Rapid Design of LCC Current-output Resonant Converters with Reduced Electrical Stresses

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Abstract

The paper presents and validates a straightforward design methodology for realising LCC current-output resonant converters, with the aim of reducing tank currents, and hence, electrical stresses on resonant components. The scheme is ideally suited for inclusion in a rapid iterative design environment e.g. part of a Graphical User Interface.

Introduction

Resonant converters are becoming preferred candidates for applications requiring high switching frequencies to facilitate reduced volume/mass of reactive circuit components. Through the use of zero-voltage switching, and simple snubbers, switching losses can be significantly reduced [1] compared with hard-switched converter counterparts. Conduction losses, however, can remain significant, particularly due to high resonant-tank currents, that can occur when operating far above the resonant frequency at full load. Here then, a design procedure is proposed that assumes the converter is operated about a specified resonant frequency for a given load and gain. The aim is to provide a converter design to reduce tank currents—in general one would use the design procedure and ultimately operate slightly above resonance, the gain at resonance has therefore to be slightly larger than required. Many of the underlying equations have been taken from previously reported design methodologies—the reader will be referred to these publications wherever possible, for brevity. The design process utilises the accuracy of Fundamental Mode Approximation (FMA) at resonance, and the potential for rapid analysis it provides.

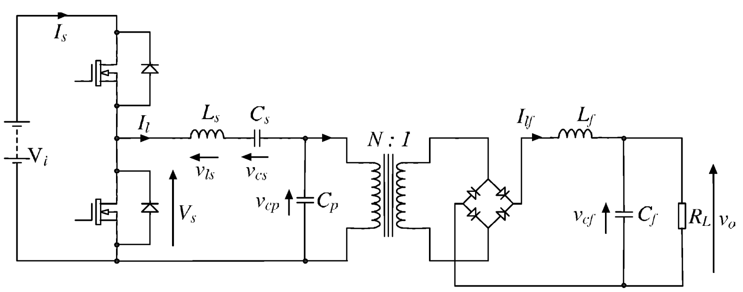

Fig. 1: LCC Current-Output resonant converter

Operation close to resonance

Operation at resonance minimises the resonant current, thereby reducing electrical stresses and improving efficiency. Figure 2 shows the excitation voltage and tank current for two resonant converters, both operated with constant output power and input voltage, at, and above, the resonant frequency, respectively. Current only flows to, or from, the supply when the switching voltage is positive (i.e. upper MOSFET reverse biased/diode conducting, in Fig. 1). To maintain a constant output power, and hence, constant input power, for an efficient converter the time integral of the supply current both at, and above resonance, must be the same. This can be seen from Figs. 2(c)(d) for operation at, and above resonance, respectively. Note that during operation above resonance, the supply current can become negative during part of the switching cycle, implying that the peak current needs to be higher to maintain a constant time-integral.

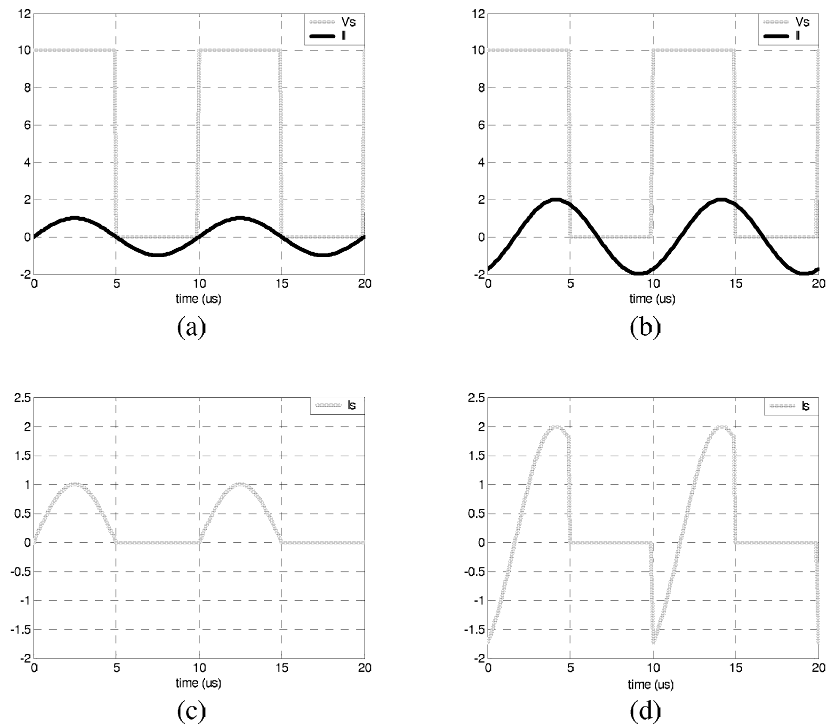

Fig. 2: Resonant current, supply current and switching voltage for operation at resonance (a),(c) and above resonance (b)(d).

