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A Distributed control framework for the optimal operation of DC microgrids

Zao Fu, Michele Cucuzzella, Carlo Cenedese, Wenwu Yu and Jacquelien M. A. Scherpen

Abstract— **In this paper we propose an original distributed control framework for DC mcirogrids. We first formulate the (optimal) control objectives as an aggregative game suitable for the energy trading market. Then, based on the dual theory, we analyze the equivalent distributed optimal condition for the proposed aggregative game and design a distributed control scheme to solve it. By interconnecting the DC mcirogrid and the designed distributed control system in a power preserving way, we steer the DC microgrid's state to the desired optimal equilibrium, satisfying a predefined set of local and coupling constraints. Finally, based on the singular perturbation system theory, we analyze the convergence of the closed-loop system. The simulation results show excellent performance of the proposed control framework.**

I. INTRODUCTION

As an important part of the (actual and future) energy system, the direct current (DC) microgrids are widely deployed in several applications, such as renewable energy sources, trains, aircraft, ships and charging stations for the more and more popular electric vehicles [1]. To improve the energy dispatch efficiency and trading fairness for the DC microgrids, one of the most effective options is the adoption of a distributed control and optimization framework (DCOF) [2]. Within such a framework, energy trading, control, and optimization processes will operate in a fully distributed way offering power stability, information privacy, plug-and-play capabilities, and market adaptability for large-scale power networks. However, compared with the opponent centralized framework, the DCOF requires to pay more attention to the design of the control objectives and deal with the constraints (especially with the coupling constraints). In general, the control objectives for the DC microgrids focus on system-level requirements, e.g., system stability [3] and convergence rate. However, the optimization objectives might focus also on economic aspects, such as maximizing the profit from selling power, minimizing power costs, and reducing power losses. For the control objectives, there are several results (see for instance [4], [5] and the references therein). We can divide the research on the optimization into two parts: modeling and algorithm design. In the modeling part, convex optimization (e.g. quadratic programming) and non-cooperative games (e.g. aggregative games [6], [7]) are the most commonly used (see e.g. [8] and the references therein for further details). In the algorithm design process, the challenges come from dealing with the following three aspects: local constraints, coupling constraints, and global information [9]. We mainly have three different methods for dealing with the local constraints. The first one is called the *penalty method*, and it uses a penalty function to embed the local constraints into the objective function [10]. The second method is called the *projection method*, and it restricts the descent direction within the feasible direction [11]. The last method is the *Lagrange multiplier method*, which dualizes the local constraints such that the corresponding dual problem does not have local constraints [12]. On the other hand, one of the most effective methods for handling the coupling constraints is the *multiplier consensus method* [13]. Such a method employs the Lagrange dual method to first deal with the coupling constraints, and then converts the resulting dual problem into an optimization problem suitable for the design of distributed algorithms [13]. For the aggregative information, one of the most common approaches is to design a (faster) estimation system (such as dynamical average consensus algorithms) to estimate the global information in a fully distributed fashion [14].

After modeling the control and optimization objectives and designing the control system, the next step is to connect the control system with the dynamics of the considered DC microgrid. Since the DC microgrid's dynamics can be shown to be passive, ensuring passivity of the controller as well, implies that, through a suitable interconnection, the closed-loop system is still a passive system. Inspired by such an idea, we design the control system based on the Lagrange dual theory, and prove that the closed-loop system converges to the desired (optimal) equilibrium, maximizing the profit while satisfying both the local and coupling constraints.

We organize the rest of the paper as follows. Section II introduces the dynamics of the DC microgrid and formulates the control and optimization objectives. In Section III, we analyze the distributed optimal condition, and in Section IV, we design the distributed control scheme, interconnect it with the DC microgrid and analyze the closed-loop stability. Section V shows the simulation results, and Section VI concludes the paper.

Notations: $\mathrm{col}\{x, y, \cdots\} = [x^\top, y^\top, \cdots]^\top$, where the

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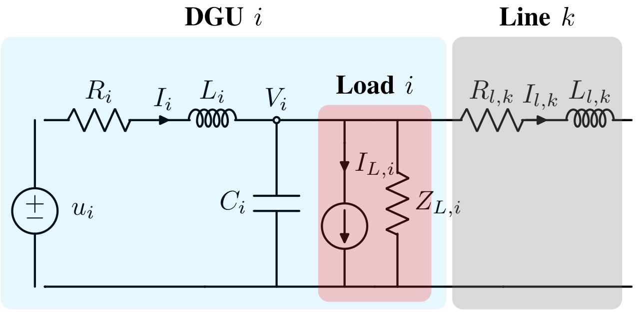

Fig. 1. Electrical scheme of DGU $i \in \mathcal{N}$ and transmission line $k \in \mathcal{E}$.

