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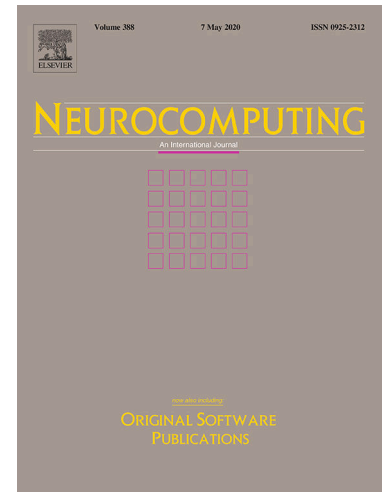
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# A Trajectory and Force Dual-incremental Robot Skill Learning and Generalization Framework using Improved Dynamical Movement Primitives and Adaptive Neural Network Control \*

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## Abstract

Due to changes in the environment and errors that occurred during skill initialization, the robot's operational skills should be modified to adapt to new tasks. As such, skills learned by the methods with fixed features, such as the classical Dynamical Movement Primitive (DMP), are difficult to use when the using cases are significantly different from the demonstrations. In this work, we propose an incremental robot skill learning and generalization framework including an incremental DMP (IDMP) for robot trajectory learning and an adaptive neural network (NN) control method, which are incrementally updated to enable robots to adapt to new cases. IDMP uses multi-mapping feature vectors to rebuild the forcing function of DMP, which are extended based on the original feature vector. In order to maintain the original skills and represent skill changes in a new task, the new feature vector consists of three parts with different usages. Therefore, the trajectories are gradually changed by expanding the feature and weight vectors, and all transition states are also easily recovered. Then, an adaptive NN controller with performance constraints is proposed to compensate dynamics errors and changed trajectories after using the IDMP. The new controller is also incrementally updated and can accumulate and reuse the learned knowledge to improve the learning efficiency. Compared with other methods, the proposed framework achieves higher tracking accuracy, realizes incremental skill learning and modification, achieves multiple stylistic skills, and is used for obstacle avoidance with different heights, which are verified in three comparative experiments.

Keywords: Incremental skill learning and generalization; Learning from demonstration; Dynamic movement primitive(DMP); Adaptive neural network (NN) control; Multiple stylistic skill generalization

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## 1. Introduction

In recent decades, learning from demonstrations (LfD), a technique that develops strategies from example states to action mappings [1], has attracted considerable attention along with the development of robotics and AI technologies. A recent survey on LfD concluded that current limitations of LfD include representation of complex behaviours, reliance on labelled data, and suboptimal and inappropriate demonstrators [2]. To solve this problem, this paper proposes an incremental skill learning and generalization framework to enable robots to modify simple initial actions to complex cases. This method is based on motion primitive (MP) technology, in which a long-term complex motion is divided into multiple sub-actions. Then, the sub-actions are extracted

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into MPs and finally these MPs are reprogrammed and generalised to fit a new task [2].

The MP can be presented in many forms, e.g. Kernelized Movement Primitive (KMP) [3], Compliant Movement Primitive (CMP) [4] and Dynamical Movement Primitive (DMP) [5]. DMP was proposed by Ijspeert et al [6], [7] and then improved by many researchers. In addition to the classical DMP, there are a number of improved methods such as discrete DMP, periodic DMP, etc., and some scholars combined DMP with reinforcement learning (RL) [8], [9], deep learning [10], life-long learning and various control methods [11]-[13] to expand the scope of DMP. DMP has a very concise expressions that is a second-order function with only three variables and a forcing function. And the applications of DMP contain trajectory tracking in Euclidean space [14], EMGs signal prediction [21], force control in a contact manipulation [22], motion and state monitoring [23] and special tasks such as obstacle avoidance [15], [17], cooperative manipulations [16], [24] and multi-modal skill learning.

The limitation of the classical DMP is that once the skills are learned, the characteristics expressed by the forcing function are fixed. Even though some variables, e.g. position, velocity, can be generalized in space and time by modifying the starts, goals and scaling factors. Some improvements of DMP in [15]-[17], [24] are made by adding additional terms for obstacle avoidance and cooperative manipulation. However, the terms are specially designed by using time-related variables such as position, angle, and velocity, etc., which cannot be generalized in phase space, like 's' in the forcing function. If operational requirements keep changing, the newly learned skills and added terms should update, which costs a lot of time and increases the complexity of computation. Reinforcement learning is used to achieve DMP-based incremental skill learning. For example, Matteo et al. proposed an incremental point-to-point motions learning method based on a dynamical system. For a new demonstration, the original dynamical system will be redesigned to approach the new task [25]. Lemme et al. proposed a bootstrapping cycle to build a suitable primitive library [26]. In this library, the old primitives are refined and new ones are added, while the unused ones are deleted. Yuan et al. [6] and Li et al. [9] followed the similar technique and updated weights by integrating probability-weighted RL to realize skill modification. Wang and Wu et al. [27], [29] proposed DMP plus (DMP+) method to realize efficient skill modifications by using truncated kernels and local biases to achieve two contributions. One is preserving the desirable properties of the original skill and achieving lower mean square errors (MSEs). The other is the reusability of existing primitives, which can reduce human fatigue in imitation learning and correcting errors in demonstration without requiring further demonstration. Compared with RL-based methods, DMP+ requires less computation and retains the original features, which is used as a benchmark method for comparison in this method.

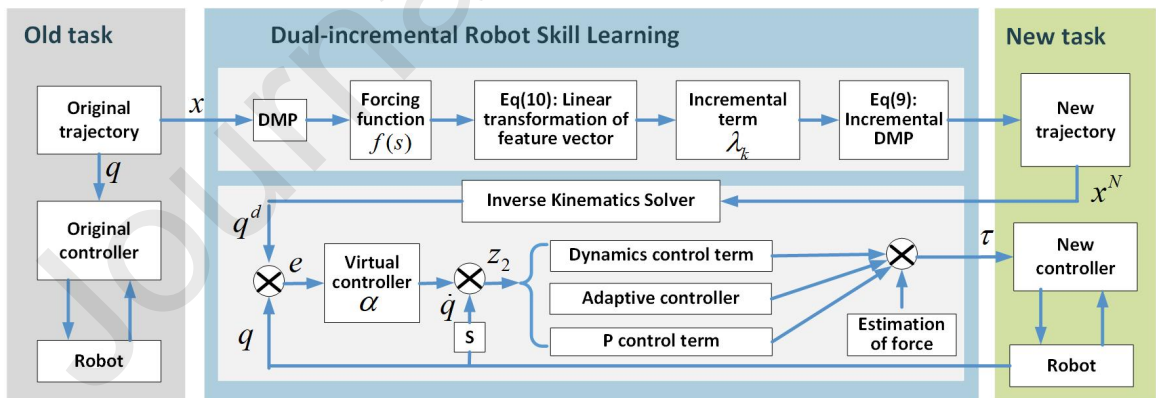


Figure.1 Diagram of trajectory and force dual-incremental robot skill learning and generalization

The combination of DMP and robot control is another topic that attracts much attention. Schaal et al., [6], [31], proposed a framework for motor control combining DMP. In our previous research, we combined DMP and adaptive NN control [21], admittance control [33], and neural networks [34]-[37] for robot control. In this paper, inspired by DMP+, we propose a novel incremental skill learning and generalization method called incremental DMP (IDMP). The forcing function of IDMP can be incrementally updated by adding new features and weights to track new trajectories. Considering uncertain dynamics parameters and state tracking limitations, an adaptive NN controller is designed, in which the NN term is also incrementally updated and can accumulate and reuse the learned knowledge to improve the learning efficiency. System stability is ensured by building a barrier Lyapunov function (BLF). The proposed framework is shown in Figure. 1: First, an old trajectory is expressed by DMP with a forcing function  $f(s)$ . After linear transformation of the feature vector, we can obtain the incremental term  $\lambda_k$  to compose a IDMP function to achieve a new trajectory  $x^N$ , which is then transferred by an inverse kinematic solver to obtain the joint information as the input of the control part. The real-time joint tracking errors are applied to create a virtual controller by BLF. We consider dynamics uncertainties and contact force estimation errors and use adaptive control term to estimate and compensate the errors based on incremental adaptive NN control to ensure system stability.

Compared to DMP-based trajectory planning and various control methods, the proposed framework offers three advantages:

#### a) Skill incremental learning and original skill preservation

Similar to DMP+ and Acnmp [39], the old skills can be preserved during the gradual adaptation process to new situations, so they can be easily recovered for the old situations. The difference in the computation from DMP+ is the skill adaptation is realized by adding new linear transformations of the existing kernels rather than changing kernels, which gives the forcing function with a stronger nonlinear adaptability and makes it more suitable for the dynamic skill learning process without adding new kernels.

#### b) High-accuracy trajectory tracking and multi-style skill transformation

According to the board learning in [20], the preliminary NN can achieve better performance after inserting additional extension nodes, which have a similar function as the linear transformations of IDMP. Therefore, we use IDMP to improve trajectory tracking performance and realize multi-style skill transformation. Multi-style skill transformation suits the situation that the original skill is ambiguous and leads to different styles in motion sequence [38]. An example, like the following second experiment, is a letter recognized as an 'a' first, and it is probably a 'u' or a 'v' after confirmation. We show the skill transformation process from one shape to multi-stylistic shapes based on the same original and extended features, but with different weights.

#### c) Adaptive NN control with constraints on the transient tracking errors

After renewing trajectory using IDMP, robot system controller should be improved to minimize tracking errors to the updated trajectory. Additionally, the controller should process the uncertain dynamics parameters, force estimation errors, and limitations on the transient state errors. In this paper, we proposed an adaptive NN and BLF-based controller, where the NN nonlinear fitting part can increase the number of neurons and update the weights, so that it can accumulate and reuse the learned weights to improve the learning efficiency.

The reminder of the paper is organized as follows: Section II briefly introduces DMP. In Section III, we present the details of IDMP and extend it to the multi-style skill learning. In Section IV, we propose the new adaptive NN controller with constraints on transient tracking errors. In Section V, three experiments are conducted to verify the above advantages. Section VI provides a final conclusion.

