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Comptes Rendus
Mécanique

Xinhua Xue

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Volume 350 (2022), p. 159-170

<https://doi.org/10.5802/crmeca.109>

Comptes Rendus
Mécanique
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https://doi.org/10.5802/crmeca.109

Short paper / *Note*

A novel model for prediction of uniaxial compressive strength of rocks

Xinhua Xue [a]

[a] State Key Laboratory of Hydraulics and Mountain River Engineering, College of Water Resource and Hydropower, Sichuan University, Chengdu 610065, PR China
E-mail: xuexinhua@scu.edu.cn

Abstract. This paper presents an empirical model for predicting the uniaxial compressive strength (UCS) of rocks using gene expression programming (GEP). A total of 44 datasets collected from the literature was used to construct the GEP model. The GEP model developed is evaluated using four conventional regression models and an artificial neural network (ANN) model in terms of three statistical indices. The comparison results confirmed that the proposed GEP model has the lowest root mean square error (RMSE) and the highest coefficient of determination (R^2) and correlation coefficient (R) values compared to the four conventional regression models and the ANN model in the literature. It is concluded that the proposed GEP model can be applied to predict the UCS of rocks.

Keywords. Gene expression programming, Uniaxial compressive strength, Artificial neural network, Rocks, Regression models.

Manuscript received 21st June 2021, revised 30th March 2022, accepted 7th April 2022.

1. Introduction

For rock engineering design, it is important to have a reasonable determination of uniaxial compressive strength (UCS) of rocks [1–6]. The traditional experiments of UCS have some disadvantages, e.g., these are time consuming, expensive, resulting in only a limited number of UCS assessments being carried out [7–10]. Clearly, it is of importance to accurately predict the UCS of rocks using less expensive and more reliable methods.

With recent developments in computational software and hardware, many artificial intelligence (AI) methods have been widely applied in predicting the UCS of rocks [11–20]. Previous studies have confirmed that the prediction performance of artificial neural network (ANN) techniques is better than that of the existing empirical models. However, the ANN model may face some issues, e.g., slow convergence rates and convergence to local minima [21, 22]. As a branch of genetic programming (GP), gene expression programming (GEP) was first introduced by Ferreira [23, 24] and can overcome the aforementioned shortcomings of the ANN approach. The most noticeable difference between GP and GEP is that GEP uses a linear fixed length expression tree (ET), which is a representation of mathematical expressions arranged in a tree, similar to a data structure. In other words, the GEP is a tree with leaves as operands

ISSN (electronic) : 1873-7234

of the mathematical expression, and the nodes contain the operators. By using an ET, the GEP can solve relatively complex problems with high performance [25, 26].

In this study, a new empirical equation was proposed for prediction of the UCS of rocks using GEP technique. Different from regression techniques and other empirical formulas, the GEP does not need to specify a predefined function and it only needs to consider the parameters that can best fit the experimental results of UCS of rocks. Besides, the forecasting performance of the proposed GEP model was compared with some regression models and an ANN model in the literature, and the use of GEP may provide a reference if it is found to be feasible and reliable.

2. Background

Some researchers have used different indices such as the block punch index (BPI), point load strength ($I_{s(50)}$), Schmidt rebound hardness (SRH), ultrasonic p-wave velocity (USV), etc. to estimate the UCS of rock materials.

The BPI test has been developed during the last decade and provides a practical index in assessing the UCS of intact rock. It mainly involves loading of a rock disc specimen by a punching block in the middle of the specimen. The compression can induce a double shear failure in the specimen. Although this test has been less explored than other index tests, the BPI has been widely used in evaluating UCS of various rock materials. A summary of empirical equations relating BPI and UCS can be found in Ref. [4].

To date, the point load strength ($I_{s(50)}$) is considered to be the best proxy for UCS of rock materials. This test mainly involves loading cylindrical, prismatic or irregular rock specimens between conical platens and subsequently failing them [5]. The estimated point-load strength values of specimens of varying sizes and also the values corrected to a standard thickness of 50 mm, and the resultant point-load strength values ($I_{s(50)}$) have been used to estimate the UCS of rock materials which correlates well with actual recorded UCS test results.

