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Optimal design of trigeneration systems for buildings considering cooperative game theory for allocating production cost to energy services

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A B S T R A C T

In the design of trigeneration plants for buildings, two fundamental issues must be addressed: the synthesis of the plant configuration (installed technologies and capacity, etc.) and the operational planning. Given the variety of technology options available and great diurnal and annual fluctuations in energy demands, finding the optimal supply system of energy services is a complex task.

Cost allocation in multi-product systems requires special attention because the way in which allocation is made will affect the prices of the final products and, consequently, the consumers' behaviour. When a polygeneration plant is designed to serve different products, it is possible to achieve a lower total cost. However, if potential consumers are free to participate, the system's management should ensure that every participant shares the benefit of joint production. In trigeneration systems this implies that all consumers should achieve, at least, a lower cost for their demanded energy services than operating separately.

The present work proposes a Mixed Integer Linear Programming model to determine the optimal configuration of trigeneration systems that must cover the energy demands of electricity, heating and cooling of a residential complex located in Zaragoza, Spain. The model considers the possibility of using a set of proposed alternative technologies within a superstructure and considers the optimal operation throughout a typical meteorological year. The objective function to be minimized is the total annual cost.

The results indicate that compared to consumers standing alone, the optimal trigeneration system can achieve 10.6% cost saving. Ten different cost assessment methods to the three final energy products of the analyzed trigeneration system are rigorously compared. Cooperative game theory shows that all consumers benefit. Using the Shapley values as the distribution criterion, the savings for electricity, heating and cooling consumers are 4.8%, 20.9% and 11.1%, respectively.

1. Introduction

The energy consumption in residential and commercial buildings in developed countries continues to grow. Thus, central plants providing energy services (utilities) for new and existing urban complexes and large buildings present a significant opportunity for polygeneration systems. The advantages include greater overall energy efficiency, improved quality of energy supply and, above all, lower cost for the building utility services. From a system performance perspective, there are three essential factors which favour installing integrated energy supply systems: (i) the correct combination of certain types of technologies reduces the primary energy input and the fuel bill between 30 and 60%; (ii) natural gas utilization as fuel allows the introduction of new technologies such as gas engines, low temperature and condensing boilers, gas engine driven chillers and gas fired absorption chillers; and (iii) the installation of engine generators and energy efficient gas-fired refrigeration technologies reduce electrical demand during peak cooling times. Next, from a market economic viewpoint, electrical deregulation has in general contributed to an increase of electricity prices during the periods of higher electrical demand during seasons of the year requiring cooling.

The commercial availability of a large variety of energy conversion technologies and the need to integrate them in the most energy efficient way makes the process of selecting the best combination of such technologies a difficult and tedious task [1]. In addition, for a given site, there are often several technically feasible combinations. In the industrial sector, this problem has been dealt successfully [2] using Mixed Integer Linear Programming (MILP) techniques; with the advantage that

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most industrial facilities show less variation in energy demand than their residential or commercial counterparts. The mathematical foundation of applied MILP can be found in Nemhauser and Wolsey [3]. Iyer and Grossmann [4], Yokoyama et al. [5], Lozano et al. [6–8], and Pina et al. [9] have applied such techniques to polygeneration systems design for (a) the selection of the number and size of equipment to be installed and (b) to find the optimal way of operating the equipment.

The allocation of the cost in joint production process resulting in the output of several products is a complex task because of nonseparable (fixed and/or variable) costs. The need for allocation schemes arises in many applications and the potential benefits from a good cost allocation between products that are jointly produced have been the topic of a great variety of research works [10]. Cost allocation in multi-product systems requires special attention because the way in which allocation is made will affect the prices of the final products and, consequently, the consumers' behaviour. So, when a polygeneration plant is designed to serve different products, it is possible to achieve a lower total cost; however, if potential consumers are free to participate, the system's management should ensure that every participant shares the benefit of joint production. In trigeneration systems this implies that all consumers should achieve, at least, a lower cost for their demanded energy services than operating separately. Thermoeconomics has been used to explain the cost formation process of internal and final products in complex energy systems [11–13].

Cooperative game theory is concerned primarily with players who coordinate their actions and pool their winnings [14–16]. Fiestras et al. [16] provide general view of applications of cooperative games to cost allocation problems in three specific areas: transportation, natural resources and power industry. Young [15] presents four interesting examples of the application of game theory to the sharing of common costs in engineering projects. The first of these has to do with the creation in the 1930s of the Tennessee Valley Authority where the water reservoirs to be built would serve various purposes: navigation, flood control and electricity production. Lozano et al. [17] applies a cooperative game theory approach to allocating benefits of horizontal cooperation identifying and exploiting win-win situations among companies at the same level of the supply chain. Numerous applications can be found in electrical power production and distribution systems. For example, Songhuai et al. [18] propose how to apply cooperative game theory to distribute power losses in an electricity supply network. Erli et al. [19] address the problem of how to expand transmission capacity and charge users with costs in the face of a congested electricity grid. Sauhats et al. [20] show how the collaboration of two small hydroelectric plants with a manager that facilitates the sale of their electricity generated to the grid, at market prices instead of a fixed rate, changes the optimal load curve of the plants in their daily operation and benefits the three participating agents. Neimane et al. [21] address the problem of cost sharing between the two products (heat and work) of a cogeneration plant. Wu et al. [22] propose a MILP model for the synthesis of a cogeneration system capable of meeting the demand of three buildings and analyze different cost sharing criteria based on cooperative game theory. Another application of interest is to establish acceptable rules for an operator to be in charge of commanding a community of prosumers that have their energy production systems (photovoltaic panels, micro-cogeneration plants, etc.) seeking to obtain a benefit for all of them thanks to a favourable exchange between prosumers and the electricity grid [23,24].

This paper proposes an optimization model for the synthesis of energy supply systems in buildings, based on the comparison of the economic annual balances for all feasible plant configurations contained in a superstructure. The model was applied to the design of a trigeneration system for a residential complex in Zaragoza, Spain. The effects of financial market conditions in the optimal structure of the system are analyzed. The results indicate that compared to consumers standing alone, the optimal trigeneration system can achieve 10.6% cost saving. Ten different cost assessment methods to the three final energy products of the analyzed trigeneration system are rigorously compared. Cooperative game theory shows that all consumers benefit. Using the Shapley values as the distribution criterion, the savings for electricity, heating and cooling consumers are respectively 4.8%, 20.9% and 11.1%, respectively.

