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A hybrid modelling approach for prediction of UCS of rock materials

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Abstract. It is necessary to make an accurate assessment of uniaxial compressive strength (UCS) for rock mass classification and rock engineering design. However, there are many shortcomings in the conventional tests for UCS of rocks. The aim of this study is to present a hybrid model by integrating the genetic algorithm (GA) into the least-squares support vector machine (LSSVM) to predict the UCS of rock materials. The GA technique was utilized to improve the forecasting accuracy of the proposed LSSVM. To develop the proposed hybrid GA–LSSVM model, four main factors including the block punch index, point load strength, Schmidt rebound hardness and ultrasonic P-wave velocity were considered as input variables, while the UCS of rock materials was the output. A comparison was conducted among the proposed GA–LSSVM, the adaptive neuro-fuzzy inference system, the fuzzy inference system, the artificial neural network and the statistical method in accordance with three statistical indexes. The results of the comparisons show that the developed GA–LSSVM model has great potential to accurately estimate the UCS of rock materials.

Keywords. Uniaxial compressive strength, Genetic algorithm, Statistical model, Rocks, Least-squares support vector machine.

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1. Introduction

The accurate assessment of uniaxial compressive strength (UCS) is fundamental to rock mass classification and rock engineering design [1–5]. The conventional tests for UCS, however, have some shortcomings in that they are time-consuming and expensive and use restricted sampling. In addition, it may be difficult to conduct sampling and sample preparations for weak rocks [6, 7]. Therefore, an alternative method is needed to provide a better estimation of UCS of rocks.

In recent years, artificial intelligence (AI) methods such as artificial neural networks (ANNs) and adaptive neuro-fuzzy inference systems (ANFISs) have been widely used in rock engineering including the prediction of UCS. For example, Singh *et al.* [8] proposed an ANN model to predict

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the strength properties of some schistose rocks from petrographic properties. Yilmaz and Yuksek [9] studied rock parameters using an ANN model. Canakci *et al.* [10] investigated the strength of basalts using ANN and gene expression programming methods. Cevik *et al.* [11] proposed a neural network model to predict the UCS of some clay-bearing rocks. It was concluded that the performance of the proposed neural network model was quite satisfactory. Yagiz *et al.* [12] studied the influence of slake durability cycles on the prediction of UCS and modulus of elasticity of carbonate rocks using ANN and nonlinear regression techniques. Mishra and Basu [13] predicted the UCS of rock materials including granite, schist and sandstone using regression analysis and the fuzzy inference system (FIS). Yesiloglu-Gultekin *et al.* [14] estimated the UCS of certain granitic rocks from their mineral contents using the ANFIS technique. They concluded that the ANFIS is a suitable method for predicting the UCS of rocks. Yesiloglu-Gultekin *et al.* [15] proposed some prediction models to estimate the UCS of six different granitic rocks and compared their prediction performances. They confirmed that the ANFIS model is a better tool for the prediction of UCS. Barzegar *et al.* [16] evaluated the performance of several AI models to predict the UCS of travertine rocks. Jalali *et al.* [17] compared the prediction performance of several soft computing methods for estimating the UCS of some sedimentary rock types from a set of index test results. They concluded that the ANFIS model provides better predictive results. Saedi *et al.* [18] predicted the UCS and elastic modulus of migmatites using the FIS.

As can be seen from the above, the ANN and ANFIS models outperform the existing empirical models; however, they may encounter some issues such as trapping in local minima [19–21]. Besides the ANN and the ANFIS, at present, the least-squares support vector machine (LSSVM) is one of the widely used AI techniques. The objective of this study is to present a hybrid model by integrating the genetic algorithm (GA) into the LSSVM to predict the UCS of rock materials. To the best of the authors' knowledge, this research work is the first study to develop a hybrid model combining the GA with the LSSVM to predict the UCS of rock materials. The novelty of this work lies in the complete description of the proposed GA–LSSVM model that considers four input parameters—the block punch index (*BPI*), point load strength, Schmidt rebound hardness (*SRH*) and ultrasonic P-wave velocity (*USV*)—and one output. A comparison was conducted among the proposed GA–LSSVM, the ANFIS, the FIS, the ANN and the statistical method in terms of three statistical indexes.

2. Methodology

2.1. *Least-squares support vector machine*

A brief description of the LSSVM is as follows [22].