If one assumes that the inductor current is sinusoidal at and above resonance, power is only carried on the fundamental of the switching voltage. For the same real power transfer, the peak inductor current above resonance is related to the peak inductor current at resonance by the power factor. It can be shown that:

$$\hat{I}_{L(above\ resonance)} = \frac{\hat{I}_{L(at\ resonance)}}{\cos(\theta)} \quad \text{where } \theta \text{ is the phase shift above resonance} \tag{1}$$

Consequently, in general, the further above resonance one operates, the larger the peak resonant current needs to be—operation close to resonance therefore results in reduced component stresses for a given power transfer.

Derivation of design methodology

From [2], the input-output voltage transfer function ($G(\omega)$) of the LCC converter, shown in Fig.1, and the relationship between the resonant frequency ω_r and the undamped natural frequency of the tank ω_o, are given, respectively, by (2)(3) where $A=C_p/C_s$ is the ratio of parallel and series capacitances, and Q_L is the loaded quality factor at the corner frequency.

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$$G(\omega)=\frac{4}{N\pi^2\sqrt{(1+A)^2\left(1-\left(\frac{\omega}{\omega_o}\right)^2\right)^2+\frac{1}{Q_L{}^2}\left(\frac{\omega}{\omega_o}-\frac{\omega_o A}{\omega(1+A)}\right)^2}} \quad (2)$$

$$\omega_r=\sqrt{\frac{Q_L{}^2(1+A)^2-1+\sqrt{\left(Q_L{}^2(1+A)^2-1\right)^2+4Q_L{}^2A(1+A)}}{2Q_L{}^2(1+A)^2}}\,\omega_o \quad (3)$$

Substituting (3) into (2) (i.e $\omega = \omega_r$) provides the gain at resonance G_r, and subsequently, by rearrangement of the gain expression, for Q_L, gives:

$$Q_L=\frac{NG_r\pi^2}{4}\sqrt{\frac{\left(G_rN\pi^2-4\right)\left(G_rN\pi^2+4\right)}{(1+A)\left(AG_r{}^2N^2\pi^4+G_r{}^2N^2\pi^4-16\right)}} \quad (4)$$

The overall gain of the converter can be written as a combination of the resonant tank gain and transformer gain:

$$G_r=\frac{G_t}{N}\quad \text{where } G_t \text{ is the resonant tank gain} \quad (5)$$

Also, substituting (5) into (4) gives:

$$Q_L=\frac{\pi^2G_t}{4}\sqrt{\frac{\left(G_t\pi^2-4\right)\left(G_t\pi^2+4\right)}{(1+A)\left(AG_t{}^2\pi^4+G_t{}^2\pi^4-16\right)}} \quad (6)$$

From (6), a constraint on the minimum value of tank gain can be found that ensures Q_L is real:

$$G_t>\frac{4}{\pi^2} \quad (7)$$

Substituting (6) into (3), the expression for ω_r then becomes:

$$\omega_r=\frac{\omega_o\sqrt{AG_t{}^2\pi^4+G_t{}^2\pi^4-16}}{G_t\pi^2\sqrt{1+A}} \quad (8)$$

Now, assuming a lossless converter, operating at resonance with a 50% duty cycle and sinusoidal current, all the power supplied to the converter is *Re*al. The input- and output-power can therefore be equated, as follows:

$$P_{in}=\left(\frac{V_i}{2}\right)\left(\frac{2\hat{I}_L}{\pi}\right), P_{out}=\frac{(V_iG_t)^2}{N^2R_L}, P_{in}=P_{out} \quad (9)$$