The SRH test is an indirect method and it provides a quick and inexpensive measure of surface hardness that is widely used for estimating the mechanical properties of rock materials. The Schmidt hammer consists of a spring-controlled mass that slides on a plunger within a tubular housing. The plunger is brought into contact with the rock's surface. This provides a spring-controlled mass with a constant potential energy to hit the rock surface. Once it hits the surface, it rebounds. Previous studies have investigated a number of empirical correlations between SRH and UCS and a thorough list of such correlations can be found in Ref. [27].

The ultrasonic test is considered as a non-destructive testing technique based on the propagation of ultrasonic waves in the object or material tested. The modulation of the ultrasonic waves by microstructural variables (e.g., mineralogy, size, density, and orientation of pores and cracks) is reflected in the wave velocity, and consequently it is possible to characterize rock materials by the velocity measurements. However, limited number of research studies have focused on the correlations between USV and UCS.

In addition to the above indices, some physical properties such as porosity and density are also widely used for characterizing the physico-mechanical parameters of rock materials, and relations between the porosity and UCS of rock materials have been studied by many researchers [28–30]. These studies indicated that there exists a negative linear or curvilinear correlation between the porosity and UCS of rock materials.

3. Data collection

The UCS data from 44 samples from Ref. [8] were used to develop the proposed GEP model. Each sample contained values of all input and output parameters required for the models.

Table 1. Statistical analysis of datasets (data from Ref. [8])

Variable	Maximum	Minimum	Mean	Standard deviation
BPI (MPa)	35.36	2.53	16.04	10.158
$I_{s(50)}$ (MPa)	11.73	1.15	5.75	3.05
SRH (%)	66.51	25.89	50.23	10.477
USV (m/s)	6250	2725	5010.23	1216.119
UCS (MPa)	182.33	17.55	80.75	52.706

Table 2. Pearson's correlation coefficients between different parameters

	BPI (MPa)	$I_{s(50)}$ (MPa)	SRH (%)	USV (m/s)	UCS (MPa)
BPI (MPa)	1				
$I_{s(50)}$ (MPa)	0.878^{**}	1			
SRH (%)	0.878^{**}	0.884^{**}	1		
USV (m/s)	0.697^{**}	0.533^{**}	0.683^{**}	1	
UCS (MPa)	0.922^{**}	0.937^{**}	0.883^{**}	0.565^{**}	1

Note: **Correlation is significant at the 0.01 level (2-tailed).

According to Ref. [8], the 44 UCS data contain three kinds of rock types including granite, schist and sandstone [5]. To construct the GEP model, four main parameters were used as input variables. These parameters are the (i) BPI, (ii) $I_{s(50)}$, (iii) SRH, and (iv) USV. The UCS of rocks is used as the output variable. In order to compare with the experimental and predicted results in Ref. [8], 30 out of 44 samples were randomly selected for model training, while the remaining 14 data points were used in testing. The statistical results of data collected are summarized in Table 1. The histogram frequencies of the input and output parameters are shown in Figure 1. The Pearson's correlation coefficients between different parameters are listed in Table 2.

4. Methodology

4.1. *Artificial neural network (ANN)*

The ANN is a computational model inspired by the biological neural structure of the human brain. It mainly consists of three layers (i) the input layer, (ii) the hidden layer, and (iii) the output layer. The neighboring layers are fully interconnected by weights. In Ref. [8], the multilayer perceptron neural network was used for prediction of UCS, and the structure of the multilayer perceptron neural network used in Ref. [8] is illustrated in Figure 2.

4.2. *Gene expression programming (GEP)*

As one of the evolutionary algorithms, GEP was first introduced by Ferreira [23, 24], and it inherits many of the characteristics from genetic programming (GP) and genetic algorithms (GAs). Generally, the GEP is mainly composed of five parts; they are the (i) function set, (ii) terminal set, (iii) fitness function, (iv) control parameters, and (v) terminal condition. In GEP, the genome or chromosomes many include one or more genes, and one gene can be divided into two parts, that is, head and tail. The head part contains both functions and terminals (e.g., variables, functions, and mathematical operators), while the tail part is composed of terminals only (e.g., constants and variables). The ET diagram of chromosome is illustrated in Figure 3, and it can be written mathematically as $c(a+b)(b/a)+((b/c)-ab)$.

