METHODOLOGY OF FORCE PARAMETERS JUSTIFICATION OF THE CONTROLLED STEERING WHEEL SUSPENSION

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Resume

The methodology for selecting the main force parameters of nonlinear non-conservative suspension for the main performance characteristics of wheeled vehicles has been developed based on the longitudinal-angular oscillations. The following has been established: a) change limits of static deformation of the elastic suspension system and its other parameters at which the basic operational characteristics for various ranges of amplitude change are satisfied; b) the amplitude of the initial perturbation of oscillations depending on the shape of the inequality and its entry speed; c) the influence of the main characteristics of damping devices on the amplitude-frequency characteristics. The results of this work can be a basis for the choice of such parameters of the considered controlled suspension, which ensure the stability of the vehicle along the curved sections of the road, its controllability, etc.

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1 Introduction

Modern wheeled vehicles (WV) are increasingly using the so-called controlled (active or semi-active) suspension system (CSS) [1-4]. It, in contrast to traditional suspension systems with linear or nonlinear force characteristics, has the ability to “adjust” the force characteristics to the most optimal conditions of movement along the road with irregularities, its curvilinear sections or maneuvering. It is primarily a question of ensuring the smoothness of the course, the stability of the movement to overturn or shift. The specified “adjustment” of CSS by change of static deformation, rigidity characteristics and damping properties of the suspension system (SS) is carried out [2-4]. To design such CSS, more precisely, to create a software product for controlling the determining force parameters of the SS, the problem is the response of the sprung mass (SM) to changes in the magnitude of the force parameters of SS and various external perturbations caused by WV motion. The latter, within one or another physical and corresponding mathematical models of motion can be obtained based on the analytical dependences that follow from the above mentioned models. As for the mathematical models of the dynamics of WV motion, which are adequate to the force characteristics of the studied SS, they are usually ordinary nonlinear differential equations. It is a problematic task to obtain the analytical dependences on their basis, for creation of a software product. The use of numerical or real experiments for these dynamic models [5-8] does not lead to the desired results due to limited for many cases information about SM dynamics. Only in some cases (for linear and some nonlinear mathematical models of SM dynamics [9-15]) it is possible to obtain analytical dependences that can be the basis for creating a software product for controlling SS force parameters and kinematic parameters of WV motion with CSS [16-17]. Obtaining more general analytical dependences for the SS with non-conservative power characteristics, which would serve as a basis for creating a software product of controlled or semi-controlled SS is the subject of this work, hence its relevance.

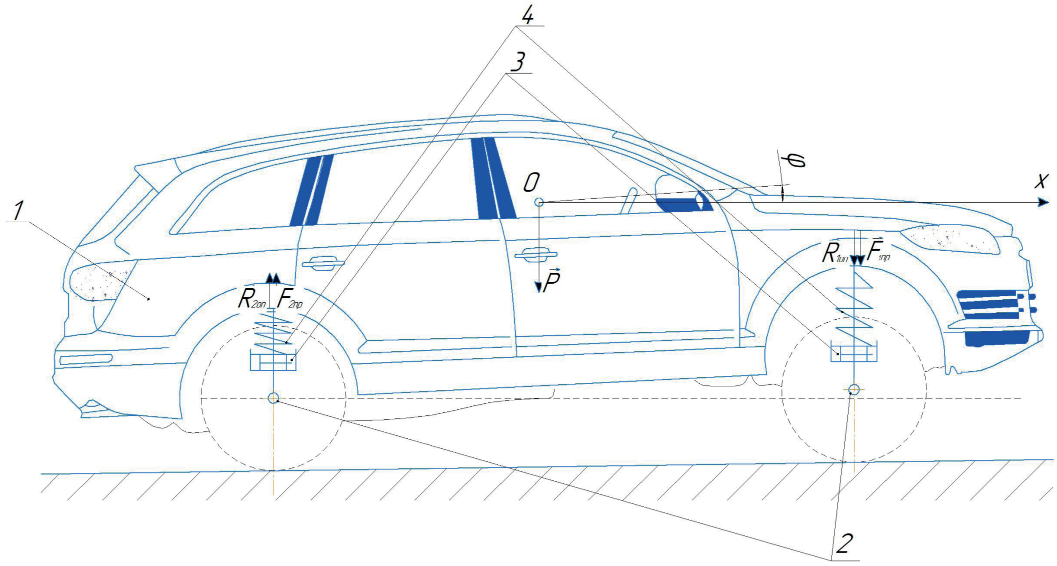


Figure 1 *Calculation scheme for the study of WV longitudinal oscillations with the distribution of forces acting on the SM*

2 Methodology

Longitudinal - angular oscillations of the SM are considered, which, considering the controllability, smoothness, passability are the most important for the vehicle. It has been stated that the restorative forces SS of the front (F_1) and rear (F_2) suspensions are described by more complex dependencies as in [9], namely

$$F_i(\Delta_i,\dot{\Delta}_i) = (\alpha_i + \beta_i(\dot{\Delta}_i)^{\nu_1})(\Delta_i)^{\nu_2+1}, \quad (1)$$

where Δ_i and $\dot{\Delta}_i$ are deformation and deformation rate, respectively, of the front ($i = 1$) and rear ($i = 2$) suspensions; $\alpha_i, \beta_i, \nu_1, \nu_2$ - constant, but ν_1, ν_2 are determined by dependencies $\nu_1 + 1 = (2m+1)/(2n+1)$, ($m,n,p,\ q$ = 0,1,2,...). These conditions are necessary and sufficient conditions for the existence of oscillatory processes in the relevant mechanical systems [18] In particular, the case $\beta_i = 0$ corresponds to the conservative law of change of SS restorative force (progressive - $\nu_2 > 0$ and regressive $-1 < \nu_2 < 0$)] and the case $\beta_i = 0, \nu_2 = 0$ - corresponds to the classical linear law of its change. Various aspects of the SM dynamics for these individual cases have been considered, for example, in [8, 11-12, 19-20]. As for the resistance forces of damping devices (shock absorbers), they, as in most cases of linear or nonlinear systems, depend on the rate of deformation of the elastic elements SS and are described by the dependences $\mathrm{R}_{\mathrm{iooп}} = \chi_i \dot{\Delta}_i^{2s+1}$, ($\chi_i$ and s - constants). In addition, the maximum values of these forces are small values compared to the maximum value of the restorative forces. This is a necessary condition for the oscillating motion of the SM during the motion of WV way with irregularities and to ensure the smoothness of WV stroke. At the same time, these conditions reduce the dynamic loads on the driver, passengers or transported goods. The task is to obtain such analytical dependences that describe the effect of the whole set of parameters $\nu_1, \nu_2, \alpha_i, \beta_i, \chi_i$ (including static deformation of the suspension system $\Delta_{cm.}$, as a derivative) on the amplitude-frequency characteristic (AFC) SM oscillations and which would also be the basis for creating a software product for controlling the power parameters of the suspension.