Consider the training dataset $\{x_i, y_i\}_{i=1}^{N}$, where x_i and y_i are the input and the output, respectively. N denotes the total sample size. The minimization of the loss function of LSSVM can be written as

$$\min \Im(w,b,\xi) = \frac{1}{2}\left(w^{\mathrm{T}} w + \gamma \sum_{i=1}^{N} {\xi_i}^2\right). \tag{1a}$$

This is subject to the equality constraints

$$y_i[w^{\mathrm{T}}\Phi(x_i) + b] = 1 - \xi_i, \tag{1b}$$

where ξ_i and γ are the error variable and the regularization parameter, respectively. b is the bias; w is an adaptive weight vector. $\Phi(x)$ is the nonlinear transformation that maps the input data to a high-dimensional feature space.

C_{γ_1}	...	C_{γ_i}	...	C_{γ_n}	C_{σ_1}	...	C_{σ_j}	...	C_{σ_n}

Figure 1. The chromosome comprises two parameters, γ and σ.

To resolve the optimization problem of LSSVM, the Lagrangian is introduced and expressed as

$$L(w,b,\xi,\alpha) = \Im(w,b,\xi) - \sum_{i=1}^{N} \alpha_i\{y_i[w^{\mathrm{T}}\Phi(x_i)+b]-1+\xi_i\}, \tag{2}$$

where α_i is the multiplier.

The conditions for optimality can be written as

$$\begin{cases} \dfrac{\partial L}{\partial w} = 0 \to w = \sum_{i=1}^{N} \alpha_i y_i \Phi(x_i) \\ \dfrac{\partial L}{\partial b} = 0 \to \sum_{i=1}^{N} \alpha_i y_i = 0 \\ \dfrac{\partial L}{\partial \xi_i} = 0 \to \alpha_i = \gamma \xi_i \\ \dfrac{\partial L}{\partial \alpha_i} = 0 \to y_i[w^{\mathrm{T}}\Phi(x_i)+b]-1+\xi_i = 0. \end{cases} \tag{3}$$

According to Mercer's condition, the output of LSSVM can be written as [23]

$$y(x) = \sum_{i=1}^{N} \alpha_i F(x,x_i) + b, \tag{4}$$

where $F(x,x_i)$ denotes the kernel function. Here, the radial basis function is utilized. It is expressed as [24]

$$F(x,x_i) = \exp\left(-\frac{1}{2\sigma^2}\|x-x_i\|^2\right), \tag{5}$$

where σ is the kernel parameter.

2.2. *LSSVM optimized by GA (GA–LSSVM)*

2.2.1. *Chromosome and genetic operators*

In this study, the chromosome of GA was composed of two LSSVM parameters, γ and σ, as shown in Figure 1. The two genetic operators, crossover and mutation, are illustrated in Figure 2.

2.2.2. *GA–LSSVM approach*

Step 1: Generate the initial population.

Step 2: Define the fitness function. In this study, the root-mean-square error (*RMSE*) is adopted to assess the performance of each chromosome. It is expressed as

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(y-\hat{y})^2}, \tag{6}$$

where y and $\hat{y}$ represent the actual and predicted values, respectively.

Step 3: Generate a new population by some genetic manipulations.

Step 4: When the termination criteria are satisfied, the procedure of calculation ends; otherwise, return to Step 3. The main flowchart of GA–LSSVM is presented in Figure 3.

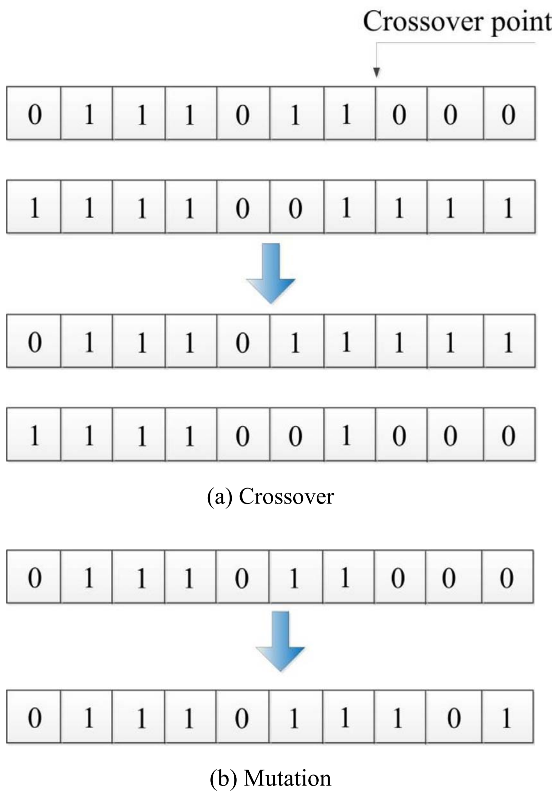


Figure 2. Genetic crossover and mutation operation.