2. Trigeneration system

The basis for this research is the superstructure shown in Fig. 1 and the fundamental mathematical model implicit in Table 1. More information about the model presented in this section can be consulted in Lozano and Ramos [8].

The technology behind trigeneration is fundamentally based on the coupling of a cogeneration module with an absorption chiller. The cogeneration module includes a thermal motor (a gas turbine or reciprocating engine, for example) that converts the fuel's energy into mechanical energy, an alternator that converts the mechanical energy into electrical energy, and a set of heat exchangers to recover useful heat. The absorption chiller can produce cooling by means of using recovered heat. There are different types of trigeneration systems, which are distinguished by the incorporation of additional equipment. Usually, the trigeneration system is complemented by hot water boilers and vapor compression chillers. Both technologies are used to guarantee supply and to avoid oversizing the cogeneration module and the associated absorption chiller. The key in operating trigeneration systems is to satisfy the demand with a minimal economic cost. The idea is that the cogeneration module, jointly with the absorption chiller, satisfies the average thermal demand for the different services (heating and cooling), while the conventional units (boilers and compression chiller) are utilized in an auxiliary way to make up for the peak demands. Therefore, supply is guaranteed, and the installation is reliable, since the existence of conventional equipment assures the satisfaction of the thermal demand. The residual heat flows that are not used must be evacuated to the environment through the use of cooling towers or other devices, which are therefore important elements of trigeneration plants.

To solve the fundamental issue of synthesizing the plant's configuration, a reducible structure (known as superstructure) was created to embed all feasible process options and interconnections that are candidates for the optimal design structure [4–9]. Redundant features were built into the superstructure to ensure that all features that could be part of an optimal solution were included. This approach has several advantages: (a) many different design options can be considered at the same time, (b) the complex multiple trade-offs usually encountered in energy supply systems design can be handled, and (c) the entire design procedure can be automated and is capable of producing designs quickly and efficiently.

The advantage is that with a single calculation model and limiting with binary variables the technologies present, the demands to be met or the optimization criteria to be used, the desired results can be obtained. This allows not only to reach the global optimum but also to compare the results corresponding to various system configurations as we will see in this paper.

Fig. 1 depicts the superstructure of the energy supply system considered in this study, showing the proposed technologies that can be selected, the available energy utilities that can be present, and the interactions between technologies and utilities. The utilities are natural gas ***NG***, electrical energy ***EE***, high-temperature water ***HTW*** (90 °C), domestic hot water ***DHW*** (60 °C) for sanitary and heating services, cooling tower water ***CTW*** ($T_0 + 5$ °C), atmospheric air-cooling ***AA*** (T_0), and chilled water ***CW*** (5 °C) for air conditioning. The candidate technologies in the superstructure can satisfy the energy demands (*D*) of the consumer centre by consuming (*C*) natural gas and electricity purchased from the market. We also consider other energy utilities that may be needed to produce the previously defined demanded utilities, such as the ***HTW***. In addition, the ***CTW*** and ***AA*** utilities are used to dissipate (*L*) heat to the environment. The possibility that a fraction of cogenerated

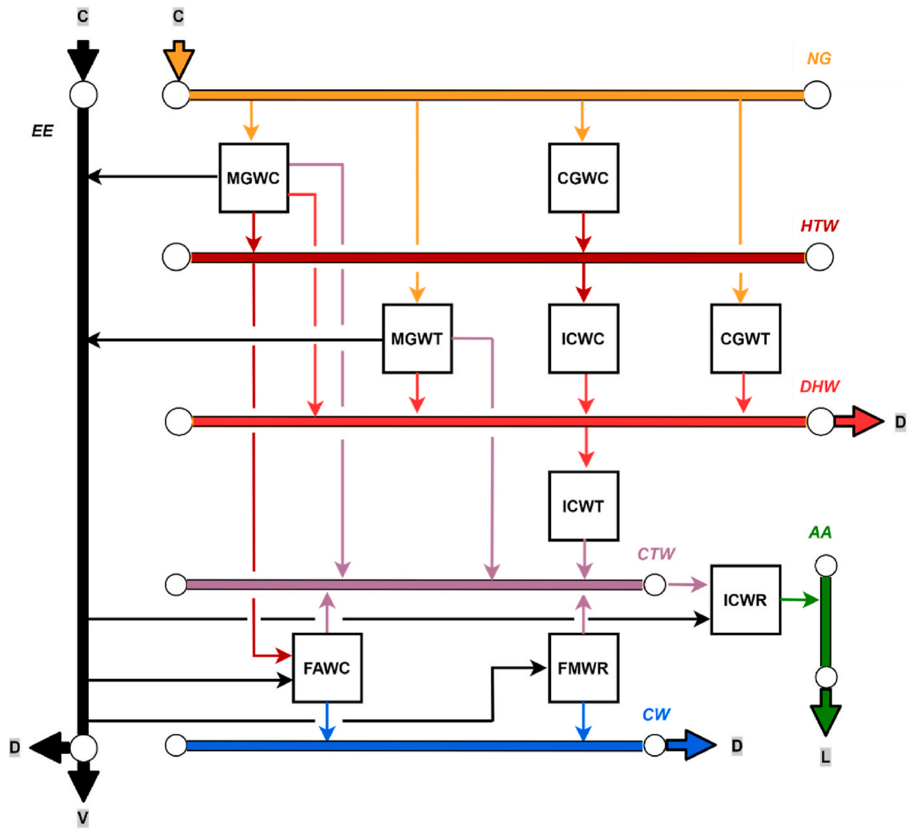

Fig. 1. Superstructure of the energy supply system.

Table 1
Technical coefficients of production (*TCP*) and technology data matrix (TDM).