To solve this problem, the calculation model WV is a flat system of two bodies separated in [14] - sprung - 1 and unsprang mass - 2, which are connected by a suspension system (elastic shock absorbers - 3 and damping devices - 4, see Figure 1). In the process of WV movement SM performs longitudinal-angular oscillations. These oscillations play a crucial role in the study of such characteristics of movement as controllability, stability, smoothness, passability of the vehicle [21-26] and are the subject of the study). Therefore, to unambiguously determine the relative position of the SM at two moments of time, it is sufficient to choose the angle of rotation of it around the transverse axis, which passes through the centre of mass (point O) and perpendicular to the velocity vector of the specified point (angle)- $\varphi(t)$).

Notes.

1. The positive direction of the SM angle rotation is taken as in [26].
2. It is considered that during the movement by elastic deformations of tires caused by external factors are much smaller than the deformations of the suspension elements and therefore they are neglected in the mathematical model.

As for the static deformation of elastic elements, which determines the position of the centre of mass relative to the unsprang mass and hence to some extent the return path, it is, as follows from equation (1), determined as:

$$\Delta_{cm.} = \sqrt[\nu_2+1]{\frac{mg}{\alpha_1+\alpha_2}}, \quad (2)$$

where m - the mass of the sprung part.

The above generally allows deformation of the SS elastic elements at any time to be presented in the form $\Delta_1 = a\varphi(t) - \Delta_{cm.}, \Delta_2 = b\varphi(t) + \Delta_{cm.}$. As a result, from the basic ratio for the relative motion of a body around an axis passing through its centre of mass is as following

$$I_0\ddot{\varphi} = -a(F_1\cdot(a\varphi - \Delta_{cm.}, a\dot{\varphi}) - R_{1on.}(\dot{\Delta}_1)) - b(F_2\cdot(b\varphi - \Delta_{cm.}, b\dot{\varphi}) - R_{1on.}(\dot{\Delta}_2)), \quad (3)$$

where I_0 - the moment of inertia of the sprung mass relative to the above transverse axis; a and b - the WV base.

Taking into account Equation (1), the last ratio is transformed into a form

$$I_o\ddot{\varphi} = -a\begin{pmatrix}(\alpha_1 + \beta_1(a\dot{\varphi})^{\nu_1})(a\varphi - \Delta_{\tilde{n}\grave{o}.})^{\nu_2+1} \\ +\chi_1(a\dot{\varphi})^{2s+1}\end{pmatrix} - b\begin{pmatrix}(\alpha_2 + \beta_2(a\dot{\varphi})^{\nu_1})(b\varphi - \Delta_{\tilde{n}\grave{o}.})^{\nu_2+1} \\ +\chi_2(b\dot{\varphi})^{2s+1}\end{pmatrix}. \quad (4)$$

Below, for simplicity, the angle $\varphi(t)$ is subtracted from the horizontal equilibrium position of SM and therefore the parameters α_1, α_2 associated with WV base and static deformation of the SM dependence $a\alpha_1(\Delta_{cm.})^{\nu_2+1} = b\alpha_2(\Delta_{cm.})^{\nu_2+1}$. In this case, the differential equation of longitudinal-angular oscillations for the first approximation can be represented as

$$\ddot{\varphi} + \frac{1}{I_o}(\beta_1 a^{\nu_1+\nu_2+2} + \beta_2 b^{\nu_1+\nu_2+2})\dot{\varphi}^{\nu_1}\varphi^{\nu_2+1} = \frac{1}{I_o}\begin{Bmatrix}-(\chi_1 a^{s+2} + \chi_1 b^{s+2})\dot{\varphi}^{2s+1} - \\ (\alpha_1 a^{\nu_2+2} + \alpha_1 b^{\nu_2+2})\varphi^{2\nu+1} + (\nu_2+1)\Delta_{cm.} \\ (\beta_1 a^{\nu_1+\nu_2+1} + \beta_2 b^{\nu_1+\nu_2+1})\dot{\varphi}^{\nu_1}\varphi^{\nu_2}\end{Bmatrix}. \quad (5)$$

Thus, the solution of the problem has been reduced to finding and studying the solution of the nonlinear Equation (5). The above restrictions on the recovery force of the suspension system and the resistance force of the damping devices (shock absorbers) will help to find it. From these limitations it follows that the maximum value of the right-hand side of Equation (5) is a small value in comparison to the main part of the nonlinear reducing force SS (arc term of the left-hand side of this equation). This is the basis for application of the general ideas of perturbation methods [26-27] adapted in [28] to construct an asymptotic solution of a similar class of nonlinear equations. According to those, first of all it is necessary to describe the undisturbed motion of SM. These SM oscillations correspond to the differential equation

$$\ddot{\varphi} + \frac{1}{I_o}(\beta_1 a^{\nu_1+\nu_2+2} + \beta_2 b^{\nu_1+\nu_2+2})\dot{\varphi}^{\nu_1}\varphi^{\nu_2+1} = 0, \quad (6)$$

and they are described using the periodic Ateb functions [28-30] in the form

$$\varphi(t) = a_\varphi ca(\nu_2 + 1, (1-\nu_1)^{-1}, \omega(a_\varphi)t + \theta)), \quad (7)$$

where a_φ - of amplitude, $\omega(a_\phi)t + \theta$ - phase, θ - initial phase, $\omega(a_\varphi)$ - natural frequency of oscillations. The latter, as for the most nonlinear oscillatory systems, depends on the amplitude and is determined by the ratio

$$\omega(a_\varphi) = \frac{\nu_2+2}{2}\left(\frac{\frac{1}{I_o}(\beta_1 a^{\nu_1+\nu_2+2} + \beta_2 b^{\nu_1+\nu_2+2})}{\frac{2-\nu_1}{(1-\nu_1)(\nu_2+2)}}\right)^{\frac{1}{2-\nu_1}} a_\varphi^{\frac{\nu_1+\nu_2}{2-\nu_1}}. \quad (8)$$

3 Results

If it was supposed that the SM mass WV is distributed along its entire "area" evenly, then the moment of SM inertia is determined by the dependence $I_0 = \frac{m}{3}\left(\frac{a^3}{a+b} + \frac{b^3}{a+b} + c^2/4\right)$ and to describe the frequency of natural oscillations, a more convenient dependence, which takes into account the static deformation of SM can be used. Static deformation is one of the control parameters of the SM oscillatory process in order to provide the most comfortable conditions for the WV movement way with irregularities. In this case, equation (8) takes the form