notation col represents “vector stack”. Without additional explanation, we use x to denote the ‘vector stack’ of $x_1, \ldots, x_n$, that is $x = \operatorname{col}\{x_i\}_{i\in\mathcal{N}} = \operatorname{col}\{x_1, \cdots, x_n\}$. The notation $\operatorname{diag}\{x_i\}_{i\in\mathcal{N}}$ represents the (block) diagonal matrix whose diagonal entries consist of $x_1, \ldots, x_n$. Let $[x]_+ = \max\{0, x\}$. The notation $\|x\|_A$, where A is a symmetric definite or semi-definite matrix, represents the norm of semi-norm, and $\|x\|_A = \sqrt{\langle x, Ax\rangle}$. The notation $\delta_{\min}(A)$ denotes the minimal singular value of the matrix A. The notation $\{x\}_i$ represents the i-th entry of the vector x. The notation ∇ denotes the gradient, and the notation ∇_x represents the gradient with respect to x. The notation $\langle x, y\rangle$ denotes the inner product of the vectors x and y. The notations $\mathbf{0}_n$ and $\mathbf{1}_n$ denote n-dimension vectors whose entries are 0 and 1. Also, we omit the dimension when it is clear. The notation ∂_x denotes the sub-gradient with respect to x. The notation $J_{F,x}(x)$ denotes the Jacobin matrix of the function $F(x)$ with respect to x. The notations $\circ$ and $\otimes$ represent the Kronecker product and the Hadamard product, respectively. The symbols with the superscripts “r” and “$*$” denote constant references and equilibriums (or Nash equilibriums). The “ˆ” denotes the control states corresponding to the microgrid’s ones.

II. MODEL DESCRIPTION

Following [3] and the references therein, we consider a microgrid consisting of a certain number of distributed generation units (DGUs), equipped with distributed controllers and decision systems. Moreover, we consider that each DGU includes constant impedance and constant current loads, and DGUs are interconnected with each other via distribution power lines. Let the sets $\mathcal{N} \triangleq \{1, \ldots, n\}$ and $\mathcal{E} \triangleq \{1, \ldots, m\}$ denote the DGU and the transmission line index sets, respectively. For the readers’ convenience, Figure 1 shows the electric scheme of the DGU i and line k (see also Table I for the description of the used symbols).

TABLE I
PHYSICAL DESCRIPTIONS FOR THE NOTATIONS.

Description	Symbol	Description	Symbol
Control input	u_i	Filter parameters	R_i, L_i
Generated current	I_i	Line parameters	$R_{l,k}, L_{l,k}$
Line current	$I_{l,i}$	Shunt capacitor	C_i
Load voltage	V_i	Load parameters	$I_{L,i}, Z_{L,i}$

According to Kirchhoff’s law, we can write the dynamics of the DC microgrid as follows (see e.g. [3]):

$$\begin{aligned} L\dot{I} &= -V - RI + u, \\ C\dot{V} &= I + BI_l - Z_L^{-1}V - I_L \\ L_l\dot{I}_l &= -R_lI_l - B^\top V. \end{aligned} \tag{1}$$

where $B \in \mathbb{R}^{m\times n}$ is the adjacency matrix associated with the arbitrary oriented graph of the connected and undirected graph $\mathcal{G}(\mathcal{N}, \mathcal{E})$. We assume that each transmission line is under the control of a unique DGU connected to it, and we let $\mathcal{E}_i$ represent the index set of the transmission lines being under the control of the DGU $i \in \mathcal{N}$. Hence, we have

$$\bigcap_{i\in\mathcal{N}} \mathcal{E}_i = \emptyset, \quad \bigcup_{i\in\mathcal{N}} \mathcal{E}_i = \mathcal{E}. \tag{2}$$

Let now define for convenience $x_i \triangleq \operatorname{col}\{I_i, V_i, I_{c,i}\} \in \mathbb{R}^{2+|\mathcal{E}_i|}$, with $I_{c,i} \triangleq \operatorname{col}\{I_{l,k}\}_{k\in\mathcal{E}_i} \in \mathbb{R}^{|\mathcal{E}_i|}$, to denote the state vector of the DGU $i \in \mathcal{N}$. The objective of this paper is to design a distributed control scheme that stabilizes the considered DC microgrid at the desired equilibrium solving a pre-designed game problem. To achieve this goal, we use passivity theory [15] to interconnect the considered microgrid’s dynamics (1) with the distributed control system we design in the following sections.

Before formulating the game problem, we describe the feasible region of operation of the considered microgrid by introducing the following set of coupling constraints:

$$K \triangleq \left\{ (u,x) \in \mathbb{R}^{m+3n} \;\middle|\; \underbrace{\begin{array}{c} I + BI_l - Z_L^{-1}V - I_L = \mathbf{0}_n \\ R_lI_l + B^\top V = \mathbf{0}_m \end{array}}_{Ax - s_A \;=\; \mathbf{0}_{m+n}} \right\}, \tag{3}$$

where we omit the definitions of the matrix $A \in \mathbb{R}^{(m+n)\times n}$ and the vector $b \in \mathbb{R}^{m+n}$. Moreover, for every DGU $i \in \mathcal{N}$, we introduce the following set of local constraints:

$$\Omega_i \triangleq \left\{ (u_i, x_i) \in \mathbb{R}^{3+|\mathcal{E}_i|} \;\middle|\; \begin{array}{c} V_i + R_iI_i - u_i = 0 \\ V_i^{\min} \leq V_i \leq V_i^{\max} \\ I_{c,i}^{\min} \leq I_{c,i} \leq I_{c,i}^{\max} \end{array} \right\}, \tag{4}$$

where the superscripts ‘min’ and ‘max’ represent the minimum and maximum values of the the corresponding state, respectively. Based on (3) and (4), we can then define the following feasible set:

$$\pi_i\left(x_{-i}\right)=\left\{\left(u_i,x_i\right)\in\Omega_i \mid \underbrace{A_i x_i+s_{A_i}}_{\phi_i(x_i)}+\underbrace{\sum_{j=1,j\neq i}^{n} A_j x_j+s_{A_j}}_{\phi_{-i}(x_{-i})}=\mathbf{0}\right\}, \tag{5}$$

where for all $i\in\mathcal{N}$, $A_i\in\mathbb{R}^{(m+n)\times(3+|\mathcal{E}_i|)}$ and $s_{A,i}$ are constant and satisfy $A=\left[A_1,\ldots,A_n\right],\sum_{i=1}^{n}s_{A_i}=s_A$. Moreover, x_{-i} denotes the stack of all the rivals' decisions, i.e., $x_{-i}=\operatorname{col}\{x_1,\ldots,x_{i-1},x_{i+1},\ldots,x_n\}$.

