ИЗСЛЕДВАНЕ НА ВЛИЯНИЕТО НА ПАРАМЕТРИТЕ НА ДВИЖЕНИЕТО ВЪРХУ К.П.Д. НА ПРАВОТОКОВ ДВИГАТЕЛ ЗА КОЛЕСЕН МОБИЛЕН РОБОТ

A STUDY ABOUT THE IMPACT OF MOVEMENT PARAMETERS ON THE EFFICIENCY OF A DIRECT- CURRENT MOTOR FOR A WHEELED MOBILE ROBOT

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Abstract

The thematic is in the engineering (mechatronics) branch, with application in the mobile robotics. In the article have been examined: the mechanical energy at motion and its corresponding electrical consumption, as they define the energy effectiveness for the mechatronics system of a vehicle. There have been analysed dependencies, defined by the physics, in order to find out the demands, imposed on the construction "battery, electrical motor, gearing and drive wheels". The goal criterion is the achieved mileage with a single battery charge, which is determined by the dependencies "electrical consumption versus torque versus speed" at any actual motor type. The robot's dynamics parameters (weight, friction at movement, terrain slope, maximal speed and acceleration) appear as input data. Combinations between model parameters have been made, in order to be determined the optimal energy indicators as well as to choose the appropriate direct- current motor. This research uses a relatively small set of empirical physics coefficients and it can be used as an effortless methodology in some lecture notes on the mechatronics of a small wheeled mobile robot.

Keywords: Model of mechatronics system, Wheeled mobile robot, Direct current motor, Simulation and modelling, Torque and efficiency of electrical motor.

1. INTRODUCTION

Here we research the mechanical energy, consumed by a robot in motion, the necessary electrical power as well as the mechanical dynamics for units (electrical motor, gearing, drive wheels, speed sensors). It is defined the region of available solutions for design choices, where the motor has efficiency near to its maximum, at the defined robot's motion parameters.

The main goal of this research is to create an uncomplicated model for the dynamics of a mobile robot, giving the correlations between mechanical parameters (both internal and external to the robot's system) as well as consumed electrical power, in case of chosen units (electrical motor, reducer, wheels, sensors) from some commercially available sets. In this regard the following questions has been placed:

- The minimum quantity of electrical power for an electrical motor (equipped with a gearing), in order to initiate the robot's movement;
- Where the maximum efficiency appears and the „range around this point“, where the mechanical dynamics is still good at acceptable efficiency.

By an survey on a set of articles, here are discussed the most important parameters, affecting the motion dynamics and electrical consumption, alongside the analysis of main relationships, considering mechatronics drive system of a wheeled robot.

The novelty is the defining of generalized and complete set of the input limitations as well as engineering and numerical requirements. It is made in order to create a methodology, easily feasible as a numerical algorithm or a simulation model, which has sufficient accuracy, when engineering design of such class of robots has been made on the base of using the maximum number of already available construction units.

By this approach, when the set of input calculation parameters is defined, one can calculate a system which can work at near maximum efficiency and to satisfy the condition of good mechanical dynamics at small consumption from the portable battery of the robot.

2. ELECTRICAL AND MECHANICAL RELATIONSHIPS

Formulas cited below are taken from references [1, 2, 3].

Overall robot's weight G is calculated by the formula:

$$G = G_O + G_U = (1 + k_U)G_U \tag{1}$$

where: $G_O = k_U G_U$ is the own weight of the system (empty platform plus batteries); G_U is the useful load; $k_U = G_O / G_U$ is a coefficient of so called „loadability“.

When the mobile system is moving, the hysteresis losses in the wheels (friction at rolling) are obtained by the friction force at movement, which in case of path elevation (slope) is:

$$F_f = fGcos(\alpha_{max}) \tag{2}$$

where: α_{max} is the maximal angle of the path elevation; f is the friction coefficient at movement by rolling, it is given empirically [1, 3].

In case of uphill movement: the weight force is projected on the movement plane and this is called the reaction force due to the elevation:

$$F_{\mathrm{E}} = Gsin(\alpha_{max}) \tag{3}$$

These two forces F_f and F_E give a common force, called path resistance force:

$$F_\psi = [fcos(\alpha_{max}) + sin(\alpha_{max})]G \tag{4}$$

At the movement we have a turbulent resistance due to the air: a “single point” reaction when the „platform face“ with area A_F is subjected to a „wind“, at speed V_{max} of the movement. The friction of the air (in dimensions of) is given by an empirical low:

$$F_T = k_{\mathrm{T}} A_F \frac{V_{max}^2}{13} \tag{5}$$

where: $A_F = BH$ is the face area of the robot (base width B and height H); k_{T} is a coefficient of turbulence friction due to the air, and its value is empirically given [1, 3].