## 2. Related work to dynamical movement primitive

The DMP model proposed by Ijspeert et al., [6], [7] is

$$\begin{cases} \tau \dot{v} = K(g-x) - Dv + (g-x_0)f(s), \\ \tau \dot{x} = v \end{cases}, \quad (1)$$

where  $K, D > 0$  are stiffness and damping factors and  $\tau > 0$  is a timing parameter for adjusting duration of the trajectory,  $x_0$  and  $g$  are start and end of the trajectory.  $f(s) = \theta^T \Psi(s)$  is a linear combination of the normalized Gaussian functions  $\psi_i$ , where  $\theta = [w_1, w_2, \dots, w_n]^T$ ,  $\Psi(s) = [\psi_1, \psi_2, \dots, \psi_n]^T$ , and  $w_i$  is the weight of  $\psi_i$  and the Gaussian functions  $\psi_i$  is expressed as

$$\psi_i = \frac{\varphi_i(s)s}{\sum_{i=1}^n \varphi_i(s)}, \varphi_i(s) = \exp(-h_i(s-c_i)^2), \quad (2)$$

where  $c_i$  and  $h_i > 0$  are the centre and width of the radial basis function  $\varphi_i(s)$ . The transformation function (or forcing function)  $f(s)$  is expressed by the phase variable  $s$  and a canonical system

$$\tau \dot{s} = -os, \quad o > 0. \quad (3)$$

The converging time is modified by factor  $o$  to ensure  $s \rightarrow 0$  at the end state for erasing the influence of  $f(s)$  in (1). The  $\theta$  is estimated by minimizing the function

$$\min \left( \sum_{k=1}^N (f_k^{Tar} - f(s))^2 \right) \quad (4)$$

where  $f_k^{Tar}(s)$  the target value of  $f(s)$  that is calculated by the  $k$ th,  $k = 1, 2, \dots, K$  demonstrated trajectory  $x_d^k$  and velocity  $v_d^k$ :

$$f_k^{Tar} = (\tau \dot{v}_d - K(g-x_d^k) - Dv_d^k) / (g-x_0) \quad (5)$$

**Remark 1:** DMP method is also applied multi-style skill learning from multi-demonstrations, named Stylistic DMP (SDMP) [38]. The SDMP modifies (1) into

$$\begin{cases} \tau \dot{v} = K(g-x) - Dv + (g-x_0)\tilde{f}(s) \\ \tau \dot{x} = v \end{cases} \quad (6)$$

where  $\tilde{f}(s)$  is then expressed as  $\tilde{f}(s) = \sum_{j=1}^J f_j(s_j)$ ,  $f_j(s_j) = \theta_j^T \Psi_j(s_j)$ ,  $\theta_j = [w_{j1}, w_{j2}, \dots, w_{jn}]^T$ ,  $\Psi_j(s_j) = [\psi_{j1}, \psi_{j2}, \dots, \psi_{jn}]^T$ ,  $\psi_{ji} = \frac{\varphi_{ji}(s_j)s_j}{\sum_{i=1}^n \varphi_{ji}(s_j)}$ ,  $\varphi_{ji}(s_j) = \exp(-h_{ji}(s_j-c_{ji})^2)$ . Set  $\mathbf{s} = [s_1, s_2, \dots, s_J]$  as a style parameter vector and  $\Theta = [\theta_1, \theta_2, \dots, \theta_J]$  as a parameter matrix, the calculation purpose is to acquire the optimal parameter vector  $\Theta^* = [\theta_1^*, \theta_2^*, \dots, \theta_J^*]$ .

Considering different skills expressed by DMP have the same expression as in (1), and the main difference focuses on the forcing function  $f(s)$ , we can update the  $f(s)$  to realize skill incremental learning. Following the idea of board learning system (BLS) [20], an efficient incremental learning system without the need for deep architecture, incremental learning algorithm has a promising performance in calculation accuracy and learning speed. Especially, with the increase of the enhancement nodes, the network can approach a nonlinear function with any accuracy, which inspires us to build an incremental updating forcing function that the vectors  $\theta$  and  $\Psi(s)$  can be extended to change the learned skills and fit new trajectories.

The main challenge is how to reshape  $\theta$  and  $\Psi(s)$  to enable the new added features and extended terms to satisfy the properties of DMP. For example, in (2), the feature variables  $\psi_i, i=1,2,\dots,m$  are normalized and satisfy

$$\begin{aligned} \sum_{i=1}^n \psi_i &= \frac{\varphi_1(s)s}{\sum_{i=1}^n \varphi_i(s)} + \frac{\varphi_2(s)s}{\sum_{i=1}^n \varphi_i(s)} + \dots + \frac{\varphi_n(s)s}{\sum_{i=1}^n \varphi_i(s)} \\ &= \frac{\sum_{i=1}^n \varphi_i(s)s}{\sum_{i=1}^n \varphi_i(s)} \\ &= s \end{aligned} \quad (7)$$

If we set an extended function as  $\Phi_j, j=1,2,\dots,m$  and  $\hat{\psi}_i$  as the modified term to  $\psi_i, i=1,2,\dots,n$ , they will be normalized and satisfy:

$$\sum_{i=1}^n \hat{\psi}_i + \sum_{j=1}^m \Phi_j = s \quad (8)$$

Then the main question in IDMP is how to generate  $\Phi_j$  and modify  $\hat{\psi}_i$  to enable (8) is satisfied. A lemma is presented for the following deduction process.

**Lemma 1:** For matrices  $A \in R^{n \times m}$ ,  $W \in R^{m \times 1}$  and  $Y \in R^{n \times 1}$  satisfying  $Y = AW$ , if  $A$  is extended to  $\bar{A} = [A|a] \in R^{n \times (m+k)}$ , and  $a \in R^{n \times k}$ , then the new weight vector  $\bar{W} \in R^{(m+k) \times 1}$  is calculated based on the  $W$  as

$$\bar{W} = \begin{bmatrix} W - db^T Y \\ b^T Y \end{bmatrix} \quad (9)$$

where  $d = (A)^+ a$ ,  $b^T = \begin{cases} c^+ & \text{if } c \neq 0 \\ (1 + d^T d)^{-1} d^T (A)^+ & \text{if } c = 0 \end{cases}$ ,  $c = a - Ad$ .

### 3. Incremental Dynamical Movement Primitive

#### 3.1. Basic Incremental Dynamical Movement Primitive

Similar to (1), we define a new skill expressed by DMP in (10) that is different from the skills learned from the original demonstration. Given new position  $x^N$  and velocity  $v^N$ , the new skill is expressed as

$$\begin{cases} \tau \dot{v}^N = K(g - x^N) - Dv^N + (g - x_0) f^N(s) \\ \tau \dot{x}^N = v^N \end{cases}, \quad (10)$$

where  $f^N(s) = (\theta^N)^T \Psi^N(s)$  is a new nonlinear function to be learned, and  $\tau$ ,  $K$  and  $D$  are as the same as those in (1).

For a new trajectory, the previous method will redefine a new pair of vectors  $\theta^N$  and  $\Psi^N(s)$  or add new kernels to DMP to adapt to novel situations [29]. DMP+ smartly reused and modified the kernels and weights for skill efficient adaptation to avoid re-calculation [27], [29]. In IDMP, we create extended feature terms  $\eta_j(s)$  by making a linear transformation to the feature vector  $\Phi(s) = [\varphi_1(s), \varphi_2(s), \dots, \varphi_n(s)]$ :

$$\eta_j(s) \equiv \zeta_j(\Phi(s)W_{ej} + \beta_{ej}), j = 1, 2, \dots, m, \quad (11)$$

where  $\zeta_j$  represents the  $j$ th transformation function of  $\Phi(s)$ , and  $W_{ej}$  and  $\beta_{ej}$  are random weights and bias terms. Then the new feature terms  $\eta_j(s)$  are used in combination with  $\hat{\psi}$  to achieve  $\lambda_k$  in (12) to contribute to the output  $f^N(s)$  (see in Figure. 2).

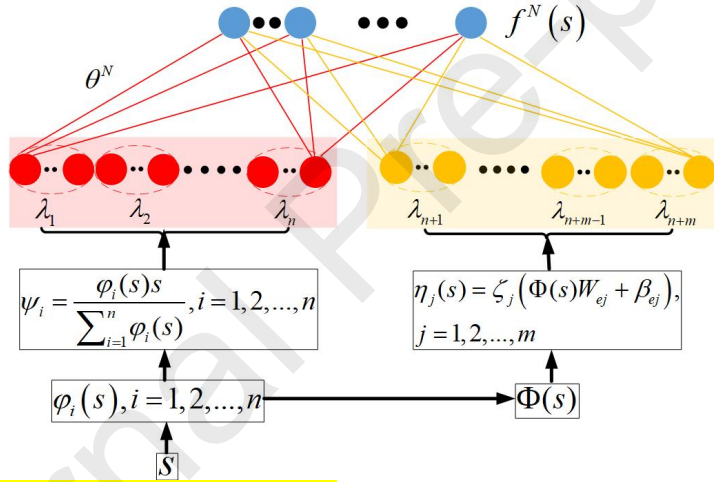


Figure.2 Diagram of incremental dynamical movement primitive

It is obvious that  $\sum_{j=1}^m \eta_j(s) + \sum_{i=1}^n \psi_i \neq s$ , such that the property (8) is not satisfied, but the property (8) is the insurance that the final state value  $x$  converging monotonically to  $g$ . Therefore, using the new term  $\eta_j(s)$  and  $\psi_i$ , we build a new term  $\lambda_k$  as

$$\lambda_k = \frac{q_k(s)\varphi_i(s) + (1 - q_k(s))\eta_j(s)}{\sum_{i=1}^n \varphi_i(s) + \sum_{j=1}^m \eta_j(s)} s, \quad k = 1, \dots, m + n, \quad (12)$$

where  $q_k(s) = 1$ , if  $k \in [1, n]$  and  $q_k(s) = 0$ , if  $k \in [n+1, n+m]$ .