We can now introduce the main goal of the paper, which can be formulated as an aggregative game probelem, i.e., for all $i\in\mathcal{N}$

$$\begin{aligned}\min_{u_i,x_i}\quad & f_i\left(u_i,x_i,x_{-i}\right)\\ \text{s.t.}\quad & \left(u_i,x_i\right)\in\pi_i\left(x_{-i}\right),\end{aligned} \tag{6}$$

with

$$\begin{aligned} f_i\left(u_i,x_i,x_{-i}\right)&=f_{1,i}\left(u_i,x_i\right)+f_{2,i}\left(x_i,x_{-i}\right)\\ f_{1,i}\left(u_i,x_i\right)&\triangleq\frac{\alpha_{u_i}}{2}\left(u_i-u_i^r\right)^2+\frac{1}{2}\|x_i-x_i^r\|^2_{A_{x_i}}\\ f_{2,i}\left(x_i,x_{-i}\right)&\triangleq-\Big(l-p_r\underbrace{\sum\nolimits_{i=1}^{n}I_i}_{s_{p_r}}\Big)V_i^r I_i,\end{aligned} \tag{7}$$

where $f_{1,i}(u_i,x_i)$ represents the cost associated with the deviation of the i-th DGU's state and input with respect to the corresponding references, while $f_{2,i}(x_i,x_{-i})$ represents the profit of DGU i, where $(l-p_r s_{p_r})>0$ is the selling price of the generated power $V_i^r I_i$. Moreover, α_{u_i} is a positive constant and the matrix $A_{x_i}\triangleq\operatorname{diag}\left\{\alpha_{I_i},\alpha_{V_i},\operatorname{diag}\left\{\alpha_{I_{l,k}}\right\}_{k\in\mathcal{E}_i}\right\}$ is positive definite; the parameters p_r and l are positive constants ensuring that the price of power is always positive, i.e., $(l-p_r s_{p_r})>0$ for all the feasible currents $I_1,\ldots,I_n$. According to the constraint (3), we can guarantee such a condition by introducing the following assumption:

Assumption 1 (**Parameter setting**) Let the following condition

$$0<l-p_r\sum_{i=1}^{n}\left(\frac{V_i^{\max}}{Z_{L,i}}+I_{L,i}\right) \tag{8}$$

hold for all $i\in\mathcal{N}$.

Note that the increase of the generated currents' sum s_{p_r} implies a reduction of the power price $l-p_r s_{p_r}$ (and vice versa), as usual in the energy trading market.

III. PROBLEM ANALYSIS

In this section, we analyze the optimality conditions associated with the game problem (6). First, we introduce the definition of the generalized Nash equilibrium (GNE) [9, Definition 1].

Definition 1 (**Generalized Nash equilibrium**) The point (u^*,x^*) is a GNE for the game (6) if and only if it solves the following problem:

$$\min_{u_i,x_i}\quad f_i(u_i,x_i,x_{-i}^*)\quad\text{s.t.}\quad(u_i,x_i)\in\pi_i(x_{-i}^*). \tag{9}$$

for all $i\in\mathcal{N}$.

Now, before formulating the dual problem of (9), for all $i\in\mathcal{N}$, we define the following penalty function (distance function):

$$\begin{aligned} g_i(x_i)=&\underbrace{\rho_{V_i}\big([V_i^{\min}-V_i]_+ +[V_i-V_i^{\max}]_+\big)}_{=\,g_{1,i}(V_i)}\\ &+\sum\nolimits_{k\in\mathcal{E}_i}\underbrace{\rho_{I_{l,k}}\big([I_{l,k}^{\min}-I_{l,k}]_+ +[I_{l,k}-I_{l,k}^{\max}]_+\big)}_{=\,g_{2,k}(I_{l,k})},\end{aligned}$$

where the positive constants ρ_{V_i} and $\rho_{I_{l,k}}$, $k\in\mathcal{E}_i$, represent the penalty parameters. According to [10, Lemma 4], there exist positive penalty parameters for the sub-problems in (9) such that its solution and the solution to the corresponding penalized problem coincide. For all $i\in\mathcal{N}$, we can then define the following Lagrange function

$$\begin{aligned}\mathcal{L}_i(u_i,x_i,x_{-i}^*,\gamma_i,\lambda_i)=&\,f_i(u_i,x_i,x_{-i}^*)\\ &+g_i(x_i)+\gamma_i(D_i^\top x_i-u_i)\\ &+\langle\lambda_i,\phi_i(x_i)+\phi_{-i}(x_{-i}^*)\rangle,\end{aligned} \tag{10}$$

