

# FRACTAL BEHAVIOR IN BULGARIAN DAY-AHEAD PRICES BASED ON DETRENDED FLUCTUATION ANALYSIS

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## Abstract

*This study investigates the hourly prices of the electricity day-ahead market nonstationary characteristics and long-range correlation. Using the detrended fluctuation analysis (DFA) approach, we show evidence of long memory for the Bulgarian day-ahead market between January 20<sup>th</sup>, 2016 and December 31, 2019. Furthermore, the results from the DFA methodology shows that behavior of the hourly electricity spot prices returns were long term positively correlated. DFA methods can be used as powerful tools for analyzing very volatile series like electricity prices considering the fact that in recent years prices become more volatile due to increased integration of renewables that are intermittent and far more volatile than other commodities normally considered with extreme volatility. Day ahead electricity prices are crucially important for forecasting, derivatives pricing and risk management and therefore in this paper we give a brief introduction on DFA method.*

**Keywords:** Volatility, Electricity price forecasting, Day-ahead market, Detrended fluctuation analysis

## 1. INTRODUCTION

Electricity is a product that is physically and economically non-storable and as a commodity has a set of characteristics that are uncommon to other market traded products. Therefore, prices in power markets are with complex nature, extremely variable and usually they are a function of weather conditions, market and system operation, supply and demand, transmission cost, congestion and government interventions. At the day-ahead market, prices are determined by supply and demand. For grid stability and reliability, supply and demand must be matched at all times. However, the disbalance between the supply and demand may cause the price spike. Human behavior is characterized with hourly, daily and weekly routines that reflects the pattern of domestic electricity demand and intensity of business and industry. On the other hand, electricity supply depends on whether temperature, wind speed, solar radiation, hydrology and available production units. Peak demand and shortage in supplies is connected

with extreme high price volatility. Therefore, energy producers, utility companies or large industrial consumers that are able to forecast the volatile wholesale prices with a reasonable level of accuracy can adjust its bidding strategy and its own production or consumption schedule in order to reduce the risk or maximize the profits in day-ahead trading.

Day ahead electricity prices are crucially important for forecasting, derivatives pricing and risk management [1]. Price mostly acts as a reference price for the intraday market price, reference price for forward and futures contracts as well as many other derivatives contracts and is used as a price formula in the bilateral market contracts. In the recent years, volatility and price spikes in the electricity market become an important challenge for researchers. Some forecasting models are able to capture the hourly behavior of electricity price and provide forecasts for the time horizon of one month to one year as mid-term and to the time horizon of more than one year as long-term. To address these questions, in this paper will be presented an electricity forecasting (EF) approach that will empirically analyze the electricity price time series from the Bulgarian day ahead electricity market. We will use the Detrended Fluctuation Analysis (DFA) method for fractal analysis.

## **2. METHODOLOGY**

### **2.1. Long-range dependence in Bulgarian spot market**

Trough description of the temporal patterns, distributional properties and the correlation structure of the price time series the long-term memory of prices may be analyzed. Detrended fluctuation analysis may be used to analyze time series with such a complicated structure [9] as electricity spot prices. Usually, prices are non-stationary and strongly persistent, but remain strongly mean reverting, which distinguishes them from other financial prices such as stock prices and exchange rates, which follow a random walk pattern [2]. Electricity prices are highly volatile, due to the uncertainty factors as generation or transmission outages, weather changes or from sudden and unanticipated changes in the demand and supply. High volatility of electricity prices is mainly due to the non-storability of electricity itself, except the possibility of pumped hydroelectric storage facilities, so reserves could be hardly never used in case of sudden increase in demand or weather change [3]. As the prices are highly volatile, the volatility has also a tendency to cluster and volatility is characteristic by an inverse leverage effect which means that positive shocks increase price volatility more than the negative ones [4]. Electricity prices tend to “jump” very frequently and these jumps are usually “price spikes” that is typical by a sharp increase followed by a slower decrease causing pronounced asymmetry. Due to the characteristic described above, for electricity prices may be considered that they are often treated as non-stationary. Therefore, the method of multifractal fluctuation analysis is used in this paper in order to determine the fractal characterization of non-stationary and stationary time series [5]. This paper is the first detailed analysis of Bulgarian day-ahead market electricity prices and their dynamics.

## 2.2. Detrended Fluctuation Analysis

The method that will be used to measure long-range dependence is the Detrended Fluctuation Analysis (DFA) introduced by Peng [6], [7] as a special case of the multifractal detrended fluctuation analysis (MF-DFA) introduced by Kantelhardt [8]. By using the MF-DFA analysis, the generalized Hurst and the Renyi exponents will be estimated for price fluctuations. By deriving the singularity spectrum from the exponents, the multifractality of a financial time series can be quantified and the multifractal properties compared. For this purpose, logarithmic return of the time series can be used.