Technology	Utility							Cost data		
	NG	HTW	DHW	CTW	AA	CW	EE	CY 10^3 $	cx $/kW	co $/kWh
MGWC	−2.6	+1.1	+0.1	+0.1	–	–	+***1***	600	1800	0
MGWT	−2.6	–	+1.2	+0.1	–	–	+***1***	600	1800	0
CGWC	−1.2	+***1***	–	–	–	–	–	90	120	0
CGWT	−1.1	–	+***1***	–	–	–	–	60	90	0
FMWR	–	–	–	+1.17	–	+***1***	−0.17	90	240	0
FAWC	–	−1.6	–	+2.6	–	+***1***	−0.01	120	360	0
ICWC	–	−1	+***1***	–	–	–	–	15	15	0
ICWT	–	–	−1	+***1***	–	–	–	15	15	0
ICWR	–	–	–	−1	+***1***	–	−0.02	30	60	0.003

Nomenclature: MGWC: Gas Engine + Heat recovery for HTW and DHW
MGWT: Gas Engine + Heat recovery for DHW
CGWC: HTW boiler
CGWT: DHW boiler
FMWR: Electrically driven chiller with water cooled condenser
FAWC: Hot temperature water heated single-effect absorption chiller
ICWC: Heat exchanger, HTW → DHW
ICWT: Heat exchanger, DHW → CTW
ICWR: Cooling tower to reject heat

electricity could be sold (*V*) to the grid exists.

All technology and equipment considered in the optimization were commercially available. Although only one unit was drawn for each type of technology in Fig. 1, there could be several pieces. The mathematical models that represent the operation of the equipment assume that the energy utilities consumption and production are directly proportional to one another. In this case, the proportionality constant is known as the technical coefficient of production (*TCP*), whose value was calculated from information available in the literature, equipment catalogues, and consultations with manufacturers. The detailed assessment of the *TCP* values for each technology can be found in Ramos [24].

The algebraic model in Table 1 is expressed in terms of the TCPs, which are defined as follows. The rows indicate potential energy conversion technologies and the columns primary and demanded energy utilities. The highlighted *TCP* (boldface **1**) shows the flow that defines the capacity of the technology. A positive technical coefficient indicates a produced utility and a negative one a consumed utility. For instance, using a **MGWC** technology (see technology definitions at the bottom of

Table 1) we have electrical energy (***EE***) as its main output, since its *TCP* is **+1**. Next, to produce x MW of electricity (***EE***), MGWC needs to consume 2.6x MW of natural gas (***NG***), it recovers 1.1x MW of high temperature hot water (***HTW***) and 0.1x MW of domestic hot water (***DHW***), and rejects 0.1x MW of heat to the environment through the cooling tower water (***CTW***). It was considered that the production coefficients were constant and independent of the operation load, so that the production $P \leq \Pi$ installed (see Section 2.2) of the equipment at any given moment.

2.1. Demand and economic data

The proposed optimization model was applied to a trigeneration system that supplies electricity, heating, and cooling to a residential complex building in Zaragoza (Spain). The annual demand data are characterized by using the energy consumption of a typical day for each of the 12 months of the year. Each day is divided into 12 2-h periods. This results in a total of 144 periods, each with a set of different heating, cooling and power demands. More information about the energy demands data can be consulted in Ramos [24]. Table 2 shows the demands for the residential building complex for three significant months: December (high heating consumption), April (no heating and no cooling) and August (high cooling consumption).

The investment cost for technologies is given in Table 1 expressed as a linear function of capacity installed. Non-energy operating costs are only considered for cooling towers. The price of natural gas is $ccNG =$ 0.045 $/kWh. The prices for purchased and sold electricity during flat-demand hours are $ccEE$ = 0.150 $/kWh and $cvEE$ = 0.135 $/kWh, respectively. Such prices are multiplied by 1.50 during peak-demand hours (Oct–Mar: 18:00–22:00, Apr–Sep: 10:00–14:00); and by 0.75 during low-demand hours (Jan–Dec: 00:00–08:00). Note that flat-demand hours are all those which are not peak or low demand hours. These data are only approximate.

2.2. MILP model for technology selection

The MILP model minimizes the total annual cost *CA* of the system (expressed in $/y):

$$CA = fa \cdot \sum_i CI_i + \sum_k NH_k \cdot CH_k \tag{1}$$

Where

- fa capital amortization factor [y^{-1}]
- CI_i cost of investment or first cost of technology i [$]
- NH_k number of hours in period k [h/y]
- CH_k plant operation cost in each period k [$/h]

The objective function is subject to the following restrictions:

The power Π (kW) installed for technology i:

$$P_i^{min} \cdot y_i \leq \Pi_i \leq P_i^{max} \cdot y_i \tag{2}$$

Where

- P_i^{min} and P_i^{max} minimum and maximum installable capacities, respectively [kW]
- $y_i \in \{0, 1\}$ binary variable related to the existence of technology i

The investment cost for technology i:

$$CI_i = CY_i \cdot y_i + cx_i \cdot \Pi_i \tag{3}$$

Where

- CY_i fixed cost related to the investment of technology i [$]
- cx_i variable cost related to the investment of technology i [$/kW]

For each period k, several restrictions apply as explained below.
Production limit for technology i:

$$P_{i,k} \leq \Pi_i \tag{4}$$

Where $P_{i,k}$ is the production in kW of the technology i in the period k.
Operating cost for technology i:

$$CO_{i,k} = co_i \cdot P_{i,k} \tag{5}$$

Where co_i is the unit operation cost of the technology i expressed in $/kWh.
Flow-product relationship for each utility j:

$$F_{i,j,k} = TCP_{i,j} \cdot P_{i,k} \tag{6}$$

Where $TCP_{i,j}$ is the technical coefficient of production given in Table 1.
Energy balance for each utility j:

$$C_{j,k} + \sum_i F_{i,j,k} - D_{j,k} - L_{j,k} - V_{j,k} = 0 \tag{7}$$

Where C represents purchases (***NG***, ***EE***), D demand (***DHW***, ***CW***, ***EE***), L losses to the surroundings (***AA***) and V sales (***EE***).
Finally, the plant hourly operation cost is defined by:

$$CH_k = \sum_j cc_j \cdot C_{j,k} - cl_j \cdot L_{j,k} - cv_j \cdot V_{j,k} + \sum_j CO_{i,k} \tag{8}$$

Where cc_j, cl_j and cv_j are the unit costs in $/kWh attributed to the purchase, loss and sale of the utility j, respectively.