It is obvious  $\sum_{k=1}^{m+n} \lambda_k = s$ , and if  $k \in [n+1, n+m]$ , (12) can be simplified as

$$\lambda_k = \frac{\eta_j(s)}{\sum_{i=1}^n \varphi_i(s) + \sum_{j=1}^m \eta_j(s)} s, \quad (13)$$

which is a modified term of  $\eta_j(s)$  and equals to the  $\Phi_j$  in (8). If  $k \in [1, n]$ , (12) can be simplified as

$$\begin{aligned} \lambda_k &= \frac{\varphi_i(s)}{\sum_{i=1}^n \varphi_i(s) + \sum_{j=1}^m \eta_j(s)} s, \\ &= \frac{\varphi_i(s)s}{\sum_{i=1}^n \varphi_i(s)} \cdot \frac{\sum_{i=1}^n \varphi_i(s)}{\sum_{i=1}^n \varphi_i(s) + \sum_{j=1}^m \eta_j(s)}, \\ &= \frac{\sum_{i=1}^n \varphi_i(s)}{\sum_{i=1}^n \varphi_i(s) + \sum_{j=1}^m \eta_j(s)} \psi_k \end{aligned} \quad (14)$$

which means the term  $\psi_k$  is scaled up  $\frac{\sum_{i=1}^n \varphi_i(s)}{\sum_{i=1}^n \varphi_i(s) + \sum_{j=1}^m \eta_j(s)}$  times and can be seen as the  $\hat{\psi}_k$  in (8) to

represent the modified  $\psi_k$ . Furthermore, we set a new scaling variable  $\Gamma$  as

$$\Gamma = \frac{\sum_{i=1}^n \varphi_i(s) + \sum_{j=1}^m \eta_j(s)}{\sum_{i=1}^n \varphi_i(s)} \quad (15)$$

Then, we can get  $\sum_{i=1}^n \varphi_i(s) + \sum_{j=1}^m \eta_j(s) = \frac{\Gamma}{\Gamma-1} \sum_{j=1}^m \eta_j(s)$ , and  $\lambda_k$  in (12) is rewritten as

$$\begin{aligned} \lambda_k(s) &= \frac{q_k(s)\varphi_i(s)s}{\sum_{i=1}^n \varphi_i(s) + \sum_{j=1}^m \eta_j(s)} + \frac{(1-q_k(s))\eta_j(s)s}{\sum_{i=1}^n \varphi_i(s) + \sum_{j=1}^m \eta_j(s)} \\ &= \frac{q_k(s)\varphi_i(s)s}{\Gamma \sum_{i=1}^n \varphi_i(s)} + \frac{(\Gamma-1)(1-q_k(s))\eta_j(s)s}{\Gamma \sum_{j=1}^m \eta_j(s)} \\ &= \frac{q_k(s)\psi_i(s)}{\Gamma} + \frac{(\Gamma-1)(1-q_k(s))\eta_j(s)s}{\Gamma \sum_{j=1}^m \eta_j(s)} \end{aligned} \quad (16)$$

Here, we set

$$\gamma_j(s) = \frac{\eta_j(s)s}{\sum_{j=1}^m \eta_j(s)}, \quad (17)$$



Then  $\lambda_k$  in (16) is further expressed as

$$\begin{aligned}\lambda_k(s) &= \underbrace{q_k(s) \frac{\psi_i(s)}{\Gamma}}_{\text{modified original skill}} + \underbrace{(1-q_k(s)) \frac{(\Gamma-1)\gamma_j(s)}{\Gamma}}_{\text{incremental skill}} \\ &= \underbrace{q_k(s) \psi_i(s)}_{\text{original skill}} + \underbrace{q_k(s) \frac{(1-\Gamma)\psi_i(s)}{\Gamma}}_{\text{old skill modification}} + \underbrace{(1-q_k(s)) \frac{(\Gamma-1)\gamma_j(s)}{\Gamma}}_{\text{incremental skill}}.\end{aligned}\quad (18)$$

**Remark 2:** Different from trajectory modification methods by adding extra terms, e.g.  $\gamma RV \varphi \exp(-\beta\varphi)$  in [15] ( $V$ ,  $\varphi$  are physical variables to represent velocity and joint), to the DMP function,  $\lambda_k$  and  $f^N(s)$  are updated by adding new feature terms  $\eta_j(s)$ , which have nonlinear relationship with  $\psi_i$  in (2). The process is somewhat similar to the truncating kernels in DMP+. While the difference is, after adding  $\eta_j(s)$ ,  $f^N(s)$  can keep updating and the learned skills can approach the desired trajectory with any accuracy, which can be explained by the principles of incremental learning in [20]. However, DMP+ depends on but is constrained by the limited kernels. The compare of two methods will be further performed through the following experiment.

Seen from (18), if  $m=0$ , we have  $\lambda_i(s) = \psi_i(s)$ , which is a standard DMP term. After adding more new terms of  $\gamma_j(s)$ , the effect of  $\psi_i(s)$  changes and the number of  $\lambda_k(s)$  increases. Here, we set  $\theta^N$  and  $\Psi^N(s)$  in (10) as  $\theta^N = [w_1, w_2, \dots, w_n, w_{n+1}, \dots, w_{m+n}]^T$  and  $\Psi(s) = [\lambda_1, \lambda_2, \dots, \lambda_{m+n}]^T$ . Each  $\lambda_k$  consists of three parts that are marked as the original skill  $q_k(s)\psi_i(s)$ , the old skill modification  $q_k(s)\psi_i(s)$  with a coefficient  $(1-\Gamma)/\Gamma$  and incremental skill generated by the normalized Gaussian function  $\gamma_j(s)$ . Setting  $Y(s) = [\gamma_1(s), \dots, \gamma_m(s)]$ , then the new forcing function  $f^N(s)$ :

$$\begin{aligned}f^N(s) &= \sum_{k=1}^{m+n} (w_k)^T \lambda_k(s) \\ &= (\theta^N)^T \Psi^N(s) \\ &= \underbrace{\theta^T \Psi(s)}_{\text{original skill}} + \underbrace{(1/\Gamma-1)(\theta^C)^T \Psi(s)}_{\text{old skill modification}} + \underbrace{(1-1/\Gamma)(\theta^U)^T Y(s)}_{\text{enhancement skill}}\end{aligned}, \quad (19)$$

where  $\theta^N$  and  $\Psi^N(s)$  are new weight and state vectors, which are expressed as

$$\begin{aligned}\theta^N &= [w_1, \dots, w_n | w_1^c, \dots, w_n^c | u_1, \dots, u_m]^T \\ &= [\theta | \theta^C | \theta^U]^T\end{aligned}\quad (20)$$

$$\begin{aligned}\Psi^N(s) &= [\psi_1(s), \dots, \psi_n(s) | (1/\Gamma-1)\psi_1(s), \dots, (1/\Gamma-1)\psi_n(s) \\ &\quad | (1-1/\Gamma)\gamma_1(s), \dots, (1-1/\Gamma)\gamma_m(s)]^T \\ &= [\Psi(s) | (1/\Gamma-1)\Psi(s) | (1-1/\Gamma)Y(s)]^T\end{aligned}\quad (21)$$

where  $\theta^C = [w_1^c, w_2^c, \dots, w_n^c]$  is the weight of  $(1/\Gamma - 1)\Psi(s)$  and  $\theta^U = [u_1, u_2, \dots, u_m]$  is the weight of  $(1 - 1/\Gamma)\Upsilon(s)$ .

Moreover,  $f^N(s)$  in (19) can be expressed as

$$\begin{aligned} f^N(s) &= \theta^T \Psi(s) + (1/\Gamma - 1) \left( (\theta^C)^T \Psi(s) - (\theta^U)^T \Upsilon(s) \right) \\ &= f(s) + (1/\Gamma - 1) \left( (\theta^C)^T \Psi(s) - (\theta^U)^T \Upsilon(s) \right) \end{aligned} \quad (22)$$

The desired value of  $(1/\Gamma - 1) \left( (\theta^C)^T \Psi(s) - (\theta^U)^T \Upsilon(s) \right)$  is  $\Delta f^N(s) = f^{N-Tar} - f(s)$ , where  $f(s)$  is calculated based on (1), and  $f^{N-Tar}$  is calculated based on the new trajectory  $x^N$  as

$$f^{N-Tar} = (\tau \dot{v}^N - K(g - x^N) - Dv^N) / (g - x_0), \quad (23)$$

From (20) and (21), the old vectors  $\theta$  and  $\Psi(s)$  are preserved in  $\theta^N$  and  $\Psi^N(s)$  and easy to be recovered by reducing new added terms. On the other hand, the skills can keep updating by extending  $\theta^N$  and  $\Psi^N(s)$  by the new features. Equation (22) shows that the new forcing function  $f^N(s)$  is calculated based on the old  $f(s)$ , and we can use  $f^{N-Tar}$  and  $f(s)$  to compute the desired value of the skill modification  $\Delta f^N(s)$  and further to obtain  $\theta^C$  and  $\theta^U$  by pseudo-inverse calculations.

Setting the initial value of  $\theta^C$  as  $\Delta f^N(s)(\Psi(s))^+$ , the weights  $\theta^C$  and  $\theta^U$  are updated by **Lemma 1** as

$$\begin{cases} \theta^C = \theta^C - \frac{db^T \Gamma}{1 - \Gamma} \Delta f^N(s) \\ \theta^U = \frac{b^T \Gamma}{1 - \Gamma} \Delta f^N(s) \\ d = -(\Psi(s))^+ \Upsilon(s) \\ c = (\Psi(s))^+ \Psi(s) \Upsilon(s) - \Upsilon(s) \\ b^T = \begin{cases} c^+ & \text{if } c \neq 0 \\ (1 + d^T d)^{-1} d^T (\Psi(s))^+ & \text{if } c = 0 \end{cases} \end{cases} \quad (24)$$

### 3.2. Incremental Dynamical Movement Primitive for multiple stylistic skill generalization

IDMP can be applied to generalize multiple stylistic skills when the initial learning skill is not accurate and there are several possible generalization solutions. Considering that all the possible skills are generated based on common initialized features, it is better to allow these skills to share the same features but with different weights. We first assume that the position and velocity terms in the multiple demonstrations are  $x_i^N$  and  $v_i^N$ ,  $i = 1, 2, \dots, m$  and the variables are  $x_i$  and  $v_i$  in the initial demonstration. By using (1), we can get an initial skill  $x_i$  and  $v_i$  as well as the common feature nodes  $\Psi(s)$  and the forcing function  $f(s)$  in the standard DMP. The next step is to compute the common extended terms  $\eta_j(s)$  and the multiple sets of  $\theta^C$  and  $\theta^U$  for

different trajectories simultaneously.