Rearrangement of (9) leads to the following expression for the magnitude of the resonant current:

$$\hat{I}_L=\frac{\pi V_iG_t{}^2}{N^2R_L} \quad (10)$$

Moreover, an alternative expression for Q_L can be obtained, previously presented in [2] :

$$Q_L=\frac{\pi^2N^2R_L}{8\omega_oL_s} \quad (11)$$

Rearranging (11) in terms of L_s and multiplying by $\omega_r/(2\pi f_r)$ gives:

$$L_s=\frac{\pi N^2R_L}{16Q_Lf_r}\frac{\omega_r}{\omega_o} \quad (12)$$

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and substituting (5),(6),(8) into (12) leads to:

$$L_s = \frac{N^2 R_L \left(A G_t^2 \pi^4 + G_t^2 \pi^4 - 16\right)}{4\pi^3 f_r G_t^2 \sqrt{G_t^2 \pi^4 - 16}} \tag{13}$$

It can be shown that [2]

$$\omega_o = \sqrt{\frac{1+A}{L_s C_P}} \tag{14}$$

and substituting (14) into (11) and rearranging, provides an expression for C_P:

$$C_P = \frac{64 Q_L^2 (1+A) L_s}{N^4 \pi^4 R_L^2} \tag{15}$$

or, alternatively, substituting (5),(6),(13) into (15):

$$C_p = \frac{\sqrt{G_t^2 \pi^4 - 16}}{\pi^3 N^2 R_L f_r} \tag{16}$$

An expression for C_s immediately follows from $C_S = C_P / A$:

$$C_s = \frac{\sqrt{G_t^2 \pi^4 - 16}}{\pi^3 N^2 R_L f_r A} \tag{17}$$

An important consideration for designs is the loaded quality factor at the resonant frequency:

$$Q_r = \frac{\omega_r L_s}{R_s} \tag{18}$$

where R_s is given by [2]:

$$R_s = \frac{R_i}{1 + \left(\omega_r C_p R_i\right)^2} \tag{19}$$

and R_i for the current output converter is:

$$R_i = \frac{\pi^2 N^2 R_L}{8} \tag{20}$$

Substituting (19),(20) into (18) gives:

$$Q_r = \frac{\omega_r L_s \left(64 + \pi^4 \omega_r^2 C_P^2 R_L^2 N^4\right)}{8\pi^2 N^2 R_L} \tag{21}$$

and substituting (5),(13),(16) and $\omega_r = (2\pi f_r)$ into (21) provides:

$$Q_r = \frac{A G_t^2 \pi^4 + G_t^2 \pi^4 - 16}{4\sqrt{G_t^2 \pi^4 - 16}} \tag{22}$$

For FMA to accurately estimate the output voltage, the inductor current should be predominantly sinusoidal. In [2] it is specified that a $Q_r > 2.5$ will provide near sinusoidal waveforms. Figure 3 provides the minimum value of A for a given G_t value in order that $Q_r > 2.5$.

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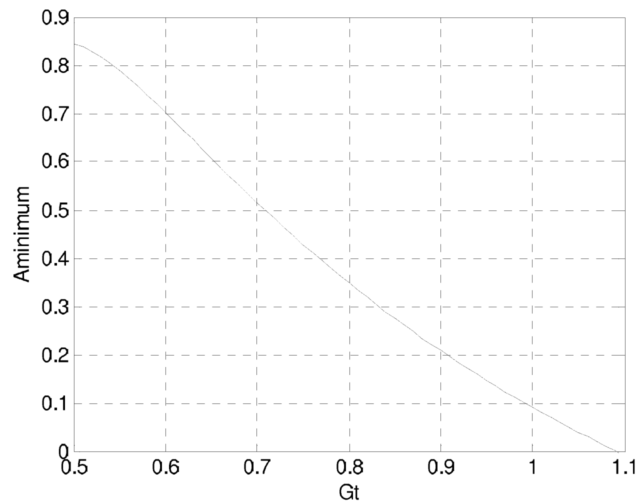

Fig. 3: Plot of $A_{minimum}$ vs G_t for Q_r>2.5

As discussed, an important consideration for resonant converters is the electrical stresses that components are exposed to, since they are often higher than for hard-switched counterparts. The resonant inductor voltage stress is found from:

$$V_{Ls} = 2\pi f_r L_s \hat{I}_L \tag{23}$$

and, by substituting (10),(13) into (23):

$$V_{Ls} = \frac{V_i\left(AG_t^2\pi^4 + G_t^2\pi^4 - 16\right)}{2\pi\sqrt{G_t^2\pi^4 - 16}} \tag{24}$$