There is an additional condition when gas engines are installed: the annual equivalent electrical efficiency of cogeneration modules must be larger than 55%.

To solve the MILP model, we have used the mathematical

Table 2
Energy demands for three significant months of the year. Adapted from Ref. [24].

Period	December			April			August		
	Heating kW	Cooling kW	Electricity kW	Heating kW	Cooling kW	Electricity kW	Heating kW	Cooling kW	Electricity kW
0–2	60	0	120	20	0	140	20	100	100
2–4	640	0	100	0	0	100	0	80	80
4–6	1220	0	100	100	0	100	80	60	60
6–8	1380	0	120	220	0	120	180	100	100
8–10	2440	0	180	180	0	180	140	140	140
10–12	1320	0	260	180	0	260	140	180	180
12–14	800	0	240	300	0	260	240	180	180
14–16	700	0	180	260	0	200	200	140	140
16–18	620	0	220	100	0	220	80	160	160
18–20	1040	0	240	160	0	260	140	180	180
20–22	1700	0	240	320	0	260	260	180	180
22–24	140	0	180	6	0	180	60	140	140

programming software LINGO [25].

2.3. Optimal results

One of the most remarkable findings of this research is the dependency between the optimal plant configuration and the capital recovery factor *fa*. Logically, a lower cost of capital favors more efficient technologies which tend to have higher installed costs. We obtained the optimal solutions shown in Table 3 with the MILP model described above. *fu* is the utilization factor: quotient of dividing the energy that a technology has produced or consumed in the year by what it would have produced or consumed if it had operated at nominal power all the year.

With $\boldsymbol{fa} = \mathbf{0.20\ y^{-1}}$ the optimal solution corresponds to install gas fired hot water boilers ($\Pi_{CGWT} = 2780$ kW) and mechanical chillers ($\Pi_{FMWR} = 2320$ kW).
With $\boldsymbol{fa} = \mathbf{0.15\ y^{-1}}$ we must add gas fired engines with DHW heat recovery ($\Pi_{MGWT} = 550$ kW, $\Pi_{CGWT} = 2120$ kW, and $\Pi_{FMWR} = 2320$ kW).
With $\boldsymbol{fa} = \mathbf{0.10\ y^{-1}}$ gas engines with cogenerated hot water HTW are installed, and hot-water heated absorption chillers become feasible ($\Pi_{MGWC} = 1283$ kW, $\Pi_{CGWT} = 1240$ kW, $\Pi_{FMWR} = 1445$ kW, and $\Pi_{FAWC} = 875$ kW.

In what follows we will consider the case with an amortization factor $\boldsymbol{fa} = \mathbf{0.10\ y^{-1}}$. Fig. 2 shows the installed power of the technologies present in the optimal solution as well as the annual magnitude of the different energy flows. A greater detail of the hour-by-hour operation and the optimal operation strategies throughout the year can be found in Ramos [24].

Table 4 shows the optimal solution corresponding to the systems that would meet separately the demands for heat {H}, cold {C} and electricity {E}; as well as the combined demands of two of the services: heat and cold {H, C}, heat and electricity {H, E}, and cold and electricity {C, E}; and finally, all the demands together {H, C, E}. As can be seen, the collaboration between C and E does not bring benefits compared to separate production, the collaboration between H and E leads to savings of 24.14·10^3 \$/y, and the collaboration between H and C entails a saving of 34.83·10^3 \$/y. When all the services are produced with the same installation, the maximum annual saving of 58.97·10^3 \$/y is achieved. As can be seen, the facilities for {H, C} and {H, C, E} have the same equipment. The difference is that in {H, C} it is not allowed to sell electricity, and in {H, C, E} it is allowed.

Conclusion: designing an installation capable of producing the three energy services demanded and operating it allowing the sale of electricity to the grid entails maximum economic savings for the applicants.

Table 3
Optimal energy system configuration based on financial conditions.

	$fa = 0.20$ y^{-1}		$fa = 0.15$ y^{-1}		$fa = 0.10$ y^{-1}	
	Π [kW]	fu [%]	Π [kW]	fu [%]	Π [kW]	fu [%]
MGWC	–	–	–	–	1283	61.6
MGWT	–	–	550	74.9	–	–
CGWC	–	–	–	–	–	–
CGWT	2780	15.8	2120	7.0	1240	2.4
FMWR	2320	10.6	2320	10.6	1445	7.3
FAWC	–	–	–	–	875	16.2
ICWC	–	–	–	–	1412	45.5
ICWT	–	–	520	39.4	1380	23.1
ICWR	2714	10.6	2769	19.3	4094	21.6
Capital cost [10^3 \$/y]	229.97		406.07		430.09	
Operation cost [10^3 \$/y]	521.79		272.16		75.85	
CT Total cost [10^3 \$/y]	751.76		678.23		505.96	

But how should they distribute said savings among them?

3. Cooperative game theory

3.1. Cost accounting

If an energy supply system covers three services, they form a cooperative game $G = (\boldsymbol{J}, \boldsymbol{c})$ where the participants set is recorded as $\boldsymbol{J} = \{$H, C, E$\}$. Each alliance between participants is represented as $\boldsymbol{S}$, where $\boldsymbol{S} \subseteq \boldsymbol{J}$ and the characteristic function $c(\boldsymbol{S})$ represents the annual cost under alliance $\boldsymbol{S}$. The vector $\boldsymbol{x} = (x_H, x_C, x_E)$ denotes the game results: the portion of the total annual cost assigned to each participant.

The basic assumption of the cooperative game is that participants are free to collaborate with each other. Thus, three participants can form 7 alliances $\boldsymbol{S}$: {H}, {C}, {E}, {H, C}, {H, E}, {C, E}, and {H, C, E}.

Given an allocation scheme $\boldsymbol{x}$, if the sum of the costs assigned to some participants in an alliance $\boldsymbol{S}$ is greater than the cost forming another alliance, these participants will break the current alliance and proceed to be part of another.