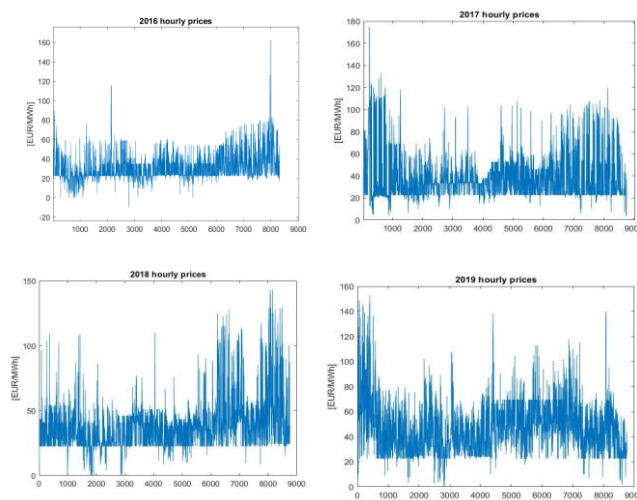
To estimate the DFA we divide a time series of returns of length  $L$  into  $d$  subseries of length  $n$ . For each subseries then  $m=1, \dots, d$  we create a cumulative time series:

$Y_{i,m} = \sum_{j=1}^i x_{j,m}$  for  $i = \dots, n$ , where  $X_{j,m}$  the  $j$  element of the  $m$  subseries we fit a least squares line  $Y_m(x) = a_m x - b_m$  to  $\{Y_{1,m}, \dots, Y_{n,m}\}$  and finally calculate the standard deviation  $F(m)$  of the integrated and detrended time series:

$$F(m) = \sqrt{\frac{1}{n} \sum_{i=1}^n (Y_{i,m} - a_m i - b_m)^2}$$

Where  $Y_{i,m}$  the cumulative series generated from  $X_{j,m}$ ,  $a_m$  and  $b_m$  the coefficient of the fitted linear regression model, used to detrend the series.

The mean value of the fluctuation for all subseries of length  $n$  is:  $\bar{F}(n) = \frac{1}{d} \sum_{m=1}^d F(m)$



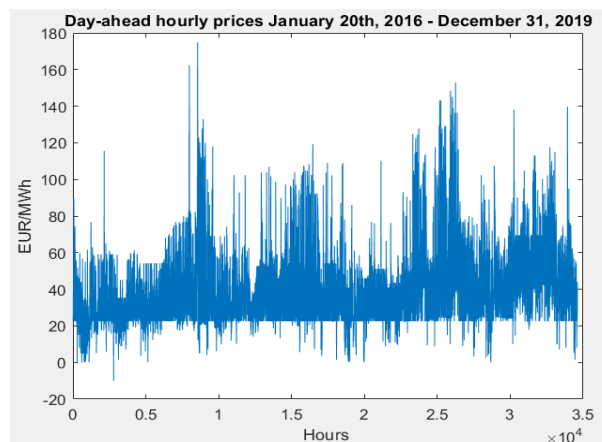
**Figure 1.** Day-ahead market clearing prices between January 20th, 2016 and December 31, 2019.  
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The linear relationship on a double-logarithmic paper of  $\bar{F}(n)$  against the interval size  $n$  indicates the presence of a power-law scaling of the form  $F(n) \sim cn^H$ , where  $H$  is the fractal DFA exponent. If  $0.5 < \alpha < 1$ , then the scaling exponent  $\alpha \approx H$ . If the slope is  $> 0.5$  the process is persistent while for an anti-persistent process the slope is  $< 0.5$ . A value of  $\alpha = 0.5$  thus indicates that there are no (or only short-range) correlations. If  $\alpha > 0.5$  for all scales, the data are long-term correlated. The higher  $\alpha$ , the stronger the correlations in the signal are.  $\alpha > 1$  indicates a non-stationary local average of the data; in this case, fluctuation analysis fails and yields only  $\alpha = 1$ . But if this parameter is equal to 1.5, then the signal is defined as Brownian noise. The case  $\alpha < 0.5$  corresponds to long-term anti-correlations, meaning that large values are most likely to be followed by small values and vice versa.

