Analytical and Experimental study on FVA of Isotropic and Orthotropic Laminates with FEM Validations

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Abstract: *Several approaches such as two-dimensional shear deformation and higher order theories, three-dimensional exact elasticity method and 2-D, 3-D finite element procedures are available in predicting the natural frequencies of composite laminates. In all, a 3-D FEM assists in developing a model for structures with complex constraints, materials and complex geometries. But FE directs an approach that obtains an approximation in solution that depends on discretization type and the polynomials method desirable for field variable limitations. Hence, a convergence test is performed in addition to FE in order to improvise the finite element results accuracy and validating with other benchmark outcomes. The current investigation involves the solving for several boundary conditions of a thick rectangular plate's free vibration analysis [FVA] for isotropic and orthotropic materials by the application of 3-D finite element methods and convergence results are compared with available analytical results. In addition, experiments are conducted on thick isotropic and orthotropic materials along with experimental modal analysis in order to verify 3-Dimensional finite element process. From this study it can be known that, the values of finite element method highly converges with various experimental and analytical results of isotropic material. When comparing 3-D FEM with the experimental results of the orthotropic sample, a deviation of 5-8% was observed.*

Key Words: *3D FEM, Isotropic materials, orthotropic materials, modal analysis, innovation*

1. INTRODUCTION

Metals are supplanted by material composites in numerous modern industrial implementations, and in other sectors such as aviation, transportation, naval, automation industries, production technology and pressing factors due to their immense and explicit strength, economical characteristics, and precise modulus and due to many other desirable qualities they exhibit over the conventional metal constructions. These constructions made of composites exhibit a unique contrast between the properties of isotropic materials because of their orthotropic nature upon considering their individual layers. The investigation of free

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vibrations developed is a piece of examination in terms of dynamic characters in which the normal frequencies of the construction could be assessed. Here exists an accompanying section that gives a concise survey over different exploration commitments involved in the investigation of the free vibrations of constructions manufactured of fiber reinforced composites of plastics. These investigations contribute to a wider extent in the analysis of these composite structures under several applications of the composites involving analysis of free vibrations, effectiveness, economical aspects and several other factors that are required to approve a construction. This evaluation includes the principles of finite element methods and its comparison with the conclusions drawn from shear deformation theory criterion. Ajay S. Patil's [1] paper on "free vibrations analysis of thin isotropic rectangular plate" gave a peculiar view of utilizing Modified Discrete Kirchhoff quadrilateral supported by classical plate theory for examining free vibrations of the specimen. This FE formulation is assessed for performance over different test cases obtained for various boundary constraints. This Kirchhoff quadrilateral interpolation functions are used to determine the effect of free vibrations over the constructions. It can be seen that at each node the degree of freedom is 3 in this element. It has also proven to be effective in performing the desired objective. C. C. Chao [2] done research over natural free vibration and frequencies of rectangular plates that are simply supported for numerous boundary constraints developed of fixed pin, hinge-roller supported edges and derived least possible frequencies when compared to earlier methods through a 3–Dimensional energy variation approach. C. Erdem Imrak [3] had presented a report on deriving an exact solution for a four edge clamped isotropic rectangular plate's governing equation in terms of hyperbolic and trigonometric functions. It was found to be effective and easier compared to previous practice

Ehab N. Abbas [4] presented a paper, explaining the thin isotropic and orthotropic CCCC plates analysis both statically and dynamically using ANSYS. Ezeh J. C. [5] had done research regarding replacing the earlier solutions by Ordinary Finite Difference Method for free vibration analysis of thin rectangular flat plate. Evaluation of replaced differences was done for each nodal point in order to get the algebraic constraint equations that had been solved using various boundary constraints of SSSS, CCCC and CSCS models. They derived results much closer to the earlier values of the research. Kanak Kalita [6] studied the free vibration of isotropic plates using the ANSYS at various mode frequencies to study the natural frequencies effect on the rectangular isotopic plates, which contributed a lot to the effective application of composites in several industrial, aeronautical, structural and construction purposes.