3.2. Classic methods

Some classic methods used for allocating total cost $c(\boldsymbol{J}) = \boldsymbol{CT}\{H, C, E\}$ are the following:

M1: Equal repartition of the total gain:

$$x_i = c(i) - \frac{1}{3}\left[\sum_{j=1}^{3} c(j) - c(\boldsymbol{J})\right] \tag{9}$$

M2: Proportional repartition of the total gain:

$$x_i = \frac{c(i)}{\sum_{j=1}^{3} c(j)} c(\boldsymbol{J}) \tag{10}$$

The method ***M2***, also known as the Moriarity method [26], has been applied by the authors as a rational proposal for the allocation of joint production costs in Thermoeconomics [13].

M3: Equal repartition of the non-marginal cost:

$$x_i = cm(i) + \frac{1}{3}\left[c(\boldsymbol{J}) - \sum_{j=1}^{3} cm(j)\right] \tag{11}$$

The marginal cost, $cm(i) = c(\boldsymbol{J}) - c(\boldsymbol{J}\backslash\{i\})$ is the additional cost when participant *i* joins to form the grand coalition $\boldsymbol{J}$. In Table 4 the marginal costs corresponding to the energy services have been noted.

M4: Proportional repartition to marginal cost:

$$x_i = \frac{cm(i)}{\sum_{j=1}^{3} cm(j)} c(\boldsymbol{J}) \tag{12}$$

M5: Proportional repartition of the non-marginal costs:

$$x_i = cm(i) + \frac{c(i) - cm(i)}{\sum_{j=1}^{3} c(j) - cm(j)}\left[c(\boldsymbol{J}) - \sum_{j=1}^{3} cm(j)\right] \tag{13}$$

The five classic methods recommend different allocations as can be seen in Table 5.

Method ***M2*** does not meet property ***b*** as electricity consumers pay a price 229.28·10^3 \$/y less than its marginal cost 231.86·10^3 \$/y (see Table 4). Therefore, they will be subsidized by the heat and cold consumers, who will not accept it. Method ***M4*** does not meet property ***a*** because electricity consumers pay a higher price 262.45·10^3 \$/y than the market price 256.00·10^3 \$/y (see Table 4) and therefore subsidize

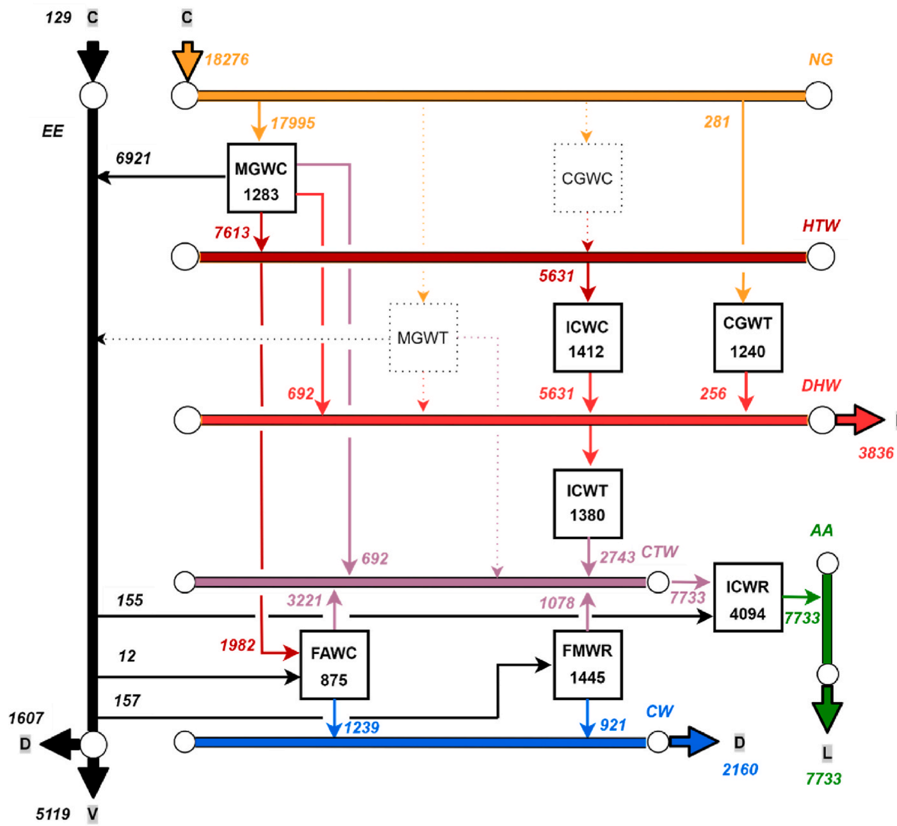

Fig. 2. Superstructure of the optimal energy supply system ($\boldsymbol{fa} = \mathbf{0.10\ y^{-1}}$) with annual energy flows in MWh.

Table 4
Optimal solutions for the different combinations of energy services served ($\mathbf{fa} = \mathbf{0.10\ y^{-1}}$).

	{H}	{C}	{E}	{H, C}	{H, E}	{C, E}	{H, C, E}
MGWC: Π [kW]	–	–	–	1283	–	–	1283
MGWT: Π [kW]	983	–	–	–	983	–	–
CGWC: Π [kW]	–	–	–	–	–	–	–
CGWT: Π [kW]	1600	–	–	1240	1600	–	1240
FMWR: Π [kW]	–	2320	–	1445	–	2320	1445
FAWC: Π [kW]	–	–	–	875	–	–	875
ICWC: Π [kW]	–	–	–	1412	–	–	1412
ICWT: Π [kW]	1040	–	–	1380	1040	–	1380
ICWR: Π [kW]	1138	2714	–	4094	1138	2714	4094
Capital cost [10^3 \$/y]	270.29	83.97	–	430.09	270.29	83.97	430.09
Operation cost [10^3 \$/y]	−121.22	75.89	256.00	−155.99	110.64	331.90	75.87
CT: Total cost [10^3 \$/y]	149.07	159.86	256.00	274.10	380.93	415.86	505.96
***cm*: Marginal cost [10^3 \$/y]**	90.10	125.03	231.86	–	–	–	–
***CA*: Cost avoidance [10^3 \$/y]**	–	–	–	34.83	24.14	0.00	58.97

heat and cold consumers.