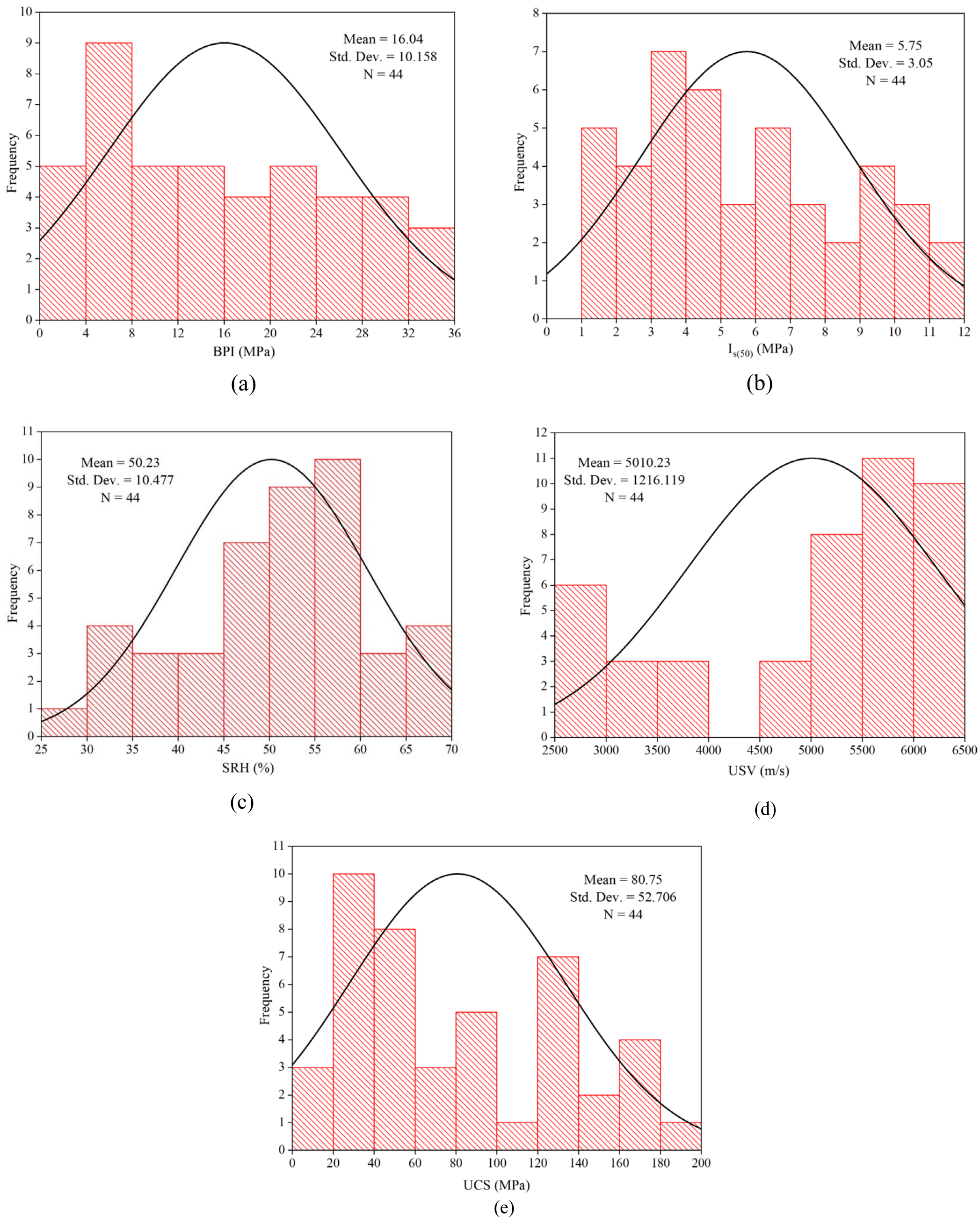

Figure 1. Histograms of the input and output parameters.