In case of acceleration movement, we have an inertial force, plus an additional resistance caused by the rotating masses in the reducer, thus the total inertial resistance is:

$$F_I = \delta_i \frac{G}{g} \frac{V_{max}}{t_A} \tag{6}$$

where: g is the gravity acceleration; t_A is the time for acceleration from „rest“ up to the speed V_{max} ; δ_I is a coefficient of rotating masses influence, empirically defined as:

$$\delta_I = 1{,}04 + 0.0025 i_G^2 \tag{7}$$

where: i_G is the reducer (gearing) ratio.

When passive wheels are presented, there is an additional drive's loading, which in case of rectilinear movement uphill on a elevation with some acceleration, is defined as:

$$F_P = f_P G_P cos(\alpha_{max}) + G_P sin(\alpha_{max}) + F_{IP} \tag{8}$$

where: f_P is the coefficient of movement (by rolling) for the passive wheels (usually it is the same as the friction coefficient f for the driving wheels); G_P is the weight force applied on the passive wheels (a portion of the weight, according to the ratio of the passive wheels number to the total wheels number); F_{IP} is the inertial force due to the passive wheels and likewise (to the inertial resistance formula), it is given as:

$$F_{IP} = \delta_{IP} \frac{G_P}{g} \frac{V_{max}}{t_A} \tag{9}$$

where: $\delta_{IP} = 1.0425$ is the coefficient of the passive wheels influence (the passive wheels do not have gearings, thus $i_{GP} = 1$)

In general case, all these forces must be overcome by the driving (motors) force:

$$F_M = F_\psi + F_{\mathrm{T}} + F_I + F_P \tag{10}$$

Check for skid (slip) condition: It is given by the maximal torque before skid, which can be translated by the wheel (because of its traction/cohesion to the path surface):

$$T_{\mathrm{MS}} = \phi G cos(\alpha_S) r_W \tag{11}$$

where: α_S is the elevation angle; r_W is the driving wheels radius;$\phi$is a cohesion coefficient, empirically defined by the adhesion between the wheel and the path surface.

If the motor torque is greater than the calculated maximal torque before skidding, then the robot will not move, but it will skid or roll instead, thus it defines a condition for lack of skidding, this way limiting the maximal force F_{MS} , supplied at the presence of elevation:

$$F_{MS} \le \frac{T_{\mathrm{MS}}}{r_W} \tag{12}$$

The necessary electrical power of the motor is given by the formula:

$$P_{Vmax} = \frac{F_M V_{max}}{3.6 \eta_E \eta_G} \tag{13}$$

where: η_G is the gearing mechanical efficiency; η_E is the overall electrical efficiency of the motor plus the connected to it electrical drive equipment [1, 5, 6].

It has to be chosen an appropriate motor, having the following electrical power:

$$P_{Vmax} < P_E \le 1.3 P_{Vmax} \tag{14}$$

The reserve of 30% , is in order to overcome elevation or to accelerate quickly [1, 5].

After the motor is chosen, the needed reducer (gearing) ratio is calculated:

$$i_G = \frac{\omega_E r_W}{V_{max}} \tag{15}$$

where: ω_E is the nominal angular speed of the electrical motor.

After the choosing of actual gearing ratio, a check for overcoming the maximum elevation is done: In this case the forces due to air, inertia, acceleration and dragging are neglected, thus the driving force at great elevation becomes (Form.10):

$$F_{\mathrm{M}} \approx F_\psi \mathrm{in}\alpha \rightarrow \alpha_{max} \tag{16}$$

This way, the necessary motor torque in presence of elevation, can be calculated, after translating the total force due to the path F_ψ to the motor shaft torque:

$$T_{\alpha_{max}} = \frac{r_W}{\eta_G i_G} F_\psi \tag{17}$$

The electrical motor has to have torque, greater than the calculated above value:

$$T_{max} \ge T_{\alpha_{max}} \tag{18}$$

If this condition is not satisfied, a more power motor has to be used or one has to increase the reducer ratio i_G , in the latter case, the maximal speed is decreased.