Here, we set  $\theta_k^C$  and  $\theta_k^U, k=1,2,\dots,m$  as the vectors of the  $k$ th trajectory and set  $\Theta^C = \{\theta_1^C, \theta_2^C, \dots, \theta_m^C\}$  and  $\Theta^U = \{\theta_1^U, \theta_2^U, \dots, \theta_m^U\}$  as vectors of the combination of weights. Similar to (23), we set the  $i$ th new learned skill expressed by DMP as

$$\begin{cases} \tau \dot{v}_i^N = K(g - x_i^N) - Dv_i^N + (g - x_0) f_i^N(s) \\ \tau \dot{x}_i^N = v_i^N \end{cases} \quad (25)$$

The extended term  $\eta_j(s)$  and new variable  $\lambda_k$  are computed in the same way as in (11) and (18) to achieve  $\Upsilon^c(s) = [\gamma_1(s), \gamma_2(s), \dots, \gamma_m(s)]$  and  $\gamma_i(s) = \eta_i(s) s / \sum_{j=1}^m \eta_j(s)$ . Then the  $i$ th target value of  $f_i^{N-Tar}(s)$  is

$$f_i^{N-Tar} = (\tau \dot{v}_i^N - K(g - x_i^N) - Dv_i^N) / (g - x_0). \quad (26)$$

and the  $f_i^N(s)$  in (25) has a similar expression to  $f^N(s)$  in (19) and (22) as

$$\begin{aligned} f_i^N(s) &= (\theta_i^N)^T \Psi(s) \\ &= \theta^T \Psi(s) + (1/\Gamma^c - 1) (\theta_i^C)^T \Psi(s) + (1 - 1/\Gamma^c) (\theta_i^U)^T \Upsilon^c(s) \\ &= f(s) + (1/\Gamma^c - 1) \left( (\theta_i^C)^T \Psi(s) - (\theta_i^U)^T \Upsilon^c(s) \right) \end{aligned} \quad (27)$$

where  $\Gamma^c$  is defined as same as  $\Gamma$  in (15). The desired value of the term  $(1/\Gamma^c - 1) \left( (\theta_i^C)^T \Psi(s) - (\theta_i^U)^T \Upsilon^c(s) \right)$  in (27) is

$$\Delta f_i^N(s) = f_i^{N-Tar}(s) - f(s), \quad (28)$$

where  $\Upsilon^c(s)$  represents the common extended feature vector, then the term  $\theta_i^j, i=1,2,\dots,m, j=C,U$  in a vector and  $\Theta^j, j=C,U$  are calculated based on the common terms  $\Upsilon^c(s)$  and  $\Gamma^c$ , similar to  $\Upsilon(s)$  and  $\Gamma$  in (19), as

$$\begin{cases} \theta_i^C = \theta_i^C - \frac{db^T \Gamma^c}{1 - \Gamma^c} \Delta f_i^N(s) \\ \theta_i^U = \frac{b^T \Gamma^c}{1 - \Gamma^c} \Delta f_i^N(s) \\ d = -(\Psi(s))^+ \Upsilon^c(s) \\ c = (\Psi(s))^+ \Psi(s) \Upsilon^c(s) - \Upsilon^c(s) \\ b^T = \begin{cases} c^+ & \text{if } c \neq 0 \\ (1 + d^T d)^{-1} d^T (\Psi(s))^+ & \text{if } c = 0 \end{cases} \end{cases} \quad (29)$$

Using (29), we can get  $m$  group vectors of the weight set  $[\theta_i^C, \theta_i^U]$  to express  $m$  stylistic skills. The detailed calculation procedure is realized by the pseudo code shown in Algorithm 1, for single and multiple

stylistic incremental skill learning.

### Algorithm 1: Incremental Dynamic Movement Primitive

**Input:** parameters  $K, D, \tau$ , number of kernels  $n$ , Gaussian function  $\psi_i$ ,  $i = 1, 2, \dots, n$ , transformation function  $\xi_j(\cdot)$ ,  $j = 1, 2, \dots, m$ , start  $x_0$  and end  $g$  of the trajectory  
 Demonstrations: 1)  $x, x^N$  (single style) or 2)  $x, x_i^N$  (multi-style) and convergence condition  $\delta$   
**Output:** single style skill learning: return the output weight  $\theta^N$  (including  $\theta, \theta^c$  and  $\theta^U$ ) and new skill in (10)  
 multi-style skill learning: return the output weight  $\Theta^N$  (including  $\theta, \Theta^c$  and  $\Theta^U$ ) and multiple learned skills in (25)

#### Step 1: Initial skill learning using DMP

- 1) Initialize  $s, h_i, c_i, i = 1, 2, \dots, n$  and  $\theta$  and  $\psi_i$ ;
- 2) Calculate  $f(s)$  by using (4) and  $f_k^{tar}$  in (5)
- 3) Finalize  $\theta$  and  $\varphi_i(s)$

#### Step 2: Calculate terms of the extensive skill features

- 1) Set the feature mapping group  $\Phi(s) = [\varphi_1(s), \dots, \varphi_n(s)]$
- 2) **For**  $j = 1, j < m$ , do
- 3) Random  $W_{e_j}, \beta_{e_j}$  and calculate  $\eta_j(s) \equiv \zeta_j(\Phi(s)W_{e_j} + \beta_{e_j})$
- 4) Calculate  $\Gamma$  in (15) based on  $\varphi_i(s)$  and  $\eta_j(s)$
- 5) Calculate  $\gamma_j(s)$  in (16) and new feature term  $\lambda_k$  in (17)
- 6) **End**

#### Step 3: New single or multiple stylistic skill learning

- 1) Initial the basic functions  $\Psi(s)$  and  $\Gamma^c, m = 1$
- 2) Calculate  $f^{N-Tar}$  in (23) and  $\Delta f^N(s)$  in (22) for single style skill learning or  $f_i^{N-Tar}$  in (26) and  $\Delta f_i^N(s)$  in (28) for multi-style skill learning
- 3) **While** the error term  $e = \sum_{i=1}^k |x - x_i^N|$  doesn't reach the threshold  $\delta$ , do
- 4) Random  $W_{e(j+1)}, \beta_{e(j+1)}$ ;
- 5) Calculate  $\eta_{j+1}(s) = \zeta_{j+1}(\Phi(s)W_{e(j+1)} + \beta_{e(j+1)})$  and  $\gamma_{j+1}(s)$ , add  $\gamma_{j+1}(s)$  to extend  $\Psi^N(s)$  in (21) and reinitialize  $\theta^N$  in single style skill learning or  $\Theta^U$  in multi-style skill learning
- 5) Use (24) to update  $\theta^c$  and  $\theta^U$  and related terms or use (29) to calculate  $\theta_i^c$  and  $\theta_i^U, i = 1, 2, \dots, m$
- 6) Use  $f^N(s) = (\theta^N)^T \Psi^N(s)$  or  $f_i^N(s) = (\theta_i^N)^T \Psi^N(s)$  to get  $f^N(s)$  in single style skill learning or  $f_i^N(s)$  in multi-style skill learning
- 7)  $m = m + 1$ ;
- 8) **End**
- 9) Use (10) with  $f^N(s)$ , or (25) with  $f_i^N(s)$  and new starting points, goals and scales to achieve new trajectories  $x^N$  and  $x_i^N$

**Remark 3:** Since the IDMP-based multi-skill learning are based on the common features, the learned multiple skill can be transformed between each other by only changing the weight vectors. For example, we set the common state vector for two stylistic skills as  $\Psi^N(s) = [\Psi(s)|(1/\Gamma - 1)\Psi(s)|(1 - 1/\Gamma)\Upsilon^c(s)]^T$  and weight vectors are  $\theta_1^N = [\theta|\theta_1^C|\theta_1^U]^T$  and  $\theta_2^N = [\theta|\theta_2^C|\theta_2^U]^T$  for two skills with the same length separately. The transformation of the two skills can be realized by linear interpolation [30] as

$$\begin{cases} \tau \dot{v}^N = K(g - x^N) - Dv^N + (g - x_0^N)f^N(s) \\ \tau \dot{x}^N = v^N \\ f^N(s) = (\theta^N)^T \Psi^N(s) \\ \theta^N = \alpha \theta_1^N + (1 - \alpha) \theta_2^N \end{cases}, \quad (30)$$

where  $\alpha \in (0,1)$  is an adaptive factor to enable  $\theta^N$  to change from  $\theta_1^N$  to  $\theta_2^N$ .

#### 4. Incremental Adaptive Neural Network Control

The trajectory is replanned using IDMP method in the Section above. However, as the trajectory changes, the controller's setpoints and performance limits also need to be changed. In this Section, we will propose a new incremental adaptive NN control method to accumulate and reuse the learned skill, considering the limitations of tracking errors and robot dynamics estimation errors.

##### 4.1. System dynamics model and control objectives

The dynamic model of robot system is expressed in a Lagrange-Euler form as

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau + \tau_e \quad (31)$$

where  $q \in R^n$  is the simplification of  $q(t)$  at time  $t \in R^+$  and represents the joint information of robot arm,  $M(q) \in R^{n \times n}$  is the inertia matrix,  $C(q, \dot{q}) \in R^{n \times n}$  is the Coriolis and centrifugal torque matrix, and  $G \in R^n$  is the gravitational torque. The control torques is  $\tau$  and  $\tau_e$  is a torques calculated by  $\tau_e = J_e^T(q)F_e$ , and  $F_e$  is forces exerted by the environment, and  $J_e^T(q)$  is a Jacobian matrix. Setting the position of the end effector is  $x$ , then the relationship of  $q$  and  $x$  is  $x(t) = \phi(q(t))$ ,  $\dot{x}(t) = J(q(t))\dot{q}(t)$ , where  $\phi(\cdot)$  is a function for joint and position transformation and  $J(q(t))$  is a Jacobian matrix for robot system. The desired value of  $x$  is set as  $x^d$ , which is achieved by using traditional DMP or I-DMP. Using  $x^d$ , we can calculate  $q^d$  and set the tracking error of  $q$  to  $q^d$  as  $e = q^d - q$ . The desired tracking performance is to enable  $e$  to keep within the predesigned performance  $-k_1 v(t) \leq e(t) \leq k_2 v(t)$ , where  $k_1$  and  $k_2$  are constants and  $v(t)$  is usually set as an exponential decaying performance function.