where $D_i=\operatorname{col}\{1,R_i,\mathbf{0}\}\in\mathbb{R}^{2+|\mathcal{E}_i|}$ is a constant vector and $\gamma_i\in\mathbb{R}$ represents the Lagrange multipliers associated with the constraints $D_i^\top x_i-u_i=0$ (see the first equality constraints in (4)). Similarly, $\lambda_i\in\mathbb{R}^{m+n}$ represents the Lagrange multipliers associated with the coupling constraints in (5). Note also that the penalty function $g_i(x_i)$ is not differentiable at some points. Hence, we need to introduce the sub-gradient of $g_i(x_i)$ consisting of the sub-gradients of the penalty functions $g_{1,i}(V_i)$ and $g_{2,k}(I_{l,k})$. For all $i\in\mathcal{N}$, the sub-gradient of $g_{1,i}(V_i)$ is as follows [16]

$$\partial g_{1,i}(V_i)\in\begin{cases}-\rho_{V_i} & \text{if } V_i<V_i^{\min},\\ [-\rho_{V_i},0] & \text{if } V_i=V_i^{\min},\\ 0 & \text{if } V_i^{\min}<V_i<V_i^{\max},\\ [0,\rho_{V_i}] & \text{if } V_i=V_i^{\max},\\ \rho_{V_i} & \text{if } V_i>V_i^{\max}.\end{cases} \tag{11}$$

The sub-gradient of $g_{2,i}(I_{l,k})$ can be obtained as in (11), thus we omit it for the sake of simplicity. To guarantee that a GNE exists for the game problem (6), we introduce the following assumption:

Assumption 2 (**Non-empty feasible set**) The feasible set given by the intersection of K in (3) and $\Omega_1,\ldots,\Omega_n$ in (4) is non-empty.

Since the constraints of each sub-problem in (6) are affine, Assumption 2 guarantees that π_i satisfies Slater's constraint qualification. Therefore, the following KKT conditions are a necessary and sufficient condition for the optimal condition of the problem (6) (refer to [12, Section 5.2.3] for details):

$$\forall\, i \in \mathcal{N}, \quad \begin{cases} \nabla_{u_i} \mathcal{L}_i(u_i^*, x_i^*, x_{-i}^*, \gamma_i^*, \lambda_i^*) = \mathbf{0}_n, \\ \partial_{x_i} \mathcal{L}_i(u_i^*, x_i^*, x_{-i}^*, \gamma_i^*, \lambda_i^*) \ni \mathbf{0}_{m+2n}, \\ \nabla_{\lambda_i} \mathcal{L}_i(u_i^*, x_i^*, x_{-i}^*, \gamma_i^*, \lambda_i^*) = \mathbf{0}_{m+n}, \\ \nabla_{\gamma_i} \mathcal{L}_i(u_i^*, x_i^*, x_{-i}^*, \gamma_i^*, \lambda_i^*) = 0, \end{cases} \tag{12}$$

where $(u_i^*, x_i^*, \gamma_i^*, \lambda_i^*)$ is the saddle-point of the Lagrange function (10). Since the multipliers $\lambda_1^*, \dots, \lambda_n^*$ can vary from each other, the solution of the KKT condition (12) may not be unique. To shrink the solution set of the KKT condition (12) to a convex set (or a singleton), such that we can develop a fully distributed algorithm, we need to introduce the following definition [17, Definition 3.2], [18].

Definition 2 (**Normalized Nash equilibrium**) A GNE (u^*, x^*) is a normalized Nash equilibrium (NNE) associated with the given $r_1, \dots, r_n > 0$, if there exist the Lagrange multipliers γ^* and λ^* such that $(u^*, x^*, \gamma^*, \lambda^*)$ solves the KKT condition (12) and satisfies the additional condition

$$r_1 \lambda_1^* = \cdots = r_n \lambda_n^*. \tag{13}$$

Remark 1 The values of the Lagrange multipliers $\lambda_1^*, \dots, \lambda_n^*$ concerning the coupling constraints represent the shadow price of all the DGUs. From a trading market point of view, the values of $r_1, \dots, r_n$ can be designed by a higher level decision system (for example, the government) in order to model different market scenarios.

For the sake of analysis, let $A_r \triangleq \operatorname{diag}\{r_i\}_{i\in\mathcal{N}} \otimes \mathbb{I}_{m+n}$, $\lambda \triangleq \operatorname{col}\{\lambda_i\}_{i\in\mathcal{N}}$, $\boldsymbol{L} \triangleq \mathbb{L} \otimes \mathbb{I}_{m+n}$, where $\mathbb{L}$ represents the Laplacian matrix associated with $\mathcal{G}$. Since $\mathcal{G}$ is undirected and connected, then the condition (13) is equivalent to $\boldsymbol{L} A_r \lambda = \mathbf{0}_{n(m+n)}$. Then, we introduce the following proposition playing a crucial role in the later controller design, as in [6].

Proposition 1 There exist $\upsilon^* \triangleq \operatorname{col}\{\upsilon_i^*\}_{i\in\mathcal{N}}$, $\nu^* \triangleq \operatorname{col}\{\nu_i^*\}_{i\in\mathcal{N}} \in \mathbb{R}^n$ satisfying

$$\begin{cases} -(\mathbb{I} + \mathbb{L})\upsilon^* - \mathbb{L}\nu^* + nI^* = \mathbf{0}_n, \\ \mathbb{L}\upsilon^* = \mathbf{0}_n \end{cases} \tag{14}$$

if and only if

$$\upsilon_1^* = \cdots = \upsilon_n^* = \sum_{i=1}^n I_i^*, \tag{15}$$

where $I_i^* \in \mathbb{R}$ for all $i \in \mathcal{N}$.