3.3. Cooperative game theory methods

The *core* of the game is the set of imputations such that $\sum_{i \in S} x_i \le c(S), \forall S \subseteq J$. Conditions ***a*** and ***b*** also define the core of the game in the 3-player case. The core of a game may be void. In that case, there is at least a subset of participants ***S*** who have interest to separate from the rest of ***J***.

Fortunately, in most of the applications in polygeneration systems, economies of scale and/or scope are so large that a non-empty core exists. In our case the application of the conditions ***a*** and ***b*** allow us to affirm that the cost distributions of the optimal energy supply system belonging to the core of the game are given by: 17.8% $< x_H <$ 29.5%, 24.7% $< x_C <$ 31.6%, and 45.8% $< x_E <$ 50.6%. Fig. 3 shows the core of the game in a triangular diagram. It can be verified again in Table 5 that the distributions of the classic methods ***M1***, ***M3***, and ***M5***, belong to the core of the game.

Although the stability of cost allocation is a necessary condition for the existence of a cooperative game, it is also necessary to compare the *fairness* of the different distributions belonging to the core of the game and find the one that convinces the players not only of the interest of participating in it but also in the justice of the distribution [27]. Moreover, since the core does not always exist, and there are infinite solutions even if it exists, it is necessary to find a method to select the distribution recognized as fair by all participants.

The ***Shapley*** value method [28] of the cooperative game theory may solve these problems on the basis of each participant's marginal contribution to the different subsets ***S*** of ***J*** in which it participates. The

Table 5
Distribution of costs according to the different methods ($\boldsymbol{fa} = \mathbf{0.10\ y^{-1}}$).

Method	Heating (H)			Cooling (C)			Electricity (E)		
	x 10^3 $/y	%	cu $/kWh	x 10^3 $/y	%	cu $/kWh	x 10^3 $/y	%	cu $/kWh
M1	129.42	25.6	0.0337	140.20	27.7	0.0649	236.34	46.7	0.1470
M2	133.51	26.4	0.0348	143.17	28.3	0.0663	***229.28***	***45.3***	***0.1426***
M3	109.75	21.7	0.0286	144.69	28.6	0.0670	251.52	49.7	0.1565
M4	101.99	20.1	0.0266	141.52	28.0	0.0655	***262.45***	***51.9***	***0.1633***
M5	119.58	23.6	0.0312	142.45	28.2	0.0660	243.93	48.2	0.1518
Shapley	119.58	23.6	0.0312	142.45	28.2	0.0660	243.93	48.2	0.1518
Nucleolus	114.24	22.6	0.0298	147.79	29.2	0.0684	243.34	48.2	0.1518
Proportional nucleolus	119.24	23.6	0.0311	142.38	28.1	0.0659	244.34	48.3	0.1520

To check the *stability* of these or other allocations we must verify that they meet the following two conditions.
a. Individual rationality: $x_i \le c(i)$. For a participant to be willing to collaborate, it is necessary to receive a cost reduction.
b. Collective rationality: $x_i \ge cm(i)$. No participant will be allowed to collaborate with a cost lower than its marginal cost.

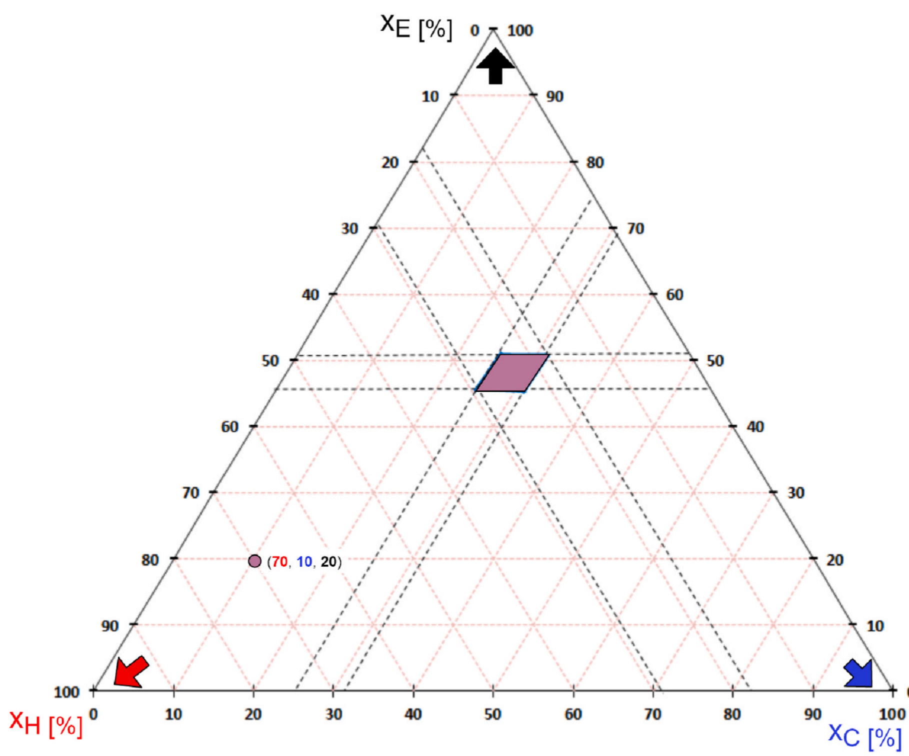

Fig. 3. Triangular diagram showing possible distributions of total cost **CT** and the **core**.

only cost allocation that satisfies the axioms of symmetry, inessential players, and additivity, formulated by Shapley is

$$x_i = \frac{1}{n!}\sum_{S \subseteq J}(s-1)!\cdot(n-s)![c(\boldsymbol{S}\backslash\{i\}) + c(i) - c(\boldsymbol{S})] \quad (14)$$

where *n* is the total number of participants in the game and *s* the number of participants in the coalition ***S*** in which participant *i* is present. An interpretation of the Shapley value is the mathematical expectation of the admission cost when all orders of formation of the grand coalition are equiprobable.

