MATH taxonomy [4] was developed to conceptualise a broad range of skills that students might show in mathematics examinations. It was prompted by the realisation that although students can retain substantial amounts of information and replicate known procedures to pass exams, most of them cannot apply that information to problems requiring conceptual understanding.

This issue has roots in traditional high-school mathematics teaching and manifests itself in complete form in higher education. For example, in 2014, an overview of A-level mathematics papers in the UK concluded that, on average, 72.6% of the questions could be solved with simple recall and replication of procedures, 21.2% prompted students to apply their existing knowledge in different contexts, and only 6.4% required critical thinking and evaluation [3].

The MATH taxonomy assumes that: (i) students should be encouraged to engage with deep approaches to learning because (ii) they have been exposed to surface approaches in high school, but (iii) students can change how they learn. Accordingly, the taxonomy defines three groups of activities that constitute tasks in examinations. Activities that only require a surface approach to learning appear at the left end of Table 1 in Group A, while activities that demand a deep approach appear at the right end in Group C.

TABLE I: SUMMARY OF THE MATH TAXONOMY. ADAPTED FROM [4].

Group	A	B	C
1	Recall of knowledge	Information transfer	Justifying and interpreting
2	Basic comprehension	Application in new situations	Implications and comparisons
3	Replication of procedures	-	Evaluation, synthesis and critical analysis

1) Group A Activities

These activities consist of factual recall of knowledge, basic comprehension of formulae and conditions, and algorithmic replication of procedures.

Group A.1 activities only require the *recall and memorisation of information*. These might range from simple formulae to complex theorems. However, factual recall is the only skill necessary to obtain full marks. For example, "State the first form of the fundamental theorem of calculus". Moreover, Group A.2 activities require some *comprehension of factual knowledge*. These activities include deciding whether conditions for a definition are satisfied, understanding the role of symbols in formulae or statements, or simply recognising examples and patterns. Examples of such activities are: "Define the order/degree of the following ODEs" or "decide whether this ODE is linear". Finally, Group A.3 activities require factual recall of knowledge and the *replication of an algorithm* in a particular way. Most of the engineering mathematics assessment is capped at this level, with questions such as "Calculate the eigenvalues and eigenvectors of matrix B", "Solve the ODE $ay'' + by' + cy = 0$" where a, b and c are real-valued constants, or "Evaluate the derivative/integral of $f(x)$" where $f(x)$ is any linear transformation, translation, multiplication or composition of a mix of polynomial, exponential, trigonometric, logarithmic or rational functions. The key feature of this type of problem is that it can be answered correctly by memorising algorithms, but not necessarily by understanding them.

2) Group B Activities

Group B activities involve transferring information between mathematical and verbal forms, explaining to non-expert audiences, and applying knowledge in new situations.

Group B.1 Activities involve translating knowledge from one form to another. This involves switching with ease between verbal and numerical form in arguments, recognising the applicability of formulae and procedures in different contexts, summarising procedures and results for a non-specialist audience, composing a mathematical problem from a verbal definition or vice-versa, explaining processes, relationships between different parts of material or assembling an argument in logical order. For example, "Find the general solution of the differential equation $f(y', y, t) = 0$ or explain why no analytical solution exists", "Describe the effects of the affine transformation A on the vector $\boldsymbol{v}$", or "Discuss the effects of the ratio $\frac{m}{a}$ on the solutions of the ODE $ay'' + my = 0$". Group B.2 activities will test the student's ability to choose and apply suitable methods to new situations or real-life problems, such as: "Calculate the maximum amplitude and natural frequency of an oscillator defined by the equation $ay'' + my = 0$". These questions will naturally introduce some visualisation and application of pure mathematics, but students will be naturally prompted to revert to Group A activities once the initial visualisation phase is completed.

3) Group C Activities

Group C activities require justifying and interpreting results, making comparisons and drawing implications of results and methods.

Group C.1 activities will require discussing the significance of results, the statement of implicit assumptions, an ability to reflect and find errors in their reasoning, or recognising the applicability and limitations of a model. Group C.2 activities will involve comparing methods and algorithms, deducing implications from a mathematical result and exemplifying conclusions. Finally, Group C.3 will assess the ability of the student to go beyond what is asked, apply their creativity and inquiry into a process of making judgements, decisions, selections, and outline a cohesive and coherent argument. Primary examples of these activities for engineering mathematics assessment are discussed in the following section.

B. *Example Question for Calculus and Linear Algebra Assessment*

Furnaces are challenging, costly engineering projects. Furnaces need to be long-lasting, optimise energy usage and waste, and have minimal emissions of greenhouse gases. Therefore, it is essential to understand how engineering design and appropriate material choice can optimise the efficiency of a furnace. First, we will consider the furnace walls with three layers with distinct features.