Proof It holds that

$$\mathbb{L}\upsilon^* = \mathbf{0}_n \;\Leftrightarrow\; \upsilon_1^* = \cdots = \upsilon_n^*. \tag{16}$$

By substituting the second equality of (14) in the first equality and multiplying both sides by $\mathbf{1}_n^\top$, we can obtain the condition (15). From (15) and $\operatorname{rank}(\mathbb{L}) = n - 1$, we deduce that there exists ν^* satisfying

$$\upsilon^* - nI^* = \Big(\sum_{i=1}^n I_i^*\Big) \mathbf{1}_n - nI^* = \mathbb{L}\nu^*. \tag{17}$$

By combining (17) and (16), we obtain the condition (14), which completes the proof. ■

According to [17, Proposition 3.2], the NNE associated with a given $r > 0$ of the problem (6) corresponds to the solution of the following variation inequality:

$$x^* \in K \cap \Omega, \;\; \langle F_r(u^*, x^*), x - x^* \rangle \geq 0, \;\; \forall\, x \in K \cap \Omega, \tag{18}$$

where $\Omega \triangleq \bigcap_{i=1}^n \Omega_i$, and the vector function $F_r(u, x)$ is the pseudo-gradient (refer to [17]) defined as follows

$$F_r(u, x) = \operatorname{col}\{r_i \nabla_{(u_i, x_i)} f_i(u_i, x_i, x_{-i})\}_{i\in\mathcal{N}}.$$

To ensure that the variational inequality (18) (as well as the problem (6)) has a unique NNE (u^*, x^*) for a fixed $r > 0$, we need to introduce the following assumption:

Assumption 3 (**Bound for parameters**) For all $i \in \mathcal{N}$, the parameter r_i satisfies the following condition:

$$2 r_i \alpha_{I_i} + (6 - n)\, r_i p_r V_i^r - \sum_{i=1}^n r_i p_r V_i^r > 0. \tag{19}$$

Under Assumption 3, one can verify that the Jacobian matrix $J_{F_r}(u, x) \succ 0$ and thus $F_r(u, x)$ is strict monotone for all $(u, x) \in K \cap \Omega$ (refer to [19, Theorem 2.3.3]). Therefore, the variational inequality (18) has a unique solution under Assumptions 2 and 3. All the parameters in (19) have to be designed, and thus Assumption 3 is not a strict condition. Following from the analysis in Proposition 1, we can deduce that the following constraint

$$\begin{cases} A_r(\operatorname{col}\{A_i x_i^* - s_{A_i}\}_{i\in\mathcal{N}} - \boldsymbol{L} A_r \lambda^* - \boldsymbol{L}\theta^*) = \mathbf{0}_{n(m+n)}, \\ \boldsymbol{L} A_r \lambda^* = \mathbf{0}_{n(m+n)} \end{cases}$$

is equivalent to the constraint $Ax^* - s_A = \mathbf{0}_{m+n}$. Therefore, by involving the constraint (13) and Proposition 1, we can rewrite the condition (12) is a distributed form as

$$\forall\, i \in \mathcal{N}, \quad \begin{cases} -\upsilon_i^* - \mathbb{L}_i \upsilon^* - \mathbb{L}_i \nu^* + nI_i^* = 0, \\ \mathbb{L}_i \upsilon^* = 0, \\ r_i \alpha_{u_i}(u_i^* - u_i^r) + \gamma_i^* = 0, \\ r_i \bar{F}_i(x_i^*, \upsilon_i^*) + r_i A_i^\top \lambda_i^* + \gamma_i^* D_i \ni \mathbf{0}_{m+3n}, \\ D_i^\top x_i^* - u_i^* = 0, \\ r_i(A x_i^* - s_{A_i}) - r_i \boldsymbol{L}_i A_r \lambda^* - r_i \boldsymbol{L}_i \theta^* = \mathbf{0}_{m+n}, \\ \boldsymbol{L}_i A_r \lambda^* = \mathbf{0}_{m+n}, \end{cases} \tag{20}$$

where for all $i \in \mathcal{N}$, the vector $\theta_i^* \in \mathbb{R}^{m+n}$ denotes the dual variables associated with the consensus constraint $\boldsymbol{L}_i A_r \lambda^* = \mathbf{0}_{m+n}$. We use the vector $\mathbb{L}_i^\top \in \mathbb{R}^n$ and matrix $\boldsymbol{L}_i \in \mathbb{R}^{(m+n)\times n(m+n)}$ to denote the rows of $\mathbb{L}$ and $\boldsymbol{L}$

associated with the DGU $i \in \mathcal{N}$, respectively. Moreover, $\bar{F}_i\left(\hat{x}_i^*, \upsilon_i^*\right)$ is defined as

$$\bar{F}_i\left(\hat{x}_i^*, \upsilon_i^*\right) = \begin{bmatrix} \alpha_{I_i}(I_i^* - I_i^r) - (l - p_r V_i^r \upsilon_i^*) + p_r V_i^r I_i^* \\ \alpha_{V_i}(V_i^* - V_i^r) + \partial g_{1,i}(V_i^*) \\ \operatorname{col}\left\{(I_{l,k}^* - I_{l,k}^r) + \partial g_{2,k}(I_{l,k}^*)\right\}_{k \in \mathcal{E}_i} \end{bmatrix}.$$