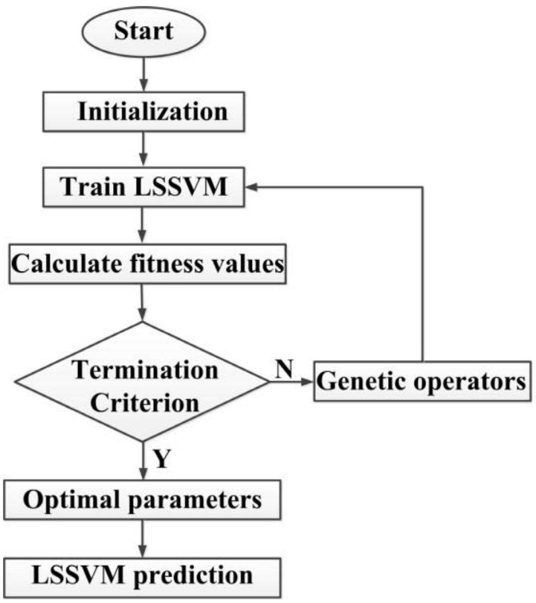


Figure 3. Flowchart of the GA–LSSVM algorithm.

Table 1. Statistical analysis of datasets (data from Ref. [4])

Variable	Maximum	Minimum	Average
BPI (MPa)	35.36	2.53	16.04
$I_{s(50)}$ (MPa)	11.73	1.15	5.75
SRH (%)	66.51	25.89	50.23
USV (m/s)	6250	2725	5010.23
UCS (MPa)	182.33	17.55	80.75

2.3. *Statistical analysis*

In conventional methods, multiple regression is often used to determine the relationships between different variables. In this study, UCS is considered to be the outcome of four rock parameters including *BPI*, point load strength ($I_{s(50)}$), *SRH* and *USV*. To generate a multivariate relation based on the main data (44 datasets), a statistical equation is obtained using MS Excel. The equation is expressed as

$$UCS = -14.02 + 2.47 \cdot BPI + 8.06 \cdot I_{s(50)} + 0.59 \cdot SRH - 0.004 \cdot USV. \tag{7}$$

2.4. *Performance evaluation*

To assess the performance of five prediction models, that is, the proposed GA–LSSVM, ANN, ANFIS, FIS and the statistical method (Equation (7)), three statistical indexes including *RMSE*, correlation coefficient (R) and coefficient of determination (R^2) were utilized in this study. The definition of R and R^2 can be given as follows:

$$R^2 = 1 - \frac{\sum_{i=1}^{N}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{N}(y_i - \overline{y}_i)^2} \tag{8}$$

$$R = \frac{\sum_{i=1}^{n}(y_i - \overline{y}_i)(\hat{y}_i - \overline{\hat{y}}_i)}{\sqrt{\sum_{i=1}^{n}(y_i - \overline{y}_i)^2 \sum_{i=1}^{n}(\hat{y}_i - \overline{\hat{y}}_i)^2}}. \tag{9}$$

Here y_i and $\hat{y}_i$ are the actual and predicted values, respectively; $\overline{y}_i$ and $\overline{\hat{y}}_i$ are the mean values of the actual and predicted results, respectively.

3. Case study

3.1. *Data collection*

In this study, a total of 44 sets of data points were collected from Ref. [4] to develop the proposed GA–LSSVM model. Among the 44 sets of data points, there are three rock types including granite, schist and sandstone. The granite is an igneous crystalline rock, and it is virtually isotropic in nature. The sandstone is a sedimentary rock, and it is porous in nature. The schist is a metamorphic rock, and it is anisotropic in nature. Four main factors including *BPI*, $I_{s(50)}$, *SRH* and *USV* were considered as input variables, while the UCS of rocks was the output. Out of 44 data points, 29 were randomly selected for model training while the remaining 15 data points were utilized for testing. The statistical results of data collected are summarized in Table 1.