The ***nucleolus*** measures the "happiness degree" of each coalition ***S*** with a proposed cost by the difference between the cost it can secure and the proposed cost. Define the excess $e(\boldsymbol{x},\boldsymbol{S}) = c(\boldsymbol{S}) - \sum_{i \in S} x_i$. If the excess is negative for one or more coalitions, the proposed allocation is outside of the core and unacceptable. If it is positive for all coalitions, the allocation is acceptable but all of them will want to achieve the highest degree of happiness. The nucleolus is the allocation that maximizes lexicographically the minimal excess [29]. To calculate the nucleolus of the game, the following linear program is solved [15]:

$$\text{Max } d \quad subject\ to \quad e(\boldsymbol{x},\boldsymbol{S}) \ge d, \forall \boldsymbol{S} \subseteq \boldsymbol{J} \quad and \quad \sum_{i=1}^{n} x_i = c(\boldsymbol{J}) \quad (15)$$

The ***proportional nucleolus*** [30] is obtained when the measures the "happiness degree" of each coalition ***S*** is defined by the relative excess $e_p(\boldsymbol{x},\boldsymbol{S}) = 1 - \sum_{i \in S} x_i / c(\boldsymbol{S})$. The linear program to be solved is

$$\text{Max } d_p \quad subject\ to \quad e_p(\boldsymbol{x},\boldsymbol{S}) \ge d_p, \forall \boldsymbol{S} \subseteq \boldsymbol{J} \quad and \quad \sum_{i=1}^{n} x_i = c(\boldsymbol{J}) \quad (16)$$

The three previous methods, which are among the most recommended by specialists in cooperative game theory, provide the distributions of total cost shown in Table 5. It can be verified that the three distributions are within the core of the game.

4. Discussion

We agree that there is no single solution to the problem of cost allocation. In fact, according to the principles of management cost accounting, the method chosen for cost allocation should consider above all the objective pursued by the allocation. Our stated objective is that once it has been demonstrated that consumers of energy services (*heating*, *cooling*, and *electricity*) are interested in the joint production in an energy integrated system, it must be verified that the established cost allocation meets the conditions of individual rationality and collective rationality, i.e. that consumers of the three services find it economically profitable to join forces with the other consumers, thus obtaining benefits from the coalition for all of them.

We have compared the results of five classical methods proposed by cost accounting theory and three methods based on the theory of cooperative games that directly seek the satisfaction of the stated objective (that cost sharing belongs to the core of the game, see Fig. 3). We have been able to verify (Table 5) that three of the classical methods (*M1*, *M3*, and *M5*) and the three methods based on cooperative game theory (Shapley, nucleolus, and *proportional nucleolus*) result in satisfactory cost shares. The advantage of the latter is that their application requires the confirmation of a set of conditions (*stability*, belonging to the *core of the game*, *fairness*). Lemaire [14] and Young [15] analyze in detail some properties (collective rationality, monotonicity in costs, additivity, consistency or stability, and staying in the core) presenting theoretical arguments to elucidate the merits of these and other methods of the cooperative game theory. An advantage of the Shapley method is that it always provides a cost allocation even when the core is empty. On the contrary, it has the disadvantage that the proposed cost allocation can be outside the core when it is not empty. In particular, Lemaire [14] ends up recommending the proportional nucleolus method. Young [15] in his study of the methods and principles concludes: "In sum, there is no all-embracing solution to the cost allocation problem. Which method suits best depends on the context, the computational resources, and the amount of cost and benefit information available".

In conclusion, it is important to emphasize that, more than a necessity, the application of cooperative game theory to the cost allocation of multi-product energy systems is about proposing alternative approaches to achieving equitable and cost-and-benefit apportionment of the joint production costs. This is done with the intent of promoting widespread deployment of polygeneration systems in buildings applications, which can only be achieved by getting the buy-in from end-consumers and by providing companies with better investment opportunities. On the one hand, end-consumers are the ones who will ultimately select which suppliers will provide them with the energy services they require, so they must be offered cheaper energy services prices relative to other alternatives available in the market. On the other hand, energy services companies, which will install and operate the plants, will aim at increasing their market share while maintaining cost competitiveness.

Previous cost allocation results could be compared with those suggested by the methods based on energy and exergy analysis.

The case of energy analysis leads to establishing that the total cost should be imputed to the energy services in proportion to the energy flows produced by the trigeneration system; that is, the unit energy cost should be equal for heating, cooling, and electricity, as shown in Table 6.

As can be seen, a cost sharing for the optimal trigeneration system that intends to apply the same unit cost per unit of energy to the three energy services would be drastically rejected by the heat consumers because if they were to collaborate, they would end up slightly subsidizing the cold consumers and exaggeratedly subsidizing the electricity consumers.

It should be taken into consideration that the exergy analysis requires the application of an ambient temperature for the calculation of the exergy of the flows and a specific temperature for the heat and cold flows produced. The relationship between exergy B and energy E is given by the Carnot factor:

Table 6
Annual costs of final energy products assigned in energy basis and average unit vs reference costs.

Energy service	Energy (MWh/y)	Annual cost ($/y)	Unit cost ($/MWh)	Reference cost[a] ($/MWh)
Heating	3836	(3836/7603) · 505960 = 255,280	66.5	38.9
Cooling	2160	(2160/7603) · 505960 = 143,740	66.5	74.0
Electricity	1607	(1607/7603) · 505960 = 106,940	66.5	159.3
TOTAL	7603	505,960		

[a] Reference cost: refers to the optimal separate production of the energy service.

$$\frac{B}{E} = 1 - \frac{T_0}{T_b} \tag{17}$$

where T_0 and T_b are the ambient temperature and the average thermodynamic temperature of the energy flow, respectively, both expressed in Kelvin.

