Forest Ecology and Management 514 (2022) 120209

Contents lists available at ScienceDirect

Forest Ecology and Management

journal homepage: www.elsevier.com/locate/foreco

Mixed-effects generalized height-diameter model: A tool for forestry management of young sweet chestnut stands

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ARTICLE INFO

Keywords:
Castanea sativa Mill.
Forest management
Modeling
h-d equations
Silviculture

ABSTRACT

Height is a key variable for forest management. However, tree height measurements are expensive and time-consuming, requiring more effort to measure in the forest than diameter breast height measurements. Indeed, height-diameter (*h-d*) models are increasingly used to overcome the difficulty in measuring tree heights. Therefore, more accurate *h-d* models are increasingly needed. The mixed-effects modeling approach is a mainstream method to estimate *h-d* models. This technique was used to model the *h-d* relationship in the first 24 years of growth of sweet chestnut (*Castanea sativa* Mill.) high-forest stands for timber production. A dataset of 10,868 *h-d* observations and 57 plots of local-inventory data were considered individually. Simple mixed-effects models considering a grouping structure in the data (plot-level) were obtained, and generalized mixed-effects models were developed by expanding the fixed structure of simple mixed-effects models with stand-level variables. Several alternative model forms were tested in terms of accuracy, applicability and measurement effort. Different alternatives for calibrated predictions of tree height at plot level were analyzed, and considerations on the trade-off between easy-to-use equations in the field practice and high-accuracy equations for forest inventory were tested. The selected Richards M1a generalized mixed-effects model simultaneously provides fixed and random parameters to estimate the chestnut tree height from tree diameter and stand-level variables using the same model. The analysis showed that the inclusion of dominant height and dominant diameter as predictors improved the accuracy of the Richards model. The Draudt method was one of the best approaches to improve tree-level height prediction accuracy using mixed-effects. The applied approach is quite feasible in 100–500 m^2 plots. The use of these models and the suggested calibration process will significantly reduce the effort and costs of fieldwork teams to measure heights for forest management planning while ensuring high accuracy. This effort is greater the greater the forest density and, therefore, greater for young stands than for adult stands.

1. Introduction

The sweet chestnut (*Castanea sativa* Mill.) is one of the most important broadleaf Mediterranean species found either in natural or semi-natural forests accompanied by other plant and animal species or forming traditional orchards, often with centennial trees (Patrício et al., 2020).

In Portugal, the sweet chestnut (henceforth, chestnut) can be found in inland mountain areas, mostly in the North and Center of the country. The species covers an area larger than 48,000 ha (ha) (ICNF, 2019), including both orchards and forest woodlands (high-forest and coppice). The area covered has been progressively increasing more expressively since the 1990 s, under the Community Support Frameworks that have served as the primary funding sources for afforestation. A considerable number of recent plantations (almost 10,000 ha in the Bragança region, NE Portugal, between 1994 and 2000) have been established with the purpose of being managed as high-forest stands for high-quality timber production. Furthermore, when combined with nonwood forest products (provisioning services) and other ecosystems services such as regulating and supporting, timber production can become a profitable investment for forest owners in mountain areas. These stands promote the forest compartment, reduce the risk of wildfires, increase the diversity of the forest mosaic and habitats and improve landscape aesthetics (Patrício and Nunes, 2017), thus fulfilling the recommendations

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https://doi.org/10.1016/j.foreco.2022.120209
Received 3 December 2021; Received in revised form 2 March 2022; Accepted 31 March 2022
Available online 8 April 2022

of the Portuguese National Forest Strategy (ENF, 2015) concerning the specialization of Portuguese forest territories. In this way, chestnut stands play an important role in the Integrated Areas of Landscape Management (IALM), provided for in law DL-n°28-A/2020 of 26th of June, which aims to provide an integrated territorial approach to respond to the need for landscape planning and management at a scale that promotes resilience to fire, the valorization of natural capital and the rural economy.

Young chestnut high-forest stands for high-quality timber production are mostly found on afforested abandoned agricultural lands (afforested farmland) belonging to private landowners. Applied research is needed to provide a timely response on ways to drive and define silvicultural management models suitable for this type of stand to obtain valuable timber and complementary forest-based products based on sustainability criteria while promoting biological and landscape diversity.

The development of simple and accurate models that allow forest managers to determine with reliability the height of the trees in a stand from diameter data is a prime objective in forest management. Knowledge of the breast-height diameter (d) and the total tree height (h) is fundamental for developing growth and yield models in forest stands (Calama and Montero, 2004) and implementing models of forestry management (Bourgeois, 1992; Monteiro and Patrício, 1996; Bourgeois et al., 2004). The tree height is often needed to estimate tree volume and describe stands and their development over time (Curtis, 1967). Measuring *d* is simpler, more accurate, and cheaper than measuring *h*. For this reason, many forest inventories save time and effort by predicting tree heights using height-diameter (h-d) models instead of direct measurements (Mehtätalo et al., 2015).

Height-diameter models can be of two types: *simple* if the model does not contain stand-level predictors in addition to *d* and *generalized* if stand-level predictors are included (Mehtätalo et al., 2015; Bronisz and Mehtätalo, 2020). The *h-d* datasets are often characterized by a grouped structure. The mixed-effects modeling approach is a mainstream method employed to analyze these forestry data types (Bronisz and Mehtätalo, 2020). Mixed models estimate both fixed and random parameters simultaneously for the same model. Introducing random parameters into the model, specific for every sampling unit, enables us to model the variability detected for given phenomena among different locations after defining a common fixed functional structure (Lindstrom and Bates, 1990; Pinheiro and Bates, 2000).

In this study, the mixed-effects modeling approach was applied to the *h-d* relationship in the first 24 years of growth of chestnut high-forest stands for timber production in a multifunctional forestry approach. An update of the data from young stands used in Patrício (2006) to develop a generalized *h-d* equation was used. The updated dataset fills the information gap on heights in the 7–15 m range, which was lacking. Plot-specific and marginal predictions of tree heights are compared. Different alternatives for calibrated predictions of tree height at plot level are analyzed. Obtaining easy-to-use and high-precision equations for processing the forest inventory information is our goal. We also present considerations on the trade-off between these objectives in the field practice.

The present study also sought to provide *h-d* growth models for forest managers. These tools, suitable for juvenile phase and first stages of maturity of high-forest chestnut stands, will be available and can be used to monitor height growth and, according to the growth phase, proceed with the implementation of the respective forestry management model, whose application is based on the height, namely the mean or dominant height, as a reference for thinning applications and other tending operations, instead of the age.

2. Materials and methods

2.1. Data

Data used in this study were collected in the region of Bragança, NE Portugal, in permanent sample plots established in high-forest stands of private landowners (afforested abandoned agricultural lands) (Fig. 1). In 2002, 15 plots with an area of 3,000 m^2 each were set up in very young chestnut stands ranging from 3 to 7 years of age. The referred plot area is justified to ensure an adequate number of adult trees per hectare after ending the prescribed silvicultural management plan to produce mainly high-quality timber. In each plot, all trees were measured for the diameter at breast height *d* (1.3 m above ground) (if applicable) and total height *h*. Additional measurements of these plots were made in 2008, 2011 and 2019, so the dataset contains information about stand growth dynamics in the 3–24 years age interval. These stands were not thinned during this period. The reduction of density observed was due to plant failure in the early years of plantation. Plot coordinates, altitude, slope, soil and climatic information are described in Patrício (2006). This dataset fills the information gap on heights in the 7–15 m range, which was lacking in the database when a generalized *h-d* equation was developed for the species. Data collected in a research trial established in 1981 to study the dynamics of sweet chestnut and Douglas fir in different mixtures were also used. This trial is described in Luís and Monteiro (1998). Measurements of *d* and *h* in all the trees were available for each of three plots with pure chestnut (area of 512 m^2). This dataset contained information on the stands dynamics in the 7–19 years age interval (measurements in 1988, 1992, 1996, 1998 and 2000).