S. D. Kulkarni [7] had presented a paper on the analysis for free vibration impacts over the rectangular isotropic thick plates by the reference and research over the Reddy's third-order theory. Qian L.F [8] used local Petrov-Galerkin method which is local , to find out 3-D infinitesimal electrodynamic deformations of a rectangular plate which is homogenous with various edge conditions. They installed a higher-order plate theory considering the transverse normal and transverse shear deformation theories together. Formulated outcomes had observed to be in close acceptance with those results of 3-D problem analysis by either analytical or FE Method. Analysis on free vibrations related to thin rectangular plates of isotropic type which is having various edge constraints is performed by Neffati M. Werfalli [9] by the utilization of Galerkin-based finite element method. Introducing a cubic quadrilateral with 12 degrees of freedom has played a vital role in their research. They used a least order possible equation and stated the various effects of elements, aspect ratio and sampling points on accuracy of the solution. The thin isotropic plates free transverse vibration analysis of several shapes & boundary constraints is done by Yoshihiro Narita [10]. This work has contributed to addressing the study of numerous laminar composites of different structures and shapes. It has been effective and eminent in the analysis and determination of natural frequencies for the required composite structure study and application using the finite element approach. Khdier [11] had obtained the whole group of second-order linear equations for the laminated composites based on which he analyzed free vibration characteristics exhibited by the cross-ply and anti-symmetric angle-ply. An analytical formulations and findings to the natural frequency analysis of laminated composite with simply supported type and sandwich plates are analyzed by Kant [12] using a higher order refined theory, compared solutions with numerous theories stated on the topic of multilayer sandwich plates. The 3-D elasticity theory powered with various higher order finite element techniques was utilized by Noor [13] in order to solve free vibration challenge in an effective way that analyzed the natural frequencies of the laminated plates. S. K. Panda's [14] study on free vibrational analysis has resulted in deriving the general mathematical model using higher-order shear deformation theory (HSDT) applied in case of laminated plates. K. Srividya presented variations of natural frequencies for various bonded isotropic materials like brass, copper, stainless steel and aluminum [15], and observed that the natural frequencies decrease with increase of thickness ratio[16], analyzed variations of natural frequencies with respect to various boundary conditions for different stacking sequences [17], which is useful for the design of skew plates with cutouts for dynamic response[18].For this purpose, the governing equations are obtained from the variational approach. The current examination plans to apply three-dimensional limited component methods for free vibration investigation of isotropic and orthotropic materials of thick type. Principal regular frequencies are concentrated by shifting various material sorts.

2. FEM VALIDATIONS USING DIFFERENT THEORIES

These models of finite element analysis are justified by a wide range of theories concluded from the values obtained in the reference and found to be convenient.

2.1 Validation of FEM by higher-order shear deformation theory

Panda [14] drawn a common mathematical model utilizing the concept of higher order shear deformation theory considered for laminated composite plates. Kant [12] evaluated an investigative values in free vibration of laminated composite plates using the concept of higher order refined theory. These are examined parameter effects between side length thickness proportion of material (a/h), orthotropy degree (E_1/E_2), boundary condition and lamination scheme on natural frequency. Table 1 shows validated FEM results for a given degree of orthotropy values at a particular lamination scheme $(0/90)_2$ and thickness ratio of a/h=5. By comparing these naturally developed frequencies, the current 3-D FEM outcomes are in very good agreement with the natural frequency values given by the theories of higher order shear deformation, and maximum variation is obtained 1.51% at E_1/E_2=40.

Table 1. - Finite element values validation* (ϖ of square composite laminated plate)

$E_1/E_2(0/90)_2$	3	10	20	30	40
Present (3-D FEM)	6.4402	8.0930	9.3457	10.1003	10.6119
Panda [14]	6.3601	8.0335	9.3894	10.2143	10.7747
Kant [12]	6.4319	8.1010	9.4338	10.2463	10.7993

*$E_1= 40E_2$, $G_{12}= G_{13}= 0.6E_2$, $G_{23}= 0.5E_2$ & $\nu_{12}= 0.25$

where the normalized frequency, $\varpi=\omega a^2/h(\rho/E_2)^{1/2}$, ω= Natural frequency

2.2 Validation of FEM by second-order shear deformation theory

Khdeir [11] studied laminated composite plates second order theory with the help of a total set of its linear equations, for analysis of free vibration behaviour of anti-symmetric angle-ply and cross-ply laminated plates. Table 2 shows validated FEM results for a given thickness ratio of L/t=5 for various periphery constraints of ssss and sscc at a particular lamination scheme [45°/-45°/45°/-45°]. By comparing the natural frequencies, the present 3-D FEM outcomes are in agreement with the given natural frequency values of second order shear deformation theory and maximum variation is obtained 1.005% at ssss.

