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Ulrik Egede and Justine Serrano

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A perspective of High Energy Physics from precision measurements  
*La physique des Hautes Energies du point de vue des mesures de précision*

# Rare $b$ -hadron decays

## *Désintégrations rares des hadrons beaux*

Ulrik Egede<sup>a</sup> and Justine Serrano<sup>\*, b</sup>

<sup>a</sup> School of Physics and Astronomy, Monash University, Melbourne, Australia

<sup>b</sup> Aix Marseille Univ, CNRS/IN2P3, CPPM, Marseille, France.

*E-mails:* [ulrik.egede@monash.edu](mailto:ulrik.egede@monash.edu) (U. Egede), [serrano@cppm.in2p3.fr](mailto:serrano@cppm.in2p3.fr) (J. Serrano).

**Abstract.** Rare  $b$ -hadron decays provide a rich environment to search for beyond Standard Model physics effects thanks to numerous observables. In the recent years, several tensions with the SM expectations have appeared. A review of the most important experimental results is presented together with their interpretation in the context of the effective Hamiltonian approach.

**Résumé.** Les désintégrations rares des hadrons beaux fournissent un environnement riche pour rechercher des effets de physique au-delà du Modèle Standard grâce à de nombreuses observables. Dans les dernières années, plusieurs tensions avec les prédictions du Modèle Standard sont apparues. Une revue des résultats expérimentaux les plus importants est présentée ainsi que leur interprétation dans le contexte de l'approche d'Hamiltonien effectif.

**Keywords.** Flavour physics, Rare decays, Penguin decays, Beyond standard model, B-hadron decays.

**Mots-clés.** Physique des saveurs, Désintégrations rares, Diagrammes pingouins, Physique au-delà du Modèle Standard, Désintégrations de hadrons beaux.

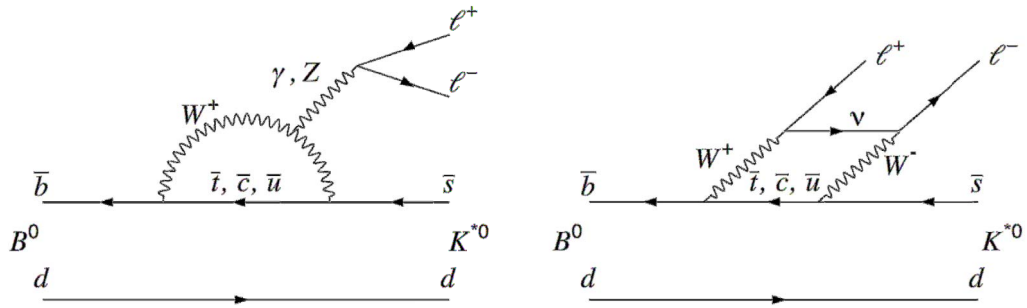
## 1. Introduction

Rare  $b$ -hadron decays are defined as flavour changing neutral current decays and have either photons or leptons in the final state. In the Standard Model (SM) these decays are proceeding through either electroweak penguin or box diagrams, with an example in Figure 1 for the decay  $B^0 \rightarrow K^{*0} \ell^+ \ell^-$ , where  $\ell = e, \mu, \tau$ . In the SM these decays are suppressed by both the small CKM factor  $V_{ts}$  (or the even smaller  $V_{td}$ ) and a loop level suppression, while in beyond the Standard Model (BSM) scenarios neither of these suppressions might be present. Combined with a very clear final state signature for many of the decays, rare  $b$ -hadron decays one of the best possible places to search for BSM physics.

Measurements of rare  $b$ -hadron decays has over the past six years shown an increasing number of discrepancies between the SM expectation and the measurements. There is no firm con-

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\* Corresponding author.



**Figure 1.** A penguin and a box diagram for the decay  $B^0 \rightarrow K^{*0} \ell^+ \ell^-$  in the Standard Model.

clusion yet if this is the result of BSM physics or a combination of experimental issues with the measurements, problems with the theoretical predictions and their uncertainties, and statistical uncertainties. After introducing the effective Hamiltonian approach and the experimental context, the sections below will outline the various experimental measurements and then move onto their interpretation in terms of effective couplings parametrising any possible BSM physics.

### 1.1. The effective Hamiltonian approach

To parametrise the branching fractions and angular distributions in rare  $b$ -hadron decays, the effective Hamiltonian [1]

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \left( \lambda_u^{(q)} \sum_{i=1}^2 \mathcal{C}_i \mathcal{O}_i^u + \lambda_c^{(q)} \sum_{i=1}^2 \mathcal{C}_i \mathcal{O}_i^c - \lambda_t^{(q)} \sum_{i=3}^{10} \mathcal{C}_i \mathcal{O}_i - \lambda_t^{(q)} \mathcal{C}_\nu \mathcal{O}_\nu + h.c. \right) \quad (1)$$

can be used, where  $q = d, s$  for processes based on the quark level  $b \rightarrow d, s$  transitions and  $\lambda_p^{(q)} = V_{pb} V_{pq}^*$ . The long-distance effects, governed by non-perturbative theory, are encoded in the operators  $\mathcal{O}_i$  while the short distance ones are proportional to the so-called Wilson coefficients  $\mathcal{C}_i$ .

The SM operators of interest for the rare  $b$ -hadron decays are the electromagnetic and chromomagnetic operators  $\mathcal{O}_{7,8}$  and the semileptonic operators  $\mathcal{O}_{9,10,\nu}$ . Under the assumption that BSM physics arise from virtual particles with a mass much above the  $b$ -quark mass, it will only affect the effective Hamiltonian by modifying the Wilson coefficients of operators that may or may not already be present in the SM. New operators could be  $\mathcal{O}'$  operators corresponding to a chirality flipped operator (right-handed current), lepton-flavour-dependent operators ( $\mathcal{O}_{9,10,\nu}^\ell$ ) in case of lepton flavour universality violation, scalar or pseudoscalar operators involving two quarks and two leptons,  $\mathcal{O}_{\mathcal{S},\mathcal{P}}$ , and lepton flavour violating operators  $\mathcal{O}_9^{\ell_i \ell_j}$ . The effective Hamiltonian approach is very powerful in the sense that it allows to combine the measured observables in a model-independent way.