The shape of the h-d relationship is not different between sites (Fig. 1), as observed in previous studies (Patrício, 2006), but there are differences in site index reflected in dominant height growth. The statistical summary of both tree and stand variables in the total dataset is presented in Table 1.

2.2. Modeling h-d relationship

2.2.1. Functions selected

Several functions have been proposed and compared to describe the relationship between tree height and tree diameter (e.g., Huang et al., 1992; Huang et al., 2000). We selected some nonlinear functions from the list presented in HD models of R-package lmfor (Mehtätalo et al., 2015). These functions were named according to Huang et al. (1992) and Zeide (1993), and we adopted the same names (Table 2). In Table 2, HD1 to HD5 are two-parameter equations, and HD6 to HD8 are three-parameter equations.

2.2.2. Modeling approach

The modeling procedures were done in the *R*-environment (R Core Team, 2019; RStudio team, 2019), basically using the nlme package (Pinheiro et al., 2019). The dataset contained 10,868 observations, and for model fitting, each combination of local-inventory date (57 combinations that we called plots) was considered individually as in Gómez-García et al. (2015). Models were fit as simple fixed-effects to each plot in an initial exploratory data analysis. The next step consisted of fitting simple mixed-effects models considering a grouping structure in the data (plot-level) and selecting the best performing models. Furthermore, generalized mixed-effects models were developed by expanding the fixed structure of the simple mixed-effects models with stand-level variables. Generalized fixed-effects models not considering hierarchy in the data were also obtained.

2.2.3. Simple fixed-effects models

The function *groupedData* from R-package nlme was used to create the grouping structure (trees inside plots). This function creates an object of *groupedData* class with extra options for data analysis, especially graphical (Pinheiro and Bates, 2000). With function *nlsList*, the *h-d* models in Table 2 were individually fit to each plot by nonlinear ordinary least squares, which provided a perspective of the variability of the models' coefficients among plots. Moreover, an indication for possible initial values of fixed parameters when fitting mixed-effects models was obtained. Also, a first visual inspection about possible

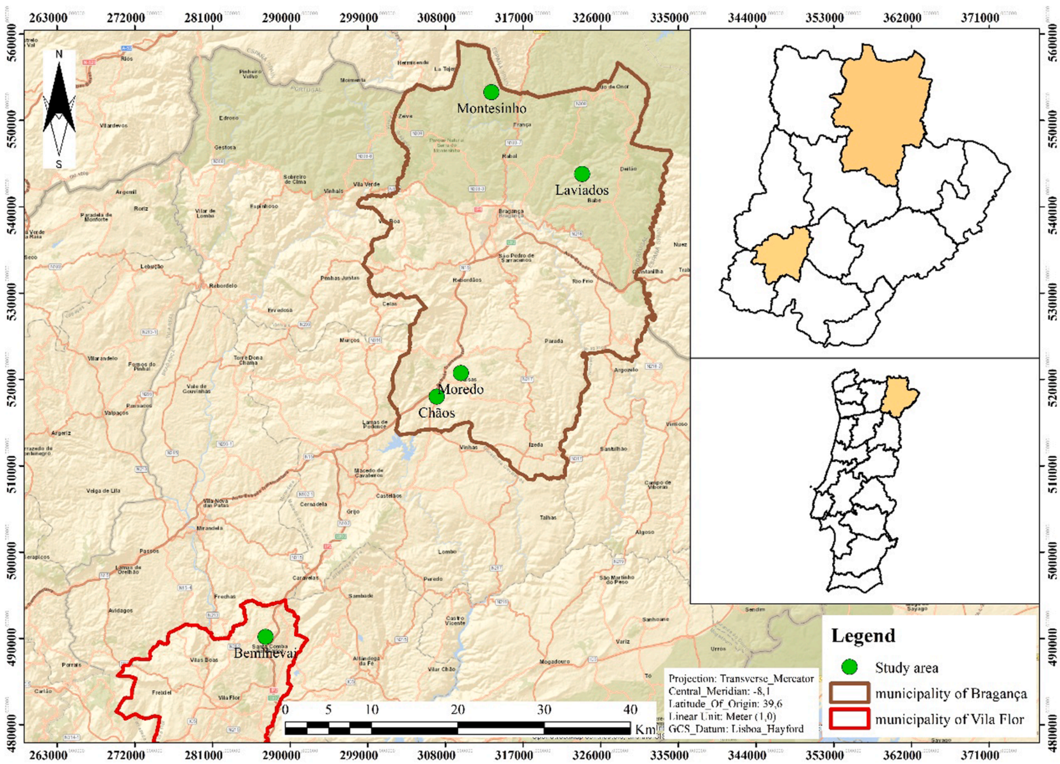

Fig. 1. Geographic location of the study area.

Table 1
Statistical summary of tree and stand variables in the total dataset.

Tree and stand variables (n° of trees = 10868, n° of plots = 57)	Mean	Minimum	Maximum	SD
d (cm)	8.1	0.3	28.3	4.8
h (m)	6.4	1.3	18.9	3.3
Age (years)	11.9	3	24	5.0
N (trees ha^{-1})	1006	220	1357	210
dg (cm)	8.9	1.5	19.1	4.0
hg (m)	6.8	2.1	15.7	3.3
G (m^2 ha^{-1})	6.9	0.1	23.0	6.1
ddom (cm)	12.2	2.3	24.1	5.3
hdom (m)	7.6	2.5	15.6	3.4
$SI_{(45)}$	24.9	15	30	2.8

d: tree diameter (1.3 m above ground); h: tree height; N: number of trees per hectare; dg: quadratic mean diameter; hg: height of the tree with d = dg; G: basal area per hectare; ddom: dominant diameter; hdom: dominant height calculated according to Assmann (1970) - the average diameter and height of the 100 thickest trees per hectare, respectively); $SI_{(45)}$: site index for the base-age 45 years. SD is the standard deviation.

Table 2
Mathematical functions selected for modeling the *h-d* relationship in young chestnut trees.

Id	Function Name	Equation
HD1	Curtis	$h = 1.3 + \beta_0 (d/(1+d))^{\beta_1}$
HD2	Naslund	$h = 1.3 + \frac{d^2}{(\beta_0 + \beta_1 d)^2}$
HD3	Meyer	$h = 1.3 + \beta_0 (1 - e^{-\beta_1 d})$
HD4	Michailoff	$h = 1.3 + \beta_0 e^{-\beta_1 d^{-1}}$
HD5	Wykoff	$h = 1.3 + exp\left(\beta_0 + \frac{\beta_1}{d+1}\right)$
HD6	Logistic	$h = 1.3 + \frac{\beta_0}{1 + \beta_1 e^{-\beta_2 d}}$
HD7	Richards	$h = 1.3 + \beta_0 (1 - e^{-\beta_1 d})^{\beta_2}$
HD8	Weibull	$h = 1.3 + \beta_0 \left(1 - e^{-\beta_1 d^{\beta_2}}\right)$

h: tree height (m); d: tree diameter (1.3 m above ground) (cm); β_0, β_1, β_2: parameters to be estimated.

heteroscedastic model errors was performed.