Table 2. - Finite element values validation* (ϖ of square composite laminated plate)

Thickness ratio of L/t	L/t=5	L/t=5
Boundary conditions	ssss	sscc
Present (3-D FEM)	12.80	13.21
Khdeir [11]	12.93	13.46

*$E_1= 40E_2$, $G_{12}= 0.6E_2$, $G_{23}= 0.5E_2$, $\nu_{12}= \nu_{13}= 0.25$ & $G_{13}= G_{12}$,
where the non-dimensional frequency parameter,$\varpi=\omega L^2/t\ (\rho/E_2)^{1/2}$

2.3 Validation of FEM using 3- D elasticity theory

Chao [2] studied the contrast of lowest natural frequencies of rectangular laminates by using the theory of 3-D elasticity, with various boundary conditions. Noor [13] solved the free vibration problem using the 3-D elasticity theory with higher order finite different schemes. Table3 shows, validated FEM results for a thickness ratio of a/h=5, given boundary condition of sliding pin supported edge (S2) and taken lamination schemes of $[0/90\ldots]_{i6}$, $[45/-45]_{ii}$. By comparing the natural frequencies, the present 3-D FEM outcomes are in very good agreement with the given natural frequency values of 3-D elasticity theories and maximum variation is obtained 3.42% at $[45/-45]_{ii}$.

Table 3. - 3-D finite element values validation* (Ωc, Ωd of rectangular composite laminated plate)

S2	$[0/90\ldots]_{i6}$	$[45/-45]_{ii}$
Ratio of side length to thickness (a/h)	a/h=5	a/h=5
Present (3-D FEM)	3.4087	8.5706
Chao... [2]	3.3495	8.2872
Noor...[13]	3.4250	---

*For composite i : $E_1= 10E_2$, $G_{12}= 0.6E_2$, $G_{23}= 0.5E_2$ & $\nu_{12}=\nu_{23}=0.25$
For composite ii : $E_1= 15E_2$, $G_{12}= 0.429E_2$, $G_{23}= 0.343E_2$, $\nu_{12}= 0.4$, $\nu_{23}= 0.456$
where the normalized frequency, $\Omega_c =10\omega h(\rho_m/E_2)^{1/2}$ and $\Omega_d =\omega a^2(\rho_m/E_2\ h_2)^{1/2}$

3. EXPERIMENTAL INVESTIGATION

3.1 Specified dimensions

An experimental modal analysis was carried out to characterize the metal plates and FRP composite plate. Aluminum and stainless steel were used to make the rectangular metal plates of 100mm long, 50mm wide and 10mm thick that have been installed in the experimentation procedures and also those for the finite element set. The laminate geometry is as shown in Figure 1. Also, the materialistic properties of the metal plates are shown in Table 4.

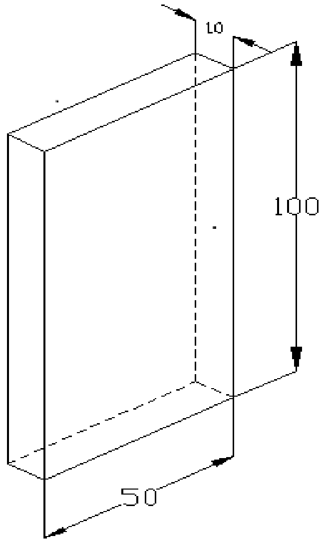

Fig. 1 - Geometry of the laminate

Table 4. - Material properties for isotropic metal plates

parameter	E (Gpa)	υ	ρ (kg/m³)
Stainless steel	200	0.34	8027
Aluminum	70	0.33	2710

3.2 Preparation of composite reinforcement

The composite used in this test was HT972 carbon fiber in LY556 epoxy resin. Resin (liquid) and hardener (pellets) are mixed in correct proportions, and then heated till they transform into liquid form.

Then the mixed liquid product is poured on the fabric, so that the product is equally distributed over the fabric with uniform thickness (brush is used for equal uniform distribution). These carbon fabrics are made of some fibers oriented along the warp and fill(weft) directions oriented perpendicular to each other.