Taken into account the intended „mileage run", the necessary capacity of the accumulator battery is defined as follows (in dimensions of h):

$$C_{Bmin} = \frac{LP_E}{V_{max}U_B\eta_E} + C_{AS} \tag{19}$$

where: L is the desired mileage (in dimensions of km), provided by a single battery charge; U_B is its nominal voltage; C_{AS} is the battery capacity, used at this „mileage run" for purpose of supplying the supporting mechatronics units.

It has to be chosen a battery with a bit larger capacity:

$$C_B \geq C_{Bmin} \tag{20}$$

Then the actual mileage is calculated as a consequence of (Form.19):

$$L_A = \frac{(C_B - C_{AS})V_{max}U_B\eta_E}{P_E} \tag{21}$$

In order to find out the motor current and efficiency, the formulae given in [5, 6] are used.

Initially the motor speed coefficient is introduced:

$$u = \omega/\omega_{max} \tag{22}$$

where: ω is the momentary motor angular speed; ω_{max} is the maximal motor angular speed (taken from the motor manufacturer's data).

The torque delivered by the motor is given by:

$$T = T_{stall}(1 - u) \tag{23}$$

where: T_{stall} is the motor torque at its stall state (zero speed). These data are available from the manufacturer, or by a test.

The current consumed from the battery by the motor is as follows:

$$i = i_{stall} + u(i_{@max} - i_{stall}) \tag{24}$$

where: i_{stall} is the motor current at its stall state; $i_{@max}$ is the current at ω_{max} ; Note that the latter current value is minimum for the motor (taken from manufacturer's catalogue or by a test, when motor runs freely, id est without mechanical load).

The input electrical power is taken from the battery:

$$P_{inp} = iV \tag{25}$$

The output mechanical power is delivered to the motor shaft:

$$P_{out} = T\omega \tag{26}$$

Thus the motor efficiency is defined as follows:

$$\eta = P_{out}/P_{inp} \tag{27}$$

It is clear from above that the dependency between the torque and its corresponding speed is linear; It must be noted the motor efficiency has non- linear behaviour as there is a peak on the “Torque vs Speed vs Efficiency” graph, where the efficiency reaches its maximum.

3. ALGORITHM USED

The goal is to research the power necessary to achieve the desired speed at given path elevation. In common case we interest in speed and acceleration of the mobile robot.

In order the robot to overcome the given elevation at given speed, appropriate parameters of the motor must be chosen. The typical ranges for the robot's design are given below in the article. Additionally, the appropriate acceleration time must be chosen, in order to ensure that the necessary torque to be less then the limiting torque where the skid occurs.

Experimental simulations have been made by means of the following algorithm:

A. Find the following mechanical values: overall robot's weight, friction at movement, force due to the elevation, path resistance, driving force at great elevation (Form.1- 4, 16).
B. Initially we choose the reducer ratio to be unit, id est $i_G = 1$.
C. Calculate the mechanical values, which depend on the maximal speed of the platform and the reducer ratio: air friction force, rotating masses influence, total inertial resistance, inertial force due to the passive wheels, drive resistance, driving force (Form.5- 10).
D. Calculate the necessary motor power and choose appropriate motor (Form.13- 14).
E. Find the reducer ratio number (Form.15) and choose a reducer with appropriate gearing ratio: (close to, but a bit greater than the calculated value); Usually, the reducer ratio is a fraction with denominator and numerator being integer values, namely they are the number of teeth on the reducer wheels [1, 7, 8].
F. Calculate the necessary motor torque at elevation, then check if the chosen motor has enough torque (Form.17- 18); If this condition is not satisfied, then change the reducer ratio (to a bigger value) and repeat point (F); otherwise go to point (H).
G. If the inequality from (Form.18) can not be satisfied after the gearing ratio change, we will choose a motor having greater power and return to point (E); We prefer first to change the reducer, because much power the motor, much power the batteries and much expensive is the drive control electronics, etcetera.

H. After the inequality from (Form.18) is satisfied, we return to point (C), because the parameters depending on i_G and V_{max} have already been changed; Id est we repeat all the calculations and checks, for the concrete chosen motor and reducer, as it is likely they to be re-selected; It is an iterative process, repeated until the above conditions have been met.