The dynamics system satisfies the following properties and assumptions:

**Property 1:** The matrices  $\dot{M}(q) - C(q, \dot{q})$  in (31) is skew -symmetric.

#### 4.2. Incremental Adaptive Neural Network control

In order to realize the predesigned performance, the system controller is designed as

$$\tau = M(q^d)\dot{\alpha} + C(q^d, \dot{q}^d)\alpha + G(q^d) + K_\tau(\alpha - \dot{q}) - \hat{\tau}_e - g(\bar{S}(z)) \quad (32)$$

where  $\alpha = \dot{q}^d - Le$ , and  $L$  is a factor calculated in the following equation,  $K_\tau$  is a positive constant factor, and  $\hat{\tau}_e = J_e^T(q)\hat{F}_e$  and  $\hat{F}_e$  represents the estimations of  $\tau_e$  and  $F_e$  separately. Usually, the estimation error term  $\tilde{\tau}_e = \tau_e - \hat{\tau}_e$  is coupled with uncertainties and disturbances [34],[35] to achieve a complex term  $Y(q, \dot{q}, e, \dot{e}, \tilde{\tau}_e) = \tilde{M}\dot{\alpha} + \tilde{C}\alpha + \tilde{G} + \tilde{\tau}_e$ , where  $\tilde{M} = M(q) - M(q^d)$ ,  $\tilde{C} = C(q, \dot{q}) - C(q^d, \dot{q}^d)$  and  $\tilde{G} = G(q) - G(q^d)$  are the uncertain terms caused by joint tracking errors,  $\tilde{\tau}_e$  represents the estimation error of the contact torque.  $g(\bar{S}(z))$  is an incremental neural networks term with an expression of  $g(\bar{S}(z)) = \hat{W}^T \bar{S}(z)$ , where  $\hat{W}$  is an estimated weight vector and  $\bar{S}(z)$  represents a vector consisted of multiple Gaussian functions. We use  $g(\bar{S}(z))$  to approach the error term  $Y(q, \dot{q}, e, \dot{e}, \tilde{\tau}_e) = W^{*T} \bar{S}(z) + \varepsilon(z)$  that is expressed by the compositions of the desired weight vector  $W^{*T}$  and  $\bar{S}(z)$ , where  $\varepsilon(z)$  is the approximation error of the neural network with the limitation of  $\|\varepsilon(z)\| \leq \varepsilon^*$ ,  $\varepsilon^* > 0$ .

Taking (32) into (31), we have

$$M(q^d)\dot{\alpha} - M(q)\ddot{q} = -C(q^d, \dot{q}^d)\alpha + C(q, \dot{q})\dot{q} - K_3(\alpha - \dot{q}) - \tau_e + \hat{\tau}_e - G(q^d) - G(q) + g(\bar{S}(z)) \quad (33)$$

According to definition of  $\alpha$ , we have  $\dot{\alpha} = \ddot{q}^d - \dot{L}e - L\dot{e}$  and take it into (33), then

$$\begin{aligned} M(q)(\dot{\alpha} - \ddot{q}) &= -C(q, \dot{q})\alpha + C(q, \dot{q})\dot{q} + K_\tau(\alpha - \dot{q}) + \tilde{M}\dot{\alpha} + \tilde{C}\alpha + \tau_e - \hat{\tau}_e + \\ &G(q^d) - G(q) - g(\bar{S}(z)) \\ &= -C(q, \dot{q})(\alpha - \dot{q}) + K_\tau(\alpha - \dot{q}) + \tilde{M}\dot{\alpha} + \tilde{C}\alpha + \tilde{G} + \tilde{\tau}_e - g(\bar{S}(z)) \\ &= -C(q, \dot{q})(\alpha - \dot{q}) + K_\tau(\alpha - \dot{q}) + Y(q, \dot{q}, e, \dot{e}, \tilde{\tau}_e) - g(\bar{S}(z)) \end{aligned} \quad (34)$$

To realize the predesigned performance and ensure the system stability, we set  $\delta = \alpha - \dot{q}$  as the velocity-level tracking error to the virtual control term  $\alpha$  and build the following barrier Lyapunov function as

$$V_q = V_1 + V_2 = \sum_{i=1}^n h_i \ln \left( \frac{(k_2 v(t))^2}{(k_2 v(t))^2 - e^2} \right) + \sum_{i=1}^n (1 - h_i) \ln \left( \frac{(k_1 v(t))^2}{e^2 - (k_1 v(t))^2} \right) + \frac{1}{2} \delta^T M(q) \delta \quad (35)$$

where  $V_2 = \frac{1}{2} \delta^T M(q) \delta$  and  $V_1$  are the items in the rest of (35), and  $h_i$  is defined as

$$h_i = \begin{cases} 1 & e > 0 \\ 0 & e \leq 0 \end{cases} \quad (36)$$

It is obvious that  $V_q > 0$  and the time derivative of  $V_q$  is expressed as

$$\begin{aligned} \dot{V}_q &= \frac{1}{2} \sum_{i=1}^n h_i \frac{(k_2 v(t))^2 - e^2}{(k_2 v(t))^2} \frac{2k_2 v(t) \dot{v}(t) \left( (k_2 v(t))^2 - e^2 \right) - (k_2 v(t))^2 (2k_2 v(t) \dot{v}(t) - 2e\dot{e})}{\left( (k_2 v(t))^2 - e^2 \right)^2} + \\ &\quad \frac{1}{2} \sum_{i=1}^n (1-h_i) \frac{e^2 - (k_1 v(t))^2}{(k_1 v(t))^2} \frac{2k_1 v(t) \dot{v}(t) \left( e^2 - (k_1 v(t))^2 \right) - (k_1 v(t))^2 (2e\dot{e} - 2k_1 v(t) \dot{v}(t))}{\left( e^2 - (k_1 v(t))^2 \right)^2} + \\ &\quad \delta^T M(q) \delta + \frac{1}{2} \delta^T \dot{M}(q) \delta \\ &= \sum_{i=1}^n h_i \frac{\dot{v}(t) \left( (k_2 v(t))^2 - e^2 \right) - k_2 v(t) (k_2 v(t) \dot{v}(t) - e\dot{e})}{k_2 v(t) \left( (k_2 v(t))^2 - e^2 \right)} + \\ &\quad \sum_{i=1}^n (1-h_i) \frac{\dot{v}(t) \left( e^2 - (k_1 v(t))^2 \right) - k_1 v(t) (e\dot{e} - k_1 v(t) \dot{v}(t))}{k_1 v(t) \left( e^2 - (k_1 v(t))^2 \right)} + \delta^T M(q) \delta + \frac{1}{2} \delta^T \dot{M}(q) \delta \\ &= \sum_{i=1}^n h_i \frac{k_2 e \dot{v}(t) - e^2 \dot{v}(t)}{v(t) \left( (k_2 v(t))^2 - e^2 \right)} + \sum_{i=1}^n (1-h_i) \frac{e^2 \dot{v}(t) - k_1 e \dot{v}(t)}{v(t) \left( e^2 - (k_1 v(t))^2 \right)} + \delta^T M(q) \delta + \frac{1}{2} \delta^T \dot{M}(q) \delta \end{aligned} \quad (37)$$

According to the definition of  $\alpha$ , we have  $\dot{q}^d = \alpha + Le = \dot{e} + \dot{q}$ , then  $\dot{e} = \alpha + Le - \dot{q}$ , then

$$\begin{aligned}
\dot{V}_q &= \sum_{i=1}^n h_i \frac{k_2 e(\alpha + Le - \dot{q})v(t) - e^2 \dot{v}(t)}{v(t)((k_2 v(t))^2 - e^2)} + \sum_{i=1}^n (1-h_i) \frac{e^2 \dot{v}(t) - k_1 e(\alpha + Le - \dot{q})v(t)}{v(t)(e^2 - (k_1 v(t))^2)} + \\
&\quad \delta^T M(q) \delta + \frac{1}{2} \delta^T \dot{M}(q) \delta \\
&= \sum_{i=1}^n h_i \frac{k_2 e(\alpha - \dot{q})v(t) + k_2 L e^2 v(t) - e^2 \dot{v}(t)}{v(t)((k_2 v(t))^2 - e^2)} + \sum_{i=1}^n (1-h_i) \frac{e^2 \dot{v}(t) - k_1 e(\alpha - \dot{q})v(t) + L k_1 e^2 v(t)}{v(t)(e^2 - (k_1 v(t))^2)} + \\
&\quad \delta^T M(q) \delta + \frac{1}{2} \delta^T \dot{M}(q) \delta \\
&= \sum_{i=1}^n h_i \frac{k_2 e(\alpha - \dot{q})}{(k_2 v(t))^2 - e^2} + \sum_{i=1}^n h_i \frac{k_2 L e^2 v(t) - e^2 \dot{v}(t)}{v(t)((k_2 v(t))^2 - e^2)} + \sum_{i=1}^n (1-h_i) \frac{-k_1 e(\alpha - \dot{q})}{e^2 - (k_1 v(t))^2} + \\
&\quad \sum_{i=1}^n (1-h_i) \frac{e^2 \dot{v}(t) + L k_1 e^2 v(t)}{v(t)(e^2 - (k_1 v(t))^2)} + \delta^T M(q) \delta + \frac{1}{2} \delta^T \dot{M}(q) \delta \\
&= \sum_{i=1}^n \left[ \frac{h_i k_2 e}{(k_2 v(t))^2 - e^2} - \frac{k_1 (1-h_i) e}{e^2 - (k_1 v(t))^2} \right] (\alpha - \dot{q}) + \sum_{i=1}^n h_i \left( k_2 L - \frac{\dot{v}(t)}{v(t)} \right) \frac{e^2}{(k_2 v(t))^2 - e^2} + \\
&\quad \sum_{i=1}^n (1-h_i) \left( \frac{\dot{v}(t)}{v(t)} + L k_1 \right) \frac{e^2}{e^2 - (k_1 v(t))^2} + \delta^T M(q) \delta + \frac{1}{2} \delta^T \dot{M}(q) \delta
\end{aligned} \tag{38}$$