These fibers that have been grouped and woven together describe that these weft yarns crosses over and under the warp-oriented yarns, thus following a fixed pattern. Figure 2 depicts a simple weave in which each filling fiber-yarn passes over a warp yarn, then under another warp thread and goes on.

Then the fabric is cut in to required sizes and shapes. These cut fabrics are placed on the bottom plate as per the laminate angles. After that, it is covered by the top plate. Since every individual fabric layer having 2 different and perpendicular fiber orientations (fibers at 0^o and 90^o), 29 disparate layers could be utilized to get every single ply simulated as ([0/90], [45/-45]). Release film is used on the base plate for easy removal of the fabric and also on the bottom surface of the top plate. If the release film is not present in the middle of the laminates it is called without defect.

Then these plates are clamped, by using screws, so that the laminates do not move unnecessarily. The cutting dimension of these carbon/ epoxy composite taken 105.1 mm length, 51mm width and 10.15mm thickness. The properties of this orthotropic composite plate material are shown in table5.

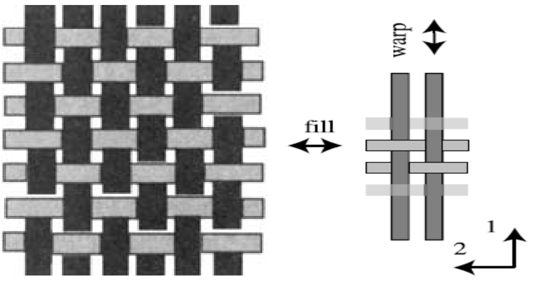

Fig. 2 - Schematic representation of woven fabric architecture

Table 5. - Material properties for orthotropic composite plate

Parameter	E_x Gpa	E_y Gpa	E_z Gpa	υ_{xy}	υ_{yz}	υ_{xz}	G_{xy} Gpa	G_{yz} Gpa	G_{xz} Gpa	ρ kg/m^3
Carbon/ Epoxy	66	67	6.9	0.3	0.25	0.25	2.58	4.5	4.5	1405

3.3 Experimental set up

Initially, a block that serves for supporting purpose is fixed on attached to the edge frame of a rotating machine with an arrangement of bolts and nuts. Up to the end of the block, the edge the edge of the metal plate is clamped between a pair of 10 mm thick stainless steel strips to achieve a perfect fit. But when it comes to carbon/epoxy composite material, it is fixed directly to the edge in 10mm surface area between the supporting and the bottom blocks. In composite plate arrangement, no strips are used because of the high strength-to-weight ratio (no deflection takes place). This configuration is designed for where one end and both ends are clamped to the edge.

3.4 Testing procedure

Initially, a Modal testing (Impact testing) had been performed to determine the frequency response. The signals of vibration are sensed by a piezo-electric strain gauge which is set over the metal plate and the structure was excited with help of an impact hammer. At the end, the FFT analyzer (signal analyzer) is used to record the output measurements, by the assistance of the accelerometer (stain gauge amplifier). Figure 3 shows the configuration of one end edge clamped case.

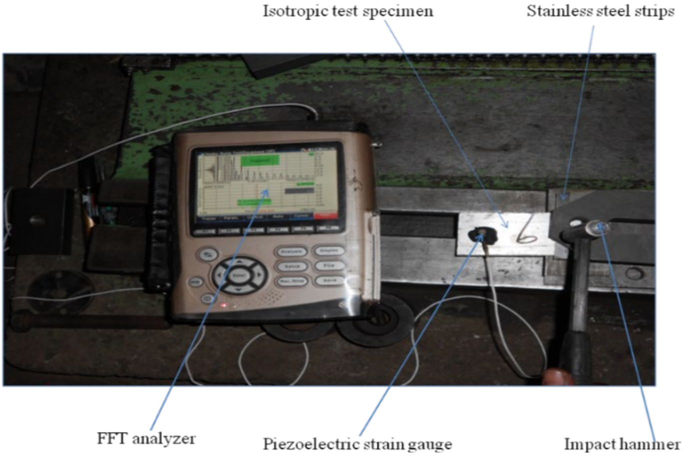

Fig. 3 - One end edge clamped metal plate configuration

4. FINITE ELEMENT MODELING

The element used here is ANSYS solid 95, developed using 3-D elasticity theory and was described by 20 nodes, each having 3 degrees of freedom and able to be translated along the x, y and z axes.