Remark 2 Based on [20, Theorem 3.4], the penalty parameters ρ_{V_i} and $\rho_{I_{l,k}}$ satisfy the following condition

$$\begin{aligned} \rho_{V_i} &\geq \alpha_{V_i}(V_i^{\max} - V_i^r) + \nabla_{V_i}\langle \lambda_i^*, (Ax - s_A)\rangle + \gamma_i^*, \\ \rho_{I_{l,k}} &\geq \alpha_{I_{l,k}}(I_{l,k}^{\max} - I_{l,k}^r) + \nabla_{I_{l,k}}\langle \lambda_i^*, (Ax - s_A)\rangle. \end{aligned} \tag{21}$$

for all $i \in \mathcal{N}$ and $k \in \mathcal{E}_i$. Hence, the penalty parameters should be large enough such that they satisfy (21).

IV. ALGORITHM DESIGN AND ANALYSIS

Based on (20), we can now design the distributed controller for each DGU $i \in \mathcal{N}$. By connecting the designed controller to the DGU $i \in \mathcal{N}$ in a passive way (see e.g. [3]), we obtain the following closed-loop system:

$$\dot{x}_i = G_{g,i}\left(u_i, x_i\right), \tag{22a}$$

$$\varepsilon \dot{\upsilon}_i = -\, \upsilon_i - \mathbb{L}_i \upsilon - \mathbb{L}_i \nu + n\hat{I}_i, \tag{22b}$$

$$\varepsilon \dot{\nu}_i = \mathbb{L}_i \upsilon, \tag{22c}$$

$$\dot{u}_i = -\, \alpha_{u_i} u_i + \gamma_i - \epsilon I_i \tag{22d}$$

$$\dot{\hat{x}}_i = -\, r_i \bar{F}_i\left(\hat{x}_i, \upsilon_i\right) - r_i A_i^\top \lambda_i - \gamma_i D_i, \tag{22e}$$

$$\dot{\lambda}_i = r_i(A_i \hat{x}_i - s_{A_i}) - r_i \boldsymbol{L}_i A_r \lambda - r_i \boldsymbol{L}_i \theta, \tag{22f}$$

$$\dot{\theta}_i = \boldsymbol{L}_i A_r \lambda, \tag{22g}$$

$$\dot{\gamma}_i = -\, u_i + D_i x_i, \tag{22h}$$

where (22a) denotes the dynamics of each DGU $i \in \mathcal{N}$, and the non-negative constants ε and ϵ denote the control system parameters. For the sake of the later convergence analysis, let $s_f \triangleq \operatorname{col}\{\upsilon, \nu\}$ and $s_d \triangleq \operatorname{col}\{u, \hat{x}, \lambda, \theta, \gamma\}$. Then we can write (22) as:

$$\dot{x} = G_g(u, x), \tag{23a}$$

$$\varepsilon \dot{s}_f = G_f(s_f, \hat{I}), \tag{23b}$$

$$\dot{s}_d = G_d(\upsilon, s_d, I). \tag{23c}$$

where we omits the detailed definitions of the maps G_g, G_f and G_d. Note that, in the framework of singular perturbation system theory [21], (23b) describes the dynamics of the fast system, while (23a) and (23c) those of the slow system. Let $h(\hat{I}) \triangleq \operatorname{col}\{h_\upsilon(\hat{I}), h_\nu(\hat{I})\}$ and $s_b \triangleq s_f - h(\hat{I})$ represent the solution of the equation $G_f(s_f, \hat{I}) = \mathbf{0}_{2n}$ and the corresponding boundary layer system state, respectively. We can write the boundary layer system and reduced-order system as follows:

$$\dot{x} = G_g(u, x), \tag{24a}$$

$$\varepsilon \dot{s}_b = G_f(s_b + h, \hat{I}), \tag{24b}$$

$$\dot{s}_d = G_d(h_\upsilon, s_d, I), \tag{24c}$$

where we abbreviate $h(\hat{I})$ as h.

Theorem 1 (**Convergence analysis**) Let Assumptions 2 and 3 hold and the initial state ν_0 satisfy $\mathbf{1}_n^\top \nu_0 = 0$. Then, there exists a $\varepsilon^* \in \mathbb{R}_+$ such that (22) converges to the largest invariant set $\varPhi_{s,f}$ for all ε satisfying $0 < \varepsilon < \varepsilon^*$, where

$$\varPhi_{s,f} = \left\{ (s_f, x, s_d) \,\middle|\, \begin{array}{l} G_g(u, x) = \mathbf{0} \\ G_f(s_f, \hat{I}) = \mathbf{0} \\ G_d(\upsilon, s_d, I) = \mathbf{0} \end{array} \right\}. \tag{25}$$