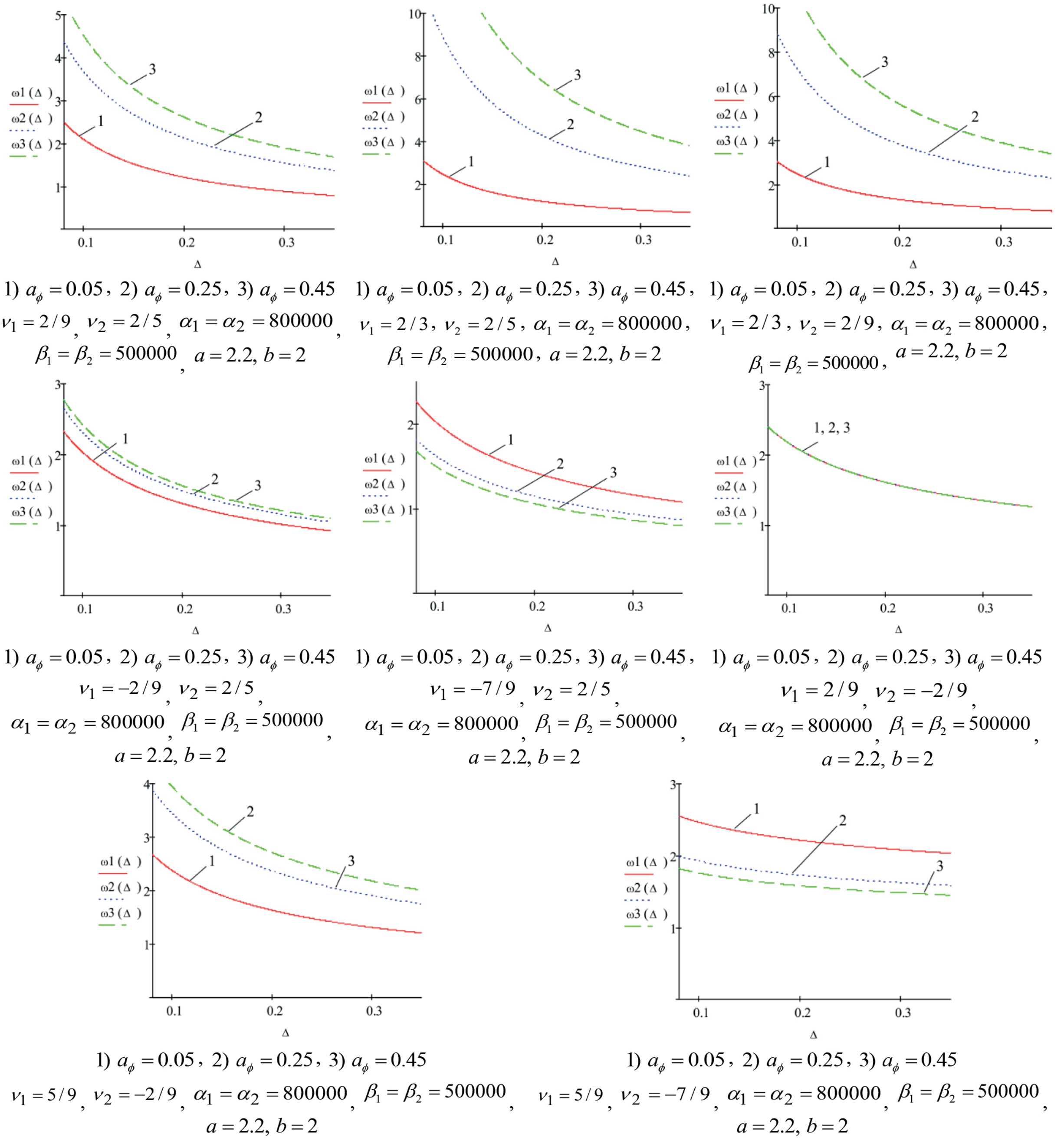
$$\omega(a_\varphi) = \frac{\nu_2+2}{2}\Theta a_\varphi^{\frac{\nu_1+\nu_2}{2-\nu_1}}, \quad (9)$$

where

$$\Theta = \left[\frac{\frac{2-\nu_1}{(1-\nu_1)(\nu_2+2)}}{\frac{3g(\beta_1 a^{\nu_1+\nu_2+2} + \beta_2 b^{\nu_1+\nu_2+2})}{(\alpha_1+\alpha_2)\Delta_{cm.}^{\nu_2+1}(a^2+b^2-ab+c^2/4)}}\right]^{\frac{1}{2-\nu_1}}.$$

Figure 2 shows the dependences of SM natural oscillation frequency in hertz $f = \frac{\omega(a_\varphi)}{2\Pi}$ from static deformation at different values of the amplitude of longitudinal-angular oscillations and parameters ν_1, ν_2 (Figure 3), Π - the half period of the Ateb functions used, i.e. $\Pi = \Gamma\left(\frac{1-\nu_1}{2-\nu_1}\right)\Gamma\left(\frac{1}{\nu_2+2}\right)\Gamma^{-1}\left(\frac{1-\nu_1}{2-\nu_1} + \frac{1}{\nu_2+2}\right)$.

Therefore, for a suspension system with a non - conservative law of change of the regenerative force of elastic shock absorbers, for the case of their greater static deformation, the frequency of natural longitudinal - angular oscillations is lower. Moreover, a larger value of the amplitude of oscillations corresponds to a larger value of the frequency, except when ν_1, ν_2, are approaching -1. At the same time in the case $\nu_1 = -\nu_2$, resulting from the Equation (9), the natural frequency is determined only by the power characteristics of the SS, that is, a similar phenomenon occurs as for linear systems (isochronous oscillations of SM). As for the dependence



1) $a_\phi = 0.05$, 2) $a_\phi = 0.25$, 3) $a_\phi = 0.45$ $\nu_1 = 2/9$, $\nu_2 = 2/5$, $\alpha_1 = \alpha_2 = 800000$, $\beta_1 = \beta_2 = 500000$, $a = 2.2,\ b = 2$

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Figure 2 *Dependence of the SM natural oscillations frequency on static deformation at different values of oscillations amplitudes and parameters* ν_1, ν_2

of the frequency (period) of oscillations on the amplitude, it is for cases $\nu_1 > 0, \nu_2 > 0; \nu_1 > 0, -1 < \nu_2 < 0$ and $-1 < \nu_1 < 0, \nu_2 > 0$ a larger value of the amplitude corresponds to a larger value of frequency (smaller period); in case $-1 < \nu_1 < 0, -1 < \nu_2 < 0$ corresponds to an increase in frequency when the amplitude remains. If it is taken into account that the dynamic loads, acting on passengers and (cargo) transported for larger values of frequency and amplitude are large, then based on the choice of SS characteristics given the dynamic loads for the case of WV movement along the road with significant irregularities, it is advisable to choose parameters adaptive suspension that satisfy the condition: $-1 < \nu_1 < 0, -1 < \nu_2 < 0$.

Equation (9) is used simultaneously to solve the inverse problem - to determine the relationship between the amplitude of perturbation of longitudinal - angular oscillations $\overline{a}_\varphi$, which are caused by the collision of WV on the inequality of the way at which the frequency of these oscillations takes values $\overline{\omega}_n$. Assuming that the vehicle is moving at a constant speed V hitting the uneven way of a smooth shape $y = g(x)$ at a point with a coordinate x_0 at inseparable contact of a wheel and a way, it has been receive that speed of perturbation of longitudinal-angular fluctuations $\hat{V}$ of the mounting points of the shock absorbers and SM is determined by