Since the use of FMA only considers the fundamental components, it can provide inaccurate estimates of the inductor voltage stress. In particular, it is notable that, at the switching instants, the inductor voltage increases by an amount equivalent to the supply voltage, thereby ultimately giving:

$$V_{Ls} = \frac{V_i\left(AG_t^2\pi^4 + G_t^2\pi^4 - 16\right)}{2\pi\sqrt{G_t^2\pi^4 - 16}} + V_i \tag{25}$$

The series capacitor voltage stress is given by:

$$V_{Cs} = \frac{A\hat{I}_L}{2\pi f_r C_p} \tag{26}$$

and, by substituting (10),(16) into (26), provides:

$$V_{Cs} = \frac{AV_i\pi^3 G_t^2}{2\sqrt{G_t^2\pi^4 - 16}} \tag{27}$$

Finally, the stress on the parallel capacitor is as follows:

$$V_{Cp} = \frac{\pi V_i G_t}{2} \tag{28}$$

Thus far, it has been suggested that G_t >≈0.4. However, a more stringent requirement is obtained when considering the reduced accuracy of FMA during operation that provides discontinuous parallel capacitor voltages. For most purposes therefore, a further constraint is required to ensure operation incurs continuous parallel capacitor voltages. From [3], the following condition must be satisfied:

$$\pi^2 N^2 C_p R_L f_s > 1 \qquad \text{where } f_s \text{ is the switching frequency} \tag{29}$$

Substituting (5),(16) into (29), equating ($f_s = f_r$) , and solving for G_t gives:

$$G_t \geq 0.52 > \frac{\sqrt{\pi^2 + 16}}{\pi^2} \tag{30}$$

It can be noted, that, in general, the load resistance and switching frequency are greater than the minimum load and resonant frequency, respectively. From (29), increasing R_L and/or f_s serves to further satisfy the constraint (i.e the minimum G_t reduces). This, therefore, implies that (29) will always ensure continuous operation.

Current-output design examples

The design procedure is therefore summarised by (5),(13),(16),(17) and is based on initial design constraints of i) selecting $G_r = V_{o\_max}/V_{i\_min}$ ii) selecting a transformer gain $1/N$ such that the resonant tank gain G_t, is as desired iii) choosing a desired nominal switching frequency f_r, iii) the minimum load resistance R_{L_min}, iv) the value of A.

To provide a degree of validation of the proposed design process, various designs have been obtained based on the requirements given in Table 1. Table 2(a) shows the percentage difference in gain G_r and resonant frequency of the converters obtained through use of the design methodology, with respect to the desired values specified in Table 1. The data is based on the results of SPICE© simulation studies. Clearly, component requirements generated by the design methodology will not provide preferred values. By judicious choice of preferred values, about those calculated by the design method, the resulting gain errors are shown to be ≤6% (see Table 2(b)), which is typical of the accuracy normally obtained when employing standard industrial grade off-the-shelf components.

TABLE I: DESIGN REQUIREMENTS

Design No.	G_t (V/V)	N(V/V)	f_r (kHz)	R_{lmin} (Ω)	A (F/F)
(1)	0.8	0.1	25	50	2
(2)	1	0.3	125	2	1.5
(3)	1.5	0.05	100	500	5
(4)	2.5	5	50	0.3	0.5
(5)	5	1	75	20	1

TABLE II: SPICE© RESULTS WHEN USING (a) IDEAL AND (b) PREFERRED VALUES

	Components			Simulation		% Error	
No.	L_s	C_p	C_s	G_t (V/V)	f_r(kHz)	G_t	f_r
(1)	6.33u	17.6u	8.8u	0.82	24.8	2.5	-0.4
(2)	293n	12.9u	8.6u	1.02	124.3	2.0	-0.6
(3)	4.08u	3.68u	736n	1.46	100.2	-2.7	0.2
(4)	7.13u	2.09u	4.18u	2.49	49.9	-0.4	-0.2
(5)	8.49u	1.06u	1.06u	4.99	74.9	-0.2	-0.1

(a)