2.2.4. Simple mixed-effects models

Closely following the notation of Mehtätalo et al. (2015), consider the height of tree *j* on plot *i* as h_{ij}, and the corresponding tree diameter at breast height as d_{ij}, where *i* is the plot index in our case. The mixed-effects model for plot *i* is defined as,

$$h_{ij} = f(d_{ij}, \boldsymbol{\beta}_i) e_{ij} \quad (1)$$

where $f(d_{ij}; \boldsymbol{\beta}_i)$ is the systematic part of the model, and e_{ij} is the unexplained residual error. If the model form is correct and parameters are known, $f(d_{ij}; \boldsymbol{\beta}_i)$ provides a conditional mean $E(h_{ij}|d_{ij})$ on the plot. The systematic part can be either linear or nonlinear with respect to $\boldsymbol{\beta}_i$. The vector $\boldsymbol{\beta}_i$ includes parameters of the mean function for plot *i* (in our case, two or three parameters). The systematic part can vary between sample plots through the inclusion of random effects. Thus, the simple mixed-effects model can be expressed as:

$$\boldsymbol{\beta}_i = \boldsymbol{B} + \boldsymbol{b}_i \quad (2)$$

where ***B*** represents the parameters for a typical plot in the whole population of plots, and plot effects $\boldsymbol{b}_i$ express the difference in the parameters of plot *i* from the typical plot. The plot effects are assumed to have a common multivariate normal distribution, i.e., $\boldsymbol{b_i} \sim N(0, \boldsymbol{D})$ where ***D*** is a variance-covariance matrix of random effects for all values of *i*. The residual errors are assumed to be independent and normally distributed with zero mean and common constant variance ($e_{ij} \sim N(0, \sigma^2)$).

When fitting nonlinear mixed models with function *nlme* of R-package nlme, we must declare the random parameters as one of the arguments. All parameters were initially assumed random, with a general positive definite variance –covariance structure for the random effects.

If the models did not converge, which only occurred with three-parameter models, all the possible alternatives with two and one random effects were analyzed, selecting the best one. This step was based on the Akaike (AIC) and the Bayesian (BIC) information criteria (Sakamoto et al., 1986; Schwarz, 1978) and the standard deviation values and confidence intervals of random-effects estimates.

Other statistics, including the mean of the residuals (Mres), the standard deviation of the residuals (Sres), quadratic total error (QTE), obtained as the sum of squared Mres and the variance of residuals (Vres), and the root mean square error (RMSE) were computed to compare model performance and select the best options besides AIC and BIC. A goodness-of-fit index similar to the adjusted R-square (R^2_{adj}) was also computed.

Because heteroscedasticity in the residuals was observed in all the models, we applied a power-type variance function with tree diameter as a predictor, such that $var(e_{ij}) = \sigma^2 \, |w_{ij}|^{2\delta}$, where $w_{ij} = d_{ij}$ (Mehtätalo et al., 2015; Bronisz and Mehtätalo, 2020).

The best models were chosen among the two groups of simple nonlinear base models in the selection procedure, with two and three parameters, respectively. The choice between models was based mainly on the AIC and BIC values of the fits with the maximum likelihood (ML) method. However, the final estimates of the model parameters were obtained with the restricted maximum likelihood (REML) method as it produces unbiased estimates of the variance components (Littell et al., 2006).

2.2.5. Generalized mixed-effects models

The random effects in a mixed-effects model represent deviations of the individual parameters from the fixed effects. In some applications, these deviations arise from unexplained intergroup variation but, frequently, they can be at least partially explained by differences in covariate values among groups (Pinheiro and Bates, 2000). The parameter vector $\boldsymbol{B}$ in expression (2) was expanded with covariates (plot-specific predictors $\boldsymbol{x}_i$) to develop generalized mixed-effects models, such that $\boldsymbol{\beta}_i$ now becomes,

$$\boldsymbol{\beta}_i = \boldsymbol{B}(\boldsymbol{x}_i; \boldsymbol{\gamma}) + \boldsymbol{b}_i \tag{3}$$

where $\boldsymbol{\gamma}$ is a vector of parameters that are estimated in model fitting. The function $\boldsymbol{B}(\boldsymbol{x}_i; \boldsymbol{\gamma})$ is assumed to be of the linear form $\boldsymbol{x}'_i \boldsymbol{\gamma}$ (Mehtätalo et al. (2015) notation). The assumptions on the residual errors and random effects are the same as for the simple models.

According to Pinheiro and Bates (2000) recommendations, plots of the estimated random effects of the simple mixed-effects models versus stand-level covariates were used, and the respective correlations were assessed to identify interesting patterns (e.g., Adame et al., 2008; Mehtätalo et al., 2015; Bronisz and Mehtätalo, 2020). Conditional F-tests for the significance of the terms in the fixed-effect specification were used. If needed, non-nested models were compared using AIC and BIC.

2.2.6. Plot-specific predictions with mixed-effects models

With mixed-effects models, two types of predictions are possible. A fixed-effect prediction is obtained by evaluating the random effects at their expected value, i.e., when $\boldsymbol{b}_i = 0$. It provides the plot-specific *h-d* curve for a typical plot of the modeling dataset. Such predictions may be useful for a sample plot with no sample tree heights available to predict the plot effects, but one wants to produce a realistic plot-specific *h-d* curve (Mehtätalo et al., 2015). Improved plot-specific prediction is provided by random-effect prediction, where random effects predictions for the plot in question are used. Of particular interest is when we need to predict height for a new plot not included in the fitting dataset, and only a sample of trees was measured. With mixed-effects models, calibrated responses in new plots can be obtained even with a small sample size (e.g., Paulo et al., 2011; Mehtätalo et al., 2015; Sirkiä et al., 2015).

We used an approach involving a linearization of the model by a first-order Taylor approximation at the current estimated best linear unbiased predictor (EBLUP) of the random effects to localize the model at the plot level (Lindstrom and Bates, 1990). The random-effects $\boldsymbol{b}_i$ was estimated using,

$$\widehat{\boldsymbol{b}}_i = \widehat{\boldsymbol{D}}\boldsymbol{Z}_i^{\hat{A}}\left(\boldsymbol{Z}_i\widehat{\boldsymbol{D}}\boldsymbol{Z}_i^{\hat{A}} + \widehat{\boldsymbol{R}}_i\right)^{-1}\left[\boldsymbol{y}_i - f(\boldsymbol{x}_i, \widehat{\boldsymbol{B}}, \widehat{\boldsymbol{b}}_i) + \boldsymbol{Z}_i\widehat{\boldsymbol{b}}_i\right] \tag{4}$$

where *f(.)* is a nonlinear function, $\widehat{\boldsymbol{B}}$ is the vector of estimates of fixed parameters, $\widehat{\boldsymbol{D}}$ is the estimate of the matrix of variance-covariance of random effects at the plot level, $\widehat{\boldsymbol{R}}_i$ is an estimate of the error matrix, $\boldsymbol{Z}_i$ includes the partial derivatives of *f* with respect to $\boldsymbol{b}_i$. Since $\widehat{\boldsymbol{b}}_i$ appears on both sides of the equation, the process must be solved iteratively (Lindstrom and Bates, 1990). We adapted the *R* code provided in Arias-Rodil et al. (2015) to our situation. After obtaining $\widehat{\boldsymbol{b}}_i$, the function can be localized at the plot level by adding the random effects to the corresponding fixed parameters.

Even one measured sample tree per plot is enough for the prediction of $\boldsymbol{b}_i$, although increasing the number of sample trees improves the accuracy of the resulting prediction (Mehtätalo et al., 2015). Several strategies have been tested for selecting a sample of trees for calibration of *h-d* mixed-effects models (e.g., Crecente-Campo et al., 2010; Gómez-García et al., 2015). We explored three alternatives. The first consisted of applying the Draudt method (Laar and Akça, 2007) to select, in each diameter class (5 cm amplitude), one tree in each nine (i.e., 1st, 10th, 19th, …) in the 3,000 m^2 plots and one tree in each five (i.e., 1st, 6th, 11th, …) in the 512 m^2 plots. Data in each plot was previously ordered by *d*, ascendingly. The second alternative selected trees corresponding to the diameter distribution quartiles (25, 50 and 75 percentiles). Finally, the third alternative selected trees corresponding to the diameter distribution deciles (10, 20, …, 90 percentiles).