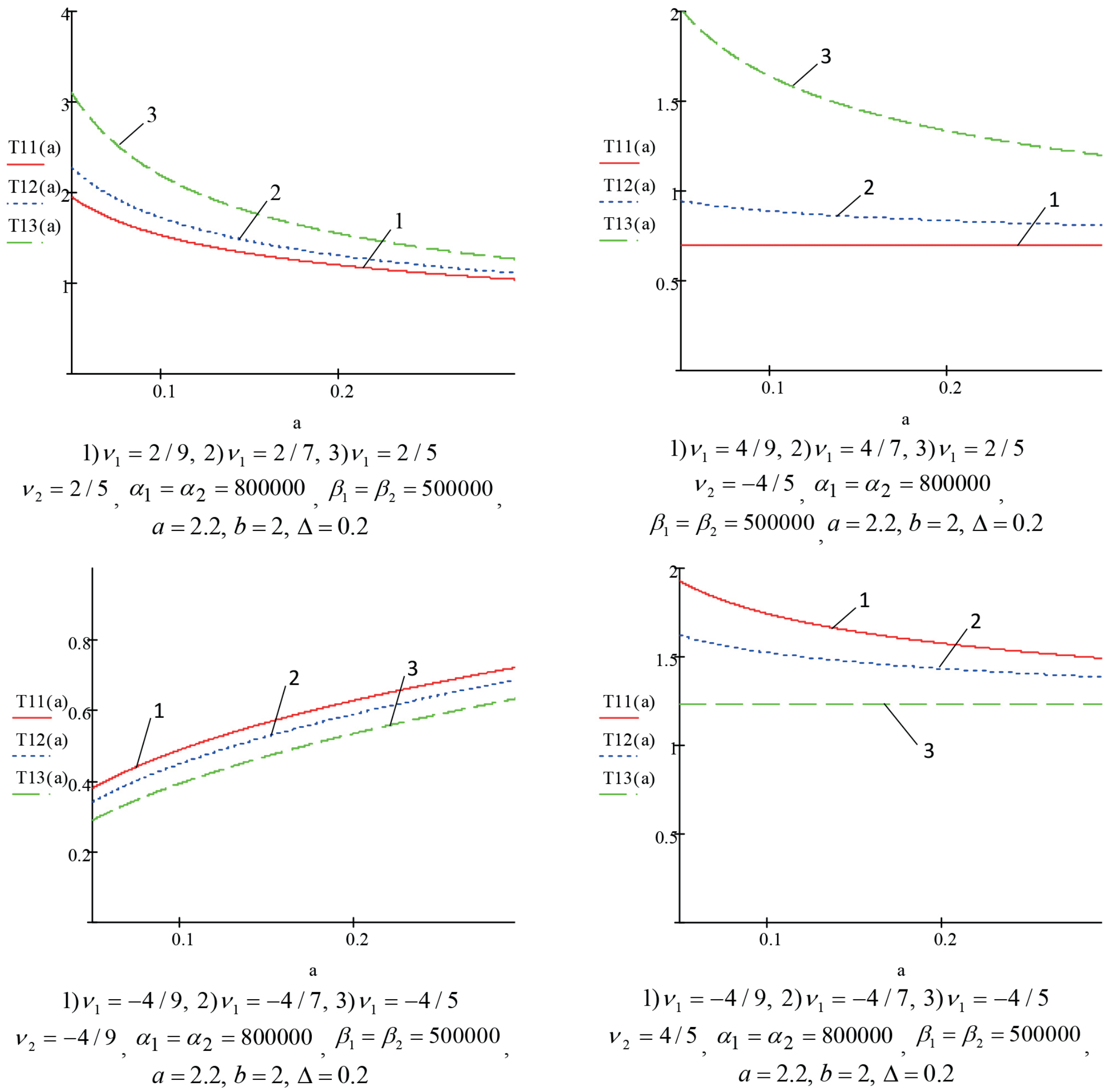


***Figure 3** Dependence of the period of the SM natural oscillations on the amplitude of longitudinal-angular oscillations at different values* ν_1, ν_2

the dependence $\hat{V} = V\sin\phi$ (ϕ - the angle of inclination of the tangent to the inequality of the path at the point with the coordinate x_0). Obviously that $tg\phi = \dfrac{dg}{dx_{|x=x0}}$. On the other hand, the value of the specified component of the speed of the mounting point of the shock absorbers and SM is equal $\hat{V} = \omega_{|x=x0} h$, h - the distance of the specified point to the axis of SM relative rotation around the point passing through the centre of mass of the specified part and, therefore, $h = \sqrt{a^2 + c^2}$.

On the other hand

$$\omega_{|x=x0} = \frac{d\varphi(t)}{dt} = \overline{a}_\varphi \times \times \frac{d(ca(\nu_2 + 1, (1-\nu_1)^{-1}, \omega(a_\varphi)t + \theta)))}{dt}_{|t-t0} = \overline{a}_\varphi \overline{\omega}_n(\overline{a}_\varphi) \frac{d(ca(\nu_2 + 1, (1-\nu_1)^{-1}, \psi))}{d\psi}_{|t-t0}, \quad \psi = \omega(a_\varphi)t + \theta, \tag{10}$$

where t_0- the moment of wheel entry on the unevenness. Limited in Equation (10) to the extreme value of the function $\dfrac{d(ca(\nu_2 + 1, (1-\nu_1)^{-1}, \psi))}{d\psi}$ the ratio that relates the required values has been obtained

$$\frac{1}{\Theta} a_\varphi^{\frac{\nu_1+\nu_2}{2-\nu_1}} - \frac{\nu_2 + 2}{2\overline{\omega}_n} \overline{a}_\varphi = 0. \tag{11}$$

It determines the amplitude of longitudinal - angular oscillations perturbation as parameters function of the suspension and frequency in the form

$$\overline{a}_\varphi = \left(\frac{\nu_2 + 2}{2\overline{\omega}_n}\Theta\right)^{\frac{\nu_1 - 2}{\nu_2 + 2}}. \tag{12}$$

If in the last expression it is passed to the velocity WV and the inequality profile, then the above is equivalent to the following

$$\overline{a}_\varphi = \left(\frac{2V\dfrac{dg}{dx}_{|x=x0}}{(\nu_2 + 2)h\Theta}\right)^{\frac{2-\nu_1}{2+\nu_2}} \tag{13}$$

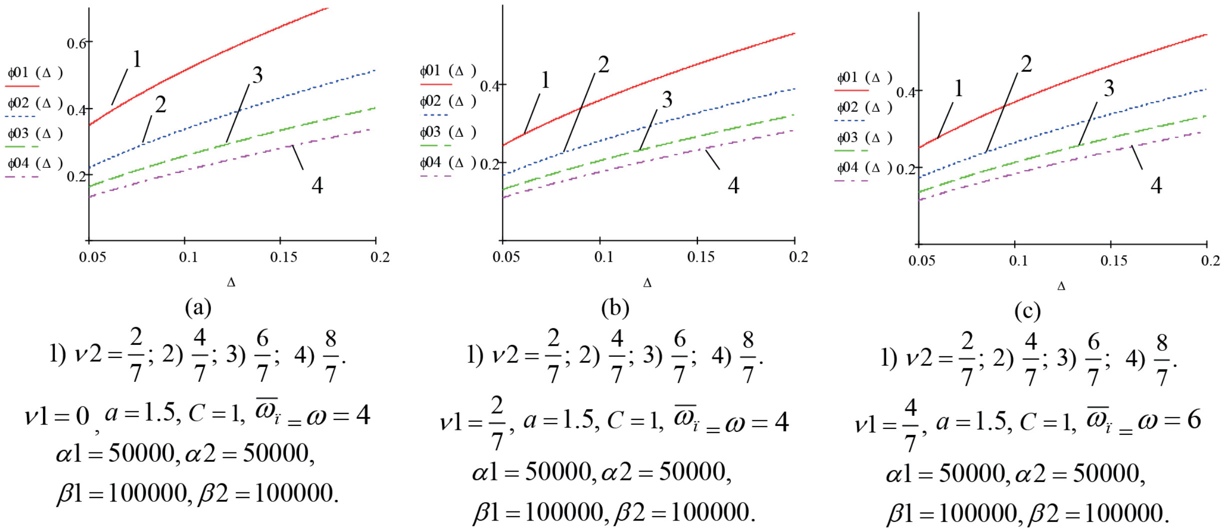


***Figure 4** Dependence of amplitudes of undisturbed motion of longitudinal - angular oscillations on static deformation of the suspension system, provided that the frequencies of these oscillations take the specified values $\overline{\omega}_n = \omega = 4$ (a), (b) and $\overline{\omega}_n = \omega = 6$ (c)???*

Figure 4 shows the dependence of the amplitude on the static deformation to the Equation (12) for various parameters that characterize WV controlled suspension, provided that the angular velocity, along the angular oscillations, takes a given value $\overline{\omega}_n$.

