

LEIBNIZ AND THE RATIONALITY OF THE INFINITE

*There is a concept which corrupts and upsets all others. I refer not to
Evil... I refer to the infinite. (Borges 1962, p.202)*

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Abstract

Part 1

Historically the term ‘infinite’ has had two apparently contrary meanings. On the one hand, it was taken by metaphysicians like Plotinus to mean that which “...has never known measure and stands outside number, and so is under no limit either in regard to anything external or internal...” (Branford 1949, V.5.11). ‘Infinite’ in this sense means ‘irrevocably complete’. On the other hand, Aristotle defined it in this way: “A quantity is infinite if it is such that we can always take a part outside what has already been taken.” (Hardie and Gaye 1941, 207a, 5-10) ‘Infinite’, in this second sense means, ‘irrevocably incomplete’. Leibniz is someone who uses both these meanings. In particular, he identifies the irrevocably complete with God and the irrevocably incomplete with the world (as we know it). Given, firstly, that what is irrevocably complete includes everything and, secondly, that it *excludes* anything incomplete, the following conclusion can be drawn: Leibniz’s philosophy of the infinite makes of the-world-as-we-know-it something that is logically dependent on God, but also something that exists in contradiction to ‘him’.

Leibniz cannot escape a kind of contradiction in what he says about God and the world but this is not a straightforward case of self-refutation. The reason turns on the consideration that to divorce the concept of the irrevocably complete from its object is to deprive this concept of its sense, specifically of its sense of completeness. For if the two are distinct, then

there is something beyond the irrevocably complete, namely, how this is independently of its concept. It follows that to deny the irrevocably complete is, in the same breath, to affirm that very thing. Yet if we cannot quite deny the irrevocable complete, neither can we as human beings quite affirm it either—for the human mind is, we do not doubt, a limited one. Thus the irrevocably complete can neither be affirmed nor denied without contradiction.

There is a strong resemblance between this paradox and the paradox of the liar: in both cases there is a thesis that says of itself that it is untrue and, in both cases, thesis and antithesis turn out to be equivalent.

Part 11

Kant offers some powerful reasons to think that the paradox discussed in Part 1 involves no real contradiction. The critical philosophy suggests that the apparent contradiction is real, only if, *per impossibile*, we have some way to positively employ the concept of the world as it is independently of our conceptions of it.

Kant's view of the infinite shares with Leibniz's the vice (if it is one) that it is paradoxical: both philosophers make use of a concept that cannot, strictly speaking, be possessed by the human mind. However each view has the significant virtue that it shows the difference between the irrevocably complete and the irrevocably incomplete to be not simply a logical difference. My overall conclusion is based on a synthesis of the Leibnizian and the Kantian philosophies of the infinite. According to Leibniz, neither the irrevocably complete, nor the irrevocably incomplete, can be eliminated from philosophy. According to Kant, infinity is—from a human perspective

at least—something *prior* to conception; putting Leibniz and Kant together, I conclude that these modes of infinity combine to produce finitude, that they are the joint conditions under which difference, and therefore finitude, is possible. In particular, I argue that the irrevocably complete is the infinity of fullness, and that the irrevocably incomplete is the infinity of emptiness, and that logic is blind to any difference there might be between these, since both are, by definition, *undifferentiated*.

Given that ethics, as well as logic, is dependent on finitude, I conclude, finally, that the perennial ambition to eliminate either the irrevocably complete or the irrevocably incomplete from philosophy is, not merely unrealisable, but potentially dangerous.

Introduction

What is the infinite? Ever since Aristotle distinguished between the actual and potential infinite in terms of the difference between "what has nothing outside it" and "what always has something outside it" (*Physics*, III, 206b35-207a1), all answers to this question have fallen into one of two basic categories. On the one hand there are those answers according to which the infinite is the irrevocably complete and, on the other, there are those according to which it is the irrevocably *incomplete*.¹

Everyone is familiar with the irrevocably incomplete. Numbers are irrevocably incomplete for, no matter how far you count in either a positive or a negative direction, there is still further to go. And some philosophers have argued that the physical world is irrevocably incomplete. Leibniz himself says that there is no limit to the number of times a material thing can be divided. "There is no atom, he says, "indeed, there is no body so small that it is not actually subdivided... *every particle of the universe contains a world of an infinity of creatures*"(Ariew and Garber 1989, pp.33-34). The irrevocable complete is, perhaps, a less familiar notion but Plotinus sums it

¹ It remains to be seen whether these answers are mutually exclusive.

up when he speaks of God as that which "...has never known measure and stands outside number, and so is under no limit either in regard to anything external or internal..." (Branford 1949, V.5.11). Cantor gives this thought a modern and pseudo-mathematical gloss: the 'absolute', he says, "...cannot in any way be added to or diminished, and is therefore to be regarded quantitatively as an absolute maximum"(Hallett 1984, p.13). However it is to be characterised, the irrevocably complete is, first and foremost, a singular thing rather than a one of a number of things.²

Aristotle denied the actuality of the actual infinite and so did not have to worry—or at least *believed* he did not have to worry—about *combining* the irrevocably complete and the irrevocably incomplete. Many later philosophers, however, wanted as it were to have their infinite cake and to eat it too. Some of these admitted that this cannot be done and therefore took irrevocable completeness to be beyond human reason. This, for example, is the idea behind the Pascal's famous defense of theism in the *Pensees* in

² William James puts the difference I have in mind—or something resembling the difference I have in mind—in terms of that between monism and pluralism:

The pluralistic world is... more like a federal republic than like an empire or a kingdom. However much may be collected, however much may report itself as present... something else is self-governed and absent and unreduced to unity. Monism, on the other hand, insists that when you come down to reality as such, to the reality of realities, everything is present to everything else in one vast co-implicated completeness—nothing can... be really absent from anything else, all things interpenetrate and telescope together in the great total conflux. (James 1977, p. 145)

which it is argued that that God cannot be the object of man's knowledge since it is impossible for the finite to know the infinite. God, says Pascal, is "...infinitely incomprehensible, since, having neither parts nor limits. He has no affinity to us..." (Popkin 1989, p.257) He goes on to argue that Christians should not be blamed for being unable to able to "...give a reason for their belief, since they profess a religion for which they cannot give a reason?" (Popkin 1989, p.257)

The philosopher on whose ideas this thesis focuses—Leibniz—wants the two modes of infinity to go together and to do so in a way that is within the reach of human reason. Where Leibniz stands out somewhat from the tradition of which he is a part is in respect of the wholeheartedness of his attempt to understand the irrevocably complete; Leibniz steadfastly upholds the rational comprehensibility of God by the mind of man. In this regard he contrasts sharply with Descartes, for whom the ways of God have a brute factual—an irrational—quality:

...knowing that my nature is extremely weak and limited, and that God's nature, on the contrary, is immense, incomprehensible, and infinite, I no longer have difficulty in recognising that there is an infinity of things in his power, the causes of which are beyond the range of my mind. And this reason alone is sufficient to persuade me that the whole class of final causes is of no use in physical or natural things; for it does not

seem to me that I can without temerity seek to discover the impenetrable ends of God. (Sutcliffe 1980, pp.134-135)

In an important passage from *On Freedom*, Leibniz says that “there are two labyrinths of the human mind, one concerning the nature of the composition of the continuum, and the other concerning the nature of freedom, and they arise from the same source, infinity.” (Ariew and Garber 1989, p.95). He goes on to criticise Descartes for refusing to even try to solve these problems, saying that “he [Descartes] preferred to slash through both these knots with a sword since either he could not solve the problems, or did not want to reveal his view” (Ariew and Garber 1989, p.95). He attributes to Descartes the view that “we can easily become entangled in enormous difficulties if we try to reconcile God’s preordination with the freedom of the will; but we must refrain from discussing these matters, since we cannot comprehend God’s nature” (Ariew and Garber 1989, p.95), *and* the view that “we should not doubt the infinite divisibility of matter even if we cannot grasp it” (Ariew and Garber 1989, p.95).

Leibniz’s view of the infinite can be given greater particularity by reference to the medieval distinction between the categorematic and the syncategorematic use of the term ‘infinite’. To use ‘infinite’ in a categorematic sense is “to say that there is something which has a property

that surpasses any finite measure; to use it syncategorematically is to say that, given any finite measure, there is something which has a property that surpasses it”(Moore 1990, p.51). The distinction is not quite a restatement of the actual/potential distinction since it leaves open the important question—closed by Aristotle’s distinction—of whether there is an actual infinite other than the irrevocably complete. Cantor’s prominent place in the history of the idea of the infinite was secured by his anti-Aristotelian argument that there are incomplete or, as he put it, ‘increasable’ actual infinities:

We certainly cannot conclude that an actual infinite is unincreasable in magnitude... Rather we must make a fundamental distinction here between: Ia Increasable actual-infinite or transfinite; IIb Unincreasable actual-infinite or Absolute. (Hallett 1984, p.41.)

Leibniz seems to me to be torn between the Aristotelian and the Cantorian account of the irrevocably incomplete. In the following note to a letter to Des Bosses, he clearly follows Aristotle:

There is a syncategorematic infinite... namely the possibility of further progression in dividing, multiplying, and adding. There is also a hyper-categorematic infinite... This is God himself. But there is no categorematic infinite, or one actually having infinite parts. (my italics) (Loemker 1969, p.31)

Elsewhere, however, he says things like this:

I am so much in favour of the actual infinite, that... I hold that nature affects it everywhere, in order the better to mark the perfection of its author. So I believe that every part of matter is, I do not say divisible, but actually divided (Moore 1990, p.79).

One thing is for sure, however, and this is that Leibniz is in his philosophy attempting to balance the irrevocably complete and the irrevocably incomplete in a way that preserves the rationality of the former.

My answer to the question of whether his balancing act succeeds is a subtle one. If it is supposed that, in order to succeed, he must combine these modes in a way that is free from contradiction, then I think there is no doubt that he fails. However, I will argue, firstly, that since there is *no* possibility of separating these modes of infinity, Leibniz's failure is a universal one; and secondly, and more significantly, that the combination of these modes is a condition of the very *possibility* of contradiction. I think therefore that the circumstances of Leibniz's 'failure' point in an unequivocal way to the inevitability *of*, and to the importance *for* philosophy, of Leibniz's overall philosophy of the infinite. That this dependence is a paradoxical one is undeniable, but I do not take this paradoxicality to mean that the irrevocably complete is beyond human reason. If it *does* mean this, I will argue, then the

same must go for the incomplete, and thus, *everything* would be beyond human reason.

PART I

1.1. Leibniz's Philosophy of the Infinite in Outline

Leibniz's philosophy of the infinite has, to my mind, two distinguishing features. The first is that it regards the-world-as-we-know-it as infinite in the incomplete sense of this term—the same sense in which numbers are infinite, or in Aristotle's terms, as that which “always has something outside it” (*Physics*, III, 206b35-207a1).

There is no portion of matter so tiny that it does not contain a sort of world of creatures infinite in number... no substance so imperfect that it does not contain the entire universe... nor is there any truth of fact... that does not depend upon the infinite series of reasons. (Ariew and Garber 1989, p.95)

The second feature is that all this incompleteness is viewed as a shadow of or—as Leibniz says—a “mark”³ of an order of reality and knowledge that is infinite in the *complete* sense of this term, in the sense in which Plotinus's God is infinite, or again in Aristotle's terms, that which “has *nothing* outside it”. (My italics.) (*Physics*, III, 206b35-207a1.)

We must realize that all creatures have impressed upon them a certain mark (character) of divine infinity, and that this is the source of many

wonderful things that amaze the human mind. (Ariew and Garber 1989, p.95)

That these modes of infinity are supposed by Leibniz to go together can be made very clear by reference to his ways out of the “two labyrinths of the human mind”.⁴ Leibniz believes these problems stem from thinking of the world as infinite in one way, but not the other, and in both cases invokes the balancing influence of this other mode of infinity as his solution: he finds that the problem of the labyrinth of the composition of the continuum arises when the world is taken to be irrevocably incomplete and not at all complete; he finds that the problem of the labyrinth concerning freedom—or contingency—to arise when the world is taken to be irrevocably complete and not at all incomplete.

1.2. The Labyrinth of the Composition of the Continuum

The first of these problems is, as Leibniz sees it, that matter is infinite in the irrevocably incomplete sense of this term, yet things that are irrevocably incomplete are unreal. “A body”, Leibniz says, “... is always further divisible, and any given part always has another part to infinity”. He

⁴ Leibniz speaks in the *Theodicy* of the “theological doctrine of predestination” and “the composition of the Continuum” that “These are, indeed, the two labyrinths which have exercised theologians and philosophers. Libertus Fromondus, a theologian of Louvain... who also wrote a book entitled explicitly *Labyrinthus de Compositione Continui*, experienced in full measure the difficulties inherent in both doctrines; and the

goes on to argue that “over and above a body or bodies, there must be substances to which true unity belongs”, things that are “one” rather than “many” (Ariew and Garber 1989, p.103). If it wasn’t so, he insists, then there would be no real beings: “where there are only beings by aggregation”, he says in a letter to Arnould, “there aren’t any real beings ... every being by aggregation presupposes things endowed with real unity” (Ariew and Garber 1989, p. 85). His solution seems at first to be that minds, not material things, form the basis of reality and, more particularly, that the infinite parts of matter are phenomena founded in an infinite number of minds. Matter is, says Leibniz, not a thing in itself, but a “well founded phenomenon”: “... properly speaking, matter isn’t composed of constitutive unities ... [it is] only a phenomenon grounded in things ... and all reality belongs only to unities” (Ariew and Garber 1989, p.79). These mindful unities Leibniz calls *monads*, after the Greek word *monas*, which means “unity” or “that which is one”.