### 1.2. Experimental context

To investigate rare  $B$  decays requires both a large number of  $b$ -hadrons produced as well as the ability to trigger and identify them with high efficiency. In the past this was achieved at the  $e^+ e^-$   $B$  Factories BaBar and Belle while it has been dominated in the past decade by the LHC experiments with the majority of results coming from LHCb [2,3] and a smaller number from ATLAS [4] and CMS [5]. The future will see upgraded detectors and increased integrated luminosity for the LHC experiments while data taking has just started at the Belle II [6] experiment.

In terms of access to rare  $b$ -hadron decays the different experiments have various strengths and weaknesses. The  $B$  Factories have just a  $B^0\bar{B}^0$  or  $B^+B^-$  pair produced in an entangled state which gives an environment with a low number of particles in the final state and with kinematics that are constrained from the well defined initial state. This allows analysis of final states that include multiple neutral particles such as  $K_S^0$  and  $\pi^0$  as well as final states with neutrinos. The weakness is in the total number of  $b$ -hadrons produced which allowed for the limit  $\mathcal{B}(B^0 \rightarrow \mu^+\mu^-) < 5.2 \times 10^{-8}$  to be set [7], and in the few  $b$ -hadron species accessible. A dedicated run above the  $B_s\bar{B}_s$  threshold has been performed by Belle with a very limited statistics though allowing for limits on rare decays such as  $B_s \rightarrow \gamma\gamma$  to be set.

The hadron collider experiments have the advantage of the large cross section for  $b$ -hadron production. The challenge is to trigger the events in the presence of the overwhelming background of events with only lighter hadrons produced. For final states involving only charged particles, background particles coming from the primary interaction vertex are not an issue but this is not the case for neutrals where the tracking system does not provide the means to distinguish between particles from a detached  $b$ -vertex and the primary vertex. In general, final states with more than one neutral particle are very difficult to reconstruct. Today the best limit set on a rare decay is  $\mathcal{B}(B^0 \rightarrow \mu^+\mu^-) < 2.1 \times 10^{-10}$  [8]. The ATLAS and CMS experiments have a much smaller trigger bandwidth devoted to the decays of  $b$ -hadrons which means that they are only competitive with LHCb for a small number of final states.

The future will allow for the experimental understanding of rare decays to be further extended. The Belle II experiment [9] has just started taking data and will over the next eight years, in an environment similar to the  $B$  Factories, acquire a dataset that is about a factor 50 larger than what Belle acquired. The ongoing upgrade I [10] of LHCb and upgrade II [11] planned for 2032 will allow for datasets of rare decays that are several orders of magnitude larger than what is analysed today. For ATLAS and CMS, the high luminosity LHC running conditions will be very challenging for rare decays but in particular for the  $B^0 \rightarrow \mu^+\mu^-$  it might provide the first observation.

## 2. Radiative decays

Radiative decays, corresponding to  $b \rightarrow s/d\gamma$  transitions, are generated by the electromagnetic dipole operator  $\mathcal{O}_7$  in the SM. The contribution from the right-handed operator  $\mathcal{O}'_7$  is suppressed by  $m_{s/d}/m_b$ , making the photon final state predominantly left-handed. Several observables such as the decay rate, photon helicity or  $CP$  and isospin asymmetries can be used to test the presence of BSM physics. From the theoretical point of view, inclusive decays, denoted  $B \rightarrow X_{s/d}\gamma$ , are much cleaner than exclusive ones, for which form factors and hadronic matrix elements have to be estimated. In particular the combined inclusive  $(s+d)\gamma$   $CP$  asymmetry is  $\mathcal{O}(10^{-6})$  in the SM [12].

The inclusive branching fractions and  $CP$  asymmetries have been measured by the  $B$  Factories using various techniques: reconstructing only the final state photon, adding leptonic or hadronic tagging of the other  $b$ -hadron, or summing together different exclusive decays. The current world averages from HFLAV are  $\mathcal{B}(B \rightarrow X_s\gamma) = (3.32 \pm 15) \times 10^{-6}$  and  $\mathcal{B}(B \rightarrow X_d\gamma) = (9.2 \pm 3.0) \times 10^{-6}$  [13], extrapolating the measurements to a photon energy larger than 1.6 GeV. These results are in good agreement with the SM predictions  $\mathcal{B}(B \rightarrow X_s\gamma) = (3.36 \pm 0.23) \times 10^{-6}$  and  $\mathcal{B}(B \rightarrow X_d\gamma) = (1.73^{+0.12}_{-0.22}) \times 10^{-6}$  [14]. The measured  $CP$  asymmetries of the  $s\gamma$  and  $(s+d)\gamma$  final states are also found to agree with the SM with absolute uncertainties of 1 and 3% respectively.

Exclusive decays  $B \rightarrow f\gamma$  are particularly interesting as they provide several methods to test the photon polarisation. The first one uses time-dependent tagged analyses to measure the  $S_f$  and  $C_f$   $CP$  observables. The Belle and BaBar experiments have analysed various  $B^0$  modes, the most precise result corresponding to the  $K^{*\gamma}$  final state with an uncertainty of  $\sim 0.2$  on  $S_{K^{*\gamma}}$  for a SM

prediction of  $\mathcal{O}(0.01)$ . LHCb has recently obtained the first measurement of these parameters in a  $B_s^0$  decay with Run1 data [15]. Another method consists in measuring the up-down asymmetry in  $B \rightarrow K\pi\pi\gamma$  decays. This was performed by LHCb in four region of the  $K\pi\pi$  system, obtaining a result inconsistent with 0 polarisation at more than  $5\sigma$  [16]. The quantitative interpretation of this measurement is nevertheless complicated by the presence of different hadronic resonances. A last method to test the photon polarisation is to use angular distribution of radiative  $\Lambda_b$  decays. The first step toward this goal has been done by LHCb that observed for the first time the  $\Lambda_b \rightarrow \Lambda\gamma$  decay [17]. The photon polarisation is also accessible from an angular analysis of  $B \rightarrow K^*ee$  decays at low masses of the di-electron system, where the rate is dominated by the electromagnetic dipole operator. Current measurements give a precision of around 0.15 on this [15].