	Components			Simulation		% Error	
No.	L_s	C_p	C_s	G_t (V/V)	f_r(kHz)	G_t	f_r
(1)	6.8u	18u	8.2u	0.83	24.6	3.8	-1.6
(2)	330n	12u	8.2u	0.94	119.4	-6.0	-4.5
(3)	4.7u	3.9u	680n	1.48	96.2	-1.3	-3.8
(4)	6.8u	2.2u	4.7u	2.60	49.3	4.0	-1.4
(5)	8.2u	1.0	1.0u	4.94	78.4	-1.2	4.5

(b)

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There are other ways in which the design equations may be utilised. For instance, it is entirely possible to specify some components in advance and solve the design equations for those remaining. The constraints G_t>0.52 and Q_r>2.5, are still necessary to maintain accuracy, however. The designer may also wish to design for a specific Q_L and/or Q_r. In this case, (6),(22) will provide further constraints on A and G_t such that a design can be obtained.

Part-load performance

In general, for this converter topology, it can be shown [2] that if Q_L is chosen to have a relatively low value, at full load, then operation at part loads restricts a reduction in tank current. Providing (30) is satisfied (to ensure a continuous parallel capacitor voltage and hence model accuracy), then, from (6), choosing the lower permissible value of resonant tank gain will result in a low $Q_{L.}$ By way of example, Fig. 4 shows a 3D plot of Q_L vs. G_t and A.

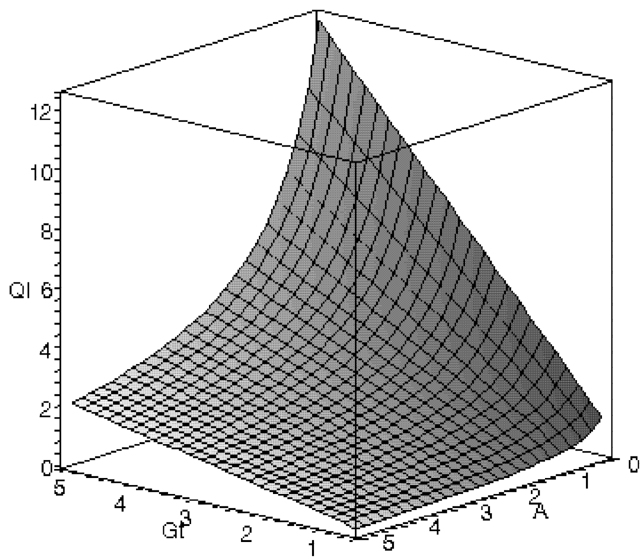

Fig. 4: 3D plot of Q_L vs. G_t and A

Figure 4 is also consistent with (16), where:

$$G_t = \frac{\sqrt{16+\left(C_p \pi^3 N^2 R_L f_r\right)^2}}{\pi^2} \tag{31}$$

From (31), it can be seen that the closer the resonant tank gain is to $4/\pi^2$, and the less sensitive the resonant frequency is to load (i.e A large), the smaller the dependence of the resonant tank gain on load. As the load varies, the phase shift above resonance necessary to maintain the output voltage will remain small, hence, from (1), the dependence of resonant current to load is much greater.

From (13), (5) the following expression can be written:

$$\frac{L_s f_r G_r^{\,2}}{R_L} = \frac{A G_t^{\,2}\pi^4 + G_t^{\,2}\pi^4 - 16}{4\pi^3\sqrt{G_t^{\,2}\pi^4 - 16}} \tag{32}$$

A 3D plot of (32) is given in Fig.5

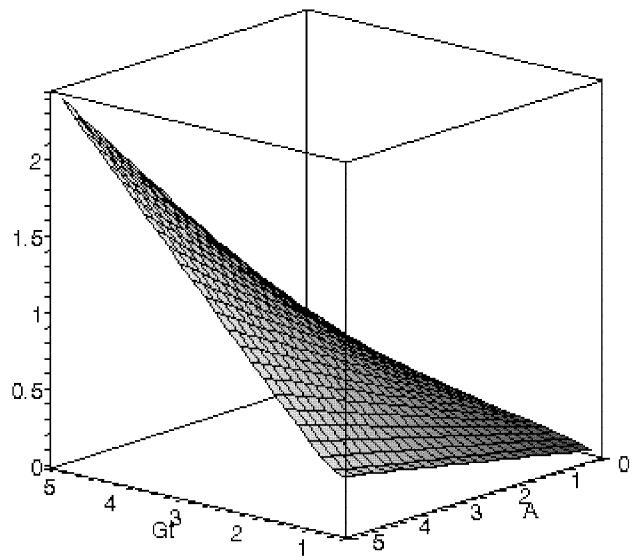

Fig. 5: 3D plot of normalised L_sf_r product vs. G_t and A

From Fig.5 it can be seen that for a given load and overall gain, the lower resonant tank gain provides a lower L_sf_r product, hence, for a given L_s, the resonant frequency is reduced, thereby facilitating a reduction in iron losses in the inductors and transformers.