- The first layer is in contact with the heat radiated by the combustion of the fuel. This layer is usually made of material that reflects heat but inevitably absorbs part of it.
- The absorbed heat is conducted onto the second layer of thermally insulating material. This material has low heat conductivity, limiting the heat the wall surface can carry.
- The final layer provides structural integrity to the furnace. This layer needs to be made from long-lasting materials with constant properties for a wide temperature range.

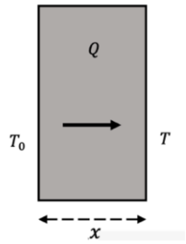

Figure 1. Schematic of heat conduction across a layer of thickness x.

The heat transferred per unit area Q [W m^{-2}] across a layer of thickness x [m] from the surface of higher temperature T_0[K] to the surface of lower temperature T [K] is given by Fourier's Law

$$Q = \frac{k}{x}(T_0 - T),$$

where k is the thermal conductivity of the layer.

a) [20 marks] Manipulate Fourier's law algebraically to obtain a formula for the internal temperature of the layer at a distance x from the left wall. What mathematical process is achieved by assuming that x takes on very small values?

b) [10 marks] If Q and k are constant, derive the units of k and explain what this physical constant represents. Discuss the effects of k on the temperature profile $T(x)$. Support your answer by plotting $T(x)$ in a 30 cm thick wall for three constant values of $\frac{Q}{k}$: 0.1, 10 and 100 [K m^{-1}].

A more realistic model would consider that k is a function of the local temperature, such that $k = k(T)$. This type of consideration can improve the accuracy of the model developed in *questions a-b*. Table A shows the thermal conductivity of three materials at temperatures 100 and 2000 K.

c) [10 marks] Use the design principles outlined in this brief to choose one of the materials in Table A for the second layer of the furnace wall and justify your decision. Use the data in Table A and your knowledge in 2x2 linear systems of equations to find a linear approximation for $k(T)$ in this layer for the temperature interval 100 – 2000 K.

Table A. Materials and their thermal conductivity k at 100 and 2000K.

	100 K	**2000 K**
Material A	1.6	3.1
Material B	3.1	6.2
Material C	45.2	45.2

The heat per unit area Q absorbed by the first layer is conducted onto the second and third layers. Each layer has a thickness x_i and thermal conductivity k_i, where i represents the layer number. Conservation of energy requires that the heat per unit area Q transferred through the first layer equals the heat transferred through the second and third layers, as illustrated in Figure 2.

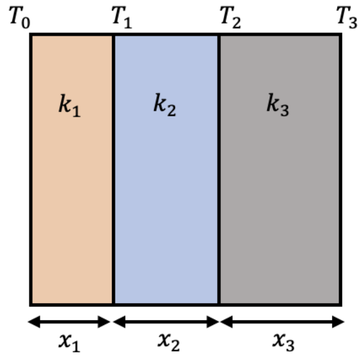

Figure 2. Heat conduction across three layers.

The heat conducted through the first layer from T_0 to T_1 over x_1 is given by

$$Q = \frac{k_1}{x_1}(T_0 - T_1),$$

which equals the heat transferred across the second layer from T_1 to T_2 over x_2 and so forth:

$$Q = \frac{k_2}{x_2}(T_1 - T_2) = \frac{k_1}{x_1}(T_0 - T_1).$$

d) [30 marks] Write a matrix model that can evaluate the temperatures T_1 and T_2 in the schematic of Figure 2. For simplicity, assume that k has a single value across the temperature range of interest and that $T_0, T_3, k_1, k_2, k_3, x_1, x_2$ and x_3 are known values.

e) [30 marks] Use your matrix model developed in *question d* and your answer to *question c* in choosing which material from Table A should be used for the first, second and third layers of thickness $x_1 = 0.1$, $x_2 = 0.7$ and $x_3 = 0.6$ m. Assume working temperatures $T_0 = 1700$ K and $T_3 = 350$ K. Again, assume that k has a single value across the temperature range but this time, justify your choice.

C. *Discussion of the example question*

The analysis of the problem exemplified in II.B will show the extensive presence of Group A and B activities. It is also clear that, given appropriate design, the use of real-life scenarios and problem-based learning introduces Group B aspects to activities that would initially belong in Group A. In these questions, we chose not to focus on rigorous mathematical definitions but favour interpretation and enquiry of those definitions within an application. Students were also encouraged to use MATLAB as often as possible to help them realise solutions to the problem.

In *question a* the students were asked to recognise a real-life example of a limit leading to the derivative $\frac{dT}{dx} = -\frac{Q}{k}$. In *question b*, students are prompted to evaluate how the ratio between the heat transferred per unit area and the thermal conductivity k influenced the temperature profile and were asked to produce evidence for their argument.

In *question c*, students are required to interpret verbal information to select a material for the second layer of the furnace wall. Although material A or B would have been good choices, material A was ideal. At this point, the marker could also choose to award full marks to students who compared both materials at the ends of the temperature interval and found the differences to be negligible.