Proof Let $E_b(s_b)$ and $E_s(x, s_d)$ denote respectively the Lyapunov functions of the boundary layer system (24b) and the reduced-order system (24a), (24c), i.e.,

$$\begin{aligned} E_b(s_b) &= \sigma \|s_b\|^2 + \frac{1}{2}\|\upsilon_b\|^2 + \frac{1}{2}\|\upsilon_b\|_{\mathbb{L}}^2 + \langle \nu_b, \mathbb{L}\upsilon_b \rangle, \\ E_r(x, s_d) &= \frac{1}{2}\left(\|\dot{I}\|_L^2 + \|\dot{I}_l\|_{L_l}^2 + \|\dot{V}\|_C^2\right) \\ &\quad + \frac{1}{2\epsilon}\|G_d(h_\upsilon, s_d, I)\|^2, \end{aligned}$$

where σ represents the largest singular value of the Laplacian matrix $\mathbb{L}$. Then, we can define the composite Lyapunov function as follows:

$$V(s_b, s_s) = (1 - e)E_r(s_s) + eE_b(s_b).$$

The convergence analysis follows from [21, Theorem 11.3]. For convenience, we define $s_s \triangleq \operatorname{col}\{x, s_d\}$ and

$$G_s(h_\upsilon, s_s, I) = \begin{bmatrix} G_g(x, u) \\ G_d(h_\upsilon, s_d, I) \end{bmatrix}.$$

Since Proposition 1 ensures that $h_\upsilon = \sum_{i=1}^n \hat{I}_i$ and it is easy to verify that $G_d(h_\upsilon, s_d, I)$ is a monotone function with respect to s_d, we can deduce that $J_{G_d, s_d}(h_\upsilon, s_d, I)$ is positive definite and

$$\begin{aligned} \frac{\partial E_r}{\partial s_d} G_d(h_\upsilon, s_d, I) \leq\ & \frac{1}{\epsilon}\langle G_d(h_\upsilon, s_d, I), \\ & J_{G_d, s_d}(h_\upsilon, s_d, I) G_d(h_\upsilon, s_d, I)\rangle \\ & - \langle \dot{u}, \dot{I} \rangle. \end{aligned} \tag{26}$$

In addition, we have

$$\frac{\partial E_r}{\partial x}\dot{x} \leq \dot{x}^\top \begin{bmatrix} -R & -\mathbb{I} & \mathbf{0} \\ \mathbb{I} & -Z_L^{-1} & B \\ \mathbf{0} & -B^\top & -R_l \end{bmatrix} \dot{x} + \langle \dot{u}, \dot{I} \rangle. \tag{27}$$

Thus we can observe that there exists a positive α_1 such that

$$\frac{\partial E_r}{\partial s_s} G_s(h_\upsilon, s_s, I) \leq \alpha_1 \|G_d(h_\upsilon, s_d, I)\|^2. \tag{28}$$

Next we proceed by taking the time derivative of $E_b(s_b)$, and based on the fact that $\mathbf{1}_n^\top s_b = 0$ (following from $\mathbf{1}_n^\top \nu_0 = 0$), we have:

$$\begin{aligned} & \frac{\partial E_b}{\partial s_b}\dot{s}_b = \\ & -\, s_b^\top \underbrace{\begin{bmatrix} (2\sigma + 1)\,\mathbb{I} + (2\sigma + 2)\,\mathbb{L} & \mathbb{L} + \mathbb{L}^2 \\ \mathbb{L} + \mathbb{L}^2 & \mathbb{L}^2 + \alpha_\nu \mathbf{1}_{n \times n} \end{bmatrix}}_{\triangleq\ A_\alpha} s_b, \end{aligned}$$

$$\left[\frac{\partial E_b}{\partial s_s} - \frac{\partial E_b}{\partial s_b}\frac{\partial h}{\partial s_s}\right] G_s(s_b + h_\upsilon, s_s, I) \le s_b^\top \begin{bmatrix} \sigma\mathbb{I} + \mathbb{I} + \mathbb{L} & \mathbb{L} \\ \mathbb{L} & \sigma\mathbb{I} \end{bmatrix} \left(\frac{\partial h}{\partial s_s} G_s(h_\upsilon, s_s, I) - \frac{\partial h}{\partial \hat{I}}(p_r V^r \circ s_b)\right). \tag{31}$$

TABLE II

PARAMETERS OF THE OBJECTIVE AND PENALTY FUNCTIONS.

Number	r_i	α_{I_i}	α_{V_i}	α_{u_i}	$\alpha_{I_{l,k}}$	ρ_{V_i}	$\rho_{I_{l,k}}$
1	1.0060	10.6569	0.7516	1.0155	1.3724	1200	1000
2	1.0399	10.6280	0.6203	1.9841	1.1981	1200	1000
3	1.0527	10.2920	0.8527	1.1672	1.4897	1200	1000
4	1.0417	10.4317	0.9379	1.1060	1.3395	1200	1000

where α_ν is a constant. Then, one can verify that all the eigenvalues of the matrix A_α are positive (we omit the proof due to space limitation). Hence, based on the property of the Rayleigh quotient, we have

$$\frac{\partial E_b}{\partial s_b}\dot{s}_b \le \delta_{\min}(A_\alpha)\|s_b\|^2 = \alpha_2\|s_b\|^2. \tag{29}$$

Now, since the function $G_s(s_b + h_\upsilon, s_s, I)$ is linear with respect to $s_b + h_\upsilon$, we can deduce that there exists a positive constant β_1 such that

$$\begin{aligned} &\frac{\partial E_r}{\partial s_s}[G_s(s_b + h_\upsilon, s_s, I) - G_s(h_\upsilon, s_s, I)] \\ &\qquad\qquad \le \beta_1\|G_d(h_\upsilon, s_d, I)\|\|s_b\|. \end{aligned} \tag{30}$$