But is the original problem even touched by such a solution? As it stands, this solution consists of the identification of matter with the minds that “perceive” it; yet if every material thing is irrevocably incomplete, and

renowned Ochino admirably presented what he calls ‘the labyrinths of predestination’.”
(Hubbard 1951, p. 89)

if every such thing is the perception of one or another mind, then every one of these minds must—after its own fashion—also be irrevocably incomplete. Given that the irrevocable incompleteness of matter is the basis of Leibniz’s insistence that it is not real, it should also form the basis of an insistence by Leibniz that mind is not real either.⁵

Leibniz gives us a strong clue that he knows that his work is not done when he goes on to say: “A plurality of things ... can neither be understood nor can exist unless one first understands the thing that is one, that to which the multitude necessarily reduces (*referatur*)” (Ariew and Garber 1989, p. 103). Admittedly he gives no further indication here that monads are not supposed to be the “one” thing(s) that the “multitude” reduces to, but elsewhere he makes this abundantly clear. Consider, for example, the cosmological argument for the existence of God given in the *Monadology*. This argument addresses itself to the problem posed for knowledge by the notion of irrevocable incompleteness. While it draws on Leibniz’s conclusions, cited above, that matter is infinitely divisible etc., it also depends on a further—epistemological—premise according to which, to

⁵ Leibniz seems to me to be within a hair’s breadth of giving the Kantian argument that an infinite (or indefinite) regress is tolerable at the phenomenal level but, unlike Kant, he is concerned that, in the final analysis, there be something “real” in which the regress of phenomena can come to an end.

fully know the truth of any matter is to have reduced one or another complex concept into its simple components⁶ When it comes to what Leibniz calls the ‘truths of reason’, this analysis can be supposedly completed in a finite number of steps—“its reasons can be found by analysis, resolving it into simpler ideas and simpler truths until we reach primitives”—but the analysis of ‘truths of fact’ involves an *infinite* number of steps. The point of the additional premise is that an infinite regress of concepts is no more tolerable than an infinite regress of objects:

Since all this detail involves nothing but other prior or more detailed contingents, each of which needs a similar analysis in order to give its reason... it must be the case that the sufficient or ultimate reason is to be found outside the sequence or *series* of this multiplicity of contingencies, however infinite it may be... that is why the ultimate reason of things must be in a necessary substance... That is what we call God.’ (Ariew and Garber 1989, pp.217–18)

That Leibniz is, at this point, trying to reduce the irrevocable incompleteness of monads themselves to the irrevocable completeness of the

⁶ The origin of this premise is Leibniz’s so called predicate-in-subject principle according to which all truths have the form $a=a$, i.e. are “primary truths” or are reducible to such:

The primary truths are those which assert the same thing of itself or deny the opposite of its opposite... all remaining truths are reduced to primary truths... through the resolution of notions... the predicate or consequent is always contained in the subject or antecedent... the connection and inclusion of the predicate in the subject is explicit in identities, but in all other propositions it is implicit and must be shown through the analysis of notions... (Ariew and Garber, 1989, p. 31)

super-monad, God, is suggested by the consideration that many commentators find this argument objectionable on the grounds that there is no reason to suppose that the process of reason-giving should not go on indefinitely. Consider in this regard the debate between Russell and Copleston (reprinted in Russell 1967, pp.138–59). Russell is apparently happy to have a never-ending chain of contingents, to which Copleston responds that, in that case there can be no “total explanation to which nothing further can be added”. Russell says that such an explanation can’t be had because to have it “ ... you have to grasp this sorry scheme of things entirely...” and, he maintains, the idea of the universe as a whole is meaningless: “I think the word ‘universe’ is a handy word in some connections, but I don’t think it stands for anything that has meaning.” The viability of Russell’s stance is dependent on his rejection of Copleston’s assumption that there is such a thing as a “universe” in the first place for, if there *is* such a thing, i.e. if the world is irrevocably complete at some level, then Copleston and Leibniz are right to argue as they do—an infinite regress of any sort is incompatible with the existence of a world of this kind. If, however, there is *no* such thing, if there is *no* level at which the world is

Essentially this principle is that, for Leibniz, concepts as well as objects may not be infinitely complex; concepts too must ultimately be reducible to the point of simplicity.

irrevocably complete, then it is Copleston and Leibniz who are wrong and Russell who is right.

This does not, by itself, show that Leibniz intends to ground the irrevocably incomplete in the irrevocably complete, only that he needs to do so if his argument is to withstand criticisms like those put forward by Russell. It is only by reference to the passage in the *Monadology* that follows the cosmological argument, that Leibniz's intentions become unambiguously clear:

Thus God alone (or the necessary being) has the privilege that he must exist if he is possible. And since nothing can prevent the possibility of what is without limits, without negation, and consequently without contradiction, this by itself is sufficient for us to know the existence of God *a priori*. (Ariew and Garber 1989, p.218)

This is Leibniz's *ontological* argument and, although brief, it is quite sophisticated. The basis of this sophistication is that Leibniz is sensitive to the consideration that the ontological argument depends upon the legitimacy of the concept of God, a minimal condition of which is that this concept contains no contradiction. In *Meditations on Knowledge Truth and Ideas* he says of the ontological argument:

An argument for the existence of God, celebrated among the Scholastics long ago and revived by Descartes... goes... whatever follows from the

definition of anything can be predicated of that thing. Since the most perfect being includes all perfections, among which is existence, existence follows from the idea of God (or the idea of the most perfect being, or the idea of that than which nothing greater can be thought). Therefore existence can be predicated of God. But one must realize that from this argument we can conclude only that, if God is possible, it follows that he exists. For we cannot safely use definitions for drawing conclusions unless we know first that they are real definitions, that is, that they include no contradictions... (Ariew and Garber 1989, p.25)

Leibniz believes the concept of God is contradiction-free because, as quoted above, “nothing can prevent the possibility of what is without limits, without negation, and consequently without contradiction...”

