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An evolving feature weighting framework for radial basis function neural network models

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Abstract

Via Granular Computing (GrC), one can create effective computational frameworks for obtaining information from data, motivated by the human perception of combining similar objects. Combining knowledge gained via GrC with a Fuzzy inference engine (Neural-Fuzzy) enable us to develop a transparent system. While weighting variables based on their importance during the iterative data granulation process has been proposed before (W-GrC), there is no work in the literature to demonstrate effectiveness and impact on Type-2 Fuzzy Logic systems (T2-FLS). The main contribution of this paper is to extend W-GrC, for the first time, to both Type-1 and Type-2 models known as Radial Basis Function Neural Network (RBFNN) and General Type-2 Radial Basis Function Neural Network (GT2-RBFNN). The proposed framework is validated using popular datasets: Iris, Wine, Breast Cancer, Heart and Cardiotocography. Results show that with the appropriate selection of feature weight parameter, the new computational framework achieves better classification accuracy outcomes. In addition, we also introduce in this research work an investigation on the modelling structure's interpretability (via Nauck's index) where it is shown that a good balance of interpretability and accuracy can be maintained.

KEYWORDS
explainable AI, feature weights, granular computing, iterative data granulation, neural network, radial basis function

1 | INTRODUCTION

The concept of iterative data granulation can be combined with Fuzzy Logic information granulation theory, by means of clustering algorithms (Mashinchi et al., 2015). Via such computational frameworks, data is simplified (via grouping/clustering) to make it easier to understand and reduce its complexity. Motivation for this work arises from the need for information simplification, summarization, information clarity, low computational cost and tolerance to uncertainty (Butenkov et al., 2017).

In the study by Rubio-Solis et al. (2019), iterative data granulation is used to determine the rulebase parameters in a GT2-RBFNN model, which is equivalent to GT2-FLS. An iterative data granulation algorithm attempts to combine information intuitively (human-like behaviour), such as two most compatible granules, iteratively until information is sufficiently compressed (Pedrycz & Bargiela, 2002). In this paper, compatibility is calculated via distance and density measures for the potential new formed granules.

In most research on iterative data granulation, all input features are considered to have equal degree of importance. However, this method is not ideal particularly if it involves data with high number of features (Huang et al., 2005), in which features may have different weight that evolves during the granulation. Intuitively, one would not include features that are less relevant to a task, when attempting to group information together.

The idea of feature weighting in data clustering is not new. In GrC in particular, in the study by Muda and Panoutsos (2022), the first attempt to integrate the evolving feature weighting algorithm is presented, referred to as weighted GrC (W-GrC), in which the current information granules play an important role to determine the weight of each feature. The concept of W-GrC is inspired by the weighted k-means (W-k-means) proposed by Huang et al. (2005) and the weighted version of Ward called Ward$_p$ (Amorim, 2015), in the area of hierarchical clustering. However, W-GrC so far is investigated in terms of its impact in forming accurate and interpretable Fuzzy Logic rulebases, only in Type-1 Fuzzy Logic systems (T1-FLS).

In this paper, we investigate and analyse the of use of W-GrC to (1) Neural-Fuzzy (NF) structures (based on RBF), (2) Type-2 Fuzzy Logic systems and (3) while also assessing for the first time the impact on rule base interpretability. A NF system leverages the learning capabilities of Neural Networks (NN) and the transparency of Fuzzy Logic systems (Rubio-Solis & Panoutsos, 2013). We present the evolving feature framework for both Type-1 and Type-2 models, known as RBFNN and GT2-RBFNN. The new framework is validated using popular datasets: Iris, Wine, Breast Cancer, Heart and Cardiotocography (UCI machine learning repository). Results show that with a proper selection of feature weight parameter, the new computational framework achieves better classification accuracy outcomes. In addition, we also investigate in this research work the impact on the modelling structure's interpretability (via Nauck's index) where it is shown that via W-GrC a good balance of interpretability and accuracy is maintained.

The rest of the paper is organized as follows. The iterative data granulation is discussed in Section 2 and the formation of fuzzy rulebase and the membership functions' parameters is described in Section 3. The fundamental of the classifiers (RBFNN and GT2-RBFNN) is presented in Sections 4 and 5, and the proposed W-GrC is discussed in detail in Section 6, together with the experiment result based on four benchmark datasets.

2 | KNOWLEDGE DISCOVERY USING GRANULAR COMPUTING

Granular computing is a computational paradigm that imitates human cognition in terms of grouping similar information together (Rubio-Solis & Panoutsos, 2013). Granular Computing's ability to find and extract knowledge from unstructured data in the form of information granules is one of its key features. The information granules, or knowledge gathered, can be utilized to build the linguistic rule-bases of a fuzzy logic-based system (Muda & Panoutsos, 2020). A variety of granulation or clustering techniques have been used in the literature to aid the design of fuzzy and Neural-Fuzzy systems including fuzzy c-means and hierarchical clustering (Panoutsos & Mahfouf, 2005).

In the study by Pedrycz and Bargiela (2002), Pedrycz suggested an iterative data granulation method which is similar to the idea of agglomerative hierarchical grouping. It starts with a set of numerical data and creation of information granules whose distribution and size accurately capture the substance of the data. These algorithms do, however, differ greatly from one another. In data granulation (referred as GrC afterwards), the original data and the information granules are closely related. This is so that each sub-granule, which each derives from the data directly, may be found within each granule (Panoutsos & Mahfouf, 2010). The transparency feature provided by GrC makes this algorithm ideal to be integrated with Fuzzy and Neural-Fuzzy systems modelling (Panoutsos & Mahfouf, 2005).