If we set  $L$  as  $L = -\sqrt{\left(\frac{1}{k_2}\right)^2 + \left(\frac{1}{k_1}\right)^2 + (k_c)^2} \left\| \frac{\dot{v}(t)}{v(t)} \right\|$ , since  $\frac{e^2}{(k_2 v(t))^2 - e^2} > 0$  and  $\frac{e^2}{e^2 - (k_1 v(t))^2} > 0$ , then we

have

$$\begin{aligned}
&\sum_{i=1}^n h_i \left( k_2 L - \frac{\dot{v}(t)}{v(t)} \right) \frac{e^2}{(k_2 v(t))^2 - e^2} + \sum_{i=1}^n (1-h_i) \left( \frac{\dot{v}(t)}{v(t)} + L k_1 \right) \frac{e^2}{e^2 - (k_1 v(t))^2} < \\
&-k_c \left( \sum_{i=1}^n h_i \frac{e^2}{(k_2 v(t))^2 - e^2} + \sum_{i=1}^n (1-h_i) \frac{e^2}{e^2 - (k_1 v(t))^2} \right)
\end{aligned} \tag{39}$$

Following the inequality  $-\frac{e^2}{(k_2 v(t))^2 - e^2} \leq -\ln \left( \frac{(k_2 v(t))^2}{(k_2 v(t))^2 - e^2} \right)$  and  $-\frac{e^2}{e^2 - (k_1 v(t))^2} \leq -\ln \left( \frac{(k_1 v(t))^2}{e^2 - (k_1 v(t))^2} \right)$ ,

(39) can be expressed as

$$\begin{aligned}
&\sum_{i=1}^n h_i \left( k_2 L - \frac{\dot{v}(t)}{v(t)} \right) \frac{e^2}{(k_2 v(t))^2 - e^2} + \sum_{i=1}^n (1-h_i) \left( \frac{\dot{v}(t)}{v(t)} + L k_1 \right) \frac{e^2}{e^2 - (k_1 v(t))^2} \\
&< -k_c \left( \sum_{i=1}^n h_i \ln \left( \frac{(k_2 v(t))^2}{(k_2 v(t))^2 - e^2} \right) + \sum_{i=1}^n (1-h_i) \ln \left( \frac{(k_1 v(t))^2}{e^2 - (k_1 v(t))^2} \right) \right) \\
&= -k_c V_1
\end{aligned} \tag{40}$$



Then (38) can be simplified as

$$\dot{V}_q \leq -k_c V_1 + \sum_{i=1}^n \left[ \frac{h_i k_2 e}{(k_2 v(t))^2 - e^2} - \frac{k_1 (1-h_i) e}{e^2 - (k_1 v(t))^2} \right] \delta + \delta^T M(q) \delta + \frac{1}{2} \delta^T \dot{M}(q) \delta \quad (41)$$

According to (34), we have  $M(q) \dot{\delta} = -C(q, \dot{q}) \delta + K_r \delta + Y(q, \dot{q}, e, \dot{e}, \tilde{\tau}_e) - g(\bar{S}(z))$ . Following **Property 1**, (41) can be simplified as

$$\begin{aligned} \dot{V}_q &\leq \sum_{i=1}^n \left[ \frac{h_i k_2 e}{(k_2 v(t))^2 - e^2} - \frac{k_1 (1-h_i) e}{e^2 - (k_1 v(t))^2} \right] \delta - \delta^T C(q, \dot{q}) \delta + \delta^T Y(q, \dot{q}, e, \dot{e}, \tilde{\tau}_e) - \\ &\quad \delta^T g(\bar{S}(z)) + \frac{1}{2} \delta^T \dot{M}(q) \delta - K_r \delta^T \delta - k_c V_1 \\ &= \sum_{i=1}^n \left[ \frac{h_i k_2 e}{(k_2 v(t))^2 - e^2} - \frac{k_1 (1-h_i) e}{e^2 - (k_1 v(t))^2} \right] \delta + \delta^T Y(q, \dot{q}, e, \dot{e}, \tilde{\tau}_e) - \delta^T g(\bar{S}(z)) - K_r \delta^T \delta - k_c V_1 \\ &= \left[ \sum_{i=1}^n \left( \frac{h_i k_2 e}{(k_2 v(t))^2 - e^2} - \frac{k_1 (1-h_i) e}{e^2 - (k_1 v(t))^2} \right) + Y(q, \dot{q}, e, \dot{e}, \tilde{\tau}_e) - g(\bar{S}(z)) \right] \delta - K_r \delta^T \delta - k_c V_1 \end{aligned} \quad (42)$$

According to the definition of  $h_i$ , we have  $\frac{h_i k_2 e}{(k_2 v(t))^2 - e^2} \geq 0$  and  $-\frac{k_1 (1-h_i) e}{e^2 - (k_1 v(t))^2} \geq 0$ , then

$$\max \left( \frac{k_2 \|e\|}{((k_2 v(t))^2 - e^2)}, \frac{k_1 \|e\|}{(e^2 - (k_1 v(t))^2)} \right) \geq \frac{h_i k_2 e}{((k_2 v(t))^2 - e^2)} - \frac{k_1 (1-h_i) e}{(e^2 - (k_1 v(t))^2)} \geq 0 \quad (43)$$

then we define  $F_i = \frac{h_i k_2 e}{(k_2 v(t))^2 - e^2} - \frac{k_1 (1-h_i) e}{e^2 - (k_1 v(t))^2}$  as a positive but limited term.

Then the updating rate of the weight vector  $\hat{W}$  is

$$\dot{\hat{W}} = \left( \Gamma_s \bar{S}(z) + u \tanh \frac{u \delta}{\varpi} \right) \delta - K_s \Gamma_s \hat{W} \quad (44)$$

where  $\Gamma_s > \|\tilde{W}\|$  is a large positive matrix,  $\tanh(*)$  is a hyperbolic tangent function and  $K_s$  is positive factor.

We further create a quadratic term  $V_m = \tilde{W}^T \Gamma_s^{-1} \tilde{W} > 0$  and the time derivative of  $V_m$  is

$$\begin{aligned}
\dot{V}_m &= \tilde{W}^T \Gamma_s^{-1} \dot{\tilde{W}} \\
&= \tilde{W}^T \Gamma_s^{-1} \left( \left( -\Gamma_s \bar{S}(z) - u_i \tanh \frac{u_i \delta}{\varpi_i} \right) \delta + K_s \Gamma_s \tilde{W} \right) \\
&= \tilde{W}^T \Gamma_s^{-1} \left( \left( -\Gamma_s \bar{S}(z) - u_i \tanh \frac{u_i \delta}{\varpi_i} \right) \delta + K_s \Gamma_s (W^* - \tilde{W}) \right) \\
&= -\tilde{W}^T \bar{S}(z) \delta - \tilde{W}^T \Gamma_s^{-1} u_i \tanh \frac{u_i \delta}{\varpi_i} \delta + \tilde{W}^T K_s (W^* - \tilde{W})
\end{aligned} \tag{45}$$

Therefore for the hybrid Lyapunov function  $V = V_q + V_m > 0$ , the time derivative is expressed as

$$\begin{aligned}
\dot{V} &\leq \left[ \sum_{i=1}^n F_i + Y(q, \dot{q}, e, \dot{e}, \tilde{\tau}_e) - g(\bar{S}(z)) \right] \delta - \tilde{W}^T \bar{S}(z) \delta - \tilde{W}^T \Gamma_s^{-1} u_i \tanh \frac{u_i \delta}{\varpi_i} \delta + \\
&\quad \tilde{W}^T K_s (W^* - \tilde{W}) - K_r \delta^T \delta - k_c V_1 \\
&= \left[ \sum_{i=1}^n F_i - \tilde{W}^T \Gamma_s^{-1} u_i \tanh \frac{u_i \delta}{\varpi_i} + \mathcal{E}(z) \right] \delta + \tilde{W}^T K_s (W^* - \tilde{W}) - K_r \delta^T \delta - k_c V_1
\end{aligned} \tag{46}$$

Following the Young's inequality and definition of  $\Gamma_s$ , we can obtain the following inequality [18]

$$F \delta - \tilde{W}^T \Gamma_s^{-1} u \delta \tanh \frac{u \delta}{\varpi} \leq \iota \varpi \tag{47}$$

Thus (46) can be further deduced by expressing the weights by the extended matrices as  $W^* = [W_{ori}^* \ W_{ex}^*]^T$  and  $\tilde{W} = [\tilde{W}_{ori} \ \tilde{W}_{ex}]^T$ , where  $W_{ori}^*$  and  $\tilde{W}_{ori}$  represent the vectors of the original weight and weight error, and  $W_{ex}^*$  and  $\tilde{W}_{ex}$  are the vectors of the extend features. Then

$$\begin{aligned}
\dot{V} &\leq -k_c V_1 + \frac{1}{2} \mathcal{E}(z)^2 + \iota \varpi - K_r \delta^T \delta - \frac{1}{2} \tilde{W}^T K_s \tilde{W} + \frac{1}{2} W^{*T} K_s W^* \\
&= -k_c V_1 - K_r \delta^T \delta - \frac{1}{2} K_s \left\| \begin{matrix} \tilde{W}_{ori} \\ \tilde{W}_{ex} \end{matrix} \right\|^2 + \frac{1}{2} \mathcal{E}(z)^2 + \iota \varpi + \frac{1}{2} K_s \left\| \begin{matrix} W_{ori}^* \\ W_{ex}^* \end{matrix} \right\|^2
\end{aligned} \tag{48}$$

Considering the completed expression of  $V$  is  $V = V_1 + V_2 + V_m = V_1 + \delta^T M(q) \delta + \tilde{W}^T \Gamma_s^{-1} \tilde{W}$ , then (48) can be expressed as

$$\dot{V} = -\eta V + \sigma \tag{49}$$

where  $\eta = \min \left( \lambda_{\min}(k_c), \frac{2\lambda_{\min}(K_r)}{\lambda_{\max}(M(q))}, \frac{2\lambda_{\min}(K_s)}{\lambda_{\max}(\Gamma_s^{-1})} \right)$  and  $\sigma = \frac{1}{2} \mathcal{E}(z)^2 + \iota \varpi + \frac{1}{2} K_s \left\| \begin{matrix} W_{ori}^* \\ W_{ex}^* \end{matrix} \right\|^2$ , and the solution is

$$V(t) \leq \left( V(0) - \frac{\sigma}{\eta} \right) \exp(-\eta t) + \frac{\sigma}{\eta} \leq V(0) \exp(-\eta t) + \frac{\sigma}{\eta} \quad (50)$$

Since the terms  $\varepsilon(z)$ ,  $\iota\varpi$  and  $K_s \left\| \left[ \tilde{W}_{ori}^* \tilde{W} \right]^T \right\|^2$  are bounded and  $\sigma/\eta$  is bounded, then  $V(t)$  is bounded and converged along with the time. This completes the proof.