The heat flow produced consists of water that is supplied at $T_2 = 70\ ^\circ\mathrm{C}$ and returns at $T_1 = 50\ ^\circ\mathrm{C}$. Therefore,

$$T_b(Q) = \frac{T_2(Q) - T_1(Q)}{ln\frac{T_2(Q)}{T_1(Q)}} = \frac{20}{ln\frac{273.15+70}{273.15+50}} = 333.05\ K \tag{18}$$

We considered as ambient temperature $T_0 = 9.3\ ^\circ\mathrm{C}$, the average for Zaragoza of the daily average temperatures during the heating season (from November to April), resulting in

$$\frac{B(Q)}{E(Q)} = 1 - \frac{T_0(Q)}{T_b(Q)} = 1 - \frac{273.15 + 9.3}{333.05} = 0.1519 \tag{19}$$

The cold flow produced consists of water which is supplied at $T_2 = 7\ ^\circ\mathrm{C}$ and returns at $T_1 = 12\ ^\circ\mathrm{C}$. Therefore,

$$T_b(R) = \frac{T_2(R) - T_1(R)}{ln\frac{T_2(R)}{T_1(R)}} = \frac{-5}{ln\frac{273.15+7}{273.15+12}} = 282.64\ K \tag{20}$$

We considered as ambient temperature $T_0 = 29.2\ ^\circ\mathrm{C}$, the average for Zaragoza of the maximum daily temperatures during the heating season (from June to September), resulting in

$$\frac{B(R)}{E(R)} = 1 - \frac{T_0}{T_b} = 1 - \frac{273.15 + 29.2}{282.64} = -0.0697 \tag{21}$$

The case of the exergy analysis leads to establishing that the total cost should be charged to the energy services in proportion to the exergy flows produced by the trigeneration system, i.e., the unit exergy cost should be equal for heating, cooling, and electricity. The obtained values are shown in Table 7.

As can be seen, a cost sharing for the optimal trigeneration system that intends to apply the same unit cost per unit of exergy to the three energy services would be drastically rejected by the electricity consumers because if they were to collaborate, they would end up slightly subsidizing the heat consumers and exaggeratedly subsidizing the cold consumers.

The cost allocation consisting in valuing equally the exergy of the products, although it seems consistent with thermodynamic principles, collides, as we have seen, with the additional difficulty of assigning the values of T_0 and T_b. A detailed thermoeconomic analysis that considers the operation hour by hour throughout the year would certainly provide more acceptable results.

Table 7
Annual costs of final energy products assigned in exergy basis and average unit vs reference costs.

Energy services	Energy (MWh/y)	Exergy (MWh/y)	Annual cost ($/y)	Unit cost[a] ($/MWh)	Reference cost[b] ($/MWh)
Heating	3836	583	(583/2341) · 505960 = 126,000	32.8	38.9
Cooling	2160	151	(151/2341) · 505960 = 32,640	15.11	74.0
Electricity	1607	1607	(1607/2341) · 505960 = 347,320	216.13	159.3
TOTAL	7603	2341	505,960		

[a] Unit cost: refers to the energy unit.
[b] Reference cost: refers to the optimal separate production of the energy service.

5. Conclusions

In the case with an amortization factor $\boldsymbol{fa} = \mathbf{0.10\ y^{-1}}$ Fig. 2 shows the installed power of the technologies present in the optimal solution as well as the annual magnitude of the different energy flows. The three energy services are produced with the trigeneration system and an annual saving of $58.97 \cdot 10^3$ $/y is achieved. So, this trigeneration system that is capable of producing the three energy services demanded and operate by selling electricity to the grid if it is economically convenient, entails maximum economic savings for the applicants. But how should they distribute said savings among them?

In the example discussed here, we see that 6 of the 8 methods provide a cost sharing that is at the core of the game. Also, there are no major differences in the costs assigned to the three energy services. For example, the unit costs proposed by the Shapley and proportional nucleolus methods are practically identical. The results indicate that compared to consumers standing alone, the optimal trigeneration system can achieve 10.6% cost saving. Cooperative game theory shows that all consumers benefit. Using the Shapley values as the distribution criterion, the savings for electricity, heating and cooling consumers are respectively 4.8%, 20.9% and 11.1%, respectively.

Additionally, the costs obtained for the final energy products applying classical thermoeconomics methods, based on cost assessment proportional to energy and proportional to exergy, have also been calculated. The interesting result obtained is that thermoeconomic cost assessment proportional to energy and proportional to exergy do not provide cost values aligned with the objective pursued of economically benefitting all consumers. Thermoeconomic cost assessment methods are mainly based on thermodynamics and their allocation methods do not implicitly encompass achieving equitable and cost-and-benefit apportionment of the joint production costs.

The application of the cost allocation methodology presented in this document is independent of the optimal design method used. The starting point is to have the results corresponding to the optimal polygeneration system capable of meeting all the energy services demanded (heating, cooling and electricity in our example) and also to have as a reference the total cost of the optimal systems capable of meeting some of these services (as shown in Table 4 in our example). Fulfilling this condition, the methodology is easy to apply and allows to obtain an acceptable distribution of the total projected costs among the consumers of the energy services. Logically, a model for design optimization will provide more accurate results to the extent that it takes into account the following factors: (i) the size of the equipment (e.g. economies of scale, technical parameters, energy prices as a function of the amount of energy resources consumed); (ii) the location of the system (e. g. policies, subsidies, power exchange regulations); and (iii) the duration of the analysis, including the temporal resolution of the model (e.g. daily, hourly or minute basis) and the time span (e.g. years, seasons, months, days). Those factors fall outside the scope of this paper because they are not strictly necessary to demonstrate the application of the cooperative game theory methodology. Regarding the uncertainties of the optimization model used, these will be transmitted to the selected cost allocation method and therefore to its results.

A question of interest to continue with the work presented in this paper is the following: assuming the proposed design and that the Shapley cost allocation method is adopted, for example, this only gives us an indication of the total annual cost to be charged to consumers for the three energy services, but the question remains as to what unit costs will be applied at different times of the year. That is, with the proposed methodology we have arrived at an indicative value of the total annual cost to be charged, but once the designed plant is commissioned it will operate under time-varying conditions (demand for services, price of commercial energy consumed, weather conditions, etc.) and it will be necessary to establish the unit costs of the three energy services hour by hour throughout the year. This problem is independent of the cost allocation methodology used and has been generally ignored so far in the literature on thermoeconomic analysis. In any case, real-time cost allocation will require a more detailed plant operating model than the one used for the design and the application of cost allocation rules that, while complying with the annual cost orientation already calculated, consider the operating state and the hour-by-hour formation process of the products. The authors have worked in this line of cost allocation with thermoeconomic analysis [12,13] and are not aware of similar works based on cooperative game theory.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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