3. Results and discussion

3.1. Simple fixed-effects and mixed-effects models

In the nonlinear least-squares fitting using the *nlsList* function, the two-parameter models presented significant coefficients and converged in all the plots. With three-parameter models, convergence did not occur in some of the plots, probably because data amplitude in these plots was not sufficiently large for a good fit. In an initial observation of simple statistics as the mean and standard deviation of the residuals, we observed that Naslund (Näslund, 1937) and Wykoff (Wykoff et al., 1982) among models with two parameters and Richards (Richards, 1959) from the group of models with three parameters presented the lowest values.

Concerning the simple mixed-effects models fitted with the ML method in *nlme* and applying the power-type variance function, Wykoff was the best performer in the two-parameter group, based on AIC, BIC, and the other computed statistics (model selected as superior in the

Table 3
Statistics for mixed-effects models fitted with ML method of *nlme* and applying the power-type variance function.

Models	R^2_{adj}	Mres	SDres	AIC	BIC	QTE
Curtis	0.939	0.020	0.820	23,760	23,811	0.672
Naslund	0.940	0.011	0.815	23,568	23,619	0.664
Meyer	0.940	0.007	0.815	23,551	23,602	0.664
Michailoff	0.938	0.027	0.826	24,063	24,114	0.682
Wykoff	0.940	0.012	0.814	23,526	23,577	0.663
Logístic	0.938	0.006	0.807	24,501	24,560	0.682
Richards	0.941	0.003	0.808	23,413	23,471	0.653
Weibull	0.941	0.005	0.812	23,394	23,452	0.646

R^2_{adj}: goodness-of-fit index; Mres: mean of the residues; SDres: standard deviation of the residues; AIC: Akaike information criterion; BIC: Bayesian information criterion; QTE: quadratic total error.

highest number of statistics) (Table 3). In this group, Meyer and Naslund models presented a very similar behavior to the Wykoff model. Indeed, this similitude of performance was observed in the same analysis but not considering the correction for heteroscedasticity.

In the three-parameter model group, despite the performance of the Weibull function in Table 3, some inconsistencies were observed in this model concerning the parameters considered as random effects and between the ML fit and the final REML fit. These discrepancies were not observed with the Richards model, which was very consistent and thus was the selected one.

The best models presented R^2_{adj} values of 0.94, and all the fitted models could explain more than 93% of the variability in the response variable. All the parameters in the two-parameter models were expanded with random effects. However, three-parameter models only converged for the solutions with one or two parameters as random effects. Thus, in the Richards model, random effects were considered for parameters β_0 and β_2, as shown in Table 2. Temesgen et al. (2008) also considered random effects in these same parameters of Richards model (representing asymptotic height and curvature, respectively).

The expressions for the selected simple mixed-effects models in their respective groups are as follows:

$$Wykoff\quad M1: h = 1.3 + exp\left((\beta_0 + b_1) + \frac{(\beta_1 + b_2)}{d+1}\right) \tag{5}$$

$$Richards\quad M1: h = 1.3 + (\beta_0 + b_1)\left(1 - e^{-\beta_1 d}\right)^{(\beta_2 + b_2)} \tag{6}$$

where b_1 and b_2 are the random effects and β_0, β_1, and β_2 are the parameters of the fixed structure. The values of the parameters are presented in Table 4 according Dias (2020).

3.2. Generalized mixed-effects models

When plotting random effects (b_1 and b_2) of models (5) and (6) against stand-level variables and analyzing correlations, it was verified that the dominant height (hdom) was the plot-specific variable most correlated (positive linear) with b_1 in both the Wykoff M1 and Richards M1 models. Moderate correlations between some stand variables with b_2 were observed for the Richards M1 model, and no correlation with any particular stand variable was found in the Wykoff M1 model. The generalized mixed-effects models resulting from expanding the fixed structure of models (5) and (6) with plot-specific variables were,

$$Wykoff\quad M1a: h = 1.3 + exp\left((\beta_0 + b_1 + \beta_{01} hdom) + \frac{(\beta_1 + b_2)}{d+1}\right) \tag{7}$$

Table 4
Estimates of parameters in the Wykoff and Richards simple mixed-effects models.

Wykoff M1			Richards M1		
Parameters	Values	Standard error	Parameters	Values	Standard error
β_0	1,985	0.050	β_0	7.711	0.367
β_1	−4.745	0.108	β_1	0.103	0.006
sd(b_1)	0.372		β_2	0.897	0.028
sd (b_2)	0.734		sd(b_1)	2.396	
r (b_1, b_2)	−0.380		sd (b_2)	0.152	
σ^2	0.196^2		r (b_1, b_2)	−0.271	
δ	0.675		σ^2	0.197^2	
AIC	23,533		δ	0.671	
BIC	23,584		AIC	23,428	
			BIC	23,486	

β_0, β_1, β_2, β_2: fixed parameters; sd (b_1) and sd (b_2): standard deviation of random effects; r: correlation of random effects; σ^2: residual variance; δ: parameter of power-type variance; AIC and BIC values are from the REML fit of the models.

$$Richards\quad M1ah = 1.3 + (\beta_0 + b_1 + \beta_{01} hdom + \beta_{02} ddom)\left(1 - e^{-\beta_1 d}\right)^{(\beta_2 + b_2 + \beta_{21} G)} \tag{8}$$

where *hdom* is the dominant height (m), *ddom* is the dominant diameter (cm) and *G* is the basal area (m^2 ha^{-1}). Parameter estimates resulting from the REML fit are presented in Table 5.

The random effects in the models described by equations (7) and (8) were tested to ensure they remained significant after expanding the fixed structure. An example of plot-specific prediction patterns of tree height using these models is shown in Fig. 2.

As observed in Fig. 2, there does not seem to be an appreciable differentiation in behavior. The AIC and BIC values of the *nlme* fitting by the ML method were lower in the Richards M1a model. Therefore, we decided to utilize this model for further analyses.

3.3. Plot-specific and marginal predictions of tree height

The estimates of random effects in the generalized mixed-effects Richards M1a model are a product of the fitting procedure and were estimated with measurements of *d* and *h* in all the trees from each plot. These values can be extracted with the *ranef* function of R-package nlme. Thus, plot-specific height predictions in the fit dataset using these estimates of random effects should be a reference for comparisons with plot-specific predictions using estimates of random effects obtained with only a sample of trees of the same plot (selection by “Draudt”, “Quartile” and “Decile” methods). Next, simple statistics, namely the mean of the errors (ME), the mean of the absolute value of the errors (MAE), the variance of the errors (VE), the variance of the absolute value of the errors (VAE) and root mean square error (RMSE), based on the model errors (observed minus predicted tree heights) were computed (Table 6).

Generalized fixed-effects models were also fit to the dataset, and the error metrics described above were computed (Table 6), using the values obtained for the mixed-effects Richards M1a model as the reference. We tested variations of the Michailoff, Stofels and Van Soest, and Prodan models proposed by Soares and Tomé (2002). Moreover, variations of the Prodan model displayed suitable results for the chestnut high-forest system (Patrício, 2006) and a modification of the Michailoff model for the chestnut coppice system (Patrício et al., 2020).

From our analysis, we arrived at the variation of the Prodan model presented below and whose parameters were obtained by generalized nonlinear least squares using the *gnls* function of R-package nlme,

Table 5
Estimates of parameters in the Wykoff and Richards models with the inclusion of covariates.