A large number of studies on GrC algorithm have been conducted in building data-driven Fuzzy models. Recent research on GrC focus on the use of GrC with Type-2 Fuzzy systems. For example, Rubio-Solis et al. (2019) implemented the iterative data granulation method to approximate the parameters of each fuzzy rule in the General Type-2 Radial Basis Function Neural Network (GT2-RBFNN) models. The neural architecture of the GT2-RBFNN is built on the representation of α-planes, and each fuzzy rule's antecedent and consequent parts employ GT2 Fuzzy Sets. In (Baraka & Panoutsos, 2019), Baraka et al. proposed long-term learning framework that employs the concepts of GrC to extract information/knowledge from raw data in the form of interval-valued sets. The structure being used in the research is Interval-Valued Radial Basis Function Neural Fuzzy (IV-RBF-NF). The integration of GrC with Type-2 systems are also the focus in Baraka et al. (2016) and Solis and Panoutsos (2015). So far, there is not any attempt to embed the feature weight concept in GrC within the Type-2 Fuzzy systems framework.

In works related to Type-1 Fuzzy systems, the idea of feature weight was first presented in Panoutsos and Mahfouf (2010), and the majority of publications on this approach employ fixed weights for each feature, such as in the studies by Baraka et al. (2014) and Muda and Panoutsos (2020). There are few studies on feature weighting for GrC; for instance, a Fast Correlation-Based Filter based on symmetrical uncertainty to find the most important aspects of a welding process is found in the study by Solis and Panoutsos (2017). However, in this pre-processing stage (which serves as a filter mechanism), the feature weights are predetermined and their values remain constant over the course of the granulation process' evolution. The progressive feature weighting algorithm is first integrated in the study by Muda and Panoutsos (2022), although W-GrC is only studied in terms of its impact on creating rulebases in Type-1 Fuzzy Logic systems (T1-FLS).

The iterative data granulation (Panoutsos & Mahfouf, 2005; Pedrycz & Bargiela, 2002) is an algorithmic method which is accomplished by repeating two important steps:

- Identify the most compatible pair of information granules and merge them.
- Repeat the previous step until an adequate level of data granulation is obtained.

Given two granules A and B, the compatibility is:

$$C(A, B) = \text{Distance}_{MAX} - \text{Distance}_{A,B} . \exp(-\alpha \times R) \tag{1}$$

in which

$$\text{Density } R = \frac{C_{A,B}/\text{Cardinality}_{MAX}}{L_{A,B}/\text{Length}_{MAX}} \tag{2}$$

Distance_{MAX}: maximum distance in dataset:

$$\text{Distance}_{MAX} = \sum_{v=1}^{d}(\text{distance}_v) \tag{3}$$

$\text{Distance}_{A,B}$: average multidimensional distance between granules weighted by feature weight w_v:

$$\text{Distance}_{A,B} = \frac{\sum_{v=1}^{d} w_v(D_1 - D_2)}{d} \tag{4}$$

where

$$D_1 = \max(\max_{Av}, \max_{Bv}) \tag{5}$$

$$D_2 = \min(\min_{Av}, \min_{Bv}) \tag{6}$$

$\max_{Av}$: upper limit in granule A, dimension v, $\min_{Av}$: lower limit in granule A, dimension v, α: parameter that balances distance and density, Cardinality_{MAX}: cardinality in data set, Length_{MAX}: maximum length of a granule, $C_{A,B}$: cardinality of merged granule, and $L_{A,B}$: length of merged granule, given by:

$$L_{A,B} = \sum_{i=v}^{d}(\max_{Xv} - \min_{Xv}) \tag{7}$$

Most previous works related to GrC set the feature importance (weight w_v) as 1 for all input variables. Feature importance could be established using expert knowledge (e.g. for known processes, heuristically, or via trial and error) as well as via mathematical and computational methods directly using the available data/information.

Figure 1 depicts the development of a two-dimensional, 400 instance synthetic data set's data granulation process. Every instance of the raw data is treated as one granule-point at the beginning. The number of granules is subsequently decreased as a result of this repeated merging process until the final information granules are established. In this example, the iterative data granulation is implemented, with the density factor of 0.2, and the number of iterations for Figure 1b,c are 390 and 395, respectively.

3 | FORMATION OF FUZZY LOGIC RULE-BASE

The information obtained from GrC provides the fuzzy rulebase initial structure, that is, the membership functions' (MFs) centre and width. Consider a system with 2 input variables (A and B) and 1 output (O). The process of granulation, across each input variable individually and across the entire data input enables us to form the rules that is equivalent to a Mamdani FIS rule-base:

$$\begin{aligned}&\text{Rule 1}: \text{IF}(\text{input } A = A_1 \text{ and input } B = B_1 \text{ and...}) \text{ THEN}(\text{output} = O_1)\\&\text{Rule 2}: \text{IF}(\text{input } A = A_2 \text{ and input } B = B_2 \text{ and...}) \text{ THEN}(\text{output} = O_2)\end{aligned} \tag{8}$$
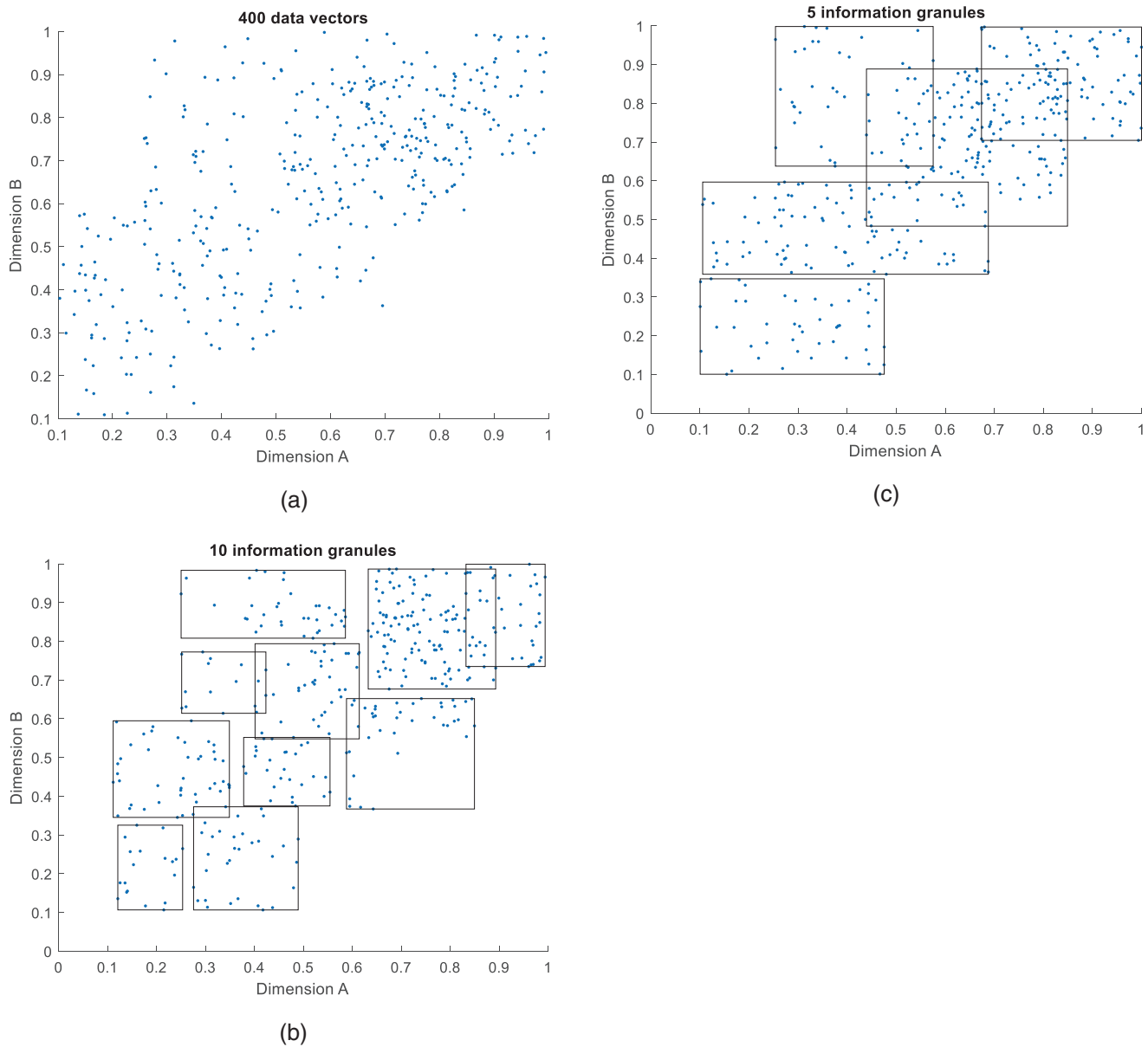
FIGURE 1 Data granulation process from (a) 400 data vectors to (b) 10 information granules and (c) 5 information granules

Figure 2 shows how five information granules in are being translated into five Fuzzy MFs. These MFs represent the rule-bases for Iris data, which consists of four inputs and one output. However, note that the number of dimensions shown in Figure 2a is only two instead of four. Gaussian membership functions (MFs) are used in this study. The formation of Gaussian MFs requires two important parameters, namely centre of MFs (c) and sigma (σ). In this paper, the MFs parameters (centre and sigma) are determined from the median and standard deviation of the data in granules, respectively.

4 | RADIAL BASIS FUNCTION NEURAL NETWORK (RBFNN)

Fuzzy logic system with singleton fuzzy output can be represented by Panoutsos and Mahfouf, (2005):

$$y = \sum_{i=1}^{p} z_i \left\{ \frac{\prod_{j=1}^{m} \mu_{ji}(x_j)}{\sum_{i=1}^{p} \prod_{j=1}^{m} \mu_{ij}(x_j)} \right\} \tag{9}$$
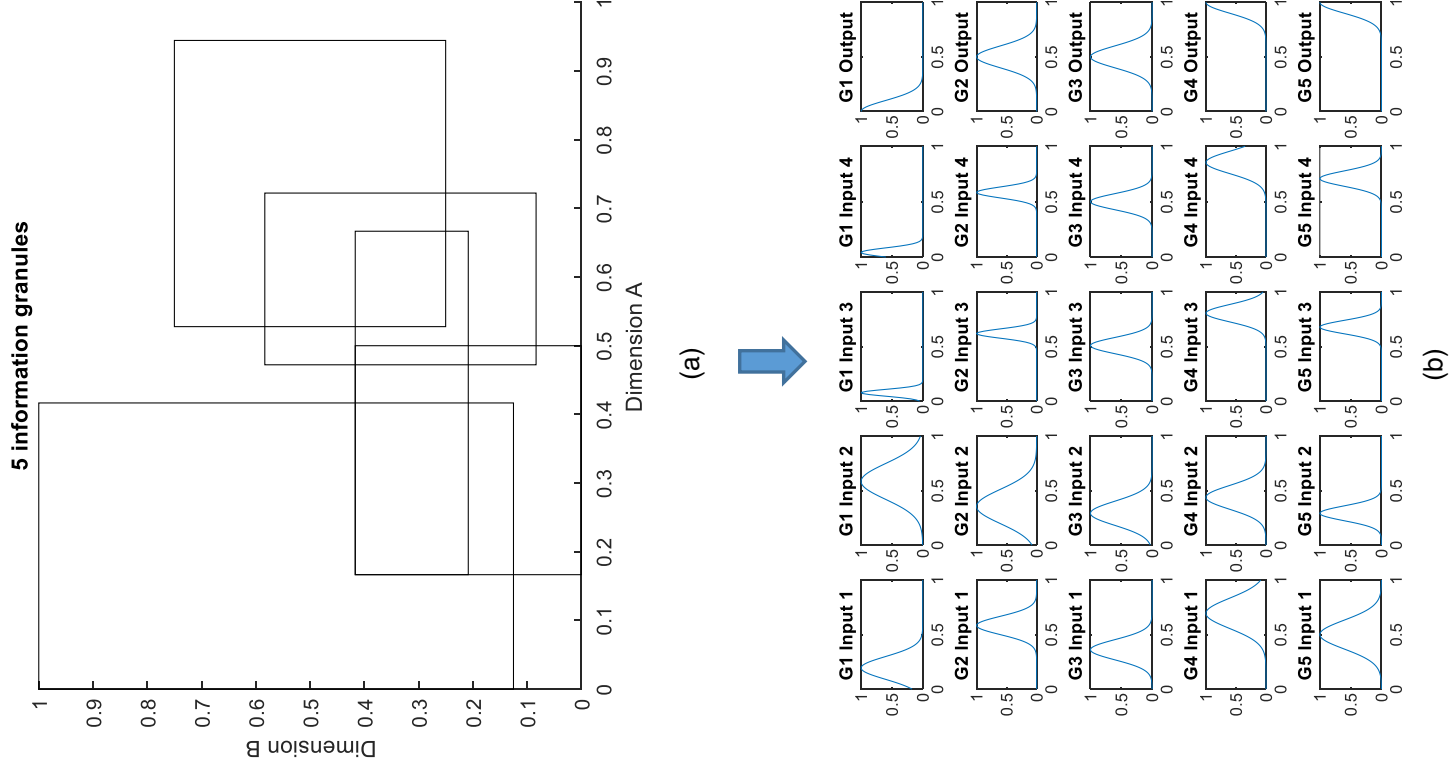
FIGURE 2 Converting (a) information granules to (b) fuzzy membership functions / rule-bases.