4.3. *Empirical models*

In the conventional methods, multiple regressions are often used to determine the relationships between different variables. According to Ref. [5], some predictive models established by simple and nonlinear multiple regression analysis is listed as follows:

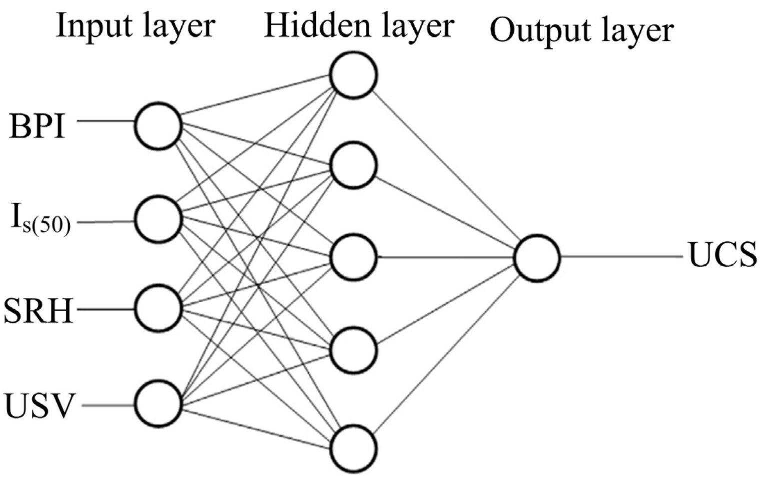

Figure 2. Structure of the multilayer perceptron neural network used in Ref. [8].

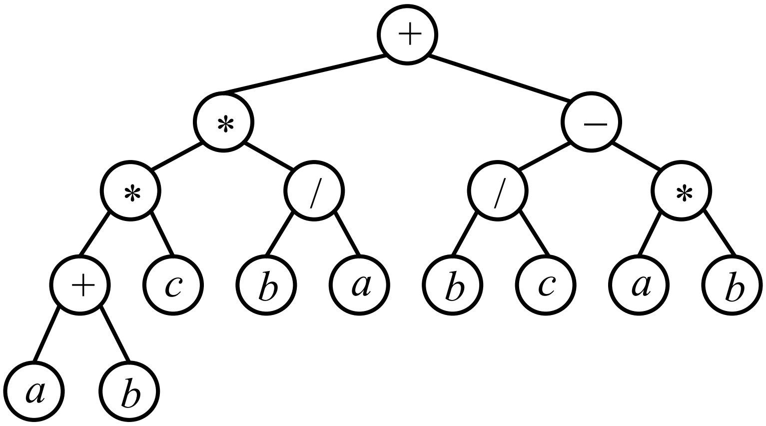

Figure 3. Example of expression tree.

(1) Simple regression analysis between BPI and UCS

$$\mathrm{UCS} = 5\mathrm{BPI}. \tag{1}$$

(2) Simple regression analysis between $I_{s(50)}$ and UCS

$$\mathrm{UCS} = 14.63 I_{s(50)}. \tag{2}$$

(3) Simple regression analysis between SRH and UCS

$$\mathrm{UCS} = 2.38\mathrm{e}^{0.065\mathrm{SRH}}. \tag{3}$$

(4) Nonlinear multiple regression (NLMR) model

$$\mathrm{UCS} = \mathrm{Exp}[0.011 \times \mathrm{BPI} + 0.065 \times I_{s(50)} + 0.029 \times \mathrm{SRH} + 0.000012 \times \mathrm{USV} + 2.157]. \tag{4}$$

4.4. *Performance evaluation*

To assess the performance of prediction models, three statistical benchmark indices including the root mean square error (RMSE), coefficient of determination (R^2) and correlation coefficient (R) were used to assess the forecasting performance of the prediction models [31,32]:

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n}} \tag{5}$$

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \overline{y}_i)^2} \tag{6}$$

$$R = \frac{\sum_{i=1}^{n}(y_i - \overline{y}_i)(\hat{y}_i - \overline{\hat{y}}_i)}{\sqrt{\sum_{i=1}^{n}(y_i - \overline{y}_i)^2 \sum_{i=1}^{n}(\hat{y}_i - \overline{\hat{y}}_i)^2}} \tag{7}$$

where y_i and $\hat{y}_i$ are the actual and predicted results, respectively. $\overline{y}_i$ and $\overline{\hat{y}}_i$ are the average of the actual and predicted results, respectively. n is the whole number of data samples.

5. Results and discussion

The best performance of the GEP is guaranteed by using the optimal setting parameters. These parameters include the number of chromosomes and genes, head size, linking function and rate of genetic operators. The main steps used in parameter identification and establishment of the GEP model are summarized as follows.

(1) First, an appropriate fitness function must be chosen. In this study, the RMSE is chosen as the fitness function and the RMSE_i of a chromosome i can be written as

$$\mathrm{RMSE}_i = \sqrt{\frac{\sum_{j=1}^{n}(P_{ij} - O_j)^2}{n}}, \tag{8}$$

where P_{ij} is the value predicted by the individual chromosome i for fitness case j, and O_j is the measured value for fitness case j.

It should be noted that (8) cannot be used directly because the fitness must increase with efficiency [24]. Therefore, the following expression is used for the fitness f_i of an individual chromosome i [24]:

$$f_i = 1000 \times \frac{1}{1 + \mathrm{RMSE}_i}, \tag{9}$$

where f_i ranges between 0 and 1000 (1000 corresponds to the ideal).

(2) A set of functions must be chosen. In this study, for the sake of simplicity, a group of straightforward mathematical functions, i.e., $\{+, -, *, /, \exp, \ln, \log, \min, \max, \mathrm{avg}\}$ was selected as the function set.

(3) The structural organization of the chromosomes, i.e., the head size and the number of genes, must be chosen. In this study, the optimal values of the numbers of genes and head size in each chromosome are determined by the trial and error strategy and set to 3 and 8, respectively. In addition, it is found that the best individuals have 30 chromosomes.

(4) The genetic operators must be selected. According to the results of the study conducted by Ferreira [24], values of 0.3, 0.3, 0.1, 0.1 and 0.044 were fixed for one-point recombination, two-point recombination, gene recombination, transposition (e.g., insertion sequence (IS) transposition, root insertion sequence (RIS) transposition, gene transposition) and mutation operators, respectively.

Table 3. The optimal parameters of GEP model

Genes	3
Chromosomes	30
Head size	8
Linking function	+
One-point recombination rate	0.3
Two-point recombination rate	0.3
Gene recombination rate	0.1
IS transposition rate	0.1
RIS transposition rate	0.1
Gene transposition rate	0.1
Mutation rate	0.044

(5) The linking function must be chosen. Different types of linking functions, including addition (+), subtraction (−), multiplication (×) and division (/), can be used in the GEP model. In this study, the linking function of addition (+) is selected because it can provide better results than other linking functions (e.g., −, ×, /).

After determining the optimal parameters in steps 1–5 (as listed in Table 3), GEP model can be established for predicting the UCS of rocks. The resulting ET of the best GEP model is illustrated in Figure 4.

In Figure 4, d_0, d_1, d_2 and d_3 denote BPI, $I_{s(50)}$, SRH and USV, respectively. The constant of the first gene c_6 is −10.62. The constant of the second gene c_6 is −9.559. The constants of the third gene c_6 and c_9 are −9.559 and 8.849, respectively. The linking function or linker is an addition and the proposed GEP model can be written as follows:

$$\begin{aligned} \mathrm{UCS} = {} & \tan\left(\frac{\mathrm{SRH}}{I_{s(50)}}\right) - \sin(\mathrm{USV}) + \max(\mathrm{SRH}, \mathrm{BPI}) + \mathrm{BPI}[0.25 I_{s(50)} + 0.25 + \cos(\mathrm{SRH})] \\ & + 2.5 I_{s(50)} + \max(8.849, \mathrm{BPI}) - 29.738. \end{aligned} \tag{10}$$

Figure 5 plots the fitting relationship between the measured and predicted UCS results using the proposed GEP model for the training, testing and total data samples, respectively.