I. By means of variation for the elevation angle (from 0 to the maximum given by the design assignment), one have to find the relationships between the elevation angle and the two values: the maximal torque before skidding and the maximal driving force translated at slope presence (Form.11- 12); This is aimed for choosing of some future control algorithms.

J. Calculate the necessary capacity of the battery, choose appropriate accumulator battery and find out the actual mileage available with single battery charge (Form.19- 21).

In order the robot to overcome given elevation with given speed, the proper parameters have to be chosen. The typical case of a robot's design is given below.

Hypothesis

For any chosen motor, there is a point of maximal efficiency, which occurs at just one value of the robot's speed, when the path elevation angle is given as a constant.

Or vice versa, at fixed speed the maximal efficiency exists at just one elevation angle.

Thus making the graphs of dependencies $\langle[\eta]vs[V]\rangle_{\alpha=const}$ (efficiency versus speed, at fixed slope) and $\langle[\eta]vs[\alpha]\rangle_{V=const}$ (efficiency versus elevation, at fixed speed) we create a visually descriptive method for obtaining the respective optimum efficiency values.

4. EXPERIMENTAL RESULTS

Typical parameters for this kind of robots have been chosen [3, 7]: overall vehicle weight $G_{gw} = 5kg$; weight on each of the wheels $G_w = 1.25kg$; radius of the wheels (with rubber) $R_{wheel} = 0.20m$; maximal speed $V_{max} = 2.5\, m/sec$; acceleration time (from “rest” up to the maximal speed) $t_a = 2sec$; maximal elevation angle $\alpha_s = 12^0$; the worst working path surface: concrete in good shape; number of drive wheels: 2 ; number of passive wheels: 2 .

After calculations, a Permanent Magnet Direct Current (PMDC) type motor, commercially available with built-in planetary gearing is chosen [5]. It has the following catalogue data:

Maximal motor rotational speed $N_{motor} \leq 2000[RPM]$, standard reducer ratio $i_G = 50$, torque $T_{stall} = 5.88[Nm]$ as well as given currents $i_{stall} = 2.60[A]$ and $i_{@max} = 0.22[A]$.

Below are given the calculated efficiency dependencies for the chosen motor; The points where the maximal efficiency occurs are noted on the graphs and their values are given. The speed in these points is often called “cruiser speed”:

(Fig.1) gives the efficiency at different speeds in case of horizontal plane motion (elevation is zero); The maximum efficiency is 0.573 (it includes both electrical and mechanical losses) and it is achieved at speed of $= 1.904[m/s]$.

(Fig.2) gives the efficiency at fixed speed $= 1.0[m/s]$, when the slope angle is varied; The maximum efficiency in this case is a bit low value and it is 0.458 ; This fact is due to the presence of additional forces, caused by the weight projection on the horizontal axle, according (Form.2)