where $\mu_{ij}(x_j)$ is the Gaussian MF of x_j in the ith rule:

$$\mu_{ij}(x_j) = \exp\left(\frac{-(x_j - c_{ij})^2}{\sigma_{ij}^2}\right) \qquad (10)$$

where m is the number of input variables and p is the number of rules.

Equation (9) can be simplified as:

$$y = \frac{\sum_{i=1}^{p} z_i m_i(x)}{\sum_{i=1}^{p} m_i(x)} \tag{11}$$

where $m_i(x)$ is the membership degree of the input vector x defined as:

$$m_i(x) = e^{\left(\|x - c_i\| / \sigma_i^2 \right)} \tag{12}$$

Based on the RBF:

$$g_i(x) = \frac{m_i(x)}{\sum\limits_{i=1}^{p} m_i(x)} \tag{13}$$

the NF output can be written as:

$$y = \sum_{i=1}^{p} z_i g_i(x) \tag{14}$$

A Radial Basis Function Neural Network (RBFNN) can be regarded as an FLS that combines the input–output space as depicted in Figure 3. In the hidden layer (or RBF layer), it resembles an additive weighted combination of the MFs of the fired-rule output sets. As a result, each hidden receptive unit in the RBFNN functions similarly to a fuzzy rule represented by a multivariable Gaussian MF (Rubio-Solis et al., 2019).

The MFs parameters c_{ij} and σ_{ij} (obtained from GrC) are optimized by using the back-error-propagation (BEP) algorithm. The BEP has been demonstrated to be efficient in the system optimization (Rubio-Solis & Panoutsos, 2013). The overview of W-GrC Neural Fuzzy (NF) is shown in Figure 4.

5 | GENERAL TYPE-2 RADIAL BASIS FUNCTION NEURAL NETWORK

T2-FLS and T1-FLS are similar in terms of their linguistic IF...THEN rules. In T2-FLSs, the premise and consequent are of type-2 fuzzy sets (T2FS). The MF in T1FS is a crisp number while the MF of a T2FS is itself a fuzzy set (Baraka et al., 2016).