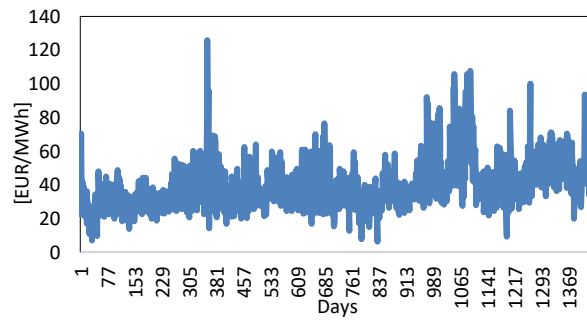
### 3. DATA ANALYSIS

We analyze hourly day-ahead spot prices of Bulgarian electricity market between January 20<sup>th</sup>, 2016 and December 31, 2019 with a total of 27 986 observations. The part of the time series consists of severe fluctuations and extreme hourly price spikes. The all-time high value occurred on Jan 10, 2017, between 12:00h-13:00h measured 174,83 EUR/MWh. Market clearing price formation is based on the aggregated energy supply and demand curve intersection point for each hour and in some hours electricity prices can increase or decrease dramatically as shown in Figure 1 form which it is visible that day ahead electricity prices are characterized as very volatile time series

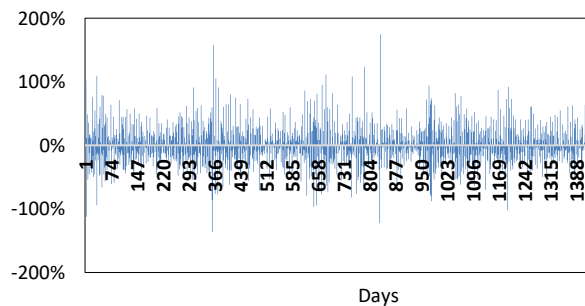
The investigated data is publicly available and can be downloaded from the Bulgarian power exchange web page. Analyzed time series data contain information of the Bulgarian day ahead market prices for every hour between January 20<sup>th</sup>, 2016 and December 31, 2019. This gives 27 986 data points for each hour, which is illustrated in Figure 2. Because the series includes a very strong daily cycle, we have created 1 442 day-long sequences of average daily prices, which is illustrated in the middle panel of Figure 3.



**Figure 2.** Day ahead market hourly clearing prices between January 20<sup>th</sup>, 2016 and December 31, 2019



**Figure 3.** Day-long sequences of average daily prices between January 20th, 2016 and December 31, 2019

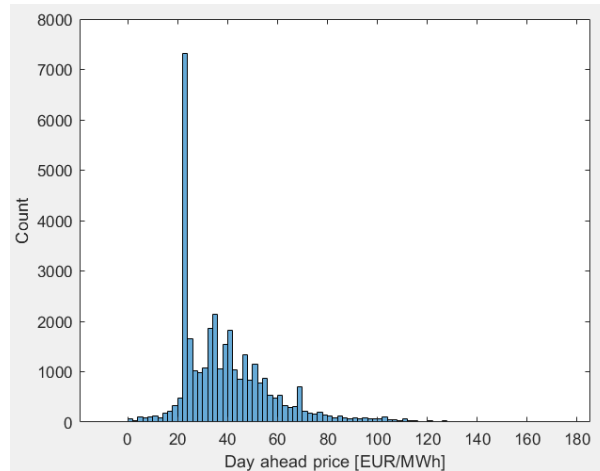


**Figure 4.** Logarithmic returns of daily average prices between January 20th, 2016 and December 31, 2019, Source-IBEX Web page

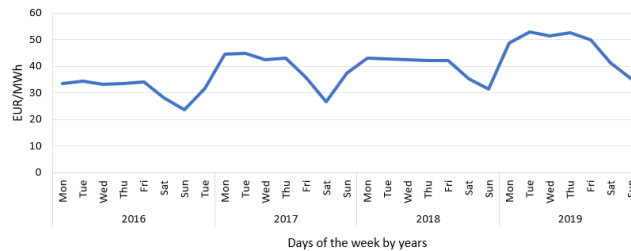
**Table 1.** Summary of IBEX hourly day-ahead prices

| <b>IBEX</b>        | <b>Prices</b> |
|--------------------|---------------|
| Count              | 27 986        |
| Mean               | 39,77648288   |
| Median             | 38,36         |
| Standard deviation | 19,4253       |
| Skewness           | 1,7609        |
| Kurtosis           | 4,3622        |
| Minimum            | (10,00)       |
| Maximum            | 174,83        |

Table 1 shows the typical characteristics of day-ahead electricity prices volatility indicators: mean, median, standard deviation, skewness, kurtosis, maximum and minimum value of the observed data of the hourly price variations for day-ahead electricity prices.