Future measurements of inclusive decay rates will be performed by Belle II. The  $\mathcal{B}(B \rightarrow X_s\gamma)$  results are systematically dominated and an uncertainty of about 3% should be achieved with the final Belle II dataset for a photon energy threshold  $E_\gamma > 1.9$  GeV. Precision on the  $CP$  asymmetries of the  $s\gamma$  and  $(s+d)\gamma$  final states will be below 0.2 and 0.5% respectively, and the one of  $S_{K^*\gamma}$  is expected to be 0.03. With the Upgrade I and II, LHCb will improve the  $CP$  measurement in the  $B_s^0 \rightarrow \phi\gamma$  channel and will also be competitive on the  $B^0 \rightarrow K_s\pi^+\pi^-\gamma$  channel. Determination of the photon polarisation will also be improved thanks to baryonic  $B$  decays and a  $B \rightarrow K^*ee$  analyses.

### 3. Purely leptonic decays

The leptonic decays  $B_q \rightarrow \ell^+\ell^-$ , where  $q = s, d$  are particularly rare in the SM. Indeed, in addition of being loop and CKM suppressed, they suffer from an additional helicity suppression, appearing when a pseudoscalar meson decay to two spin-1/2 particles. Within the framework of the effective Hamiltonian approach defined in (1), the SM branching fraction of  $B_q \rightarrow \ell^+\ell^-$  can be expressed as

$$\mathcal{B}(B_q^0 \rightarrow \ell^+\ell^-)_{SM} = \tau_{B_q} \frac{G_F^2 \alpha^2}{16\pi^2} f_{B_q}^2 m_\ell^2 m_{B_q} \sqrt{1 - \frac{4m_\ell^2}{m_{B_q}^2}} |V_{tb}V_{tq}^*|^2 |\mathcal{C}_{10}^{SM}|^2, \quad (2)$$

where  $\tau_{B_q}$ , and  $m_{B_q}$  are the  $B_q$  meson lifetime and mass,  $\alpha$  is the electromagnetic constant,  $m_\ell$  is the mass of the final state lepton, and  $f_{B_q}$  is the  $B_q$  meson decay constant. For the case of  $B_s^0$  mesons, this expression, which is valid at  $t = 0$ , has to be corrected to take into account the fact that  $B_s^0$  mesons oscillate before decay. The corresponding time-integrated branching fraction is expressed as [18]

$$\bar{\mathcal{B}}(B_s^0 \rightarrow \ell^+\ell^-)_{SM} = \frac{1 + y_s \mathcal{A}_{\Delta\Gamma}}{1 - y_s^2} \mathcal{B}(B_s^0 \rightarrow \ell^+\ell^-)_{SM}, \quad (3)$$

with  $y_s = \Delta\Gamma_s/(2\Gamma_s) = 0.065 \pm 0.005$  [13] and  $\mathcal{A}_{\Delta\Gamma} = 1$  in the SM. The SM predictions for the branching fractions are [19]

$$\begin{aligned} \mathcal{B}(B^0 \rightarrow e^+e^-) &= (2.48 \pm 0.21) \times 10^{-15}, \quad \bar{\mathcal{B}}(B_s^0 \rightarrow e^+e^-) = (8.54 \pm 0.55) \times 10^{-14}, \\ \mathcal{B}(B^0 \rightarrow \mu^+\mu^-) &= (1.06 \pm 0.09) \times 10^{-10}, \quad \bar{\mathcal{B}}(B_s^0 \rightarrow \mu^+\mu^-) = (3.65 \pm 0.23) \times 10^{-9}, \\ \mathcal{B}(B^0 \rightarrow \tau^+\tau^-) &= (2.22 \pm 0.19) \times 10^{-8}, \quad \bar{\mathcal{B}}(B_s^0 \rightarrow \tau^+\tau^-) = (7.73 \pm 0.49) \times 10^{-7}. \end{aligned}$$

Their precision is limited by the knowledge of the hadronic decay constants and the CKM matrix elements.

In case where contributions from particles beyond the SM are allowed, the  $|\mathcal{C}_{10}^{\ell SM}|^2$  factor of (2) is replaced by

$$|S|^2 \left(1 - \frac{4m_\ell^2}{m_{B_q}^2}\right) + |P|^2, \quad (4)$$

where

$$S = \frac{m_{B_q}^2}{2m_\ell} (\mathcal{C}_S^\ell - \mathcal{C}_S^{\prime\ell}), \quad \text{and} \quad P = (\mathcal{C}_{10}^\ell - \mathcal{C}_{10}^{\prime\ell}) + \frac{m_{B_q}^2}{2m_\ell} (\mathcal{C}_P^\ell - \mathcal{C}_P^{\prime\ell}). \quad (5)$$

From these equations, one can see that while  $\mathcal{C}_{10}^{(\prime)\ell}$  is affected by the helicity suppression factor  $m_\ell/m_{B_q}$ , this is not the case for the scalar and pseudoscalar contributions. It is actually a unique property of the  $B_q \rightarrow \ell^+ \ell^-$  decay to be strongly helicity suppressed in the SM but not in the presence of BSM physics scalar operators, making it a 'golden channel' to search for new physics. One may note that a pseudoscalar contribution can also suppress the branching fraction in case of negative interference with  $\mathcal{C}_{10}^\ell$ . In case of BSM physics,  $\mathcal{A}_{\Delta\Gamma}$  is expressed as

$$\mathcal{A}_{\Delta\Gamma} = \frac{\text{Re}(P^2 - S^2)}{|P|^2 + |S|^2}. \quad (6)$$