The potential of T2FS to model additional uncertainty (due to its extra dimension) have sparked the interest amongst the researchers (Hamrawi et al., 2017). The Footprint of Uncertainty (FOU) encompassed by T2FS provides more degree of freedom to the T2FS as compared against the T1FS, and hence better modelling of uncertainty (Wagner & Hagras, 2010).
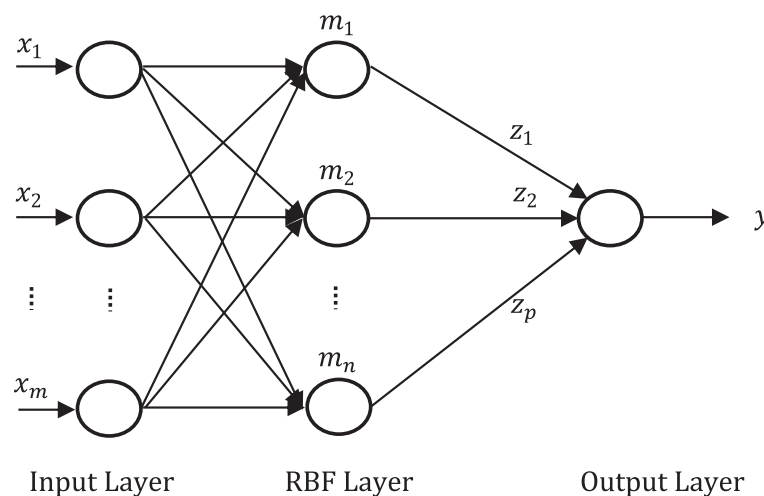
FIGURE 3 General structure of an RBFNN
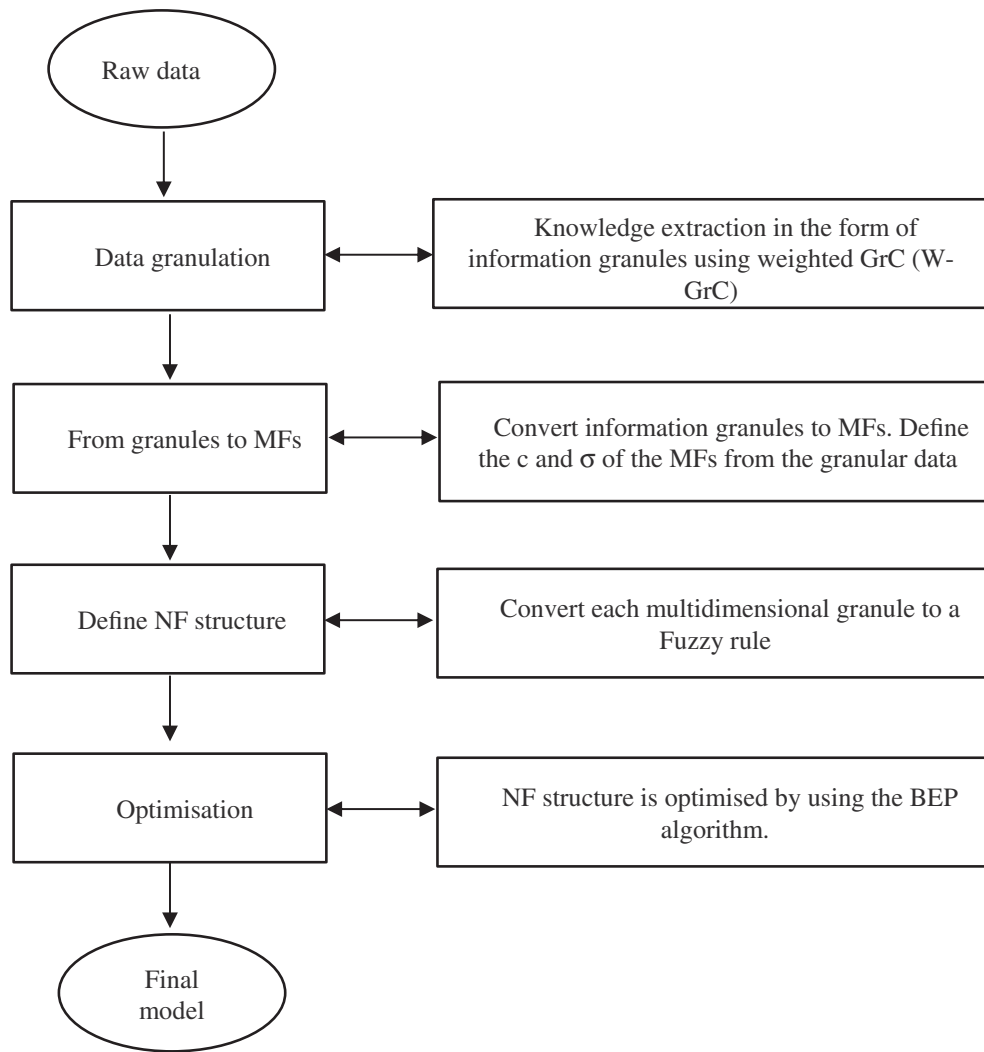
FIGURE 4 W-GrC- NF modelling framework

5.1 | α-Plane representation

GT2-FS represented by $\tilde{A}$ is characterized by a bivariate MF $\mu_{\tilde{A}}(x, u) \subseteq [0, 1]$ where the primary variable is $x \in X$. The secondary variable is characterized by $u \in Jx \subseteq [0, 1]$ as shown in Figure 5. Thus, $\tilde{A}$ is:

$$\tilde{A} = \left\{ (x, u), \mu_{\tilde{A}} (x, u) \mid \forall x \in X, \forall u \in J_x \subseteq [0, 1] \right\} \tag{15}$$

An α-plane $\tilde{A}$ is represented by $\tilde{A}_{\alpha}$, is the union of the primary MFs of $\tilde{A}$ with $\alpha (0 \le \alpha \le 1)$:

$$\tilde{A}_{\alpha} = \left\{ (x, u), \mu_{\tilde{A}} (x, u) \ge \alpha \mid x \in X, u \in [0, 1] \right\} \tag{16}$$

5.2 | GT2-RBFNN

For a Type-1 RBFNN, each receptive unit represents a fuzzy rule:

$$\begin{aligned} R^1 &: \text{IF } x_1 \text{ is } F_1^i \text{ and } ...\text{IF } x_k \text{ is } F_k^i \text{ and}... \\ &\quad \text{IF } x_n \text{ is } F_n^i \text{ THEN } y \text{ is } G^i; \; i = 1,...M \end{aligned} \tag{17}$$
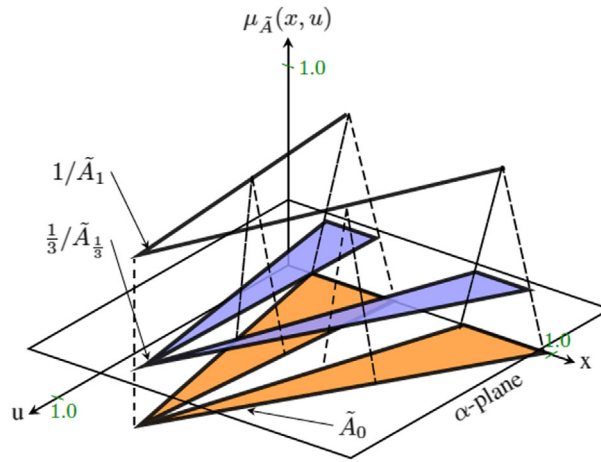
FIGURE 5 $\alpha-$ planes (reproduced via Rubio-Solis et al. (2019)).

In the form of GT2-FLS, the rule can be described as:

$$\tilde{R}_a^i : \text{IF } x_1 \text{ is } \tilde{F}_1^i \text{ and ...IF } x_k \text{ is } \tilde{F}_k^i \text{ and...}$$
$$\text{IF } x_n \text{ is } \tilde{F}_n^i \text{ THEN } y \text{ is } \tilde{g}^i\left(\vec{x}_p\right); \ i=1,...M \tag{18}$$

where $x_1,...,x_k$ are the input vectors, $\tilde{F}_1^i,..., \tilde{F}_k^i$ are the T2FS, and $i,...,M$ is the number of rules.