Wykoff M1a			Richards M1a		
Parameters	Values	Standard error	Parameters	Values	Standard error
β_0	1.281	0.027	β_0	3.299	0.286
β_{01}	0.107	0.003	β_{01}	1.305	0.056
β_1	−4.756	0.101	β_{02}	−0.371	0.035
sd(b_1)	0.080		β_1	0.094	0.004
sd (b_2)	0.685		β_2	0.957	0.026
r (b_1, b_2)	−0.599		β_{21}	−0.017	0.003
σ^2	0.197^2		sd(b_1)	0.590	
δ	0.672		sd (b_2)	0.118	
AIC	23,364		r (b_1, b_2)	0.852	
BIC	23,423		σ^2	0.198^2	
			δ	0.666	
			AIC	23,199	
			BIC	23,280	

β_0, β_{01}, β_{02}, β_2 and β_{21}: fixed parameters; sd (b_1) and sd (b_2): standard deviation of random effects; r: correlation of random effects; σ^2: residual variance; δ: parameter of power-type variance; AIC and BIC values are from the ML fit of the models.

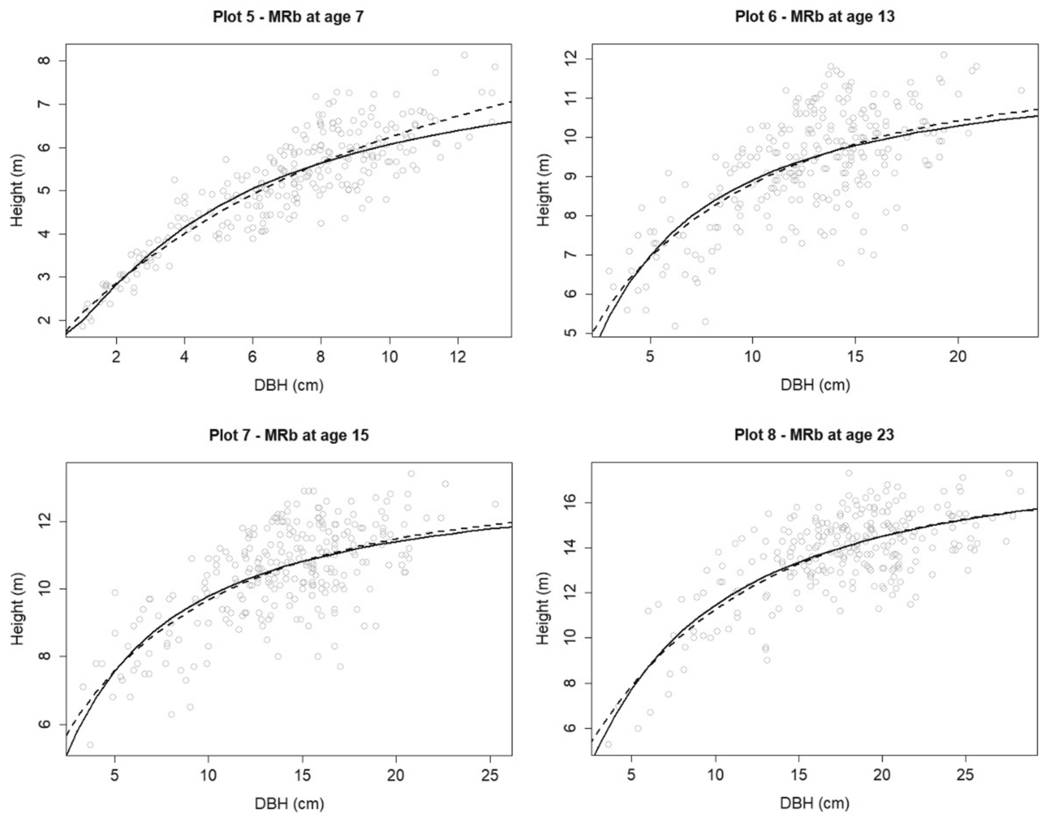

Fig. 2. Plot-specific prediction patterns of the Wykoff M1a (solid line) and Richards M1a (dashed line) models.

Table 6
Error metrics of nonlinear generalized fixed and mixed-effects models for different calibration methods (using a fit dataset).

Models	ME	MAE	VE	VAE	RMSE
(T6-A) Generalized fixed-effects models					
Richards fixed	0.0016	0.6064	0.7031	0.3354	0.8390
Prodan fixed	0.0054	0.6108	0.7246	0.3515	0.8514
(T6-B) Generalized mixed-effects models					
Richards M1a	0.0012	0.5807	0.6528	0.3156	0.8081
Richards M1b	0.0026	0.5807	0.6533	0.3161	0.8084
Richards M1	**0.0027**	**0.5806**	**0.6533**	**0.3161**	**0.8085**
(T6-C) Generalized fixed-effects models developed in previous research					
High-forest 1 (*)	0.0739	0.6584	0.8326	0.4046	0.9155
High-forest 2 (*)	0.0639	0.6560	0.8369	0.4107	0.9172
Coppice (*)	−0.0593	0.6460	0.7719	0.3581	0.8807
(T6-D) Calibration using the Draudt method					
Richards M1a	−0.0016	0.5925	0.6728	0.3216	0.8206
Richards M1b	−0.0082	0.5957	0.6833	0.3284	0.8268
Richards M1	**−0.0076**	**0.5959**	**0.6834**	**0.3284**	**0.8267**
(T6-E) Calibration using the Quartile method					
Richards M1a	−0.0170	0.5997	0.6926	0.3333	0.8326
Richards M1b	−0.1374	0.6644	0.8307	0.4081	0.9219
Richards M1	**0.1761**	**0.6358**	**0.7796**	**0.4063**	**0.9006**
(T6-F) Calibration using the Decile method					
Richards M1a	−0.0606	0.6055	0.6991	0.3361	0.8385
Richards M1b	−0.1997	0.6749	0.8211	0.4055	0.9281
Richards M1	**0.0350**	**0.6236**	**0.7627**	**0.3750**	**0.8743**

(*): These models can be found in Patrício (2006) or Patrício et al. (2020).

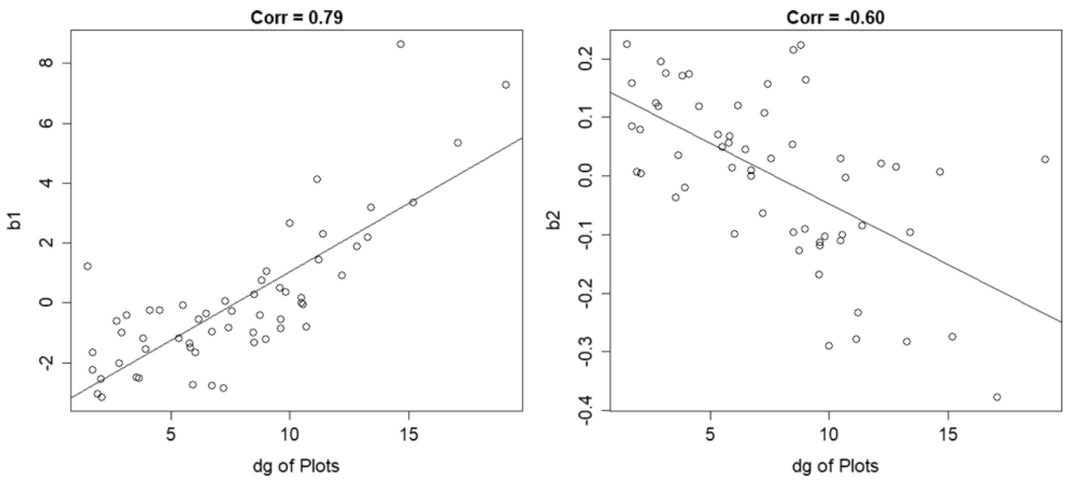

Fig. 3. Correlation of random effects of Richards M1 model (b_1 and b_2 in Eq. (6)) with the quadratic mean diameter (dg) at plot-level.

Table 7
Estimates of parameters in Richards M1b model.