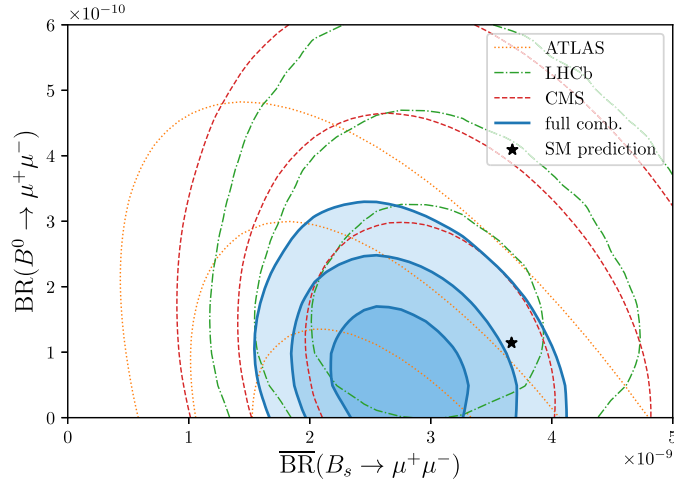
The measurement of the branching fraction and  $\mathcal{A}_{\Delta\Gamma}$ , which is accessible through the measurement of the  $B_s^0 \rightarrow \ell^+ \ell^-$  effective lifetime, can therefore provide complementary information.

The experimental search for the  $B_q \rightarrow \ell^+ \ell^-$  decays started in the eighties with the CLEO, UA1 and Argus collaborations. The limits were then improved by the Tevatron and  $B$  Factories experiments and are nowadays studied by the LHC experiments. Thanks to the high muon trigger and reconstruction efficiency of the LHC experiments, the analyses first focused on the muonic final state. The three experiments use a strategy based on a likelihood fit to the dimuon invariant mass in bins of a multivariate discriminant. The first evidence of the  $B_s^0 \rightarrow \mu^+ \mu^-$  decay has been obtained by LHCb in 2012 [20] and the first observation by a combined analysis of LHCb and CMS Run1 data [21]. The latest results from LHCb [22], ATLAS [8] and CMS [23] are based on data collected until 2016 and shown in Figure 2, where one can see a correlation between the  $B_s^0$  and  $B^0$  branching fractions due to the overlap of the two signals in the dimuon invariant mass. The highest significance is obtained by LHCb at  $7.8\sigma$  with the measurement  $\mathcal{B}(B_s^0 \rightarrow \mu^+ \mu^-) = (3.0 \pm 0.6_{-0.2}^{+0.3}) \times 10^{-9}$ . A naive combination of the two-dimensional likelihoods leads to a compatibility with the SM expectation at the level of  $\sim 2\sigma$ . The  $B^0 \rightarrow \mu^+ \mu^-$  decay is still not observed and the most stringent limit is currently obtained by ATLAS at  $2.1 \times 10^{-10}$  at 95% C.L. A measurement of the  $B_s^0 \rightarrow \mu^+ \mu^-$  effective lifetime was also obtained by the CMS and LHCb experiments, although still with a limited sensitivity to  $\mathcal{A}_{\Delta\Gamma}$ .

The electron modes are more difficult to study because of large Bremsstrahlung radiation. The current best limits are from the CDF collaboration at  $8.3 \times 10^{-8}$  ( $2.8 \times 10^{-7}$ ) for the  $B^0$  ( $B_s^0$ ) mode at 90% C.L [26], and are still about seven orders of magnitude larger than their SM prediction.

The tauonic modes are experimentally very challenging due to the  $\tau$  decay which necessarily implies undetected final states neutrinos. A first limit was obtained by BaBar on the  $B^0$  mode, by fully reconstructing the event [27]. The LHCb experiment improved this result and obtained a first limit for the  $\mathcal{B}(B_s^0 \rightarrow \tau^+ \tau^-)$  reconstructing both  $\tau$  into the  $3\pi\nu$  final state and performing a likelihood fit to the output of a boosted decision tree [28]. The corresponding limits are  $2.1 \times 10^{-3}$  ( $5.2 \times 10^{-3}$ ) for the  $B^0$  ( $B_s^0$ ) mode at 90% C.L.

Using Run3 data, LHCb will be able to measure the  $B_s^0 \rightarrow \mu^+ \mu^-$  branching fraction with a 8% uncertainty. If the Upgrade II is confirmed this number could be improved by a factor 2 at the end of the high luminosity LHC runs. Depending on their future trigger strategy, ATLAS and CMS will have measurements at 7–13% [29,30]. With this level of precision, it will be crucial to also improve the systematic uncertainties, which are up to now are dominated by the ratio of hadronisation fractions  $f_s/f_d$ . The  $B^0 \rightarrow \mu^+ \mu^-$  decay should be observed with the HL-LHC and the effective lifetime of the  $B_s^0 \rightarrow \mu^+ \mu^-$  decay could reach a precision of less than 0.1 ps, allowing to further constrain the BSM physics phase space.



**Figure 2.** Two-dimensional likelihood contours in the space of the  $B^0 \rightarrow \mu^+ \mu^-$  and  $B_s^0 \rightarrow \mu^+ \mu^-$  branching ratios from individual measurements (thin contours), the naive combination (thick solid contours). Created using FLAVIO [24, 25].