Question *d* is the most traditional problem in this example. Students are simply required to assemble a linear system of equations in matrix form, calculate the inverse of the matrix and present their results as a vector equation. Although this problem is very contextualised, it is expected that students have already defined the deliverables in this question with considerable precision. This process is also hinted at in *question c*, where they are asked to find a linear approximation for the thermal conductivity of whichever material was chosen for the second layer.

Finally, in *question e*, the overarching theme of the question becomes clear: they can apply differential calculus and matrix algebra to minimise heat loss across a furnace wall. This type of question can be more rewarding because students can see the final product of their mathematical work and be satisfied with their achievement's depth.

As an illustration of the framework, some key activities required in these problems can be mapped onto the MATH taxonomy as outlined in Table 2:

TABLE II: CLASSIFICATION OF ACTIVITIES IN II.B ACCORDING TO THE MATH TAXONOMY.

Group A	**Group B**	**Group C**
a: Recalling the definition of limits and derivatives and manipulating formulas b: Using dimensional analysis to obtain the results of a constant c, d, e: Procedural elaboration of linear systems followed by inversion of matrices	a, c, e: Applying mathematical definitions in real-life contexts b, c: Explaining the relationship between parameters and their meaning a, b, c, d, e: Extracting verbal information from a mathematical form or vice-versa	b, c, d, e: Synthesis and critical analysis; interpreting results and drawing conclusions c, e: Making judgements and selecting for relevance e: Deducing implications from results obtained

III. STUDENT FEEDBACK AND RESULTS

After our students submitted the first assessment paper in the course, we circulated an anonymous feedback survey to 884 students enrolled in ENGF0003: Mathematical Modelling and Analysis I. There were 203 respondents, 22.96% of the total students enrolled in the module. We asked our students for qualitative feedback about their perceptions of the assessment. These questions were designed as a feedback tool to understand how students perceived contextual assessment. Therefore, we advise using the following results only as question-formulating data.

ENGF0003: Mathematical Modelling and Analysis I is a cross-departmental module at UCL (University College London). Our students come from diverse backgrounds and are spread across eight programmes: Biochemical Engineering, Biomedical Engineering, Chemical Engineering, Civil Environmental & Geomatic Engineering, Engineering and Architectural Design, Electrical & Electronic Engineering, Mechanical Engineering, and Medical Sciences & Engineering. A similar question to the furnace design example discussed above was present in the first assessment paper of 2021 and contained the highest weight amongst two other contextual questions.

We started the survey by asking how challenging students thought the assessment was, where 93.1% responded that it was challenging, 4.4% thought it was neither challenging nor easy, and 2.5% responded that it was not challenging. The students were then asked how interesting they found the assessment, having 63% of responses rating the paper as interesting, 23.6% in a neutral position, and 13.3% stating that the paper was not attractive.

When asked how familiar they were with contextual questions, 70% of the students said they had not been exposed to contextual questions in high school, 23.6% had limited previous exposure to such questions, and only 15.8% of students had practised contextual mathematics before. When prompted to compare engineering questions to procedural mathematics questions, 75.8% of our students said that the engineering questions were the most challenging, 15.8% responded that they were equally difficult to pure mathematics questions, and 8.4% found the contextual questions easier. Furthermore, 53.1% of students agreed that contextual engineering questions were more exciting than procedural mathematics questions, 31.5% thought both types of questions were equally exciting, and 12.3% responded that mathematics questions were most exciting.

When asked about the most time-consuming task when engaging with the assessment, 54.19% of students responded that understanding the questions (i.e. interpreting the text and data given and deciding on appropriate mathematical models) was the most time-consuming task. Creating computational models in MATLAB to produce figures and analyse the behaviour of models was the most time-consuming task for 21.18% of respondents. Writing the coursework paper, discussing results and adjusting tables and figures was the most time-consuming task for 19.21% of students, whilst working through mathematical formulae and procedures was the most time-consuming task for only 5.42% of the students.

Finally, students were asked how much value they thought contextual assessment added to their higher education and whether they had learned new things in engaging with the paper. Again, 89% of students responded that contextual assessment adds value to their education. Moreover, 80.8% of students agreed that they learned new skills with the assessment.

IV. Conclusions

The MATH taxonomy is a useful tool in the design of engineering mathematics assessment that promotes deep learning whilst keeping mathematics at the forefront. Our data suggest that students perceive this assessment type as more challenging and exciting than purely mathematical problems.

Our data also suggests that most first-year engineering students have not been exposed to contextual mathematics questions that require conceptual understanding, indicating a need for foundational courses to move in this direction.

The fact that most students spent most of their time engaging with the assessment on a conceptual level indicates that our approach successfully fostered deep learning of the material. This is further reinforced by students being able to recognise that this type of assessment adds value to their education and that they had learned new skills in completing the assessment.

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