As expected, for an SS with a progressive law of change of regenerative force, as well as in the case of $\nu_1 > 0, \nu_2 > 0$, a larger value of static deformation corresponds to a larger value; of the amplitude of transverse-angular oscillations at which their frequency takes a given value, in addition, in the case of larger values of the parameter ν_2 it is smaller.

The above Equations (8), (11) serve as a basis for creating a CSS software product. Indeed, if for the base value of the specified suspension system to choose static deformation, it is a function of kinematic and force parameters SS is determined by the ratio

$$\Delta_{st.} = \left(\frac{\nu_2+2}{2\overline{\omega}_{n.}}\right)^{\frac{\nu_1-2}{\nu_2+1}} \times \frac{\left((\alpha_1+\alpha_2)(1-\nu_1)(\nu_2+2)\times (a^2+b^2-ab+c^2/4)\right)^{\frac{1}{1+\nu_2}}}{\left(3g(2-\nu_1)(\beta_1 a^{\nu_1+\nu_2+2}+\beta_2 b^{\nu_1+\nu_2+2})\right)^{\frac{1}{1+\nu_2}}} \times \overline{a}_{\varphi}^{\frac{\nu_2+2}{\nu_2+1}}. \quad (14)$$

Figure 5 presents the dependence of the static deformation on the amplitude of transverse-angular oscillations at which their frequency takes values $\overline{\omega}_s$.

The presented graphical dependences show that for the considered suspension system in a wide range of changes of amplitudes of longitudinal-angular oscillations the SS with small values of its static deformation satisfies ergonomic operating conditions for small amplitudes of oscillations. With increasing amplitudes of oscillations, the magnitude of static deformation should increase and the growth rate of static deformation is less for the progressive and regressive characteristics of the SS with a smaller value of the parameter ν_2; in case $\nu_1 < 0, \nu_2 < 0$ - greater for smaller values of the parameter values ν_2; in case $\nu_1 < 0, \nu_2 > 0$ - smaller for larger parameter values ν_2.

The influence of the power characteristics of the damping devices (shock absorbers) SS on the AFC oscillations SM has been determined. This can be done based on the solution of the perturbed Equation (5). The influence of these SS elements is manifested, as for all the suspensions, in time reduction of oscillations amplitude. The latter automatically causes a change in the frequency of oscillations. This change can be analytically determined based on the asymptotic solution of Equation (5). The simplest way to find it for the approximation under consideration is the method, which is based on the basic idea of the Van der Paul method [26]. According to the main idea of the abovementioned method, the perturbed motion SM is described by the dependence

$$\varphi(t) = a_\varphi ca\left(\begin{matrix}\nu_2+1, \frac{1}{1-\nu_1}, \\ \omega(a_\varphi(t))t+\theta(u)\end{matrix}\right). \quad (15)$$

The problem arises in describing the law how the unknown functions $a_\varphi(t)$ and $\theta(t)$ change depending on the form of the right-hand side of equation (5). For this purpose, by differentiating Equation (15) with respect to time, it has been obtained

$$\dot{\varphi}(t) = \dot{a}_\varphi(t) ca\left(\begin{matrix}\nu_2+1, \frac{1}{1-\nu_1}, \\ \omega(a_\varphi)t+\theta(t)\end{matrix}\right) - \frac{2}{\nu_2+2} a_\varphi(t)(\omega(a_\varphi)t)+\dot{\theta}(t))sa^{\frac{1}{1-\nu_1}} \times \left(\frac{1}{1-\nu_1}, \nu_2+1, \omega(a_\varphi)t+\theta(t)\right). \quad (16)$$