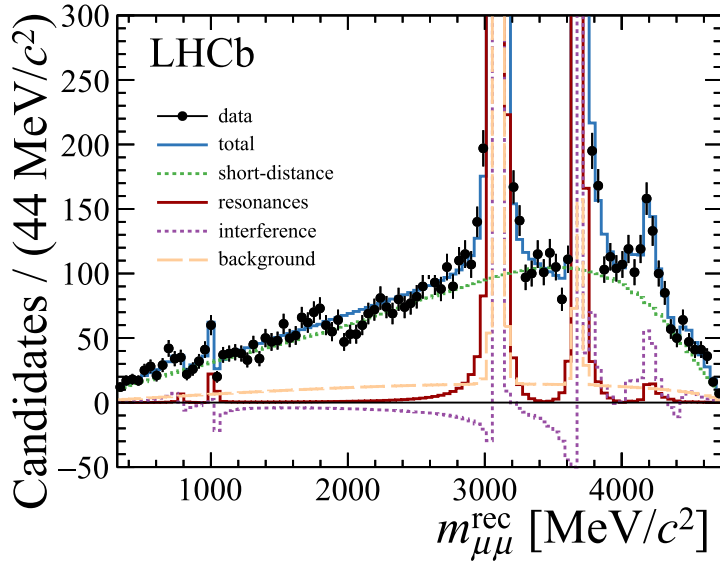
While LHCb will remain the only experiment being able to search for the  $B_s^0 \rightarrow \tau^+ \tau^-$  decay, with an expected limit at few  $10^{-4}$  with  $300 \text{ fb}^{-1}$ , Belle II could be competitive for the  $B^0 \rightarrow \tau^+ \tau^-$  decay going below  $10^{-4}$  [9].

#### 4. Semileptonic decays

Semileptonic rare  $b$ -hadron decays are FCNC decays of the type illustrated in Figure 1. While the main measurements are coming from the decays  $B^+ \rightarrow K^+ \ell^+ \ell^-$  and  $B^0 \rightarrow K^{*0} \ell^+ \ell^-$ , the other  $b \rightarrow s$  modes  $B_s^0 \rightarrow \phi \mu^+ \mu^-$ ,  $\Lambda_b^0 \rightarrow \Lambda \mu^+ \mu^-$ ,  $B^+ \rightarrow K^{*+} \ell^+ \ell^-$ ,  $B^0 \rightarrow K_S^0 \ell^+ \ell^-$  and the rarer  $b \rightarrow d$  modes  $B^+ \rightarrow \pi^+ \mu^+ \mu^-$  and  $\Lambda_b^0 \rightarrow p \pi \mu^+ \mu^-$  have been measured as well. For several of them, the differential branching fraction measured in  $q^2$  bins, where  $q^2$  is the di-lepton invariant mass squared, tend to lie below the SM prediction [31–33]. The branching fraction of the decay  $B^+ \rightarrow K^+ \ell^+ \ell^-$  is given as

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 \alpha^2 |V_{tb} V_{ts}^*|^2}{128\pi^5} |k|\beta \left\{ \frac{2}{3} |k|^2 \beta^2 |\mathcal{E}_{10} f_+(q^2)|^2 + \frac{4m_\ell^2 (m_B^2 - m_K^2)^2}{q^2 m_B^2} |\mathcal{E}_{10} f_0(q^2)|^2 \right. \\ \left. + |k|^2 \left[ 1 - \frac{1}{3} \beta^2 \right] \left| \mathcal{E}_9 f_+(q^2) + 2\mathcal{E}_7 \frac{m_b + m_s}{m_B + m_K} f_T(q^2) \right|^2 \right\}, \quad (7)$$

with  $k$  the momentum of the kaon,  $\beta = \sqrt{1 - 4m_\ell^2/q^2}$ , and  $f_0$ ,  $f_+$ , and  $f_T$  the  $B \rightarrow K$  scalar, vector and tensor form factors respectively. The expression in (7) is not the full story from an experimental point of view. The  $K^+ \ell^+ \ell^-$  final state can also be reached through decays like  $B^+ \rightarrow K^+ \psi_X$  where  $\psi_X$  represents any of the vector charmonium resonances that can subsequently decay to a pair of leptons. These decays are tree level Cabibbo favoured decays and thus several orders of magnitude more common than the  $B^+ \rightarrow K^+ \ell^+ \ell^-$  decay. The  $J/\psi$  and  $\psi(2S)$  resonances are very narrow and can be excluded from any measurements by narrow vetos in the dilepton mass, but the  $\psi$  resonances above the open charm threshold are wide, and while the resonances will predominantly decay to open charm there is still a component of decays to leptons that interfere with the semileptonic decay. In addition to this, the effect is not limited to the resonances and the influence of what is called *charm loops* is a hotly debated topic that

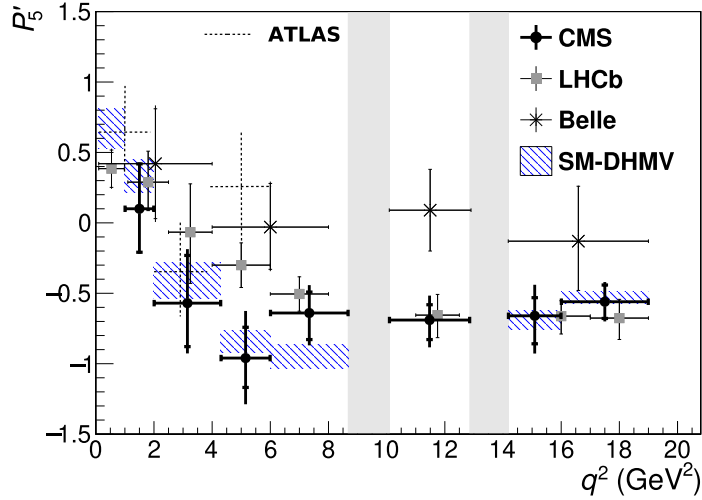


**Figure 3.** The differential branching fraction of the decay  $B^+ \rightarrow K^+ \mu^+ \mu^-$  as a function of the dimuon mass. The contribution of the charmonium resonances (that reach far out through the top of the plot), including the interference with the semileptonic decay, can clearly be seen. From [37].

will have an influence on the decay even below the kinematic limit of two charm quark masses. To measure the Wilson coefficients from the branching fraction, two approaches can be taken. Either the measurement can be made in regions well away from the charmonium resonances, such that their contribution can be ignored [34–36]; or a fit, as seen in Figure 3, can be made that try to include all knowledge of how the charmonium decays will influence (7) and a fit made to the full range of dilepton masses [37]. While experimental measurements can provide some information about the form factors as a function of  $q^2$ , the overall scaling of the form factors is a purely theoretical calculation using light cone sum rules at low  $q^2$  [38] and lattice QCD at high  $q^2$  [39]. Measurements of the  $B^+ \rightarrow K^+ \ell^+ \ell^-$  decay are thus mainly sensitive to the sum in quadrature of  $\mathcal{C}_9$  and  $\mathcal{C}_{10}$  and results in an overall uncertainty of the Wilson coefficients of around 6%.