3.2. *Results and discussion*

It should be noted that the selection of GA parameters has an important influence on the convergence rate and forecasting accuracy. For this purpose, the optimal parameters of GA

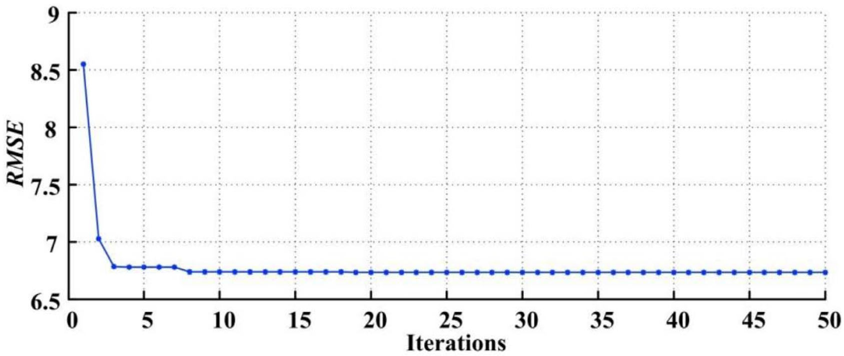


Figure 4. Convergence procedure of GA.

Table 2. Performance comparison among different models

	R			R^2			$RMSE$		
	Training	Testing	Total	Training	Testing	Total	Training	Testing	Total
GA–LSSVM	0.9920	0.9795	0.9918	0.9841	0.9595	0.9837	6.73	6.58	6.68
ANN	0.9666	0.9036	0.9611	0.9343	0.8165	0.9238	13.88	15.35	14.40
ANFIS	0.9918	0.9668	0.9904	0.9837	0.9347	0.9808	7.07	8.73	7.67
FIS	0.9841	0.9453	0.9826	0.9685	0.8936	0.9655	9.66	9.95	9.76
Equation (7)	0.9640	0.9397	0.9622	0.9293	0.88295	0.9259	14.17	14.27	14.20

were determined by the trial and error approach. They are as follows: the maximum iterations $n_{\max} = 50$, the size of the population $N_p = 20$, the probability of crossover $P_c = 0.8$ and the probability of mutation $P_m = 0.05$. After procedures of GA–LSSVM, the optimal parameters of LSSVM were selected, i.e. $\sigma = 5.5309$ and $\gamma = 100$. Figure 4 shows the convergence curves of GA.

To assess the performance of the proposed GA–LSSVM model, a comparison between the experimental results and the predictions by the ANN [4], ANFIS [4], FIS [4] and the statistical method (Equation (7)) is made. The results of the comparison are shown in Figures 5 and 6 and Table 2, respectively.

From Figures 5 and 6 and Table 2, it can be seen that the *RMSE* values of GA–LSSVM, ANN, ANFIS, FIS and the statistical method (Equation (7)) for training, testing and total samples are 6.73, 6.58 and 6.68; 13.88, 15.35 and 14.40; 7.07, 8.73 and 7.67; 9.66, 9.95 and 9.76; 14.17, 14.27 and 14.20, respectively. The R values of GA–LSSVM, ANN, ANFIS, FIS and the statistical method (Equation (7)) for training, testing and total samples are 0.9920, 0.9795 and 0.9918; 0.9666, 0.9036 and 0.9611; 0.9918, 0.9668 and 0.9904; 0.9841, 0.9453 and 0.9826; 0.9640, 0.9397 and 0.9622, respectively. The R^2 values of GA–LSSVM, ANN, ANFIS, FIS and the statistical method (Equation (7)) for training, testing and total samples are 0.9841, 0.9595 and 0.9837; 0.9343, 0.8165 and 0.9238; 0.9837, 0.9347 and 0.9808; 0.9685, 0.8936 and 0.9655; 0.9293, 0.8829 and 0.9259, respectively. Irrespective of the training, testing or all the datasets, we can see from above that the R and R^2 values of the developed GA–LSSVM are the highest while the values of the *RMSE* of the developed GA–LSSVM are the lowest among these five models. Obviously, the forecasting performance of the developed GA–LSSVM model surpasses the other four models. In addition, we can see that the forecasting performance of the traditional ANFIS model surpasses the FIS, the statistical method (Equation (7)) and the ANN model. On the whole, the results show that the forecasting performance rank is in the following order: GA–LSSVM > ANFIS > FIS > the statistical method (Equation (7)) > ANN model. It should be noted that the ANFIS model is also a suitable