With uncertain standard deviation $\sigma_i = \left[\sigma_i^1, \sigma_i^2\right]$, the GT2-RBFNN inputs the data as a multidimensional crisp vector that is represented by $\vec{x}_p = [x_1, ..., x_n]$. The firing interval $F_i^{\alpha_s}$ is used to determine the reduced set $[y_l^{\alpha_s}\left(\vec{x}_p\right), y_r^{\alpha_s}\left(\vec{x}_p\right)]$ (in the type-reduction layer) before obtaining the defuzzified output by the average of $y_l^{\alpha_s}\left(\vec{x}_p\right)$ and $y_r^{\alpha_s}\left(\vec{x}_p\right)$.

6 | WEIGHTED GRC

The work in this paper is inspired by the Weighted K-Means algorithm (also known as WK-Means) proposed by Huang et al. (2005), which is a modified version of the K-Means criterion algorithm that assigns weights to the features. Their approach resembles the wrapper approach of feature selection, where the feature weights are weighted to a set of patterns during the clustering phase. The WK-Means algorithm introduced by Huang et al. minimizes the following object function:

$$W(S, C, w) = \sum_{k=1}^{K} \sum_{i \in S_k} \sum_{v \in V} w_v^\beta d(y_{iv}, c_{kv}) \tag{19}$$

The objective function is minimized by iteratively optimizing (19) for S, C, and w, where $S = \{S_1, S_2, ..., S_k, ..., S_K\}$, $c_k \in C$ is the granule centre, y_i is an instance, β is the feature weighting parameter. During the clustering process, the feature weight w_v needs to be updated. There are two possibilities for the update of w_v, with S and C fixed, subject to $\beta > 1$, and w_v is given by:

$$w_v = \begin{cases} 0, & \text{if } D_v = 0 \\ \dfrac{1}{\sum_{j=1}^{h} \left[\dfrac{D_v}{D_j}\right]^{\frac{1}{\beta-1}}}, & \text{if } D_v \neq 0 \end{cases} \tag{20}$$

where h is the number of features where $D_v \neq 0$. D_v is defined as:

$$D_v = \sum_{k=1}^{K} \frac{1}{N-1} \sum_{i=1}^{N} |y_{iv} - c_{kv}|^2 \tag{21}$$

Algorithm 1   Weighted Granular Computing (W-GrC)

1. Initial setting

   For the first iteration, all features are assigned with feature weight of 1. Feature weight parameter, β is selected (i.e. $\beta > 1$). The rest follows the conventional GrC settings.

2. Merging of granules

   Only the first merging process is based on the conventional GrC (granules distances with equal weights), while the rest of iterations will adapt the new evolving feature weights w_v in computing the compatibility measure and selection of pair of granules to be merged.

3. Granule update

   Update the cardinality, maximum and minimum limit of the new information granule, and keep track the instances in each granule.

4. Weight update

   Update the feature weights based on Equation (20). Repeat step (2–4).

where N is the cardinality in the granule k. Based on Equation (20), a zero weight will be assigned when the vth variable only contains a single value (i.e. $D_v = 0$).

Algorithm 1 shows how the feature weighting algorithm is embedded in the GrC. Only the first merging process will treat all the features equally; the algorithm then will adapt the new evolving feature weighting in computing the distance (and hence the compatibility measure). After each merging process, the granules will be updated and all the instances in all information granules are keep in track. This is an important step in the W-GrC, since we need to monitor the within granule variance in each iteration (that is used to adjust the feature weight). Finally, the feature weights for all features are updated, and are applied in the next iteration.

The hypothesis in this feature-weighting algorithm, in that the input variable with large variance in granules is considered as less important (therefore, designed to yield less influence in forming future information granules). The mechanism to achieve this is to penalize its contribution in the compatibility index. Importance here, is addressed via having robust information granules, however other metrics can be used too (e.g. correlation to output/outcome).

6.1 | Simulations and empirical results

Five classification datasets were used to validate the proposed approach—Iris, Wine, Breast Cancer, Heart and Cardiotocography with 4, 13, 9, 13 and 21 input features, respectively. As suggested in Huang et al. (2005), we set the feature weighting parameter $\beta > 1$ (from 2 to 10) to investigate how to set the value of β to affect the model predictive performance in terms of the classification accuracy. The initial weights for all features are set to 1 and expected to evolve throughout the granulation process. Simulation results aim to show the effectiveness of W-GrC (compared to GrC) in model accuracy as well as interpretability.

While the optimization/selection of the number of granules (hence rules) is not the focus for this work, five granules were heuristically chosen, towards generating a relatively parsimonious system. For consistency, five information granules (or rules) are formed throughout the simulations, and 10 runs are conducted for each value of β. The selection of five information granules is based on the system's interpretability (as few rules as possible), system's accuracy and also with reference to the compatibility measure. In further work, one may want to optimize and further investigate the number of rules (which would impact interpretability, as well as accuracy).

Tables 1–6 summarize how W-GrC performs with different values of β in RBFNN and GT2-RBFNN, respectively. The result is benchmarked against the original GrC that is represented in the 'GrC' row.

For the Iris data, improvement in terms of classification accuracy were obtained in RBFNN at $\beta \in \{2,5,6,8,9,10\}$ with the highest accuracy of 98.33%. Correspondingly, W-GrC with GT2-RBFNN outperforms the conventional GrC at $\beta \in \{2,3,4,5,7,9,10\}$, with the maximum accuracy of 98%, that occurs at $\beta = 2$ and $\beta = 10$.

Moving on to more complex data, Wine (13 input features as opposed to 4 in Iris), the impact of W-GrC can be observed in RBFNN at $\beta \in \{2,4,10\}$. Interestingly, most values of β in GT2-RBFNN result in higher accuracy than the conventional GrC. However, it is noted that the performance of the conventional GrC in GT2-RBFNN is slightly lower than its counterpart in RBFNN.