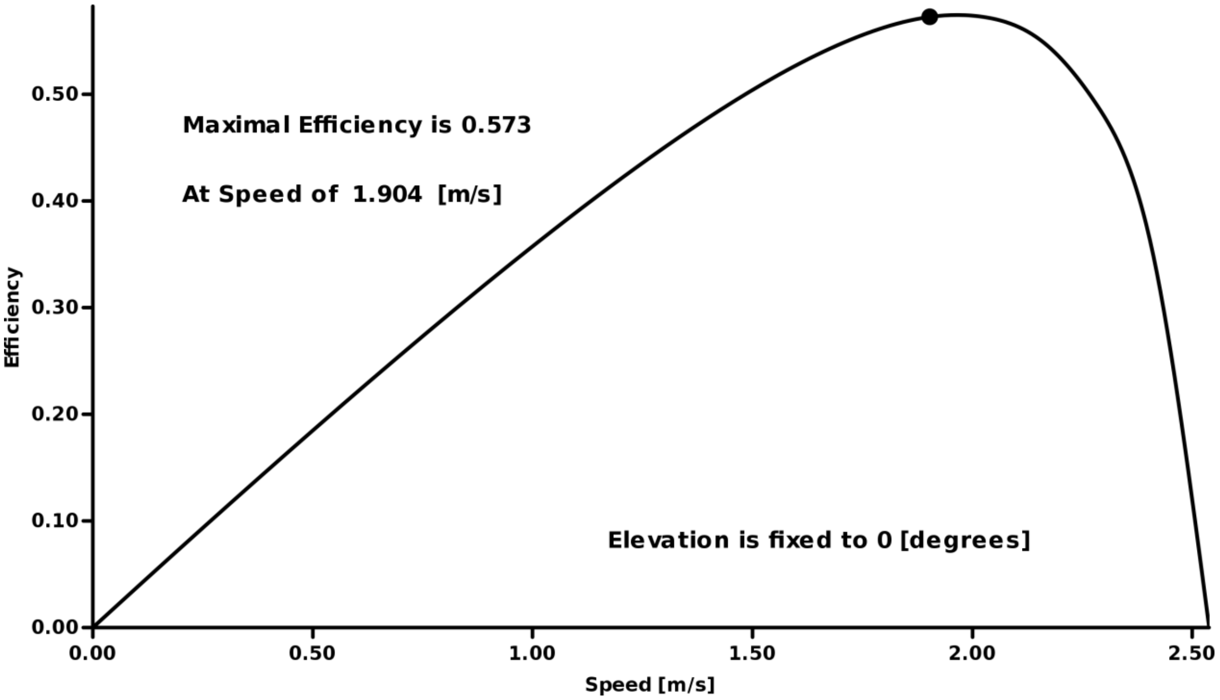

Fig. 1. *Graph of Efficiency vs Speed of the robot when the Elevation is constant.*

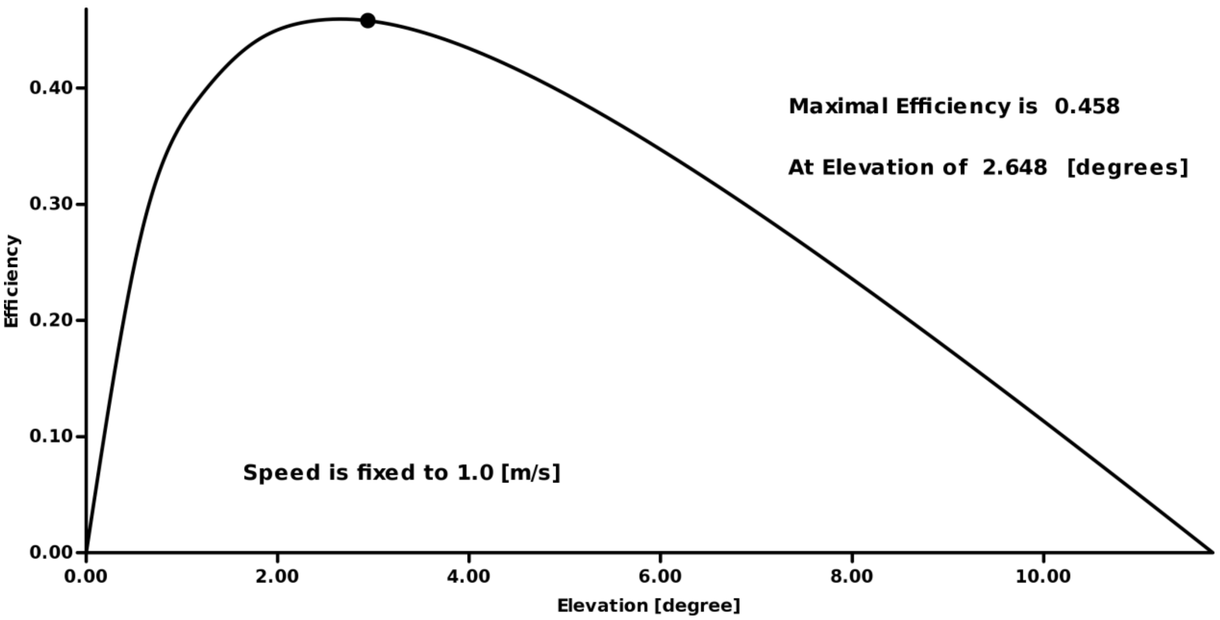

Fig. 2. *Graph of Efficiency vs Elevation when the Speed of the robot is constant.*

CONCLUSION

The variation of V_{max}, t_A and α_{max}, will give some changes in the values obtained, for example electrical power of the motor, motor angular speed and its torque.

Using the formulas about an electrical motor, given in [5, 6], in the body of the presented article, it becomes available to find out more precisely the dynamic of efficiency change, thus both the actual electrical consumption and the mileage by single battery charge (Form.21) in conditions of the concrete parameters; It gives the way how to choose the system parameters, such that either the motor to work with maximal efficiency, or it to provide a satisfactory acceleration [7, 8, 9].

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