In order to figure out what he means by this potentially odd-sounding remark, it is necessary to attend to Leibniz’s solution to the problem of evil—his *Theodicy*. (And after all, the *Monadology*, from which the quote comes is, as well as a summation of his philosophical view, a commentary on the *Theodicy*.) In this larger work Leibniz uses an argument to account for metaphysical evil, for what he calls the ‘original imperfection’ of creatures. Here is a critical passage from E.M. Huggard’s translation:

The celebrated Kepler and M Descartes... after him have spoken of the natural inertia of bodies; and it is something which may be regarded as a perfect image and even a sample of the original limitation of creatures.... Let us suppose that the current of one and the same river carried along with it various boats, which differ among themselves only

in the cargo.... That being so, it will come about that the boats most heavily laden will go more slowly than the others.... It is not, properly speaking, weight that is the cause of this retardation ... but it is the same cause which also increases the weight in bodies that have greater density.... It is therefore matter itself which originally is inclined to slowness or privation of speed ... since more matter is moved by the same force of the current when the boat is more laden, it is necessary that it go more slowly.... But that indeed would not be necessary if the matter were absolutely indifferent to repose and to movement, and if it had not this natural inertia ... to give it a kind of repugnance to being moved. Let us now compare the force which the current exercises on boats ... with the action of God, who produces and conserves whatever is positive in creatures ... let us compare ... the inertia of matter with the natural imperfection of creature, and the slowness of the laden boat with the defects to be found in the qualities and the actions of the creature.... The current is the cause of the boat's movement, but not of its retardation; God is the cause of perfection in the nature and the actions of the creature, but the limitation of the receptivity of the creatures is the cause of the defects there are in its action.... [T]he current is the cause of the boat's speed without being the cause of the limits of this speed. And God is no more the cause of sin than the river's current is the cause of the retardation of the boat. (Huggard 1951, pp.140–41)

But why did God not make the boats lighter, you want to ask? Why did he not make them 'absolutely indifferent to repose and to movement'? Leibniz's answer to this question is far less substantial than it needs to be. He says, lamely, when returning to the same analogy in his abridgment to the *Theodicy*:

If God had willed to do more then he must needs have produced either fresh natures in his creatures or fresh miracles to change their natures, and this his best plan did not allow. It is just as if the current of the river must needs be more rapid than its slope permits or the boats themselves less laden, if they had to be impelled at a greater speed. So this limitation or original imperfection of creatures brings it about that even the best plan of the universe cannot... be exempted from certain evils... (Huggard 1951, p.384)

But he has still not really said why it must be the case that there is any “limitation or original imperfection of creatures”, only that a different set of natures and/or miraculous interventions of some kind would otherwise be required. When the answer comes it is a somewhat fleeting one for something with so many repercussions: “*God could not give the creature all*”, Leibniz says quietly, “*without making of it a God.*” He goes on to say that, accordingly, “*... there must needs be different degrees in the perfection of things, and limitations also of every kind.*” (Huggard 1951, p.142). Leibniz is saying, in effect, that God could not possibly make creatures perfect, because there would in that case be no creation whatsoever, that if there is more than a single thing—God—then this thing can only be because an element of imperfection has been introduced. If this is *not* the case—if it is possible to have a perfect yet plural creation—then there is no barrier to God creating a world that is perfect; if this is *not* the case, Leibniz would

have no imaginable way to justify the existence of evil, he could only deny its reality altogether.⁷

I take this to be a *huge* claim, one that impacts profoundly on Leibniz's philosophy as a whole. Its hugeness consists in the fact that it identifies imperfection with plurality. Given that Leibniz's God is perfect it follows that 'he' is—like Plotinus's God—radically singular.

This interpretation of Leibniz receives further clarification in the *Dialogue on Human Freedom and the Origins of Evil*, where Leibniz compares the limitation of a created thing to a circle: "a circle is limited", he says, "on account of the fact that the separation of the compass used to inscribe that circle was not larger. Thus the boundaries ... of this separation determine the circle. It is the same for all other things, for they are bounded or imperfect by virtue of the principle of negation or of nothingness they contain, by virtue of the lack of an infinity of perfections in them, and which are only nothingness with respect to them." He goes on to say that "*in the same way that an infinite circle is impossible ... an absolutely perfect*

⁷ To the casual reader of Leibniz, it might seem that this is just what he *is* doing. Voltaire, for example, apparently thought this and he satirised Leibniz mercilessly in *Candide* as Dr Pangloss. (Adams 1991). It is important to remember, however, that underpinning his arguments to the effect that the world is not as bad as it seems, is Leibniz's belief that evil is a necessary condition of there being any world at all.

created thing is also impossible” (Ariew and Garber 1989, p.114) (my italics).

Now we are in a position to see what Leibniz means when he says that the concept of God is possible because it is without negation: if we did not see that Leibniz supposes God to be *super*-logical then his position would be likely to seem bizarre and obviously false—if God is a monad *among* monads, then it surely must be admitted that the concept of God contains an *infinity* of negation in the form of information about God’s distinctness from the world and its infinite elements. More importantly, we are also now in a position to see that for Leibniz the world is irrevocably incomplete, God is irrevocably complete, and that he is deliberately trying to make the former a consequence of the latter.

1.3. The Labyrinth of Freedom

But if Leibniz sees the downside of irrevocable incompleteness, he also sees the downside of irrevocable *completeness*. If everything is reducible to the ways of a necessary being—of a being that could not be other than it is—then what role is there for contingency, for a world that is other than the actual world? Leibniz provides more than one answer to this question, but the most concerted of these is, along with his attempt to solve the problem of

the composition of the continuum, the other principal sign of the alliance that exists in his philosophy between the irrevocably complete and the irrevocable incomplete. This answer turns on Leibniz's view that matter is infinitely divisible; he distinguishes between necessary and contingent truths on the basis that the latter concern a world that is infinitely complex, and are therefore not amenable to finite analysis. Accordingly, these truths must be known under a different principle than that under which necessary truths are known, namely, the principle of sufficient reason rather than that of contradiction. By virtue of the latter principle "we judge that which involves a contradiction to be false"; however, by virtue of the former principle "we consider that we can find no true or existent fact ... without there being a sufficient reason why it is thus and not otherwise" (Ariew and Garber 1989, p.217). The sufficient reasons Leibniz has in mind here, all turn on the fact that whatever is "more perfect" will come to be.

Perfection or essence is an urge for existence from which existence follows per se, not necessarily, but from the denial that another thing more perfect prevents it from existing. (Ariew and Garber 1989, p.200)

This is the point at which Leibniz's famous doctrine of possible worlds comes into focus. Whereas necessary truths are to be explained in terms of themselves—in terms of the fact that to deny them is to deny that a thing is

the same as itself—contingent truths are to be explained by reference to their place in the overall scheme of things, by reference to the question of what (perfection) they contribute to the world as a whole. In Leibniz’s view, the world-as-we-know-it was chosen by God for actualisation from among an infinity of alternatives because it was the best of these...⁸

This is interesting but unhelpful unless our inability to know contingent truths under the principle of contradiction is a theoretical rather than merely a practical one. If God knows all things under the principle of contradiction, then the principle of sufficient reason is just the principle of contradiction in disguise, and by the same argument, contingent truths are merely necessary truths in disguise. Deleuze says that Leibniz really has no interest in the question of contradiction when he talks about contingent truths because, he says, these truths are supposed by Leibniz to never fall under the principle of contradiction—God himself does not know them in this way:

⁸ Scholars have long debated the question as to what criterion of goodness Leibniz had in mind and doubtless various answers are possible but my impression is that, first and foremost, it is an aesthetic criterion. Leibniz spells this out in the dialogue at the end of the *Theodicy* in which he imagines that every possible world is like the hall of a palace, a palace through which you might take a virtually real tour:

The halls rose in a pyramid, becoming even more beautiful as one mounted towards the apex, and representing more beautiful worlds. Finally they reached the highest one which completed the pyramid, and which is the most beautiful of all... (Huggard 1951, pp.370.72)

Leibniz, in another drawer, has another formula to give you: identity governs truths of essence, but not truths of existence.... What interests him at the level of truths of existence is not identity of the predicate and the subject, it's rather that one passes from one predicate to another, from one to another, and again on from one to another, etc.... from the point of view of an infinite analysis, that is, from the maximum of continuity. *In other words, it's identity that governs truths of essence, but it's continuity that governs truths of existence.* (Stival, <http://www.imagnet.fr/deleuze/TXT/ENG/220480.html>)

He goes on to propose that Leibniz construes the principle of sufficient reason in terms of the mathematics of the infinitesimal calculus, rather than in terms of classical logic. Leibniz will admit, Deleuze says, that the relationship between the subjects and predicates of contingent propositions is not a relationship of identity, not even in the mind of God, but he will say instead that it is one of continuity. Two things are continuous, Deleuze says, when there is an 'evanescent' difference between them, a difference, that is, which 'tends to disappear'.

I would say therefore that truths of essence are governed by the principle of identity, truths of existence are governed by the law of continuity, or evanescent differences, and that comes down to the same. Thus between sinner and Adam you will never be able to demonstrate a logical identity, but you will be able to demonstrate—and the word demonstration will change meaning—you will be able to demonstrate a continuity, that is, one or several evanescent differences. An infinite analysis is an analysis of the continuous operating through evanescent differences. (Stival, <http://www.imagnet.fr/deleuze/TXT/ENG/220480.html>)

In so far as the question of whether contingent propositions are identical is concerned, Deleuze's view is in close agreement with what Leibniz says in *On Freedom*, where he denies that a contingent proposition can ever be reduced to an "equality" or an "identity" (Ariew and Garber 1989, p. 96). Moreover, Leibniz says here that "God does not see the end of the resolution ... but the connection of the terms" (Ariew and Garber 1989, p. 96). There is sense also to Deleuze's suggestion that Leibniz sees the analysis of contingent truths in mathematical rather than logical terms:

[Just as] in proportions, while the analysis sometimes comes to an end ... [yet] in other cases [it] can be continued to infinity, as happens in the comparison of a rational number and an irrational number ... So similarly truths are sometimes provable, that is, necessary, and sometimes they are free or contingent, and so cannot be reduced by any analysis to an identity. (Ariew and Garber 1989, p.97)

Nor does Leibniz suppose that God can analyse his way free of incommensurability. When the Leibniz-character is asked, in the *Dialogue on Human Freedom*, whether God would be able to find a number capable of expressing exactly the square root of two or the length of a diagonal of a square, he says:

God cannot find absurd things. That would be as if we asked God to teach us the way to divide three coins into equal parts without breaking

one, that is, without getting a one and a half or the like. (Ariew and Garber 1989, p.116)

Deleuze admits that Leibniz often says that truths of existence, like truths of essence, are reducible to identities; however, he also speculates that Leibniz says different things depending on the sophistication of his audience. He thinks accordingly that the most sophisticated understanding of Leibniz's philosophy is one according to which the principle of sufficient reason is that it is not, at any stage, to be reduced to the principle of contradiction.

The problem with which Leibniz is dealing when it comes to the labyrinth of the composition of the continuum, is attributed by him to the irrevocably incomplete nature of the world as it we know it, to which his solution is the irrevocably complete nature of God. If Leibniz declines to push his solution this far—if he rests content with regular monads—then, since these are themselves irrevocably incomplete, there is nothing to stop the slide towards the unreality that is promised by the infinity of irrevocable incompleteness. When it comes to the labyrinth of freedom the problem and solution are reversed. In order to successfully establish the conclusion he wants—that there are *bona fide* contingent truths—Leibniz's world must be infinite in the irrevocably incomplete, rather than the irrevocably complete, sense of this term. If it is infinite in the latter sense then (although the human

mind can't perform the relevant analysis) "contingent" propositions are reducible to identities, and therefore it is at the very least doubtful that there could be any genuine contingency at all.

1.4. The Paradox of the Irrevocably Complete

There can be little question that Leibniz's desire to combine the irrevocably complete and the irrevocably incomplete involves at least an apparent contradiction. Generally speaking, this is just because the very idea of completeness contradicts that of incompleteness, but there is considerably more going on here than this. If the concepts of the human mind are supposed to be incomplete and yet there *is* supposed to be something complete, and *if also* we identify ourselves with the human rather than the divine, then we are saying that this thesis itself is, not merely false, but meaningless (or non-existent). It cannot be quite meaningful since any concept that the human mind forms of the irrevocably complete is bound not to be a concept of the object it is supposed to be a concept of. If it isn't so, then it can be concluded that either the human mind isn't bedeviled by incompleteness after all, or that we are not merely human but divine. Consider, in regard to this point, the film *The Matrix*, the central premise of which is that the-world-as-we-know-it is *virtual*, a computer-generated illusion.

Everyday life is turned upside-down when Thomas Anderson discovers that the world around him is a detailed virtual reality created by a computer that has taken over the Earth of the future. When Anderson realizes his predicament he teams up with Morpheus, the leader of a gang of freedom fighters, to reclaim their individuality and “wake up” the world. (<http://www.movie-page.com/1999/Matrix.htm>)

Can you conceive of this system from an inside perspective? The film is divided on this question. On the one hand Morpheus tells Anderson: “No one can be told what the Matrix is. You have to see it for yourself.” On the other hand, Morpheus is himself inside the matrix when he makes this “statement”. The “no” answer is a fairly compelling one. To conceive of the Matrix is to conceive of the illusory nature of one’s world inside the matrix and, by hypothesis, this is possible only if one is precisely *not* inside the matrix. Leibniz proposes that everything in the human realm is an incomplete form of the divine. Given that the divine realm is *complete*, it follows straightforwardly that, like those in the Matrix, we too are under an illusion. The thing is, however, that anyone—such as Morpheus or Leibniz—according to whom the-world-as-we-know-it is systematically illusory, contradicts themselves in what is a fairly blatant way.