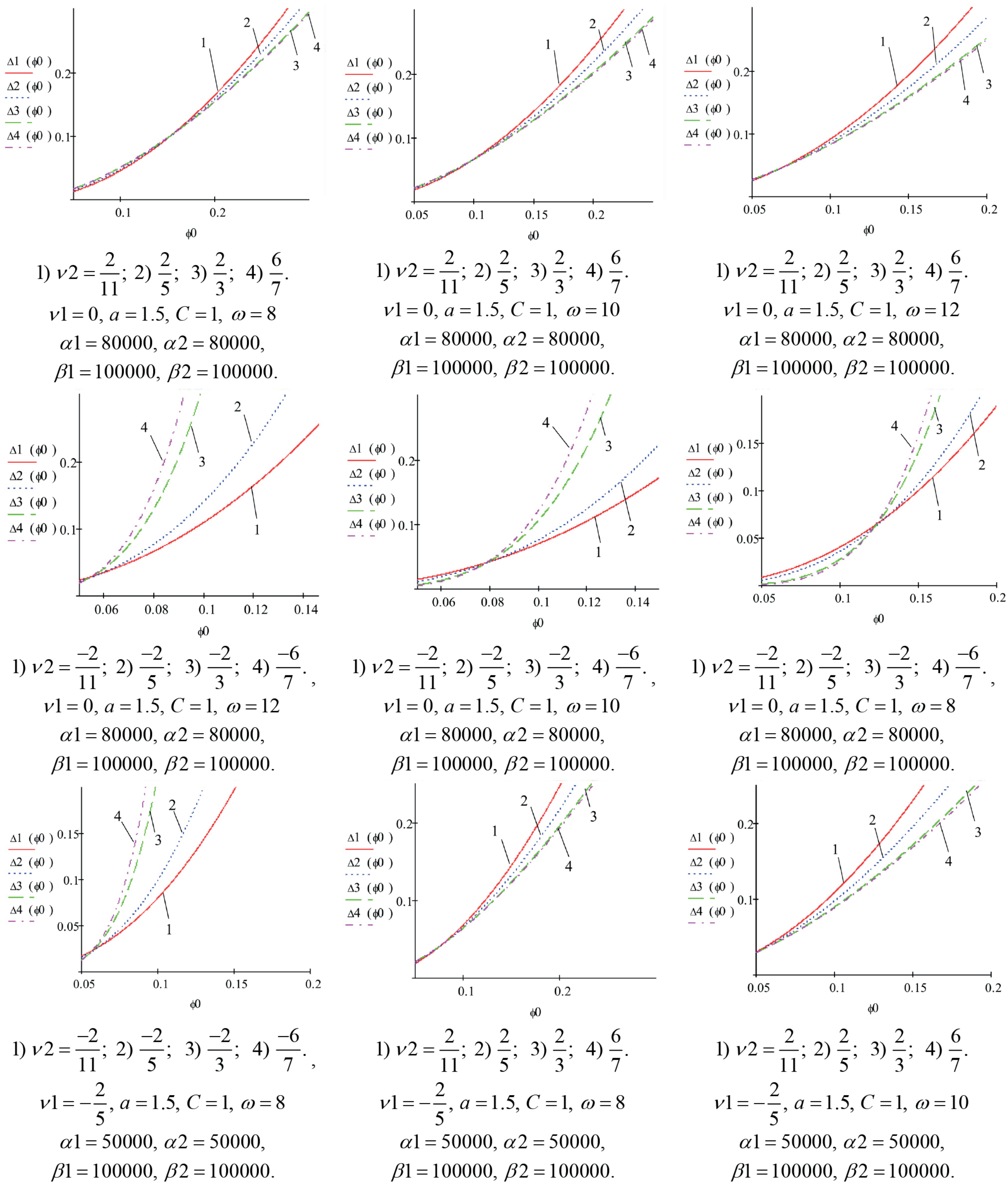


Figure 5 *Values of the SS static deformation at which the frequency of transverse angular oscillations takes set values at different oscillation amplitudes*

The following differentiation by the independent variable and taking into account the undisturbed case $\dot{\varphi}(t) = -\frac{2}{\nu_2+2} a_\varphi \omega(a_\varphi) sa^{\frac{1}{1-\nu_1}}((1-\nu_1)^{-1}, \nu_2+1, \psi))$, $\psi = \omega(a_\varphi)t + \theta(t)$, one obtains:

$$\ddot{\varphi}(t) = \frac{4}{(\nu_2+2)(2-\nu_1)} a_\varphi(t)\omega(a_\varphi(t)) sa^{\frac{\nu_1}{1-\nu_1}}((1-\nu_1)^{-1}, \nu_2+1, \psi)) ca^{\nu_2+1}\begin{pmatrix}\nu_2+1, \\ (1-\nu_1)^{-1}, \psi)\end{pmatrix} \times$$
$$(\omega(a_\varphi(t)) + \dot{\theta}(t)) - \frac{2\dot{a}_\varphi(t)}{\nu_2+2}\begin{pmatrix}\omega(a_\varphi(t)) + \\ a_\varphi \dfrac{d\omega(a_\varphi(t))}{da_\varphi}\end{pmatrix} sa^{\frac{1}{1-\nu_1}}((1-\nu_1)^{-1}, \nu_2+1, \psi). \tag{17}$$

If taking into consideration the form of the relation that describes the natural frequency of SM oscillations, then the dependence $\omega(a_\varphi) + a\frac{d\omega(a_\varphi)}{da_\varphi}$ y in Equation (17) can be replaced by a simpler one, namely: $\omega(a_\varphi) + a_\varphi\frac{d\omega(a_\varphi)}{da_\varphi} = \frac{2+\nu_2}{2-\nu_1}\omega(a_\varphi)$. Taken together allows to obtain a system of the first-order differential equations, from Equation (5), that describe the laws of change of amplitude and frequency of perturbed motion

$$\dot{a}_\varphi(t)ca(\nu_2+1,(1-\nu_1)^{-1},\psi)) - \frac{2}{\nu_2+2}\times$$
$$a_\varphi(t)\dot{\theta}(t)sa^{\frac{1}{1-\nu_1}}((1-\nu_1)^{-1},\nu_2+1,\psi)) = 0, \tag{18}$$

$$\frac{2+\nu_2}{2-\nu_1}\dot{a}_\varphi(t)sa((1-\nu_1)^{-1},\nu_2+1,\psi) + \frac{2}{(2-\nu_1)}\times$$
$$a_\varphi(t)\dot{\theta}(t)ca^{\nu_2+1}(\nu_2+1,(1-\nu_1)^{-1},\psi)) = \frac{\nu_2+2}{2\omega(a_\varphi)}\times$$
$$sa^{\frac{\nu_1}{\nu_1-1}}((1-\nu_1)^{-1},\nu_2+1,\psi))f(a_\varphi,\dot{a}_\varphi,\psi)$$

where $f(a_\varphi,\dot{a}_\varphi,\psi)$ corresponds to the value of the right-hand side of Equation (5), provided that $\varphi(t)$ and $\dot{\varphi}(t)$ take the main values. From the above system of differential equations, it has been found

$$\dot{a}_\varphi(t) = -\frac{2-\nu_1}{2\omega(a)}sa((1-\nu_1)^{-1},\nu_2+1,\psi))\times$$
$$f(a_\varphi,\dot{a}_\varphi,\psi)$$
$$\dot{\theta}(t) = -\frac{(\nu_2+2)(2-\nu_1)}{4a_\varphi\omega(a_\varphi)}ca\begin{pmatrix}\nu_2+1,\\(1-\nu_1)^{-1},\psi)\end{pmatrix}\times \tag{19}$$
$$sa^{\frac{\nu_1}{\nu_1-1}}((1-\nu_1)^{-1},\nu_2+1,\psi))f(a_\varphi,\dot{a}_\varphi,\psi).$$

It can be greatly simplified based on the following considerations: the speed (rate) of amplitude change and oscillations parameter $\dot{\theta}(t)$ are slowly changing functions of time and therefore the maximum values of their change over the period of oscillations SM are insignificant. This is the basis for averaging the right-hand sides of Equations (19) by the phase of oscillations. Thus, for the first approximation, the differential equations describing the basic parameters of the SM oscillations take the form

$$\dot{a}_\varphi(t) = -\frac{2-\nu_1}{4\omega(a_\varphi)\Pi}\left(-\frac{2}{\nu_2+2}a_\varphi(t)\omega(a_\varphi)\right)^{2s+1}$$
$$\times\frac{2\Gamma\left(\frac{2(s-\nu_1)+1}{2-\nu_1}(1-\nu_1)\right)\Gamma\left(\frac{1}{\nu_2+2}\right)}{\Gamma\left(\frac{1}{\nu_2+2}+\frac{2(s-\nu_1)+1}{2-\nu_1}(1-\nu_1)\right)} \tag{20}$$
$$\frac{d\psi}{dt} = \omega(a_\varphi).$$

Figures 6 and 7 present (for different parameters that describe the force characteristics of the suspension system) the laws of change in time of the amplitude and frequency of longitudinal-angular SM KTZ oscillations.

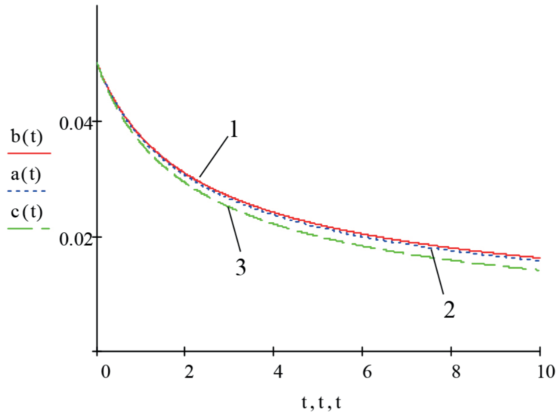
The presented graphical dependences show that the qualitative picture of the decrease (decline) of the amplitude of the longitudinal-angular parameters of the main suspension system has a negligible effect. As for the quantitative side, then, for example, for cases $\nu_1 < 0, \nu_2 > 0; \nu_1 > 0, \nu_2 < 0; \nu_1 > 0, \nu_2 > 0$ a larger value of the parameter ν_2 corresponds to a lower rate of decline in time of oscillations amplitude; for the case $\nu_1 > 0, \nu_2 > 0$ a larger value of the parameter ν_2 - lower rate of decrease of oscillations amplitude.