Finally, one can show that (31) holds. Furthermore, since $\partial h/\partial s_s$ and $\partial h/\partial \hat{I}$ are constant matrices, then we can deduce that there exist two positive constants β_2 and ξ such that the following inequality holds

$$\begin{aligned} &\left[\frac{\partial E_b}{\partial s_s} - \frac{\partial E_b}{\partial s_b}\frac{\partial h}{\partial s_s}\right] G_s(s_b + h_\upsilon, s_s, I) \\ &\qquad\qquad \le \beta_2\|G_d(h_\upsilon, s_d, I)\|\|s_b\| + \xi\|s_b\|^2. \end{aligned} \tag{32}$$

So far, we have verified all the conditions in [21, Theorem 11.3], i.e, (28), (29), (30) and (32). Hence, we can conclude that if

$$0 < \varepsilon \le \varepsilon_e^* = \frac{\alpha_1\alpha_2}{\alpha_1\xi + \beta_1\beta_2},$$

then system (22) converges to the largest invariant set $\Phi_{s,f}$. ■

V. SIMULATIONS

In this section, we assess the performance of the proposed distributed control system (22) in simulation, considering a microgrid with four DGUs in a ring topology. We set the price parameters p_r and l as 5 and 0.01, respectively. Also, we select the fast system parameter ε equal to 0.01. The parameters of the objective and penalty functions are reported in Table. II. We report the parameters of all the DGUs and transmission lines in Tables III and IV.

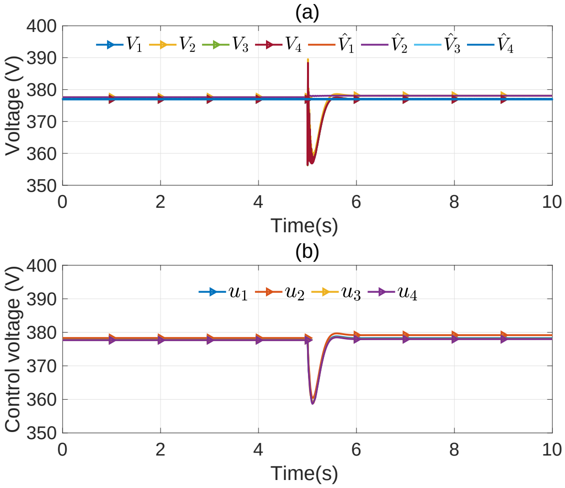

Fig. 2. (a) The loads' and decision system' voltages. (b) The DGU control voltages.

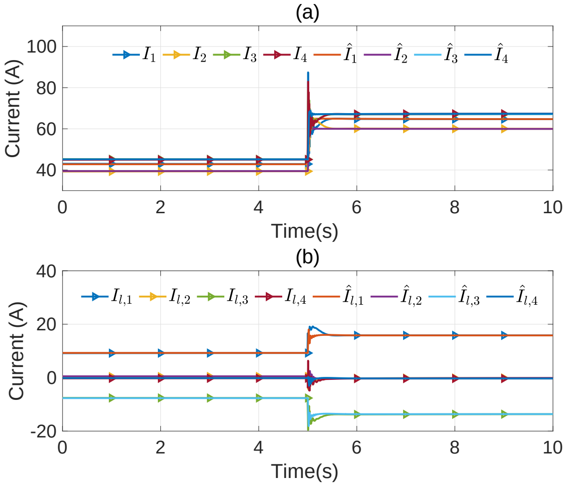

Fig. 3. (a) The DGUs' and the decision system's currents. (b) The transmission lines' and decision system's currents.

We consider that the microgrid initial conditions are within the feasible set and the system remain unperturbed for the first 5 seconds. Then, at the time instant $t = 5$ s, each current-type load $I_{L,i}$ and resistance-type load $Z_{L,i}$ is decreased by 3 units. We present the results in Fig. 2 and 3, and we observe that the microgrid's states converge to the equilibrium within a short time after the loads change. Moreover, the new equilibrium satisfies the optimal condition (20).

In general, the simulation results show excellent performance both in terms of optimality and transient response.

VI. CONCLUSION

In this paper, we first design a distributed control system to solve the power trading problem (described as an aggregative game) in a DC microgrid. Then, we interconnect the designed control system with the microgrid in a passive way and

TABLE III

PARAMETERS OF THE DGUS AND LOADS

DGU Number	L_i (mH)	C_i (mF)	R_i (mΩ)	I_i^r (A)	u_i^r (V)	V_i^r (V)	$V_i^{\min}$ (V)	$V_i^{\max}$ (V)	$Z_{L,i}$ (Ω)	$I_{L,i}$ (A)
1	1.8	2.2	20	0	0	380	377	383	16	30
2	2.0	1.9	18	0	0	380	377	383	50	15
3	3.0	2.5	16	0	0	380	377	383	16	30
4	2.2	1.7	15	0	0	380	377	383	20	26

TABLE IV

PARAMETERS OF THE TRANSMISSION LINES.

Line Number	Head node	Tail node	$R_{l,k}$ (mΩ)	$L_{l,k}$ (μH)	$I_{l,k}^{\min}$ (A)	$I_{l,k}^{\max}$ (A)	$I_{l,k}^r$ (A)	Manage agent
1	1	2	70	2.1	-20	20	0	1
2	2	3	50	2.0	-20	20	0	2
3	3	4	80	3.0	-20	20	0	3
4	4	1	60	2.2	-20	20	0	1

analyze the convergence of the overall closed-loop system. Although we prove that there exists ε^* for the fast system, we do not explicitly provide its exact bound, which is left as a future work.

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