It might be thought, therefore, that solution to this problem is to do what Leibniz would not do and give up the irrevocably complete, the irrevocably incomplete, or both. The first option is a non-starter; for to give

up the irrevocably incomplete is impossible unless you are yourself God and yet, from God's perspective—from the perspective of something whose concept contains no negation—the distinction on which our whole discussion turns could not be formed in the first place. It might be thought, next, that the irrevocably complete should go. In order to see that this initially promising proposal seems to lead down a false trail, consider that there can be no difference between the concept of the irrevocably complete and the thing itself. If there is such a difference then there is something that doesn't fall under this concept, namely, the irrevocably complete as it is *independently* of any conception of it and the irrevocably complete as it is *in itself*—this is absurd. In *Extraordinary Tales*, Borges and Casares tell of an empire in which the art of cartography had "... achieved such perfection that the map of a single province occupied the whole city, and the map of the empire, the whole province." In time, however, these "...disproportionate maps failed to satisfy and the schools of cartography sketched a map of the empire which was the size of the empire and coincided at every point with it." (Borges and Casares 1973, p.123). The concept of the irrevocably complete is like this empire-sized map of the empire—it leaves nothing out and thus, can only be a conception of itself. It follows, therefore, that to deny the irrevocably complete is in the very same breath to affirm its existence,

since you can't deny something unless you can conceive of it in the first place.

This kind of problem is supposedly solved by the Kant-inspired view that sees existence as not a property. Kant himself argues that since concepts can't function as their own objects, it can never be a matter of logic alone that a concept has application; thus, it can never be a matter of logic alone that God, or anything else, exists. In the *Critique of Pure Reason* (Kemp-Smith 1933), where his solution to the problem appears, Kant rings on a financial analogy and points out "...my financial situation is not in the least improved on account of my having the mere concept of a 'hundred thalers' at my disposal." He goes on to argue, in the same way, that God becomes no more real simply because he, Kant, has a concept of God.⁹ Hegel accuses Kant of arguing, illicitly, from the case of a finite thing which, by definition, involves a distinction between concept and object—"It is in fact this and this alone which marks everything finite: its being in time and space is discrepant from its notion."—to the case of an infinite being: "God, on the contrary, expressly has to be what can only be 'thought as existing'; his

⁹ If concepts were horses—Kant might well have said—then beggars could ride!

notion involves being. It is this unity of the notion and being that constitutes the notion of God” (Wallace, 1975, Section 51).¹⁰

This objection devastates Kant’s argument. It doesn’t help that Kant *allows* that there is a concept of a necessary being—“The analytic criterion of possibility, as consisting in the principle that bare positives (realities) give rise to no contradiction, cannot be denied to it”, he says—but, in any case, Hegel has him in trouble (Kemp Smith 1933, A 602, B 630). This is because *there is no employing the concept of something such that its concept and its*

¹⁰ It is difficult to overestimate the importance of what Hegel is saying here so it worth putting these remarks in their original context:

The uniformly favourable reception and acceptance which attended Kant’s criticism of the Ontological proof was undoubtedly due to the illustration which he made use of. To explain the difference between thought and being, he took the instance of a hundred sovereigns, which, for anything it matters to the notion, are the same hundred whether they are real or only possible, though the difference of the two cases is very perceptible in their effect on a man’s purse. Nothing can be more obvious than that anything we only think or conceive is not on that account actual; that mental representation, and even notional comprehension, always falls short of being. Still it may not unfairly be styled a barbarism in language, when the name of notion is given to things like a hundred sovereigns. And, putting that mistake aside, those who perpetually urge against the philosophic Idea the difference between Being and Thought might have admitted that philosophers were not wholly ignorant of the fact. Can there be any proposition more trite than this? But after all, it is well to remember, when we speak of God, that we have an object of another kind than any hundred sovereigns, and unlike any one particular notion, representation, or however else it may be styled. *It is in fact this and this alone which marks everything finite: its being in time and space is discrepant from its notion. God, on the contrary, expressly has to be what can only be ‘thought as existing’; his notion involves being. It is this unity of the notion and being that constitutes the notion of God.* (my italics) (Wallace 1975, Section 51)

existence are unified without granting the existence of this thing. To deny the soundness of the Ontological Argument is, in the same breath, to affirm the conclusion of this argument.

According to Russell's Kantian approach the temptation to argue 'Ontologically' because of the false assumption that statements of non-existence are naming the non-existent thing when in fact what seems to be a name is a description that might or might not apply in the relevant case. The statement 'Pegasus does not exist', for example, is not to be construed, says Russell, as making a claim about a first-order object Pegasus, but rather as making a claim about a second-order object, the description of Pegasus. 'Pegasus', as it occurs in the statement. 'Pegasus doesn't exist' is not really a name at all but the description 'winged horse that with stroke of hoof caused fountain Hippocrene to flow on Mount Helicon' in disguise: instead of Pegasus doesn't exist, say "It is not the case that there exists *something*—(x)—that is winged and horsy and with hoof stroke caused fountain Hippocrene to flow on Mount Helicon." Russell:

The whole realm of non-entities, such as 'the round square', 'the even prime other than 2', 'Apollo', 'Hamlet', etc, can now be satisfactorily dealt with. All these are denoting phrases which do not denote anything. A proposition about Apollo means what we get by substituting what the

classical dictionary tells us is meant by Apollo, say 'the sun-god'... 'The most perfect Being has all perfections; existence is a perfection; therefore the most perfect Being exists' becomes: 'There is one and only one entity x which is most perfect; that one has all perfections; existence is a perfection; therefore that one exists.'

As a proof, this fails for want of a proof of the premise 'there is one and only one entity x which is most perfect.' (Russell 1956, p.54)

But how does Russell's theory deal with the claim that there is nothing irrevocably complete or that there is no meaningful idea of such a thing (this last claim is one that Russell himself makes during the course of his debate with Copleston)? It doesn't, because the description corresponding to the disguised description 'God' is not something that can possibly be divorced from the thing it is a description of. If it could be, then it wouldn't be that thing at all. The fact is that there is no denying the existence, the knowability, or the meaningfulness of the irrevocably complete without presupposing the very thing you aim to deny. Russell says that the ontological argument fails as a proof "for want of a proof of the premise there is one and only one entity x which is most perfect". But in fact it is the theory of descriptions that fails for want of a first-order/second-order distinction in the case of a description that is the description of itself. In order to see that this problem lies at the very root of Russell's theory, consider the question of what the 'something' in 'It is not the case that there

is *something* that is winged and horsy etc.’ refers to.¹¹ The great irony is that this word ‘something’ refers, if at all, to what is devoid of any properties barring existence itself; it can refer, that is to say, only to the very thing whose irrelevance to Russell’s theory is a condition of the possibility of that theory, the world as it is independently of any distant conception of it, God, or the irrevocably complete.

1.5. Leibniz and the Liar Paradox

It will not have escaped attention that Leibniz’s alliance of the irrevocably complete and the irrevocably incomplete has a liar-like quality to it, and now I want to explain in greater detail why I think that this alliance *is* a form—the most fundamental form—of the liar paradox.¹² At its simplest, this paradox concerns the sentence ‘This sentence is false.’ If it is true that ‘This sentence is false.’ then, since it says that it is false, it *is* false; and if it is false then, since *that* is what it says, it is true. Thus, whether it is true or false, it seems that the liar sentence is true *and* false. Compare this liar sentence to the liar sentence ‘There are no truths.’ The latter sentence is not paradoxical unless there is only a single truth. If there is more than one

¹¹ And it surely must refer because, as Quine says, ‘... the burden of objective reference which had been put upon the descriptive phrase is now taken over by words... like ‘something’, ‘nothing’, ‘everything’ (Quine 1961, p.6).