**Remark 4:** In our previous research [18] and [33], we proposed a combining framework of trajectory learning and board learning-based control to approximate the unknown dynamics of the robot. The improvements of this proposed method are building a new Lyapunov function and creating a new weight estimation function (44) based on the trajectory learned by IDMP. Therefore, the controller is designed with a specific parameters as  $L$  in the definition of  $\alpha$ .

## 5. Experiments

We will verify the three contributions shown in the Introduction through three following experiments.

### 5.1. Experiment 1: Accurate trajectory approaching

In this experiment, we aim to verify the improvement of trajectory tracking accuracy by IDMP, compared with other DMP-based methods. We prepare a handwriting letter 'A' in blue in Figure. 3 and use the standard DMP, DMP+ proposed by Wang and Wu et al. in [29], and IDMP in this paper to track this trajectory with the same kernels.

First we choose 5 kernels for the forcing function for all DMP-based methods. The 5 initial kernels are chosen with random centres and widths of the radial basis functions for all three methods. The initial learning results of DMP in Figure 3(a) show a not good tracking effect.

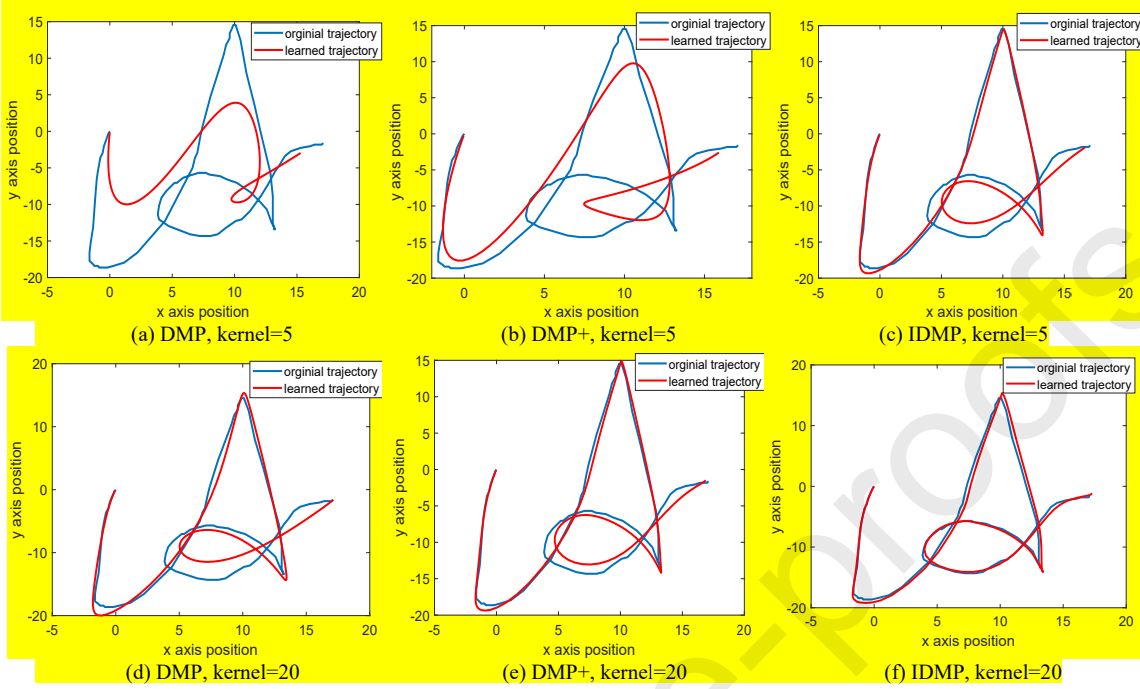


Figure 3. Handwriting trajectory learning by using standard DMP, DMP+ and IDMP with different kernels

With DMP+, the 5 kernels are modified and the mean squared error (MSE) in tracking the demonstrated trajectory is significantly reduced (see Fig.1(b)). IDMP can extend the feature vector  $\Psi(s)$  by adding new transformation terms  $\eta_j(s)$  of the 5 original kernels in (11), so that the trajectory tracking performance is further improved (see Fig.1(c)).

We further expand the number of initial kernels in three methods from 5 to 20 and the simulation results are shown in Figure.3 (d) to (f). The tracking performances of all methods are significantly better than those with 5 kernels. The MSEs to the original trajectory of IDMP are much lower than the results of the previous two cases and the trajectory almost coincides with the demonstration, which benefits from the increasing number of the extended features and certifies that IDMP has the best trajectory tracking accuracy among the three methods. But, it also shows that the tracking errors are still partially affected by the initial number of kernels.

### 5.2. Experiment 2: Multi-style skill learning and transformation

The second experiment is to examine multi-style skill learning, modification, and transformation. As shown in Figure 4, we retrofitted a PHANTOM Desktop haptic device and fixed a pen at the end of the effector. The demonstrator operates the haptic device to write letters and the device records trajectories of the end tip. The trajectories are processed (e.g., alignment and filtering) and then used for handwriting style learning and transformation under the control of the incremental adaptive NNcontroller in (32).



Figure 4. Experimental setup

As shown in Figure. 5 (a), we write five letters ‘a’, ‘z’, ‘l’, ‘k’ and ‘w’ with similar size and same start and end. The letter ‘a’ is selected as the initial writing style and the others are provided as the writing styles after skill modification and transformation. After matching and resampling the demonstrated trajectories, we use the method described in **Algorithm 1** to learn stylistic letters and realize skill modification and transformation from ‘a’ to other letters. The processes for the skill transformations are presented in Figure 6.

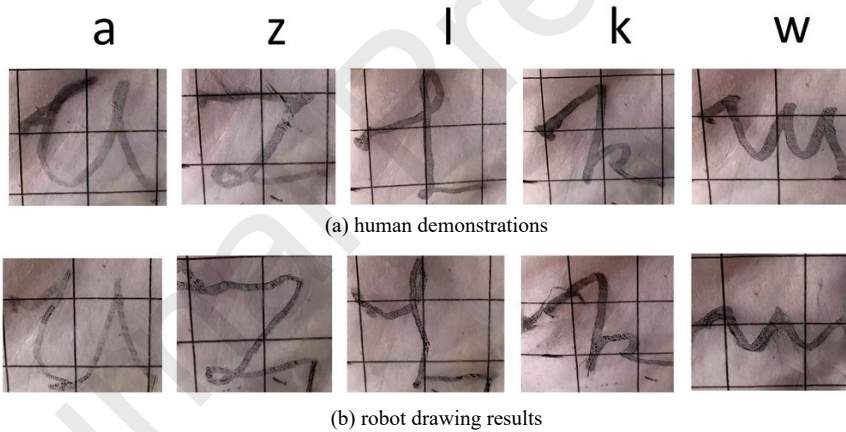


Figure 5. Human demonstrations of handwriting and robot drawing results after multi-style skill learning

The initial trajectory ‘a’ is coloured red, marked with a red square in the centre of Figure 6, and learned with the standard DMP. The targeted manuscripts are presented with black squares in the four corners. The transformation starts at centre ‘a’ to approach the handwritings in the corners, performing every 5 incremental steps. In this way, the shapes near the centre are more similar to ‘a’. With the extension of feature vector and weight vector, the learned trajectories are gradually changed from ‘a’ to other stylistic letters. After adding 30 extended feature kernels, the modified letters are clearly distinguishable from each other, resulting in the letters in the corners of Figure. 6. For some letters, such as ‘z’ and ‘k’, the modified letters from ‘a’ have been similar to the final trajectories. While, others, such as ‘w’ and ‘l’, are changed gradually to the desired states. Since the common features are determined by all the stylistic letters, the trained trajectories that are closest to

the corner demonstrations still have some differences that can be considered as compromise results. The skill transformation between different stylistic actions is realized by using (30) to change weight vectors. Finally, we utilize the haptic device as an actuator and use adaptive NN control method to draw the learning results, as shown in Figure 5(b).

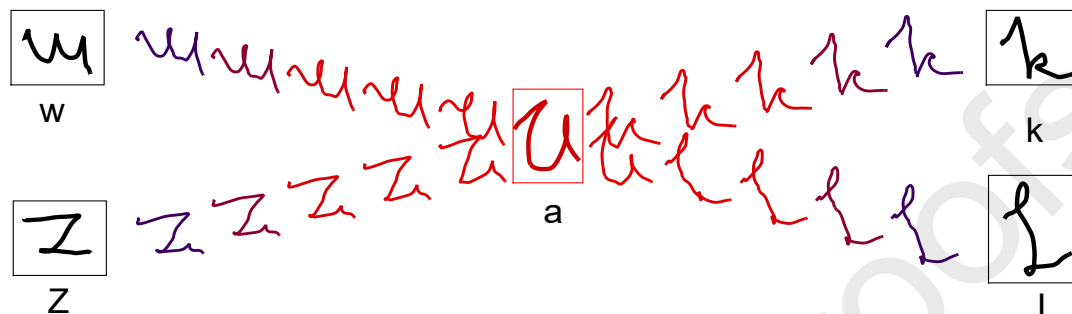


Figure 6. Incremental learning process from the letter 'a' to multiple styles of handwritings: 'w', 'z', 'k' and 'l'

### 5.3. Experiment 3: Dual-incremental skill learning and control for crossing different-height obstacles

The third experiment is also conducted with the haptic device PHANTOM Desktop and a height-adjustable obstacle to illustrate incremental skill learning process and its application of obstacle avoidance in practice. As shown in Figure 7, humans hold the joystick to cross the obstacle and put the pen tip to touch the intended target point. The current obstacle consists of three  $2\text{cm} \times 2\text{cm} \times 2\text{cm}$  cubes, each of which can be added and removed to change the height of the obstacle. On the top of the cubes, we add a  $2\text{cm} \times 8\text{cm} \times 0.2\text{cm}$  lip. On the base plane, we set one start point and nine target points on the two sides of the obstacle. The central point on the right side is used for human demonstration and the other points, which are  $2\text{cm}$  away from each other, are used for skill generalization.