The decay  $B^0 \rightarrow K^{*0} \ell^+ \ell^-$  with  $K^{*0} \rightarrow K^+ \pi^-$  provides as a four-body decay a much richer phenomenology than the  $B^+ \rightarrow K^+ \ell^+ \ell^-$  decay. In the angular distribution of the decay products, the different Wilson coefficients contribute in different ways, making it possible to measure them with much lower relative correlation as well as becoming less sensitive to the overall normalisation of the form factors. The full expression for the angular distribution in regions not affected by charmonium resonances can be found in Ref. [40]. The coefficients of the angular distribution can be measured directly, but from a theoretical point of view it is better to experimentally measure a number of observables that are formed from the coefficients. The idea in the observables is to form ratios where the uncertainty in the form factors are minimised [41–43]. The ATLAS [44], Belle [45], CMS [46] and LHCb [47] experiments have all measured these observables. The most famous one of these is the  $P'_5$  observable as it, as seen in Figure 4, have shown a significant deviation from the SM prediction as calculated in Ref. [48]. The branching fraction is measured as the differential branching fractions in the regions well away from the charmonium regions in  $q^2$ , then normalised relative to the branching fraction of





**Figure 4.** The  $P'_5$  observable as measured in bins of  $q^2$ . All experimental measurements compared to a theoretical prediction. Figure adapted from [46].

$B^0 \rightarrow K^{*0} J/\psi$  as measured at the  $B$  Factories and finally extended to the full  $q^2$  region using a theoretical model for the interpolation. For  $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ , HFLAV [13] is calculating the average

$$\mathcal{B}(B^0 \rightarrow K^{*0} \mu^+ \mu^-) = (1.05 \pm 0.07) \times 10^{-6}, \quad (8)$$

where the uncertainty is dominated by the normalisation.

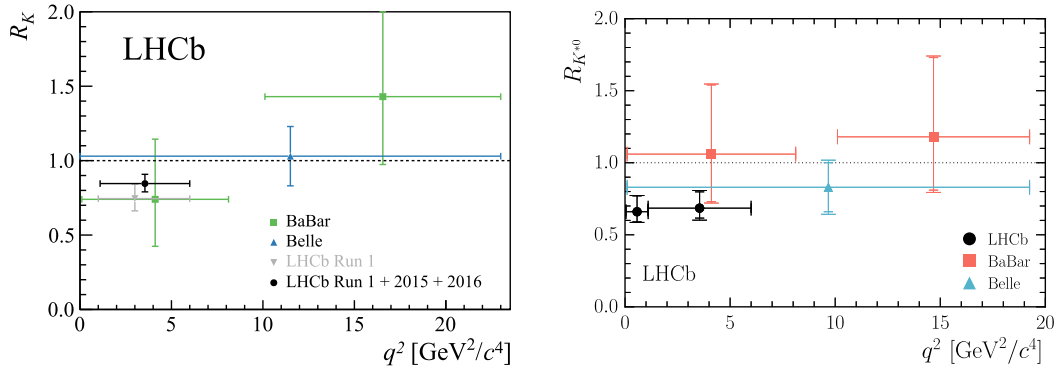
Regarding the understanding of the charm loop effects, there is promising progress on the theoretical side based on an analytical dispersion relation which can take away the need to calculate the charm loop effect directly close to the charmonium resonances [49]. There is also a proposal to fit the angular distribution of  $B^0 \rightarrow K^{*0} \mu^+ \mu^-$  in an unbinned way as a function of  $q^2$  and include the charmonium resonance regions [50].

To avoid the theoretical uncertainties from the charmonium resonances, it is also possible to look for decays of the type  $B \rightarrow h\nu\bar{\nu}$  where  $h$  represents a light-quark hadron. As the charmonium resonances only couple to  $\nu\bar{\nu}$  through the weak force, the interference from those is insignificant and a measurement of the final state is a direct measurement of the semileptonic decay, independent of  $q^2$ . However, to identify a decay with two neutrinos in the final state is a challenge and only possible at the  $B$  Factories where the full or partial reconstruction of one of the two  $B$  mesons produced allows the kinematics of the other  $B$  mesons to be fully constrained. Using this method, Belle [51] has with its full dataset put branching fraction limits in the region of  $10^{-5}$  on a number of final states. The measurement of the decays with neutrinos, in connection with the decays to charged leptons as discussed above, gives detailed information on any type of BSM physics [52].

Semileptonic decays into  $\tau$  leptons are still poorly known; only BaBar set a limit on the  $\mathcal{B}(B \rightarrow K^+ \tau^+ \tau^-)$  at  $2.25 \times 10^{-3}$  [53]. Both LHCb and Belle II should be able to study these decays in the future, reaching limits at the order of  $10^{-5}$ . The SM value of these decays should be reachable with a future high-luminosity  $Z$  factory [54].