¹² See Appendix 1.

truth then the falsity of this carries no implication that it is true, only that there are *some* truths. The argument to the effect that the first liar sentence is paradoxical is a *reductio ad absurdum* that goes like this: assume that the sentence is true; if it is true then it is false, and therefore it is false; then assume that the sentence is false; but if it is false then it is true, and therefore is true. To prove the *second* liar sentence paradoxical one would have to argue thus: assume that sentence is true; if it is true then it is false, and therefore it is false; assume then that it is false; but if it is false then it is true, and therefore it is true. The thing is, however, it doesn't follow from the assumption that the sentence is false that it is true just *so long as there is more than one truth*. If there is more than one truth, the assumption that the sentence is false implies innocuously, only that there are some truths. Now while a hope existed that Leibniz's philosophy of the infinite had the form of the second rather than the first liar sentence—while it could be seen a problem attributable to the infinity of irrevocable completeness—then there was the possibility that Leibniz was merely refuting himself in a way that 'There are no truths.' might be merely self-refuting. That there is an actual paradox here is suggested by the consideration that although irrevocable completeness cannot be affirmed without self-refutation, nor can it be denied. The problem with the proposal to give up the idea of a complete

world is that such a world clearly *is* a single truth, from which it follows that it can be denied if, and only if, it cannot.

Following a suggestion by Russell, Patrick Grim argues at length in *The Incomplete Universe* (Grim 1991), that the liar paradox and other related paradoxes arise because of the assumption that there is a totality of truths or a totality of knowledge—that the world is complete. He therefore *denies* that there is such a totality on the grounds that these concepts give rise to liar-like paradoxes. He says that ‘Far too often... it is asked what has gone wrong with the paradoxes rather than what paradoxes may have to teach us’ and that ‘What the Liar may genuinely have to teach us, I think, is that *there really can be no coherent notion of all truth or of omniscience.*’ (Grim 1991, p.46) (my italics). Although there is a great deal of sense in Grim’s approach to the paradoxes, he is fundamentally mistaken. His mistake is to suppose that the ‘notion’ of the world as something complete is the source of the contradiction, when in fact it is this notion *coupled with* the notion that it is at all *incomplete*. (There is nothing wrong with the concept of the irrevocably complete so long as you are yourself God and this is your concept of yourself!)

Grim's denial that the world is complete is equivalent to Russell's Theory of Types solution to the paradox that bears his name, and Tarski's Undefinability Theorem, and therefore it faces the same difficulties faced by these solutions. The Russell paradox concerns the set of all sets that are not members of themselves. If the set of all sets is a member of itself then, since that could only be because it is a set that is *not* a member of itself, it is not a member of itself; and if it is not member of itself then, since this is exactly what qualifies it to be a member of itself, it *is* a member of itself. Thus, the so-called Russell set apparently is, and is not, a member of itself. Russell hopes to solve, or at least to avoid this paradox, by dividing things into a hierarchy, starting with first-order things, moving up to sets of things, and then up to sets of sets of things, and so forth. Russell's theory forbids reference between things belonging to the same level of the hierarchy, and therefore there can be no statement to the effect that a set is, or is not, a member of itself. In a similar mood, Tarski points out that the fundamental assumption of the liar paradox is that:

... the language in which the antinomy is constructed contains, in addition to its expressions, also the names of these expressions, as well as semantic terms such as the term '*true*' referring to sentences of this language; we have also assumed that all sentences which determine the adequate usage of this term can be asserted in the language. A language

with these properties will be called “*semantically closed*.”(Tarski 1944, p.672)

Tarski believes on the basis of the liar that there can be no consistent semantically closed language and so proposes, like Russell, that paradox is best avoided by appeal to a hierarchy of semantically open formal languages, languages whose truth predicate must always be defined in a language belonging to a higher level of the hierarchy. The thing about Russell’s and Tarski’s approaches to the paradoxes with which they are concerned, is that they can solve—or avoid—them only if there is no totality of truths, if the world is irrevocably incomplete. Otherwise one will eventually come to the top of these hierarchies at which point self-reference is not merely possible but unavoidable, at which point the languages at lower levels of the hierarchy are revealed to have been semantically closed all the while. If the world is irrevocably complete then undoubtedly there is such a thing as a topmost level of the hierarchy. This point can be readily made by reference to the set of all sets. This is a set at the top of Russell’s hierarchy; at the bottom of the hierarchy you have things, at the next level of the hierarchy come sets of things; at the next level comes sets of sets of things... and at the top of the hierarchy there is—naturally—the set of all the sets of things. Until this point in the hierarchy is reached there must be at least one

more level to go. But the set of all sets surely must be a member of itself.¹³
By this same logic, the truth predicate of the language that oversees all the
lower level languages must be definable in this very language or not at all.

¹³ Russell says at the end of the *Principles of Mathematics* that, although his paradox is solved by “the doctrine of types”, “...there is at least one closely analogous contradiction which is probably not solved by this doctrine.”(Russell 1903, p. 528) He refers here to Cantor’s paradox which can be most simply explained by saying that there is a clash between the idea of a set of all sets and the fact that any set can be used to construct a set that is larger, specifically the set of all its subsets, its so called power set. Clearly no set can be larger than the set of *all* sets.

PART II

2.1. Possible responses

At first sight there are three possible responses to the contradiction between the irrevocably complete and the irrevocably incomplete. One is to give up the irrevocably incomplete, a possibility that is open only to God. Another is to give up the irrevocably complete, a possibility that is apparently ruled out by the consideration that the concept and the object of the irrevocably complete are the same. If it is true that one cannot give up the irrevocably complete then it surely is not possible to give up *both* the irrevocably complete and the irrevocably incomplete, but a consideration of the suggestion there is in Kant's philosophy that this is what we should do will help provide a fuller understanding of Leibniz's philosophy of the infinite.