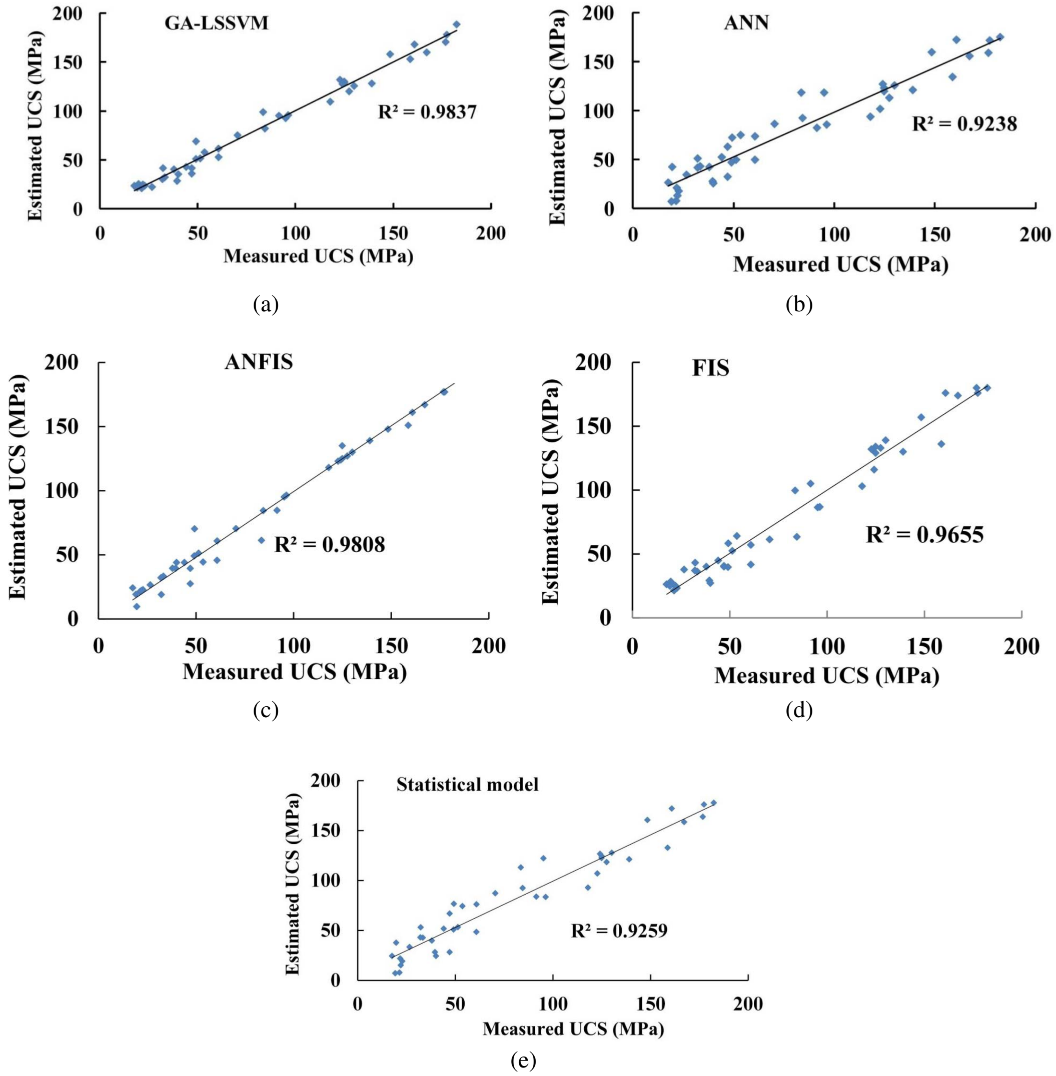
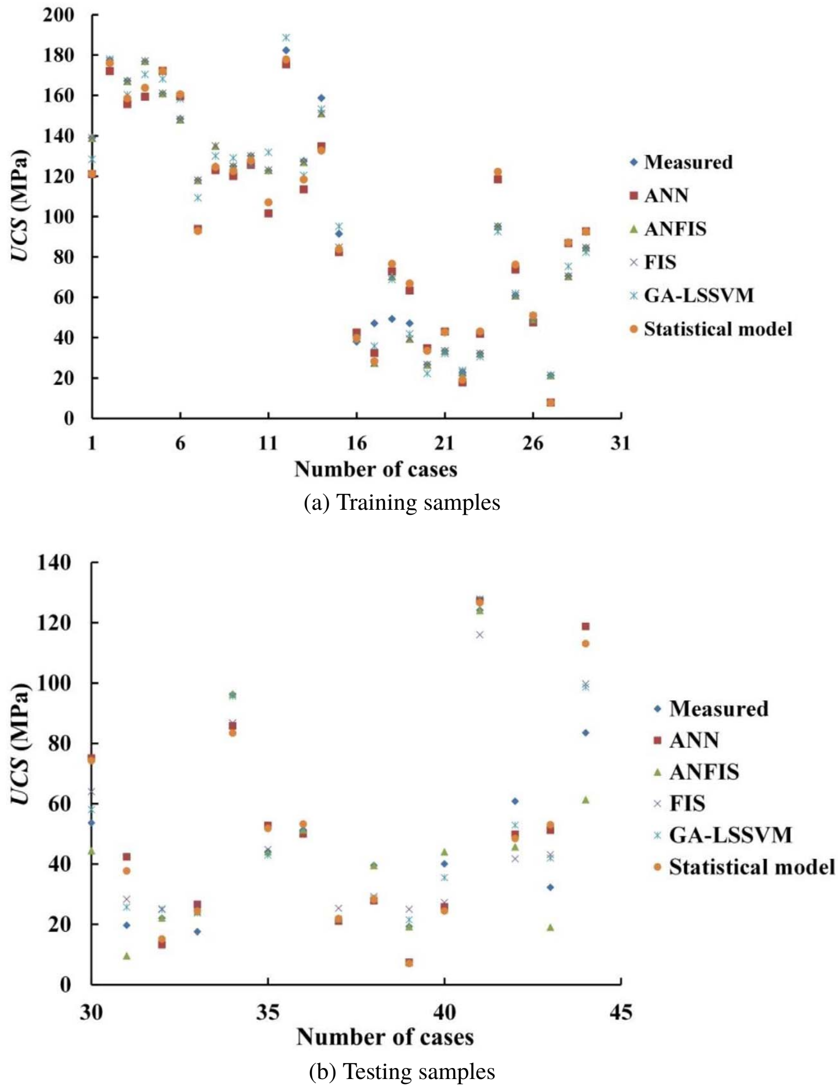