As for the change in time of the frequency of damped longitudinal-angular oscillations, then the parameters that characterize the suspension system affect not only the qualitative, as well as the quantitative characteristics of the change in the frequency of natural oscillations. So, for the case $\nu_1 < 0, \nu_1 > 0$ a larger value of ν_2 corresponds more to the value of the natural frequency and for cases $\nu_1 < 0, \nu_2 < 0; \nu_1 > 0, \nu_2 < 0; \nu_1 > 0, \nu_2 > 0$ a larger value of parameter ν_2 correspond to the lower values of natural frequencies. In particular, when $\nu_1 = \frac{2}{3}, a_\varphi = 0.25, \Delta_{st} = 0.2$ increasing the value of the parameter ν_2 from $\frac{2}{9}$ to $\frac{2}{5}$ causes a decrease in the frequency of oscillations in 2.65 times and at the amplitude of oscillations $a_\varphi = 0.05$ for all other values of the specified parameters - in 1.02 times.

4 Conclusion

The proposed research methodology of basic parameters influence, which characterize the nonlinear nonconservative system of suspension of a WV, has been proposed in the study and allows with accuracy necessary for engineering research, to receive analytical dependences of an estimation of their influence on defining parameters of longitudinal-angular fluctuations. The non-protection of the oscillatory process is a peculiarity of the latter, as for almost all the nonlinear oscillatory systems. Despite this, the methodology also allows to establish the influence of the main kinematic parameters of a vehicle, the main external factors of traffic disturbance on the amplitude-frequency characteristics of the longitudinal-angular oscillations of the sprung part. The latter is the basis for assessing and maximizing the WV performance, such as smoothness, dynamic stability along curved sections of road and during maneuvering, the passability of the load on the crew and passengers (transported goods), etc. Therefore, the obtained results can serve as a basis for creation of a software product of the adaptive suspension system with the studied SS power characteristics. In particular, based on the dynamic loads during the collision with the unevenness of the road, in order to minimize the latter, it is necessary that the parameters that characterize the nonlinear conservative force change within: $-1 < \nu_1 < 0, -1 < \nu_2 < 0$, at the same time, after overcoming the unevenness of the path, the adaptive suspension system must switch to their values within $\nu_1 > 0, \nu_2 > 0$.

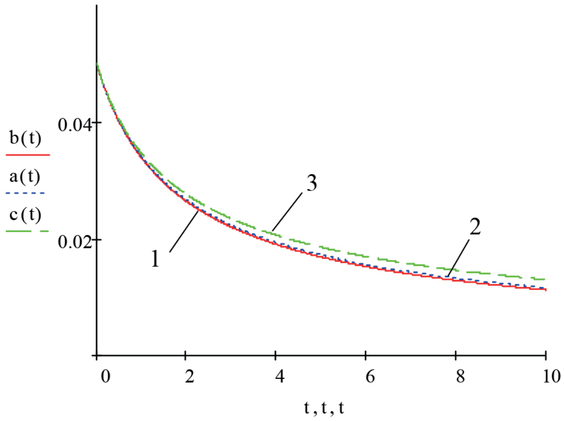
As for the specific results that follow from the obtained results and relate to the choice of another important parameter for controlling the dynamics of SM - static deformation, the ergonomic operating conditions



1) $\nu_2 = \nu 21 = 2/7$; 2) $\nu 22 = 2/9$; 3) $\nu 23 = 0$.

$\alpha_1 = \alpha_2 = 800000 ... \Delta = 0.30$

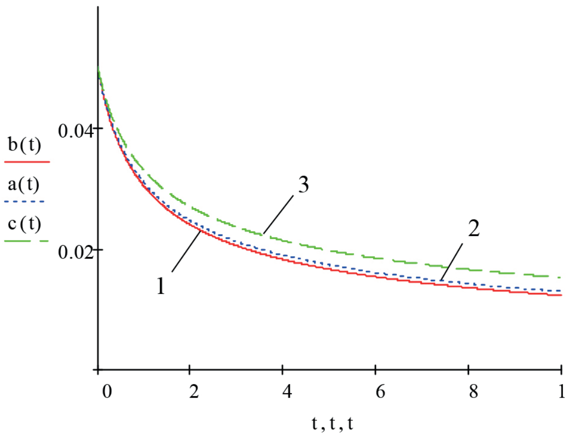
$\beta_1 = \beta_2 = 500000, \nu_1 = -2/5$



1) $\nu_2 = \nu 21 = -2/7$; 2) $\nu 22 = -2/9$; 3) $\nu 23 = 0$.

$\alpha_1 = \alpha_2 = 800000 ... \Delta = 0.25$

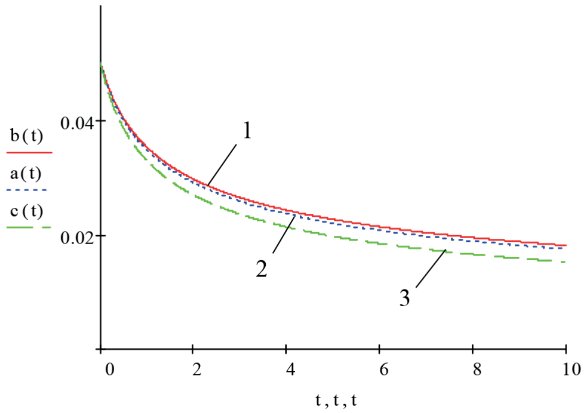
$\beta_1 = \beta_2 = 500000, \nu_1 = -2/5$



1) $\nu_2 = \nu 21 = -2/7$; 2) $\nu 22 = -2/9$; 3) $\nu 23 = 0$.

$\alpha_1 = \alpha_2 = 800000 ... \Delta = 0.25$

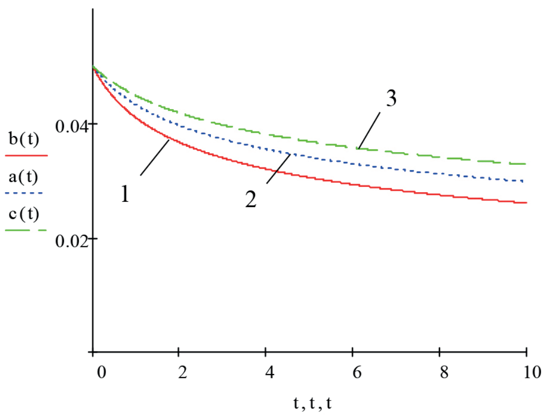
$\beta_1 = \beta_2 = 500000, \nu_1 = 2/5$



1) $\nu_2 = \nu 21 = 2/7$; 2) $\nu 22 = 2/9$; 3) $\nu 23 = 0$.