Richards M1b		
Parameters	Values	Standard error
β_0	4.186	0.524
β_{01}	0.461	0.052
β_1	0.101	0.006
β_2	1.047	0.039
β_{21}	−0.021	0.005
sd (b_1)	1.482	
sd (b_2)	0.119	
r (b_1, b_2)	0.335	
σ^2	0.197^2	
δ	0.670	
AIC	23,368	
BIC	23,441	

β0, β01, β02, β2 and β21: fixed parameters; sd (b1) and sd (b2): standard deviation of random effects; r: correlation of random effects; σ^2: residual variance; δ: parameter of power-type variance; AIC and BIC values are from the REML fit of the models.

$$Prodan\ \ fixed: h = 1.3 + (hdom - 1.3)(1 + 0.9499 - 0.0299G)hdom(1/d - 1/ddom))^{-1} \tag{9}$$

where stand-level variables were described before (RMSE = 0.851; R^2_{adj} = 0.934).

The fixed structure of the Richards M1a model was also fit with *gnls* to obtain the nonlinear generalized fixed model (RMSE = 0.839; R^2_{adj} = 0.936),

$$Richardsfixed: h = 1.3 + \left(2.1784 + 1.2431hdom - 0.2879ddom\right)\left(1 - e^{-0.1110d}\right)^{(0.9408 - 0.0093G)} \tag{10}$$

Mehtätalo et al. (2015) refer the particular situation when plot-specific aggregates of diameter (e.g., $\overline{d}$, *dg*, *G*) correlate with random effects of the simple mixed-effects model. This situation is interesting because if generalized mixed-effects models are adequately developed with their fixed structure expanded with covariates of this type, no additional information is required compared to the simple fixed-effects models. As shown in Fig. 3, random effects of the Richards M1 model correlated with the quadratic mean diameter (dg) (Fig. 3). Thus, an additional generalized mixed-effects model was considered to obtain calibrated (plot-specific) predictions of tree height (Table 7),

$$Richards\ \ M1b: h = 1.3 + (\beta_0 + b_1 + \beta_{01}dg)\left(1 - e^{-\beta_1 d}\right)^{(\beta_2 + b_2 + \beta_{21}dg)} \tag{11}$$

In Table 6, we can observe that the most accurate plot-specific predictions of tree height are obtained, as expected, with the random effects of generalized mixed-effects models estimated with measurements of all trees within the plot. The lowest values of the error metrics are thus in section T6-B of Table 6. These metrics are slightly worse for the calibrated responses obtained with the Draudt method; however, the results are promising (section T6-D). As applied, the Draudt method selected about 30 trees, on average, in the 3,000 m^2 plots and about 10 trees in the 512 m^2 plots. Measuring the height of 30 sample trees for calibration requires considerable effort in terms of costs and time. If dominant height (hdom) is necessary, additional measurements are needed, as in the Richards M1a model, and the effort needed will increase. However, since this variable is of interest, it is usually obtained in forest inventories (including NFI) and applied research to characterize the rhythm of stand growth and productivity. In 500 m^2 plots, frequently used in forest inventory in Portugal, calibration using trees selected by the Draudt method requires less effort than larger plots, becoming a more attractive alternative. If *hdom* (and *ddom*) is not available or not necessary, the Richards M1b model is very useful for improved plot-specific height predictions. Notably, its behavior does not seem inferior to the Richards M1a model (Fig. 4).

Huang et al. (2009) found that the inclusion of appropriate subject-

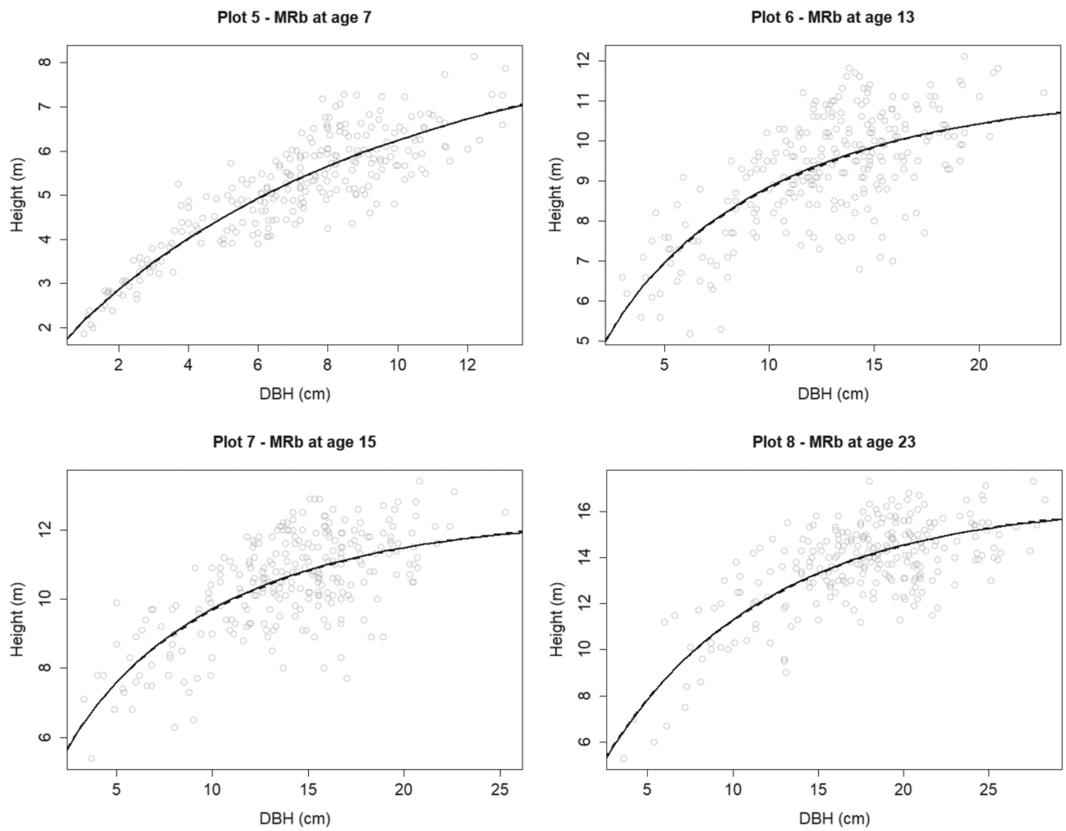

Fig. 4. Plot-specific prediction patterns of the Richards M1b (solid line) and Richards M1a (dashed line) models.

specific random parameters in the base of height-diameter models (Richards and Logistic) already allowed the plot level variations related to many known and unknown factors such as topography, soil type, nutrient status, genetics, climate, silvicultural regime, environment, and others to be accounted for without actually requiring that they be identified or measured. The results of their study have important practical implications because they demonstrate that the efforts, time, and costs associated with collecting additional variables may not be necessary. So, we calibrated the simple mixed-effects model (Eq. (6) or Richards M1 in Table 4), and the results are also presented in Table 6. It was observed that the performance of M1 is very similar to the performance of Richards M1a and M1b models.

In the other calibration methods tested (sections T6-E and T6-F), plot-specific predictions showed less accuracy than in the Draudt method, with RMSE values increasing towards the RMSE of the fixed-effect prediction of the mixed-effects model. The parameters in the fixed structure of the Richards M1a and Richards M1b models can be used to obtain plot-specific predictions of tree height when estimates of the random effects for the plot are not available.

The generalized fixed-effects models (section T6-A) fit by nonlinear least-squares present error metrics, including RMSE, close to the ones in the calibration methods, mainly the “Quartile” and “Decile” methods. These models include stand-level variables such as *hdom*, *ddom* and *G*, which can capture some of the variations in the response variable between different plots. These models assume that plots with similar values of stand-level predictors also have similar *h-d* curves (Mehtätalo et al., 2015). They predict the mean height of trees with given diameter and plot-specific predictors in the population of trees (marginal prediction or population-averaged prediction). These models do not group the observations into sample plots (Mehtätalo et al., 2015). In this sense, it is easy for a forest manager or a landowner to use this type of model in forestry practice. If used as a reference, they can provide valuable information to support silvicultural management model applications based on height growth (Bourgeois, 1992; Monteiro and Patrício, 1996; Álvarez et al., 2000; Bourgeois et al., 2004; Patrício et al., 2020), namely the thinning cycle.