The forecasting performance comparisons of these six models are listed in Table 4 and Figure 6. As shown, regardless of the training, testing or total data sets, the R and R^2 values of the proposed GEP model are the highest while the values of RMSE of the proposed GEP model are the lowest among these six models. For example, for the training data samples, the values of RMSE, R and R^2 of the proposed GEP model are 10.22, 0.9806 and 0.9651, respectively. However, the values of RMSE, R and R^2 of the ANN, Equations (1)–(4) are 13.88, 20.74, 17.17, 21.64 and 15.16; 0.9666, 0.9251, 0.9559, 0.9238 and 0.9606; 0.9343, 0.8559, 0.9137, 0.8534 and 0.9227, respectively. Obviously, the forecasting performance of the proposed GEP model surpasses the other five models. In addition, it can be confirmed that the predictive ability of ANN model is much better than that of the regression models. Among the four regression models, the predictive ability of the NLMR model (4) is better than that of the simple regression models (e.g., Equations (1)–(3)).

6. Conclusions

In this study, an empirical model for predicting the UCS of rocks is developed by using the GEP technique. A total of 44 datasets collected from the literature was used to construct the GEP model. The developed GEP model is assessed using four conventional regression models and

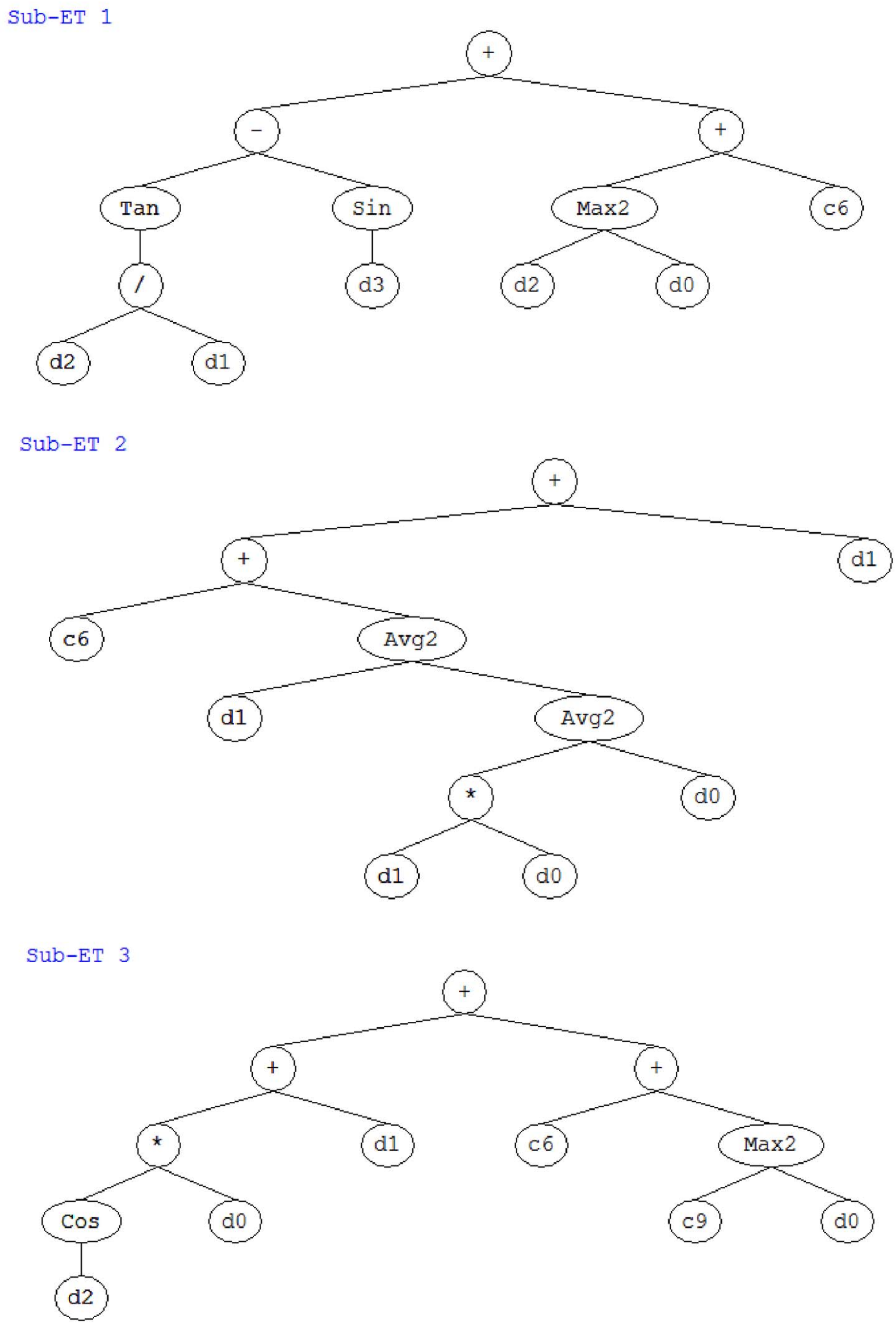

Figure 4. ET of the predictive GEP model.

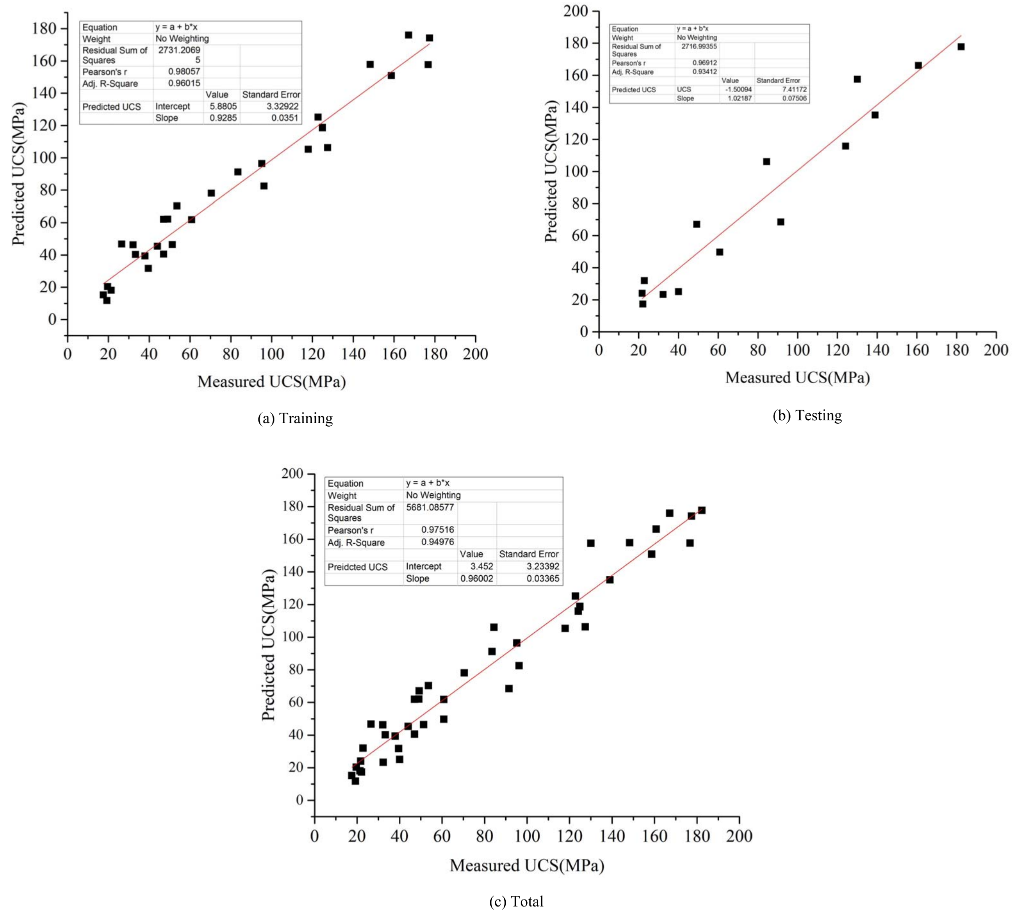

Figure 5. Relationships between the actual and predicted for the training, testing, and total data samples.