Further insights from the design methodology

From (16), the required parallel capacitor can be written:

$$C_p = \frac{f_1(G_t)}{N^2 R_L f_r} \tag{33}$$

and, from (13), the required series inductor:

$$L_s = \frac{N^2 R_L}{f_r} f_2(A, G_t) \tag{34}$$

Equating the f_r terms in (33) and (34) leaves:

$$\frac{L_s}{N^4 R_L{}^2 C_p} = \frac{f_2(A, G_t)}{f_1(G_t)} \tag{35}$$

and equating the N^2RL terms in (33) and (34) gives:

$$f_r = \sqrt{\frac{f_1(G_t) f_2(A, G_t)}{L_s C_p}} \tag{36}$$

Equations (35) and (36) can provide further insight into the component and parameter choices that are available. Nominally, converters with the same A and resonant tank gain G_t, possess similar properties, such as normalised frequency response. It is feasible that a designer may find it desirable to maintain the properties of a previous design, within a new given specification. The following section therefore demonstrates how one can simply i) change the minimum load ii) change the overall gain iii) vary the resonant frequency of an existing design to accommodate changes in a given specification.

i) From (35) it can be seen that whilst changing component parameters, maintaining the l.h.s and A ratio would result in a constant resonant tank gain. This implies that the operational load resistance could, for example, be doubled and the overall gain maintained providing N and A remain constant, and the L_s /C_p ratio subsequently quadrupled. However, in general, this will result in a change of resonant frequency. If this is not desired, maintaining the resonant frequency, from (36), requires a constant C_pL_s product. Doubling the resonant inductance and halving the parallel capacitor is therefore necessary.

ii) Alternatively, it may be desirable that the overall gain be doubled. The simplest way in which to achieve this is via the turns ratio of the transformer. Whilst maintaining the tank gain, the transformer gain simply has to double, i.e N must be halved. Since A and the resonant tank gain are to remain constant, further components on the l.h.s must change in order to maintain the balance, these being the resonant tank components. From (35) the L_s /C_p ratio must reduce by a factor of 16. In general, this will result in a change of resonant frequency. If this is not desired, maintaining the resonant frequency, from (36), requires a constant C_pL_s product. From (36) one therefore has to quadruple the parallel capacitance and quarter the resonant inductance.

iii) To vary the resonant frequency, whilst maintaining the transformer gain, resonant tank gain, A and load resistance, one can only vary the resonant components. From (35) and (36) doubling the resonant frequency is achieved by halving both the resonant inductance and parallel capacitance.

Equation (35) is also useful when considering the conversion from ideal to preferred component values, since the gain is usually the determining factor in a converter design. From (35) the preferred component should be chosen such that the L_s/C_p ratio and A ratio are as close to ideal as possible. The cost of any change in component value will be an alteration in resonant frequency. If component values cannot be found that satisfy the required ratios, A should be adjusted.

Rectifier diode compensation

The design process, as it stands, is based on the assumption that the overall converter is extremely efficient. It can, however, be readily modified to accommodate for losses due to the rectifier diodes. The diodes can simply be viewed as a parasitic load resistor. Thus, the overall load resistance will become apparently larger, and the gain reduced due to the resulting potential divider network. The following expressions can be utilised to compensate for the rectifier voltage drop:

$$R_L' = R_L\left(1+\frac{2V_d}{V_o}\right), \qquad G_t' = G_t\left(1+\frac{2V_d}{V_o}\right) \tag{37}$$

where R_L' and G_t' are the required effective load resistance and resonant tank gain to achieve specifications.

Conclusion

A simple design scheme for LCC current-output resonant converters, is proposed, that is suitable for use in a rapid iterative design environment e.g. as part of a GUI, and which aims to reduce resonant tank currents—thereby reducing the electrical stresses on components and improve tank efficiency. A comparison of results from a number of candidate designs has demonstrated accuracy comparable with that expected from the use of commercial off-the-shelf components, and their associated tolerances.

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