Figure 5. Estimated *UCS* plotted against measured *UCS*: (a) GA–LSSVM, (b) ANN, (c) ANFIS, (d) FIS and (e) statistical model.

method for predicting the UCS of rocks, and the findings of Yesiloglu-Gultekin *et al.* [14, 15] also verify this point. In addition, the predictive ability of the ANN model is slightly weaker than the statistical method (Equation (7)) in the present study. This shows that the ANN model needs to be further optimized to achieve better results.

4. Conclusions

In this study, a hybrid GA–LSSVM model is developed for the prediction of UCS of rock materials. To establish the hybrid GA–LSSVM model, four input parameters including the *BPI*, point load strength, *SRH* and *USV* are considered as the input variables, and the UCS of rock materials is the output. By comparing the results derived from the developed GA–LSSVM, ANN, ANFIS, FIS and the statistical model, it can be concluded that the R and R^2 values of the developed GA–LSSVM



(a) Training samples

(b) Testing samples

Figure 6. Comparison of the predicted and experimental results.

are the highest while the values of *RMSE* of the developed GA–LSSVM are the lowest among these five models. In other words, the forecasting performance rank is in the following order: GA–LSSVM > ANFIS > FIS > the statistical method (Equation (7)) > ANN model in the present study. Therefore, the proposed GA–LSSVM model has great potential to accurately estimate the UCS of rock materials. In addition, the results show that ANFIS is also a suitable method for predicting the UCS of rocks and that the ANN model needs to be further optimized to achieve better results.

Conflict of interest

The authors declare that they have no conflict of interest.

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