In section T6-C of Table 6, error metrics are presented for generalized fixed models developed in previous studies on chestnut high-forest and coppice systems (Patrício, 2006; Patrício et al., 2020). The coppice model was developed for the first 24 years of growth, as expected, we observed a slightly faster increase in the early ages of the coppice since these trees already have a developed root system. However, the difference is not very marked.

Yuancai and Parresol (2001) considered the Bertalanffy-Richards model (Richards, 1959) one the most flexible and versatile functions available for modeling the *h-d* relationship. However, no particular function has been identified as superior (Mehtätalo, 2004, Mehtätalo et al., 2015) for this task, and mixed models based on two-parameter equations have been reported to adequately fit the *h-d* relationship, reducing the number of parameters (e.g., Bronisz and Mehtätalo, 2020).

A generalized mixed-effects model was developed to estimate tree height from tree diameter and stand-level variables (Richards M1a). The model is based on the Richards (1959) three-parameter *h-d* equation:

$$h = 1.3 + \left(3.299 + b_1 + 1.305hdom - 0.371ddom\right)\left(1 - e^{-0.094d}\right)^{(0.957 + b_2 - 0.017G)}$$

Similar studies using Richards function are found in literature (e.g. Temesgen et al., 2008; Huang et al., 2009).

Two types of plot-specific predictions are possible with such a model: an improved calibrated response if additional tree height measurements are available to estimate random effects and a prediction with random effects (b_1 and b_2) set at its expected value $\boldsymbol{b}_i = 0$, using only the parameters in the fixed structure. If *hdom* (and *ddom*) are not available, an alternative mixed-effects model can be used, identified as the Richards M1b model (eq. (11)).

From the calibration methods tested, the Draudt method yielded more accurate predictions. The way it was applied is feasible in 100–500 m^2 plots, but in 3,000 m^2 plots, the effort in the measurements could be appreciable in young stands with high density. However, 500 m^2 or less is the most frequent situation in forest inventory in Portugal. It should also be pointed out that we conducted an exercise using only 15 plots of the 57 considered to compare the Draudt method with the other three methods: trees corresponding to the quartiles of the *d* distribution, as before, plus minimum (mind) and maximum (maxd) diameter trees; trees with a diameter equal to the center of 5 cm *d* classes plus mind and maxd; the same trees as before plus the trees with a diameter equal to the upper and lower limits of 5 cm *d* classes. The exercise was performed with the Richards M1b model. It was concluded that the Draudt method was the most accurate, closely followed by the third referred method. Bronisz and Mehtätalo (2020) detected gains in predictive power when selecting trees from the extrema of the diameter range to estimate the random effects.

The chestnut has always played an important role in the rural economy of the most disadvantaged populations in the inland regions of Portugal. Chestnut is a timber-producing utilized for the most diverse purposes. However, its timber can only be considered of quality when the silviculture is properly followed up with appropriate techniques, from its installation until the end of the rotation, which may be more or less long, depending on the objective (Monteiro and Patrício, 2007; Patrício et al., 2020).

Reference silvicultural management models are available with prescriptions to guide managers and landowners to produce high-quality wood in the contexts of sustainability and multifunctional landscape (Bourgeois, 1992; Monteiro and Patrício, 1996; Álvarez et al., 2000; Bourgeois et al., 2004; Patrício et al., 2020), as the Integrated Areas of Landscape Management (IALM), provided for in law DL-n°28-A/2020 of 26th of June, aiming at a multifunctional and resilient landscape, new economic activities and remuneration for ecosystem services. In these management models the knowledge of tree height, and thus the availability of *h-d* models, is very important to adequately apply tending operations such as pruning and thinning.

4. Conclusions

The *h-d* mixed-effects model developed in this paper (Richards M1a) is the best *h-d* model for describing the first 24 years of growth of high-forest chestnut stands in afforested abandoned agricultural lands in Portugal.

Alternatively, when *hdom* is not available, the Richards M1b model is suggested since it is possible to estimate tree height based on plot-specific aggregates of diameter, which are easier to obtain with an adequate level of precision. Similar precision can be obtained using the simple mixed-effects model (Richards M1) with similar effort.

The generalized fixed-effects models (section T6-A) fit by nonlinear least-squares present error metrics close to the ones in the calibration methods. They predicted the mean tree height with given diameter and plot-specific predictors in the population of trees. If used as a reference, they can provide useful information to support silvicultural management model application, such as pruning and thinning cycles, based on height growth.

The results of this study display that when the purpose of the *h-d* model is prediction and calibration data are not available, fixed-effect (with no random parameters) or mixed-effect (random parameters $\boldsymbol{b}_i = 0$) models should be used. However, when calibration is performed, the Draudt method is one of the best approaches to improve the height prediction accuracy at the tree level using mixed-effects. This approach helps the user make the best selection in the random-effects calculation for practical applications and scenarios.

The use of these models and the suggested calibration process will significantly reduce the effort and costs of field work teams to measure heights for forest management planning while ensuring high accuracy.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

Conception of the study, data analysis, drafting of the manuscript and critical revision: MSP, LN, CRGD. Contributed materials: MSP, LN.

The authors are grateful to the Foundation for Science and Technology (FCT, Portugal) for financial support by national funds FCT/MCTES to CIMO (UIDB/00690/2020); AGRO Program, Project 267: Sustainable Management of Chestnut Forested Areas in High-Forest and Coppice Systems; Project PTDC/AGRCFL/68186/Mixed forests: Modeling, dynamics and geographical distribution of productivity and carbon storage in mixed forest ecosystems in Portugal; Project PDR2020-101-031671 GO_FTA: Afforestation of agricultural lands with + value, financial support of FEADER and Portuguese Government.

References

Adame, P., del Río, M., Cañellas, I., 2008. A mixed nonlinear height-diameter model for Pyrenean oak (*Quercus pyrenaica* Willd.). For. Ecol. Manage. 256, 88–98. https://doi.org/10.1016/j.foreco.2008.04.006.

Álvarez P., Barrio-Anta, M., Castedo-Dorado, F., Díaz-Varela, R.A., Fernández-Lorenzo, J. L., Mansilla-Vázquez, P., Pérez-Otero, R., Pintos-Varela, C., Riesco-Muñoz, G., Rodríguez-Soalleiro, R. J., Salinero-Corral, M. C., 2000. Manual de Selvicultura del Castaño en Galicia. Universidade de Santiago de Compostela (Editor), Agrobyte. http://www.agrobyte.com/agrobyte/publicaciones/castano/indice.html (accessed 27-08-2021).

Arias-Rodil, M., Castedo-Dorado, F., Cámara-Obregón, A., Diéguez-Aranda, U., Reigosa, M., 2015. Fitting and Calibrating a Multilevel Mixed-Effects Stem Taper Model for Maritime Pine in NW Spain. PLoS ONE 10 (12), e0143521.

Assmann, E., 1970. The principles of forest yield study. English transl. S.H. Gardiner, Pergamon Press Ltd., Oxford.

Bourgeois, C., 1992. Le châtaignier un arbre, un bois. IDF, 1ère ed., Paris.

Bourgeois, C., Sevrin, E., Lemaire, J., 2004. Le châtaignier un arbre, un bois. IDF, 2eme ed., Paris.

Bronisz, K., Mehtätalo, L., 2020. Mixed-effects generalized height–diameter model for young silver birch stands on post-agricultural lands. For. Ecol. Manage. 460, 117901 https://doi.org/10.1016/j.foreco.2020.117901.

Calama, R., Montero, G., 2004. Interregional nonlinear height-diameter model with random coefficients for stone pine in Spain. Can. J. For. Res. 34, 150–163. https://doi.org/10.1139/x03-199.