$\alpha_1 = \alpha_2 = 800000 ... \Delta = 0.25$

$\beta_1 = \beta_2 = 500000, \nu_1 = 2/5$

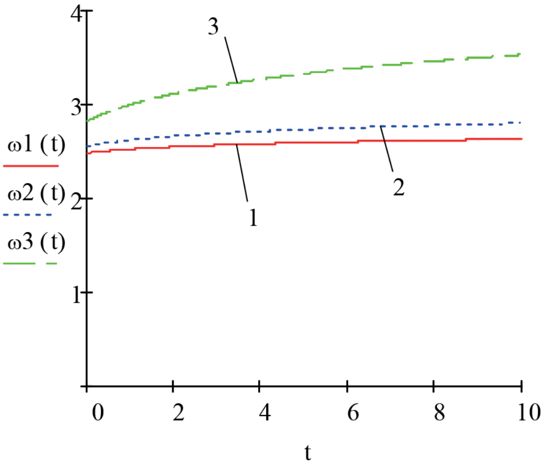


1) $\nu_2 = \nu 21 = 2/7$; 2) $\nu 22 = -2/9$; 3) $\nu 23 = 0$.

$\alpha_1 = \alpha_2 = 800000 ... \Delta = 0.25$

$\beta_1 = \beta_2 = 500000, \nu_1 = 2/5$

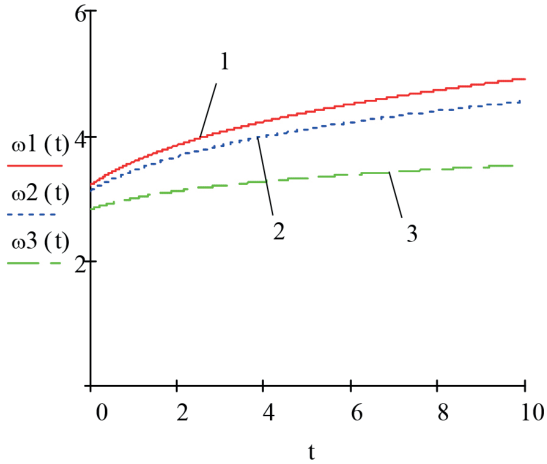
Figure 6 *Laws of decreasing amplitude of transverse-angular oscillations of the inverter at different values of its SS force characteristics*



$1)\ \nu_2 = \nu 21 = 2/7;\ 2)\nu 22 = 2/9;\ 3)\nu 23 = 0.$

$\alpha_1 = \alpha_2 = 800000...\Delta = 0.25$

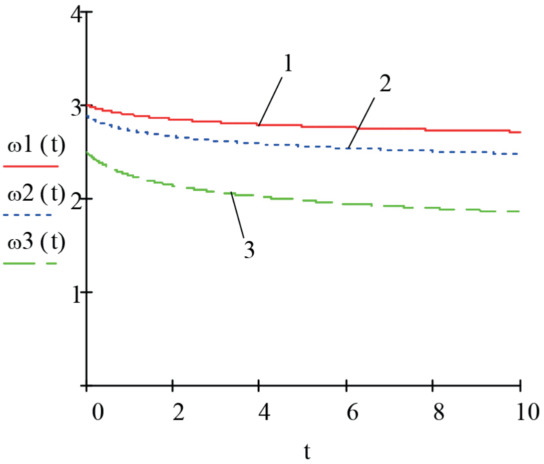
$\beta_1 = \beta_2 = 500000, \nu_1 = -2/5$



$1)\ \nu_2 = \nu 21 = -2/7;\ 2)\nu 22 = -2/9;\ 3)\nu 23 = 0.$

$\alpha_1 = \alpha_2 = 800000...\Delta = 0.25$

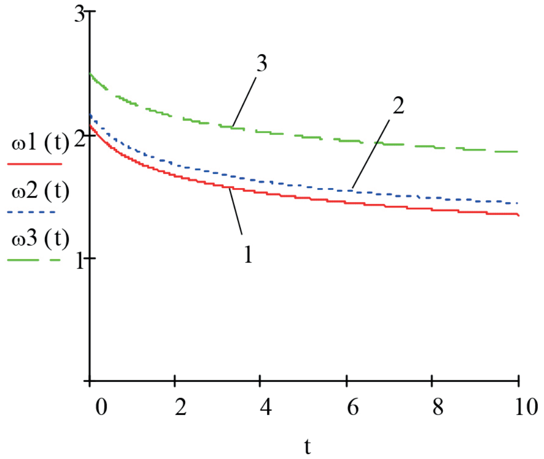
$\beta_1 = \beta_2 = 500000, \nu_1 = -2/5$



$1)\ \nu_2 = \nu 21 = -2/7;\ 2)\nu 22 = -2/9;\ 3)\nu 23 = 0.$

$\alpha_1 = \alpha_2 = 800000...\Delta = 0.25$

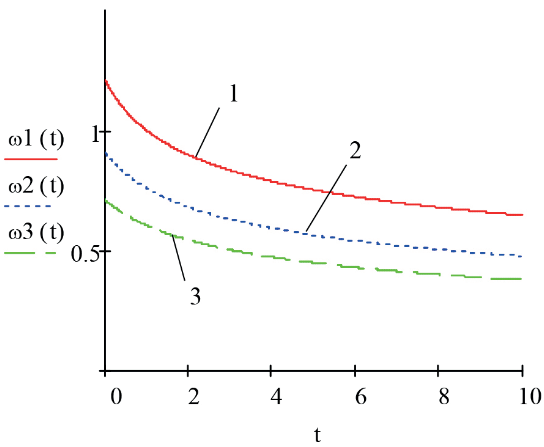
$\beta_1 = \beta_2 = 500000, \nu_1 = 2/5$



$1)\ \nu_2 = \nu 21 = 2/7;\ 2)\nu 22 = 2/9;\ 3)\nu 23 = 0.$

$\alpha_1 = \alpha_2 = 800000...\Delta = 0.25$

$\beta_1 = \beta_2 = 500000, \nu_1 = 2/5$



$1)\ \nu_2 = \nu 21 = 8/7;\ 2)\nu 22 = 8/5;\ 3)\nu 23 = 2.$

$\alpha_1 = \alpha_2 = 800000...\Delta = 0.25$

$\beta_1 = \beta_2 = 500000, \nu_1 = 2/5$

Figure 7 *Laws of change in time of the frequency of transverse-angular oscillations of the inverter at different values of its force characteristics* are more satisfied with suspensions with the following values of the nonlinearity parameters $\nu_1 > 0, \nu_2 > 0$ - for small amplitudes of oscillations and suspension deformation of elastic shock-absorbers, which change within $0.2m < \Delta_{st.} < 0.35m$ and $-1 < \nu_1 < 0,\ \nu_2 > 0$ - for large oscillation amplitudes. If the dynamic process of SM is considered as a continuous (damped oscillations after hitting the uneven path) in a wide range of changes in the amplitude of oscillations, so a controlled suspension with the following characteristics is the most favourable from the ergonomic side: $0.2m < \Delta_{st.} < 0.3m$ for $0 < \nu_1 < 2/5$ and $0 < \nu_2 < 8/7$.

The results obtained above will serve as a basis for solving no less important tasks - the study of resonant SM WV phenomena with a controlled suspension system, stability and controllability of vehicles.

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