For these rectangular composite laminate, the deformed position for the1st mode shear deformation shapes for a single end and also for both ends that are edge clamped is depicted in Figures 4 and 5.

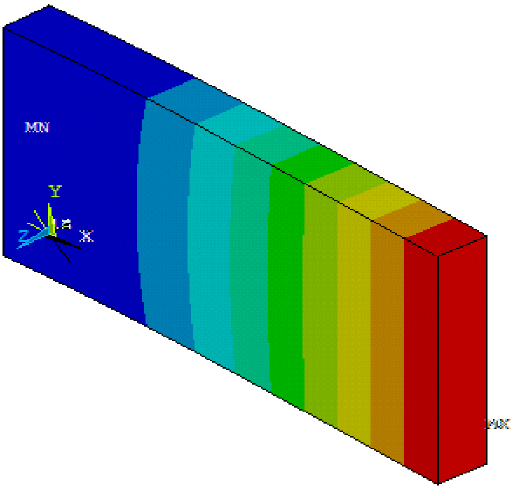

Fig. 4 - Deformed position for1stmode (shape) of one end edge-clamped laminate

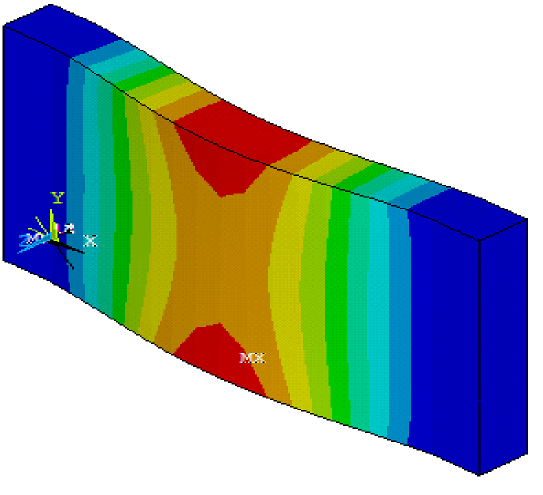

Fig. 5 - Deformed position for 1st mode (shape) of two ends edge-clamped laminate

5. FEM VALIDATIONS USING DIFFERENT EXPERIMENTAL MATERIALS

5.1 Results and discussions

The experimental results for the analysis of free vibration of different materials for single end edge-clamped and both the ends-edge clamped are presented in Tables 6 and 7.

Table 6. - For edge clamped at one end (cantilever), comparison of first mode natural frequencies (hz) of various materials from experimental modal results and finite element analysis

Plate	Experimental values	Finite element modeling (Hz)	Percentage error of experiment with FEM
Stainless steel	920.0	908.12	1.29
Aluminum	955.0	947.24	0.82
Carbon/epoxy	975.0	1035.39	5.83

For the above experimental values, the required graphs are plotted in Figures 6-8.

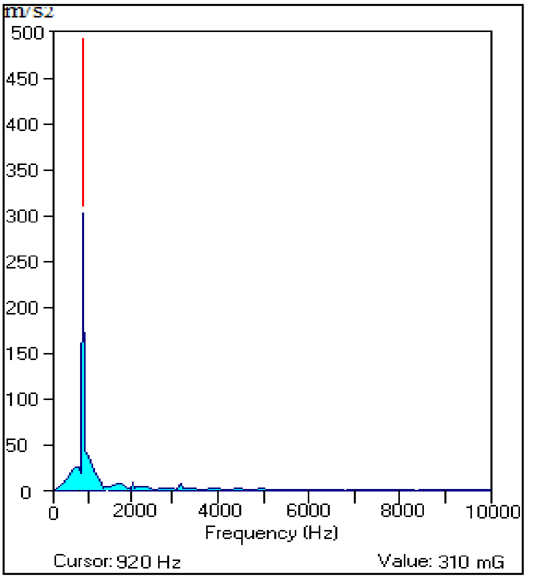

Fig. 6 -Stainless steel (920Hz)