In the case of the Breast Cancer dataset, it is obvious that larger values of β ($\beta \geq 6$) are required in RBFNN to generate good outcome. The highest accuracy is when $\beta = 7$ with 98.16% while GrC obtained 97.43%. For GT2-RBFNN, almost the similar pattern in Wine is shown here, where the W-GrC outperformed the original GrC in all values of β, except when $\beta = 2$.

TABLE 1 Performance of W-GrC in Type-1 RBFNN with different β values (Iris and Wine)

	Iris			Wine		
	RMSE	Acc. (%)	Standard deviation (%)	RMSE	Acc. (%)	Standard deviation (%)
GrC	0.0763	97.67	2.74	0.0866	98.67	1.72
$\beta = 2.0$	0.0777	98.33	2.36	0.0726	99	1.61
$\beta = 3.0$	0.0909	96	3.06	0.0904	98.33	2.36
$\beta = 4.0$	0.0859	96.33	3.67	0.0714	99.33	1.41
$\beta = 5.0$	0.0732	98	2.33	0.0797	98.67	1.76
$\beta = 6.0$	0.0783	98.33	1.76	0.0868	97.33	5.84
$\beta = 7.0$	0.0735	97.67	1.61	0.0881	98.33	2.36
$\beta = 8.0$	0.0765	98	1.72	0.0854	98.33	3.6
$\beta = 9.0$	0.0785	98.33	1.76	0.0807	98.33	4.22
$\beta = 10.0$	0.0761	98	1.72	0.0768	99.67	1.05

TABLE 2 Performance of W-GrC in Type-1 RBFNN with different β values (Breast Cancer and Heart)

	Breast cancer			Heart		
	RMSE	Acc. (%)	Standard deviation (%)	RMSE	Acc. (%)	Standard deviation (%)
GrC	0.1486	97.43	1.05	0.3339	84.75	4.99
$\beta = 2.0$	0.1676	96.91	1.95	0.3479	84.41	3.46
$\beta = 3.0$	0.1506	97.35	1.84	0.3336	86.95	2.89
$\beta = 4.0$	0.1628	96.84	1.04	0.3383	86.44	2.65
$\beta = 5.0$	0.151	97.35	1.11	0.3306	85.42	2.42
$\beta = 6.0$	0.154	97.5	1.11	0.3273	87.46	4.39
$\beta = 7.0$	0.1302	98.16	1.16	0.3263	85.08	4.51
$\beta = 8.0$	0.1473	97.5	1.35	0.3198	86.27	3.04
$\beta = 9.0$	0.1352	97.94	1.19	0.3233	85.42	3.12
$\beta = 10.0$	0.1599	97.72	1.07	0.3338	85.08	4.21

TABLE 3 Performance of W-GrC in Type-1 RBFNN with different β value (Cardiotocography)

	Cardiotocography		
	RMSE	Acc. (%)	Standard deviation (%)
GrC	0.1773	84.63	2.54
$\beta = 2.0$	0.1505	86.12	2.58
$\beta = 3.0$	0.1757	86.2	2.59
$\beta = 4.0$	0.1796	83.69	2.51
$\beta = 5.0$	0.1609	87.76	2.63
$\beta = 6.0$	0.178	84.39	2.53
$\beta = 7.0$	0.1733	85.49	2.56
$\beta = 8.0$	0.175	85.25	2.56
$\beta = 9.0$	0.1897	82.2	2.47
$\beta = 10.0$	0.1678	84.94	2.54

RBFNN shows a more consistent performance against its counterpart GT2-RBFNN in Heart, where all trials with W-GrC achieve higher accuracy than GrC, except for when $\beta = 2$. As a comparison, the higher accuracy is achieved in W-GrC with GT2-RBFNN at $\beta \in \{3,4,5,7,8\}$.

TABLE 4 Performance of W-GrC in General Type-2 RBFNN with different β values (Iris and Wine)

	Iris			Wine		
	RMSE	Acc. (%)	Standard deviation (%)	RMSE	Acc. (%)	Standard deviation (%)
GrC	0.1179	96.33	2.92	0.0971	96.33	3.67
$\beta = 2.0$	0.1101	98	1.72	0.1062	97	1.89
$\beta = 3.0$	0.1131	97.67	2.74	0.1033	97.33	2.11
$\beta = 4.0$	0.1183	96.67	2.72	0.0867	97.67	2.25
$\beta = 5.0$	0.1078	97.67	2.25	0.095	96.33	2.46
$\beta = 6.0$	0.1139	96.33	2.46	0.0853	97.67	2.74
$\beta = 7.0$	0.1094	97.33	2.63	0.088	99	1.61
$\beta = 8.0$	0.1122	95.33	2.81	0.0968	97.33	2.11
$\beta = 9.0$	0.1158	97.33	2.11	0.0933	98.67	1.72
$\beta = 10.0$	0.1101	98	1.72	0.1051	96.33	3.67

TABLE 5 Performance of W-GrC in General Type-2 RBFNN with different β values (Breast Cancer and Heart)

	Breast cancer			Heart		
	RMSE	Acc. (%)	Standard deviation (%)	RMSE	Acc. (%)	Standard deviation (%)
GrC	0.1741	96.25	2.12	0.3059	81.69	4.06
$\beta = 2.0$	0.1824	96.18	1.58	0.3063	81	5.99
$\beta = 3.0$	0.1663	96.76	1.44	0.3072	83	3.79
$\beta = 4.0$	0.1451	97.21	1.3	0.31	84	3.56
$\beta = 5.0$	0.1685	96.76	1.21	0.3034	83	3.73
$\beta = 6.0$	0.1662	96.62	1.56	0.2997	80	2.75
$\beta = 7.0$	0.1574	97.35	1.39	0.305	81.86	4.66
$\beta = 8.0$	0.1608	96.91	1.69	0.3031	81.86	5.06
$\beta = 9.0$	0.1502	97.65	0.67	0.3009	81.69	5.23
$\beta = 10.0$	0.1654	96.47	1.83	0.31	80.68	4.67