Crecente-Campo, F., Tomé, M., Soares, P., Diéguez-Aranda, U., 2010. A generalized nonlinear mixed-effects height–diameter model for *Eucalyptus globulus* L. in northwestern Spain. For. Ecol. Manage. 259, 943–952. https://doi.org/10.1016/j.foreco.2009.11.036.

Curtis, R.O., 1967. Height-diameter and height-diameter-age equations for second-growth Douglas-fir. For. Sci. 13, 365–375. https://doi.org/10.1093/forestscience/13.4.365.

Dias, C.R.G., 2020. Modelação do crescimento juvenil H-D com recurso a modelos de efeitos mistos em castanheiro de alto fuste no norte de Portugal. Master Thesis. Instituto Politécnico de Bragança. Bragança, Portugal. (in Portuguese).

Decree-Law-n°28-A/2020 of 26th of June. Diário da República n. ° 123/2020, 1° Suplemento, Série I de 2020-06-26. Ministry of Environment and Climate Action, Lisbon. https://data.dre.pt/eli/dec-lei/28-A/2020/06/26/p/dre.

ENF, 2015. Estratégia nacional para as florestas [National forest strategy]. Resolução do Conselho de Ministros [Resolution of the Council of Ministers] n.° 6-B/2015 (in Portuguese).

Gómez-García, E., Fonseca, T.F., Crecente-Campo, F., Almeida, L.R., Diéguez-Aranda, U., Huang, S., Marques, C.P., 2015. Height-diameter models for maritime pine in Portugal: a comparison of basic, generalized and mixed-effects models. iForest – Biog. Forest. 9, 72–78. https://doi.org/10.3832/ifor1520-008.

Huang, S., Titus, S.J., Wiens, D.P., 1992. Comparison of nonlinear height-diameter functions for major Alberta tree species. Can. J. For. Res. 22, 1297–1307. https://doi.org/10.1139/x92-172.

Huang, S., Price, D., Titus, S.J., 2000. Development of ecoregion-based height-diameter models for white spruce in boreal forests. For. Ecol. Manage. 129, 125–141. https://doi.org/10.1016/S0378-1127(99)00151-6.

Huang, S., Wiens, D.P., Yang, Y., Meng, S.X., VanderSchaaf, C.L., 2009. Assessing the impacts of species composition, top height and density on individual tree height prediction of quaking aspen in boreal mixedwoods. For. Ecol. Manage. 258, 1235–1247. https://doi.org/10.1016/j.foreco.2009.06.017.

ICNF, 2019. IFN6 - Termos e definições. [pdf], 22 pp, versão 1.0 Instituto da Conservação da Natureza e das Florestas (in Portuguese).

Laar, A., Akça A., 2007. Forest Mensuration, Springer, the Netherlands, 385 pp.

Lindstrom, M.J., Bates, D.M., 1990. Nonlinear mixed effects models for repeated measures data. Biometrics 46 (3), 673–687. https://doi.org/10.2307/2532087.
Littell, R.C., Milliken, G.A., Stroup, W.W., Wolfinger, R.D., Schabenberger, O., 2006. SAS® for mixed models, second ed. SAS Institute Inc., Cary, NC, USA.
Luís, J.F.S., Monteiro, M.L., 1998. Dynamics of a broadleaved (*Castanea sativa*) conifer (*Pseudotsuga menziesii*) mixed stands in Northern Portugal. For. Ecol. Manage. 107, 183–190. https://doi.org/10.1016/S0378-1127(97)00341-1.
Mehtätalo, L., 2004. A longitudinal height–diameter model for Norway spruce in Finland. Can. J. For. Res. 34, 131–140. https://doi.org/10.1139/x03-207.
Mehtätalo, L., de-Miguel, S., Gregoire, T.G., 2015. Modeling height-diameter curves for prediction. Can. J. For. Res. 45 (7), 826–837.
Monteiro, M.L., Patrício, M.S., 1996. O castanheiro: modelos de gestão. Revista Florestal ix(4):51-56 (in Portuguese).
Monteiro, M.L., Patrício, M.S., 2007. Conservação, regeneração e exploração do castanheiro, in: Público Comunicação Social SA, Fundação Luso-Americana para o desenvolvimento (Eds.), Árvores e Florestas de Portugal, Vol. 5, Do Castanheiro ao Teixo, Lisboa, pp 51-78 (in Portuguese).
Näslund, M., 1937. Skogsförsöksanstaltens gallringsförsök i tallskog (Forest research intitute's thinning experiments in Scots pine forests). Meddelanden frstatens skogsförsöksanstalt Häfte 29 (in Swedish).
Patrício, M.S., 2006. Análise da Potencialidade Produtiva do Castanheiro em Portugal. Dissertation, Universidade Técnica de Lisboa, Instituto Superior de Agronomia, Lisboa (in Portuguese).
Patrício, M.S., Nunes, L., 2017. Density management diagrams for sweet chestnut high-forest stands in Portugal. iForest 10, 865-870. Doi: 10.3832/ifor2411-010.
Patrício, M.S., Nunes, L., Monteiro, M.L., 2020. Does the application of silvicultural management models drive the growth and stem quality of sweet chestnut coppices towards sustainability? New For. 51, 615–630. https://doi.org/10.1007/s11056-019-09748-3.
Paulo, J.A., Tomé, J., Tomé, M., 2011. Nonlinear fixed and random generalized height-diameter models for Portuguese cork oak stands. Ann. For. Sci. 68, 295–309. https://doi.org/10.1007/s13595-011-0041-y.
Pinheiro, J., Bates, D., 2000. Mixed-effects models in S and S-PLUS. Springer, New York.
Pinheiro, J., Bates, D., DebRoy, S., Sarkar, D.R., 2019. nlme: Linear and nonlinear mixed effects models. R Package version 3.1-141, https://CRAN.R-project.org/package=nlme.
R Core Team, 2019. R: A Language and Environment for Statistical Computing. R Foundation for Statistical Computing, Vienna, Austria.
Richards, F.J., 1959. A flexible growth function for empirical use. J. Exp. Bot. 10 (29), 290–300. https://doi.org/10.1093/jxb/10.2.290.
RStudio team, 2019. RStudio: Integrated Development for R. RStudio Inc., Boston, MA.
Sakamoto, Y., Ishiguro, M., Kitagawa, G., 1986. Akaike Information Criterion Statistics. Reidel, Dordrecht, Holland.
Schwarz, G., 1978. Estimating the dimension of a model. Ann. Stat. 6, 461–464.
Sirkiä, S., Heinonen, J., Miina, J., Eerikäinen, K., 2015. Subject-specific prediction using a nonlinear mixed model: consequences of different approaches. For. Sci. 61, 205–212. https://doi.org/10.5849/forsci.13-142.
Soares, P., Tomé, M., 2002. Height-diameter equation for first rotation eucalypt plantations in Portugal. For. Ecol. Manage. 166, 99–109. https://doi.org/10.1016/S0378-1127(01)00674-0.
Temesgen, H., Monleon, V.J., Hann, D.W., 2008. Analysis and comparison of nonlinear tree height prediction strategies for Douglas-fir forests. Can. J. For. Res. 38 (3), 553–565. https://doi.org/10.1139/X07-104.
Wykoff, W.R., Crookston, N.L., Stage, A.R., 1982. User's guide to the stand prognosis model. USDA For. Serv. Gen. Tech. Rep. 133.
Yuancai, L., Parresol, B.R., 2001. Remarks on height–diameter modeling. Research Note SE-10. U.S. Department of Agriculture, Forest Service, Southeastern Forest Experiment Station, Asheville, NC. Doi: 10.2737/SRS-RN-10.
Zeide, B., 1993. Analysis of growth equations. For. Sci. 39 (3), 594–616. https://doi.org/10.1093/forestscience/39.3.594.