**Figure 5.** Price histogram for the day-ahead market prices between January 20th, 2016 and December 31, 2019.



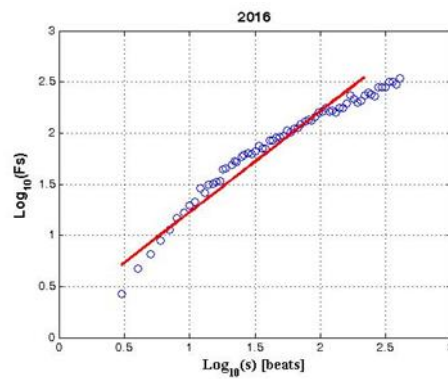
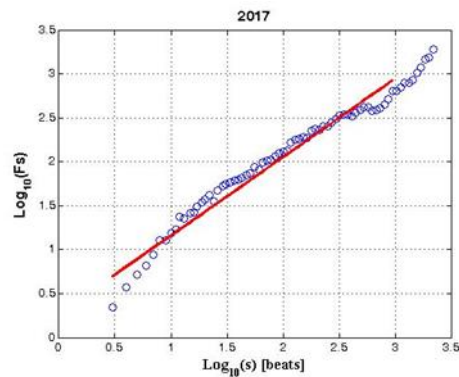
**Figure 6.** Weekly seasonality of daily spot prices, January 20th, 2016 and December 31, 2019

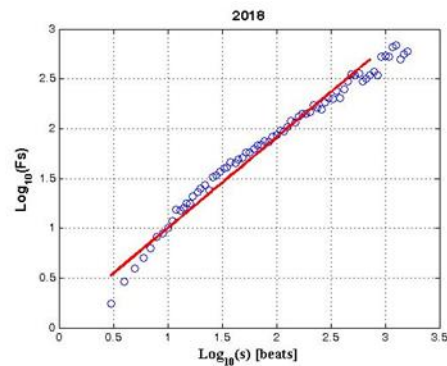
#### 4. RESULTS AND DISCUSSION

In order to analyze long-term memory for Bulgarian electricity spot prices, in this paper we used detrended fluctuation analysis. Initially it is assumed that the series contain strong cycles but might also possess long-term memory based of the analysis of the series in the Data analysis section. Detrended fluctuation analysis as a procedure that generates  $\alpha$  that is used in order to measure the fractal scaling behavior and probably evaluate how the amplitude of fluctuations in a signal depends on the scale over which they are measured. In this paper, by DFA the fractal structure of the Bulgarian hourly electricity spot prices is quantified by estimating the  $\alpha$  scaling exponent by years. The results are shown in Table 2.

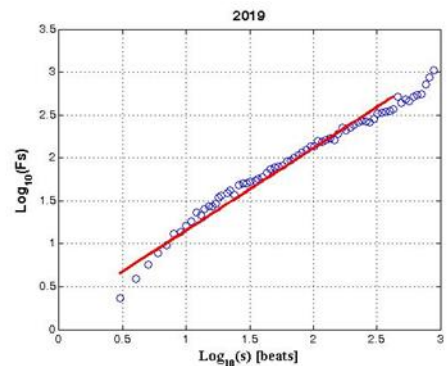
**Table 2.** Results of applying DFA

| Parameter | 2016   | 2017   | 2018   | 2019   |
|-----------|--------|--------|--------|--------|
| Alpha     | 0.9851 | 0.8924 | 0.9111 | 0.9574 |

*Figure 7.* The Plot of DFA exponent of Bulgarian hourly day-ahead market for 2016*Figure 8.* The Plot of DFA exponent of Bulgarian hourly day-ahead market for 2017



**Figure 9.** The Plot of DFA exponent of Bulgarian hourly day-ahead market for 2018



**Figure 10.** The Plot of DFA exponent of Bulgarian hourly day-ahead market for 2019.

At the figures 7, 8, 9 and 10 are plotted the results of  $\alpha$  scaling correlation exponent that quantifies the correlation properties of the price signals. The value of the exponent is found to be near to 1, for all years of data sets with small variance in 2017. Market behavior shows strong long-term correlation which means that the higher the value of  $\alpha$ , the stronger the correlations in the signal and that correlations present in the time series are larger fluctuations on longer time-scales. The data in Table 2 for  $\alpha$  shows consistency of the power spectrum and DFA methods.

## CONCLUSION

In this paper through DFA have been studied the fractal properties of the electricity spot price logarithmic increments of the hourly day ahead electricity prices in Bulgarian market. Electricity prices are a volatile and a considerable amount of the volatility is caused by the hourly, daily and weekly periodicities. Therefore, signals are analyzed by using DFA methods for analysis. The obtained results from all these methods show that the investigated signal has multifractal properties.  $F(\alpha)$  is showing how frequently events with  $\alpha$  scaling exponent may occur. The width of the fractal spectrum shows the



distinction between the maximum probability and the minimum probability. In our case, the width of spectrum reveals that there is strong multifractality in hurly day ahead prices returns.

We showed that time series price data possesses persistent behavior with long range dependence. This persistency indicates practicably the same sign for the next non-overlapping time segment in line, which means that an increase in the process is most likely to be followed by increase in the next time segment.

## ACKNOWLEDGMENTS

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