TABLE 6 Performance of W-GrC in General Type-2 RBFNN with different β values (Cardiotocography)

	Cardiotocography		
	RMSE	Acc. (%)	Standard deviation (%)
GrC	0.1584	87.36	2.62
$\beta = 2.0$	0.1661	86.52	2.59
$\beta = 3.0$	0.1648	87.77	2.63
$\beta = 4.0$	0.1545	87.98	2.67
$\beta = 5.0$	0.1544	88.92	2.63
$\beta = 6.0$	0.159	87.51	2.65
$\beta = 7.0$	0.1561	88.45	2.65
$\beta = 8.0$	0.1585	88.35	2.66
$\beta = 9.0$	0.1584	88.56	2.63
$\beta = 10.0$	0.148	87.67	2.62

However, in the case of larger dataset Cardiotocography (with 2126 instances and 21 input variables), the performance of GT2-RBFNN is higher than RBFNN. The maximum accuracies recorded by GT2-RBFNN and RBFNN are 88.92% and 87.76%, respectively. The positive impact of W-GrC with GT2-RBFNN is shown in all values of β except when $\beta = 2$. In RBFNN, the improvement can be observed at $\beta \in \{2,3,5,7,8,10\}$.

TABLE 7 Comparison of the interpretability index

| | RBFNN | | General Type-2 RBFNN | | | |
| | | | W-GrC | | GrC | |
	W-GrC	GrC	UMF	LMF	UMF	LMF
Iris	0.0252	0.0239	0.0081	0.0132	0.0083	0.0135
Wine	0.0061	0.0065	0.0027	0.0045	0.0027	0.0044
Breast Cancer	0.0065	0.0067	0.0049	0.0052	0.0054	0.0055
Heart	0.0044	0.0044	0.0017	0.0019	0.0018	0.002
Cardiotocography	0.004	0.0039	0.0022	0.0025	0.0025	0.0029

From the result, we can conclude that, W-GrC performs better than the conventional GrC. However, careful selection of β is crucial to achieve superior classification accuracy. W-GrC accomplishes the best performance when a suitable parameter β is chosen. This is due to assigning higher weights to the more important input features at a given instance during the formation of information granules.

The benchmark with other related works also shows that the results obtained in this paper is comparable with other researchers' models, particularly involving neural network and support vector machine (SVM). In neural network research, Saha and Ramanathan (2019); Zhou et al. (2020) yield performance of 98.04% and 97.66%, respectively, for Iris data. The same authors also reported accuracies of 96.72% and 98.66% in Wine. For Breast Cancer dataset, the radial basis function network (RBFN) model developed in Chaurasia et al. (2018) achieves 96.77%, while in another research (Islam et al., 2020) scores 98.51%. Even though the results obtained in Heart are not as high as in other datasets (87.46% in RBFNN and 84% in GT2-RBFNN), they are still comparable with other works, for example in Zhang et al. (2020) with 86.25% and Begum et al. (2021) with 83% accuracies. For Cardiotocography dataset, neural network research show the accuracies of 84.2% (Çapa, 2018), 86.38% (Parvathsuman et al., 2019) and 83.12% (Amin et al., 2019).

The results are also compared against another popular method, SVM. Iris results presented in Nasiri and Shakibian (2022) and Singaravelan et al. (2018) for example, show the accuracies of 96% and 98%, while Wine results in (Deng et al., 2017; Nasiri & Shakibian, 2022) are 98.73% and 100%, respectively. Comparable performance also can be seen in Breast Cancer and Heart, for example in (Nurhayati, Agustian, & Lubis, 2020) (Breast Cancer—98.07%), (Islam et al., 2020) (Breast Cancer—97.14%), (Panup et al., 2022) (Heart—82.9%) and (Khanom et al., 2020) (Heart—86.89%). Finally, the performance of SVM with Cardiotocography is also compared, for example 74.92% in Amin et al. (2019), 86.59% in Prasetyo et al. (2021) and 84.38% in Parvathsuman et al. (2019).

6.2 | Interpretability index

In the proposed research, both the interpretability and accuracy features are considered. Interpretability is defined as the attribute of a model to allow human to understand the behaviour of a system (Shukla & Tripathi, 2012). We are interested to investigate the impact on model's interpretability by the improved predictive performance.

In this paper, the interpretability is measured with Nauck's index (NI) which is proposed by Nauck (2003). It is defined as:

$$\text{Nauck index} = \text{comp} \times \overline{\text{cov}} \times \overline{\text{part}} \tag{22}$$

(the details of NI can be referred to Nauck (2003)).

Table 7 shows the NI for the W-GrC and the original GrC in both RBFNN and GT2-RBFNN. It is shown that in both models, W-GrC can produce higher accuracy without significantly sacrifice the model interpretability. The impact on NI is minor, as presented in Table 7. Note that for GT2-RBFNN, there are two values for the interpretability index: upper MF (UMF) and lower MF (LMF).

7 | CONCLUSION

W-GrC is extended to NF modelling structures based on RBF. The feature weighting algorithm is used to depict the significance of the input features (in terms of their robustness) and such weights influence the granulation as it evolves. The impact of W-GrC is demonstrated for the first time in both T1 and T2 FLS in terms of achieved data-driven modelling accuracy (UCI machine learning repository data).

In both T1 and T2 models, W-GrC performs better against the conventional GrC when suitable hyperparameter is chosen. While the performance that can be achieved is better, one has to acknowledge the introduction of a new hypermeter (that needs to be successfully identified/tuned). In terms overall Fuzzy Logic system interpretability, the Nauck's index calculated indicates that there is no major impact on the rulebase interpretability.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available in UCI Machine Learning Repository at https://archive.ics.uci.edu/ml/about.html. These data were derived from the following resources available in the public domain: Iris,

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