## 5. Test of lepton flavour universality

In the SM, the electroweak bosons couple equally to the leptons of different families. This lepton flavour universality (LFU) is an accidental property of the SM and could be violated by BSM



**Figure 5.** Comparison of the different experimental measurements of  $R_K$  (left) and  $R_K^*$  (right) as function of  $q^2$ . Figures from [55, 56]. A new preliminary measurement of  $R_K^*$  by Belle II is also available in [58].

processes. Measurement of ratios of decay rates to different final state leptons, referred to as  $R$ -ratios, are expected to be 1 modulo phase-space factors related to the lepton masses, and thus provide very clean tests of the SM. In the past years some tensions in the  $R_K$  and  $R_K^*$  ratios, defined as

$$R_K^{(*)} = \mathcal{B}(B \rightarrow K^{(*)} \mu^+ \mu^-) / \mathcal{B}(B \rightarrow K^{(*)} e^+ e^-), \quad (9)$$

have appeared at the level of  $\sim 2.5\sigma$ . This is shown in Figure 5, where it can be seen that the LHCb measurements are below the SM expectation [55, 56]. While LHCb is using only the  $K^+$  and  $K^{*0}$  modes and actually measures double ratios to the resonant  $B \rightarrow K^{(*)} J/\psi$  mode in order to mitigate the reconstruction differences between electrons and muons, the  $B$  Factories also includes the  $K_s^0$  and  $K^{*\pm}$  channels [36, 57, 58]. Nevertheless, they have quite large statistical uncertainties and their results are both compatible with LHCb and the SM.

Belle also performed the first test of LFU with angular observable [45]. The results are limited by the size of the data sample but Belle II will be able to provide more stringent constraints.

These tensions triggered quite some excitement in the flavour community since similar discrepancies are also observed in test of LFU in charged currents as reviewed in Ref. [13]. More data are however necessary to confirm these effects and if the central values stay the same, both LHCb and Belle II will be able to confirm LFU violation in  $b \rightarrow s \ell^+ \ell^-$  processes around 2025 [59]. Search for LFU violation has been carried out in the  $\Lambda_b^0 \rightarrow p K^- \ell^+ \ell^-$  [60] and  $B_s^0 \rightarrow \phi \ell^+ \ell^-$  should appear in the coming years. The ATLAS and CMS experiments have improved their trigger for  $B$  physics and plan to obtain first measurements of  $R$ -ratios with LHC Run 2 data.

## 6. Search for lepton flavour/number violating decays

Lepton number is a quantum number intrinsic to each elementary particle that is defined for each family (or flavour). In the SM and in absence of neutrino masses, lepton flavour number is conserved, even though this is not associated to a fundamental symmetry. However, the observation of neutrino mixing explicitly implies that lepton flavour is not conserved in the neutrino sector. It also implies a violation of lepton flavour in the charged sector through loop processes containing neutrinos, but at a rate far from reachable by any current and future experiment ( $< 10^{-40}$ ) as underlined in [61]. The observation of a lepton flavour violating process (LFV) in the charged sector would thus be an evident sign of BSM physics. Lepton number violating (LNV) processes such as  $B^- \rightarrow \pi^+ \mu^- \mu^-$  provide hints about the nature of the neutrinos, as they could occur if the neutrino is of Majorana type.

An exhaustive review of LFV and LNV decays can be found in [13]. Most of the  $B^0$  results were obtained by BaBar at the level  $10^{-5}$ – $10^{-7}$ , the weakest limits being for final states with  $\tau$  leptons. The LHCb experiment put more stringent constraints on dimuon final states, for example in the  $B^- \rightarrow \pi^+ \mu^- \mu^-$  decays where limits at  $\sim 10^{-9}$  are obtained. The best limits for the purely leptonic  $e\mu$  and  $\tau\mu$  final states are also from LHCb, at the level of few  $10^{-9}$  and few  $10^{-5}$ , respectively.

In the future, improvements of these limits by 1 to 2 orders of magnitudes are foreseen by Belle II and LHCb. Tauonic modes will particularly benefit from an improved tagging method at Belle II [62], and improved trigger and tracking efficiency from the LHCb upgrade.

## 7. Interpretation

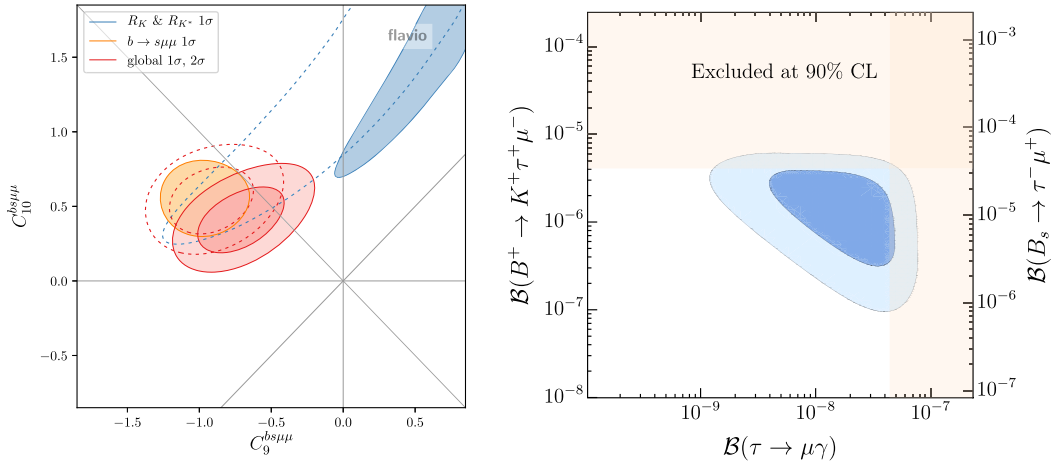
When considered all together, there are several hundred experimentally observed branching fractions, angular observables and asymmetries from rare  $b$ -hadron decays. Within the framework of the effective Hamiltonian as defined in Section 1.1, it is possible in what is called *global fits* to put all the measurements together, combine them with the theoretical uncertainties arising from QCD and fit for a consistent set of Wilson coefficients. Within the SM, the Wilson coefficients are well known, and it is thus possible in this way to ask if the data is compatible with a set of measurements from the SM or not. It can also be asked which set of Wilson coefficients are most likely to give the resulting experimental measurements.