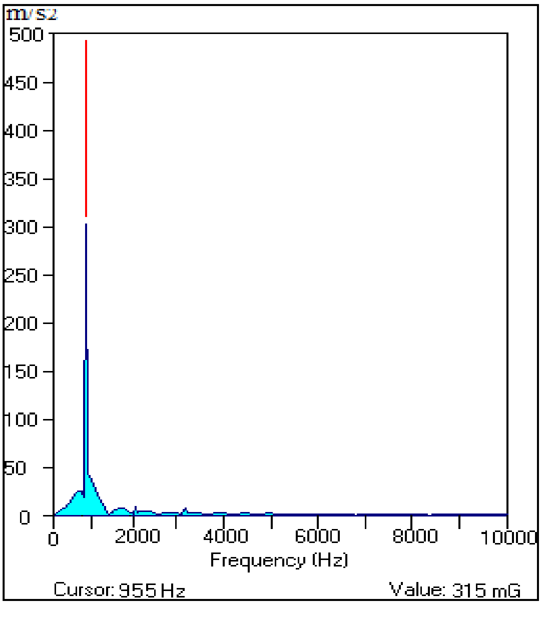

Fig. 7 - Aluminum (955Hz)

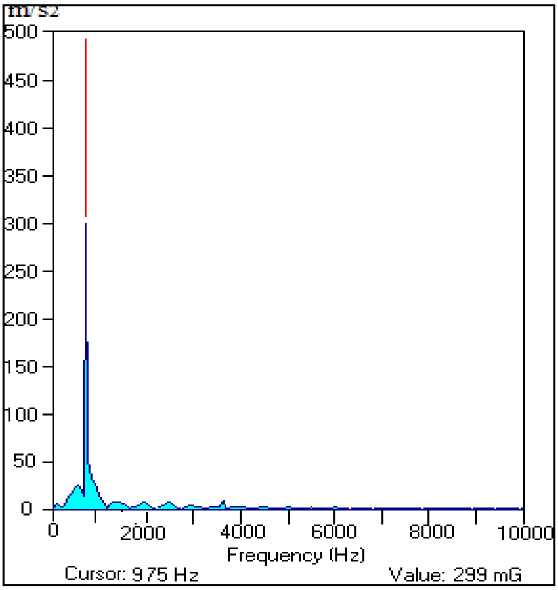

Fig. 8 - Carbon/ epoxy (975Hz)

Table 7. - For edge clamped at both ends, the comparison of first mode natural frequencies (hz) of various plates from experimental modal results and finite element analysis

Plate	Experimental values	Finite element modeling	Percentage error of experiment with FEM
Stainless steel	6325.0	6159.9	2.68
Aluminum	6750.0	6586.6	2.48
Carbon/epoxy	6825.0	6336.7	7.70

For the above experimental values, the required graphs are plotted in figures 9-11.

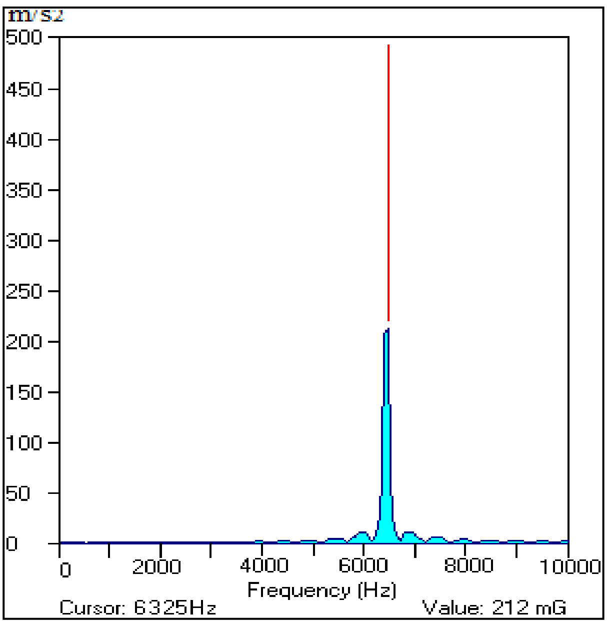

Fig. 9- Stainless steel (6325Hz)