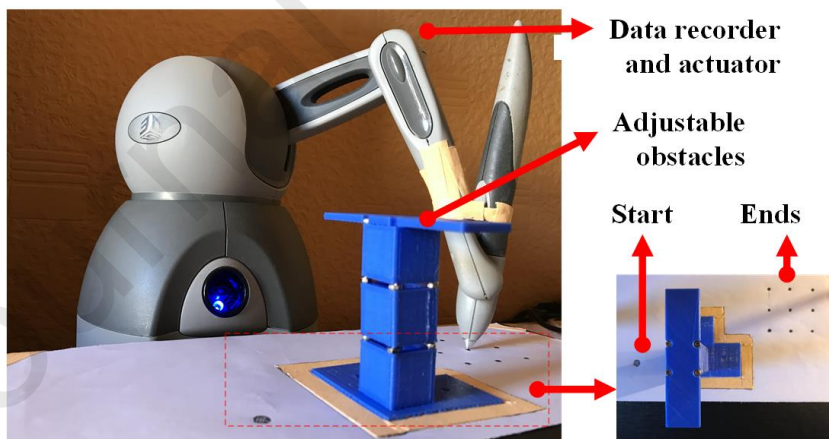


Figure 7. Experimental setup for obstacle avoidance

After collecting data from demonstrations (gray lines in Figure. 8 (a)), we use DMP to learn an initial skill of crossing an obstacle with a height of 1 block (red line in Figure. 8 (a)). In our previous work [21], [28], the

demonstrations for the height-adjustable obstacle are divided into several phases and the final positions of the internal phases are changed according to the heights of the new obstacles. In this paper, we use the IDMP to realize skill modification. As shown in Figure. 8 (b), the deep green and dark blue lines are the final learning results for crossing two and three blocks. The lines with gradient colours from red to green and from green to blue show the learning process with the increase of extended features, according to the diagram in Algorithm 1. Figure. 8 (b) also verifies the convergence of the learning results such that the degrees of curve changes are decreasing until they approach the final learning results. After gaining the ability to overcome the obstacles of different heights, we can generalize them to achieve different goals by changing factor  $g$  in (25).

In Figure. 8 (c) and Figure. 8 (d), the green lines show the learning effect for different obstacles and the red lines are the generalized trajectories of the robot end to achieve different goals.

Using the adaptive NN controller in (32), the joystick works as an actuator to follow the generalized skills with limited tracking errors. Figure. 9 (a) shows human demonstrations process. Figure. 9 (b) and Figure. 9 (c) show the joystick movement to reach the predefined goals without conflicting with obstacles.

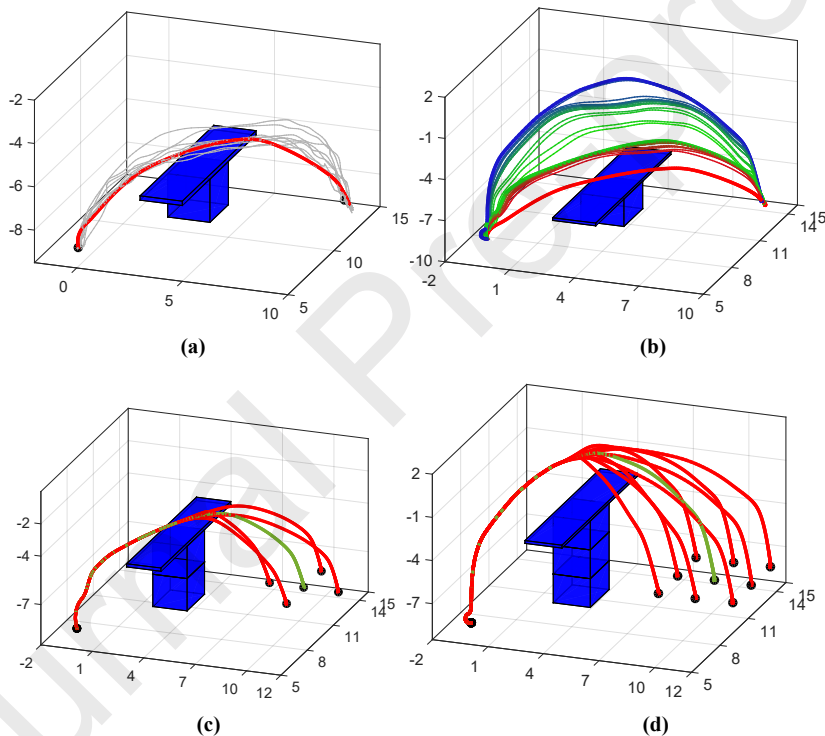


Figure 8. Skill incremental learning and generalization based on IDMP and adaptive NN-based control method (a) Demonstrations and skill learning based on DMP (gray lines are trajectories of demonstrations and the red line is the learned skill) (b) Incremental learning process for different learning skills (red line is the crossing skill for 1 block height, green line is the crossing skill for 2-block height, and blue line is for the height of 3 blocks, and the thin lines between the different skills represent the transformation of the skills) (c) Generalization of the skill for the 2-block height (green line shows the skill learning and red lines represent the skill generalizations) (d) Generalization of the skill for the height of 3 blocks (green line shows the skill learning and red lines show the skill generalizations).

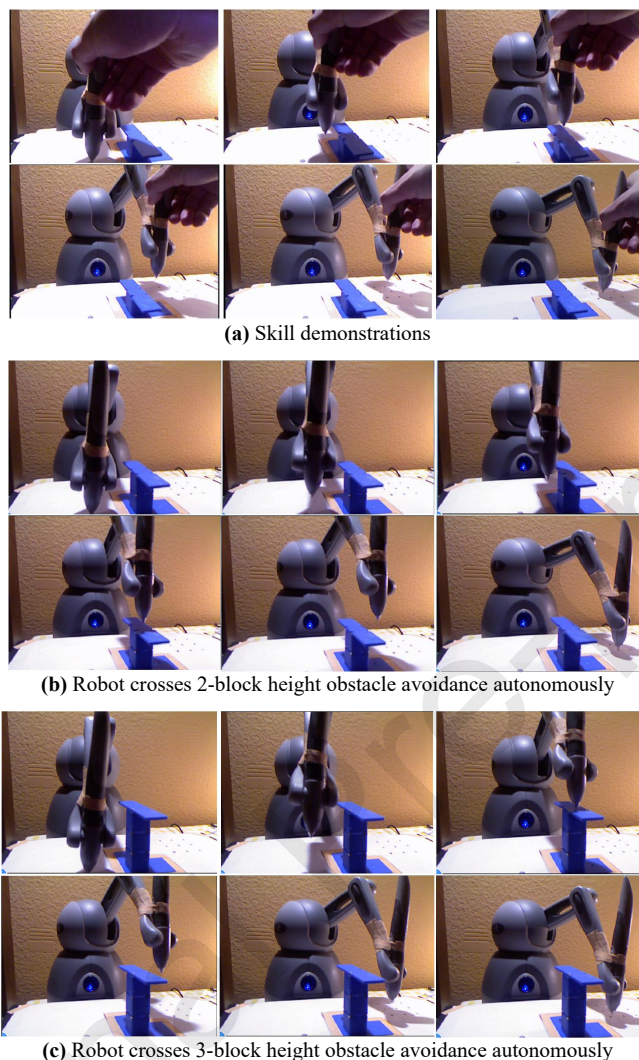


Figure 9. Skill generalization and control for height-adjustable obstacle avoidance based on human demonstrations

#### 5.4. Discussion

The three experiments verify three properties of the incremental trajectory and force learning framework for robots proposed in the Introduction. Experiment 1 shows the advantage of IDMP in terms of accuracy, compared to DMP+ and standard DMP and can realize life-long skill learning to some extent. Experiment 2 shows the versatility of IDMP in generalizing and transforming skills into multiple styles. The incremental adaptive controller ensures system stability and keeps trajectory tracking errors within performance limits. Experiment 3 shows applications of the proposed framework in obstacle avoidance. Moreover, the properties can be used in combination to achieve other goals. For example, we can first generalize the original skill to multi-style skills, and then further refine the details of a specific style. Since the old features and weights are



contained in the vectors, we can easily perform skill transformation from one to another or transformation between different skills as **Remark 3** shown, and ensure the smoothness of the transformation by choosing an appropriate  $\alpha$ . But, since IDMP is calculated based on the elements of the original DMP skills, the proposed method is still limited by the original learning outcomes.

## 6. Conclusion

In this paper, we propose a new framework for incremental trajectory and force learning and generalization to modify initially learned skills due to the environmental changes and inaccurate initialization. The trajectory learning part is based on the IDMP, and the controller uses adaptive NN control method to reduce the cost and improve the efficiency of learning. Three experiments are taken to verify the effectiveness and advantages of the proposed framework in accurate trajectory tracking, multi-stylistic trajectory tracking and application in obstacle avoidance. Compared to other DMP-based methods, this framework achieves better performance in robot trajectory tracking and greater flexibility in skill modification and transformation.

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## A Trajectory and Force Dual-incremental Robot Skill Learning and Generalization Framework using Improved Dynamical Movement Primitives and Adaptive Neural

## Network Control

### Abstract

Due to changes in the environment and errors that occurred during skill initialization, the robot's operational skills should be modified to adapt to new tasks. As such, skills learned by the methods with fixed features, such as the classical Dynamical Movement Primitive (DMP), are difficult to use when the using cases are significantly different from the demonstrations. In this work, we propose an incremental robot skill learning and generalization framework including an incremental DMP (IDMP) for robot trajectory learning and an adaptive neural network (NN) control method, which are incrementally updated to enable robots to adapt to new cases. IDMP uses multi-mapping feature vectors to rebuild the forcing function of DMP, which are extended based on the original feature vector. In order to maintain the original skills and represent skill changes in a new task, the new feature vector consists of three parts with different usages. Therefore, the trajectories are gradually changed by expanding the feature and weight vectors, and all transition states are also easily recovered. Then, an adaptive NN controller with performance constraints is proposed to compensate dynamics errors and changed trajectories after using the IDMP. The new controller is also incrementally updated and can accumulate and reuse the learned knowledge to improve the learning efficiency. Compared with other methods, the proposed framework achieves higher tracking accuracy, realizes incremental skill learning and modification, achieves multiple stylistic skills, and is used for obstacle avoidance with different heights, which are verified in three comparative experiments.

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### Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

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