A large number of papers has been published with global fits in recent years. The overall conclusion of these papers is that within our current understanding of the theoretical uncertainties, there is a tension between the SM prediction and the experimental results. The values of the Wilson coefficients that are giving the highest likelihood of the observed data are where a negative contribution to  $\mathcal{C}_9^\mu$  and/or a positive contribution to  $\mathcal{C}_{10}^\mu$  in addition to the SM contribution is allowed. An example [25] of such a global fit is seen in Figure 6(left). The measurements of  $R_K$  and  $R_{K^*}$  give a weaker indication that the Wilson coefficients for electrons and muons are different. Different global fits more or less agree on the Wilson coefficients that give the highest likelihood of the observed data and depending on the exact data used and theoretical assumptions made, make a BSM physics scenario favoured over the SM with significances of  $4$ – $7\sigma$  [25, 63–65]. This large variation is dominated by the uncertainties that are assigned to the non-factorisable effects in the decay and due to the influence of charm loops in regions of  $q^2$  where measurements are made for the semileptonic decays.

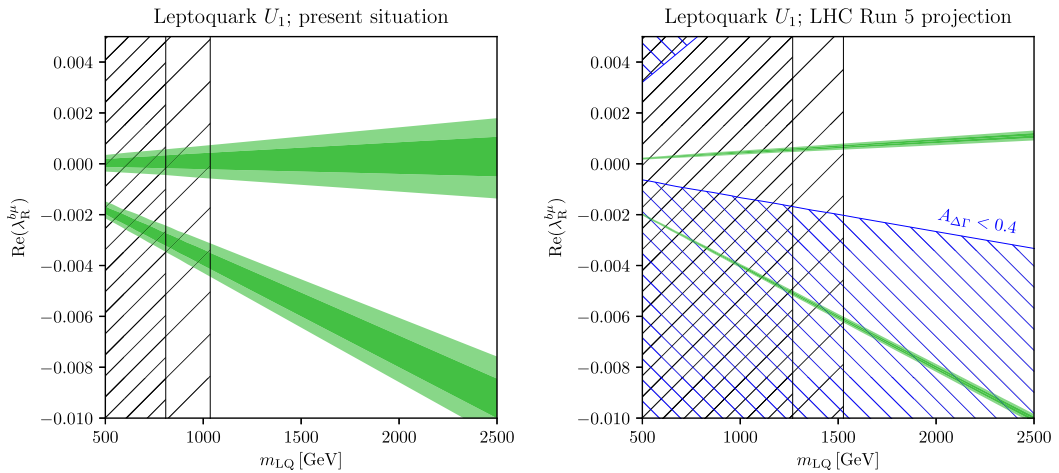
There has also been many papers that discuss which type of BSM physics could explain the observed pattern in the Wilson coefficients. These models are broadly divided up into models that introduce a new  $U(1)$  symmetry to provide a new  $Z'$  vector boson with a gauged  $L_\mu - L_\tau$  symmetry [67, 68], and models that introduce leptoquarks [66, 69]. In [66], the flavour anomalies are analysed in the context of a simplified model with a vector leptoquark  $U_1$  that can couple to both left and right-handed SM fields. Figure 6 (right) shows the preferred fit region in a plane representing different LFV decays. The current limits from  $\tau \rightarrow \mu\gamma$  and  $B_s^0 \rightarrow \tau\mu$  decays start to corner this model, demonstrating the interplay between semileptonic and LFV decays. An example of constraints from the  $\mathcal{B}(B_s^0 \rightarrow \mu^+ \mu^-)$  measurement in the  $U_1$  vector leptoquark scenario is shown in Figure 7 for current and future measurement [24]. One can see that this observable can constrain leptoquark masses well above what is reachable by direct LHC searches, and that the measurement of  $\mathcal{A}_{\Delta\Gamma}$  allows to break the degeneracies.

## 8. Conclusion

Rare  $b$ -hadron decays have been extensively studied in the past decades at different facilities. Among the large number of measured observables, some mainly related to semileptonic  $b \rightarrow s\ell\ell$



**Figure 6.** (left) Likelihood contours of the global fit and fits to subset of observables in the plane  $\mathcal{C}_{10}^\mu$  vs  $\mathcal{C}_9^\mu$  [25]. Solid (dashed) contours include (exclude) the Moriond 2019 results for  $R_K$  and  $R_{K^*}$ . (right) Preferred fit region of a  $U_1$  leptoquark model at 1 (light blue) and 2 (dark blue)  $\sigma$  as function of different LFV decays [66].



**Figure 7.** Current (left) and future (right) constraints from  $B_s^0 \rightarrow \mu^+\mu^-$  decays in the plane defined by the mass and coupling for the LQs  $U_1$ . The green bands correspond to the regions allowed by  $\mathcal{B}(B_s^0 \rightarrow \mu^+\mu^-)$  at the 1 and 2 $\sigma$  level. The black hatched regions show the exclusion from direct searches. The blue hatched region on the right plot shows the exclusion that would bring a measurement of  $\mathcal{A}_{\Delta\Gamma}$  with SM-like central value. Figures from [24].

decays have shown tensions with respect to their SM prediction. The combination of smaller statistical uncertainties from the larger datasets analysed by LHCb and Belle II, a first measurement of the decay  $B^0 \rightarrow \mu^+\mu^-$  and theoretical improvements in the understanding of the charm loop effects, it should be possible within the next decade to conclusively determine if the current indications of BSM physics are the first signs of a new sector of physics or if they are an interplay between statistical effects and issues with our current understanding of non-factorisable QCD. Im-

provements of measurements in radiative and LFV decays, as well as rare decays into  $\tau$  leptons are also expected, which will allow to further reduce the BSM physics phase space.

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