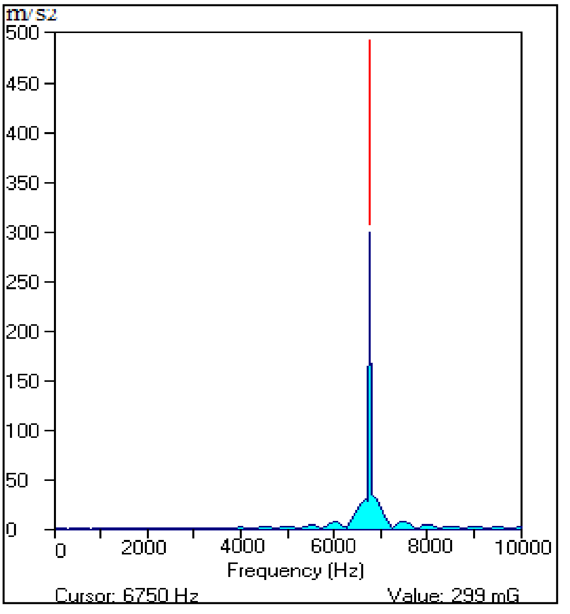

Fig. 10 - Stainless steel (6750Hz)

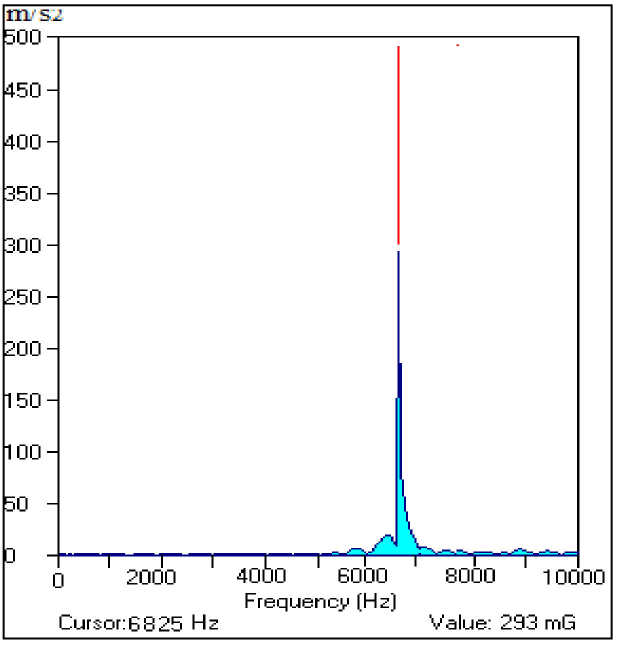

Fig. 11 -Stainless steel (6825Hz)

The deviation in frequency measured(e) in form of percentage can be shown as the ratio of analytical and experimental values difference to the analytical value multiplied by 100.

Table 6 shows the results of natural frequencies for the edge clamping of material at one end. The acceptance in the experimental and FE method outcomes was magnificently appreciable for Stainless Steel and Aluminum metal plates. The acceptance for the experimental and FE method outcomes is slightly better in the case of composite plate of carbon/epoxy, having 5.83%discrepancy.

Table 7 shows the results of natural frequencies for the both ends edge clamped. There is good correlation between experimental and FEM results for the stainless Steel and Aluminum metal plates, and it is not as good as the others for the composite plate of carbon/ epoxy with the worse case having a discrepancy of 7.7%.

The results presented in Tables 6-7 generally show a good agreement between the experimental and the FEM natural frequencies, the worst deviation being of 7.7 percent.

In configuration of both ends edge clamping, there is a potential for a slippage of the plate between the clamps due to mid plane stretching. Also, the length of the plate was shorter than for the other configuration. The natural frequencies of the type higher than that of other configurations are the consequences of shorter length and as well as boundary conditions. It may be a considerable factor for increasing the proportional variation between experimental and finite element method prediction due to non-ideal boundary constraints.

6. CONCLUSIONS

This free vibration analysis of orthotropic and isotropic materials of thick rectangular plates subjected to several edge constraints can be found by implementing the three-dimensional finite element method and comparing these convergent results with the available and investigated analytical values.

By the comparison and validation of the different theories and their results with the present3-D FEM, the present analysis is in good agreement with the natural frequency values and there is a maximum deviation of 3.42%. The experimental validating procedure is also processed, the FEM outcomes are in very well agreement for all experimental and analytical values of isotropic material. In case of comparison of 3-D FEM to the orthotropic material's experimental outcomes, there is a deviation of 5-8%. We believe that the raise in the deviation of percentage amid the experimental and finite element method values is because of the nonideal boundary conditions.

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