Table 4. Performance comparison among different models

Models	R			R^2			RMSE		
	Training	Test	Total	Training	Test	Total	Training	Test	Total
GEP	0.9806	0.9691	0.9751	0.9615	0.9392	0.9509	10.22	13.98	11.55
ANN	0.9666	0.9036	0.9611	0.9343	0.8165	0.9238	13.88	15.35	14.40
Equation (1)	0.9251	0.8651	0.9223	0.8559	0.7484	0.8506	20.74	19.22	20.26
Equation (2)	0.9559	0.8681	0.9373	0.9137	0.7536	0.8786	17.17	22.59	19.06
Equation (3)	0.9238	0.9239	0.9379	0.8534	0.8535	0.8796	21.64	12.44	19.2
Equation (4)	0.9606	0.9518	0.9554	0.9227	0.906	0.9128	15.16	15.98	15.61

an ANN model in terms of three statistical indices. The following conclusions can be drawn from this study:

(1) The proposed GEP model provides an accurate prediction of the UCS of rocks that is most fitting to the measured results compared to the available five models in the literature.

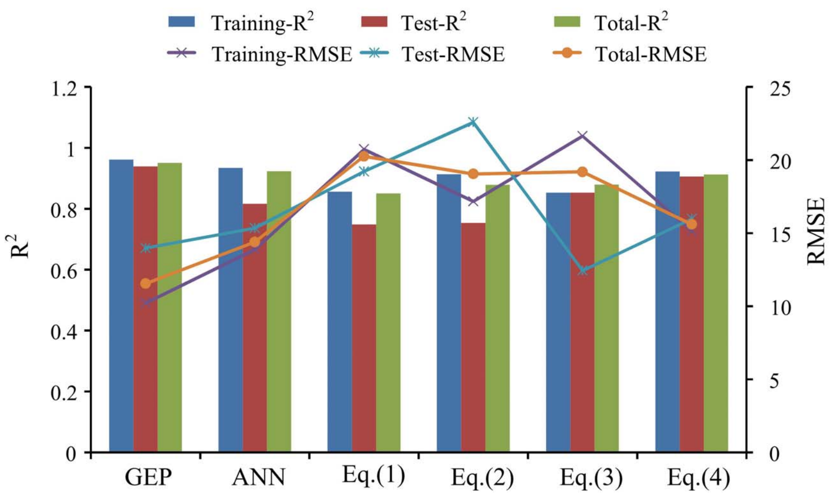

Figure 6. Performance comparison among different models.

(2) The proposed GEP model has the lowest RMSE and the highest R^2 and R values compared to the four conventional regression models and an ANN model in the literature. For the training data samples, the values of RMSE, R and R^2 of the proposed GEP model are 10.22, 0.9806 and 0.9651, respectively.

(3) The predictive ability of ANN model is much better than that of the regression models. Among the four regression models, the predictive ability of the NLMR model is better than that of the simple regression models.

Nomenclature

UCS	Uniaxial compressive strength
AI	Artificial intelligence
ANN	Artificial neural network
GP	Genetic programming
GEP	Gene expression programming
ET	Expression tree
BPI	Block punch index
$I_{s(50)}$	Point load strength
SRH	Schmidt rebound hardness
USV	Ultrasonic p-wave velocity
GA	Genetic algorithm
RMSE	Root mean square error
R^2	Coefficient of determination
R	Correlation coefficient
NLMR	Nonlinear multiple regression

Conflicts of interest

The author declares that he has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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