2.2. A Kantian Solution to the Paradox?

The most fundamental tenet of Kant's philosophy is this: although we can't say that the world must *be* a certain way—although we can't pronounce positively on the world as it is independently of how it is revealed

to us in experience—we can say that it must be *conceived* in a certain way. Accordingly Kant points to the certain concepts¹⁴ (those of space and time, quantity, quality, relation, and modality) as concepts under which *any* world must fall. The sole function of these concepts is to serve up the world in a cognitively digestible form and, accordingly, they cannot properly be applied to themselves:

The pure categories are nothing but representations of things... In a word, if all sensible intuition, the only kind of intuition which we possess, is removed, not one of these concepts can in any fashion *verify* itself, so as to show its *real* possibility. Only *logical* possibility then remains, that is, that the concept or thought is possible. That, however, is not what we are discussing, but whether the concept relates to an object and so signifies something. From all this it undeniably follows that the pure concepts of understanding can *never* admit of *transcendental* but *always* only of *empirical* employment... (Kant, 1933, A 245, B 303)

There is a very strong connection between Kant's philosophy and that of the early Wittgenstein. Each of these philosophers can, I believe, be pointed to in order to illuminate the view of the other. This connection is the suspicion with which Kant *and* Wittgenstein regard what can for want of a

¹⁴ I shall not here bother to employ the Kantian distinction between forms of intuition and concepts, but rather I shall use the word 'concept' broadly enough to cover both. Though important work is done by this distinction for Kant, none of it is crucial to my own present purposes.

better term be called ‘introspection’; the suspicion with which they regard the sorts of things the mind comes up with when it concentrates its attentions on the *means* by which it represents rather than these representations themselves. They are suspicious because their empiricist assumption that the function of the mind is to represent what is *other* than it, what is *outside* of it. Wittgenstein goes so far as to suggest therefore that the sort of introspective activity that has traditionally been the province of philosophy cannot sensibly go on, that philosophy is barely—if at all—a legitimate activity of mind:

The correct method in philosophy would really be the following: to say nothing except what can be said, i.e. propositions of natural science—i.e. something that has nothing to do with philosophy... (Wittgenstein 1961, 6.53)

He faces up to the inconsistency when towards the end of the *Tractatus* with the words:

My propositions are elucidatory in this way: he who understands me finally recognizes them as senseless, when he has climbed out through them, on them, over them. (He must so to speak throw away the ladder, after he has climbed up on it). (Wittgenstein 1961, 6.54)

Kant recognizes no such inconsistency because he thinks that introspection is possible in so far as it is—as is the case with his own philosophy—a *negative* rather than a positive thing, something that is to be done in order to

place a *limit* upon what can be accomplished in this way. He denies, in a way that at least borders on inconsistency, that the distinction between the world as it appears to be and the world as it is in itself can be given any “positive” sense:

[T]he concept of a noumenon is necessary, to prevent sensible intuition from being extended to things in themselves, and thus to limit the objective validity of sensible knowledge... The concept of a noumenon is thus a merely *limiting concept*, the function of which is to curb the pretensions of sensibility; and it is therefore only of negative employment. At the same time it is no arbitrary invention; it is bound up with the limitation of sensibility, though it cannot affirm anything positive beyond the field of sensibility. The division of objects into phenomena and noumena, and the world into a world of the senses and a world of the understanding, is therefore quite inadmissible in the positive sense... (Kemp Smith 1933, A 255, B 311)

Anyway, since knowledge is produced only by the action of these inevitable concepts upon something *other* than these concepts, it follows—Kant argues—that the philosophical ambition to pronounce on the world as a whole is an unrealisable one. This ambition has its origins in the fact that reasoning involves finding the basis for a conclusion in certain premises, the “*conditions*” under which it is true. But what about these conditions? There are in turn conditions under which *they* are true, and thus a regress of conditions is set up or, as Kant says, “set us as a task”. The task

in question is to find something that depends on nothing other than itself, “to find [for] conditioned knowledge ... the unconditioned whereby its unity is brought to completion” (Kemp Smith 1933, A 307, B 364). Kant sees the question of whether this regress terminates in something unconditioned as the one that divides rationalism and empiricism. He points to the rationalist tendency to affirm the immortality of the soul, the beginning of the world in space and in time, and the existence of God, as born of this attraction of reason to the ‘unconditioned’, and to the empiricist tendency to *deny* these things as a skeptical counter-move. More importantly, though, he regards this conflict as a false one, as one that arises on the basis of the false assumption that the regress of conditions concerns the world as it is in itself. Kant argues that since our only dealings with the world concern this world as it *appears* to us—as it is *after* having had form imposed on it by our minds—there can be no inference from the existence of something conditioned to the existence of the whole series:

If, however, what we are dealing with are appearances...I cannot say... that if the conditioned is given, all its conditions (as appearances) are likewise given, and therefore cannot in any way infer the absolute totality of the series of its conditions. (Kamp Smith 1933, A 307, B 527)

Kant says therefore that a basic conflict between rationalism and empiricism is therefore the same sort of conflict as that between the claim that all bodies have a good smell and the claim that all bodies have a bad smell *in the circumstance that bodies don't have a smell*. Kant draws a distinction between two kinds of logic, analytical logic which focuses on how the data of experience yield knowledge of the natural, phenomenal, world, and dialectical logic, which operates independently of experience and erroneously professes to give knowledge of transcendent 'noumena', of the world as it is in itself:

I beg permission to entitle this kind of opposition dialectical, and that of contradictories analytical. Thus of two dialectically opposed judgments both may be false; for the one is not a mere contradictory of the other, but says something more than is required for a simple contradiction. If we regard the two propositions, that the world is infinite in magnitude and that it is finite in magnitude, as contradictory opposites, we are assuming that the world, the complete series of appearances, is a thing in itself that remains even if I suspend the infinite or the finite regress in the series of its appearances. If, however, I reject this assumption, or rather this accompanying transcendental illusion, and deny that the world is a thing in itself, the contradictory opposition of the two assertions is converted into a merely dialectical opposition. (Kemp Smith 1933, A 504, B 532)

Kant addresses himself to the question of whether the regress of conditions is finite or infinite rather than that of whether it is irrevocably

complete or irrevocably incomplete but, in so far as he can help us remove the tension from this opposition also, it must be taken to be no less susceptible than the former to the Kantian criticism that human reason cannot legitimately make entire worlds—only the things *in* worlds—the subject of an analytic opposition. Consider, again, the good smell/bad smell opposition and consider that Kant diffuses it by appealing to the third possibility that there is no smell at all. The finite/infinite opposition is to be diffused by appeal to the third possibility that there is, in effect, no *completed series of conditions* to be the focus of such an opposition. Kant does admit, however, that the infinite/*not* infinite opposition is a genuine one:

If, therefore, we say that the world is either infinite... or is not infinite... and if the former proposition is false, its contradictory opposite, that the world is not infinite, must be true. And I should thus deny the existence of an infinite world, without affirming in its place a finite world. But if we had said that the world is either infinite or finite (non-infinite), both statements might be false. (Kemp-Smith, 1933, A 503, B 531)

The same presumably goes for finite/*not finite* as goes for infinite/*not infinite*—these are for Kant analytic, not merely dialectical, oppositions and so you have to wonder what it is that is neither infinite, *nor* finite, that Kant regards as his third option here. The answer is the indefinite. The regresses to which reason inclines are, for Kant, not infinite, not finite, but *indefinite*;

it is not a matter of these going on *ad infinitum* but *in definitum*. Here he is on the subject of this regress as a cosmological one:¹⁵

[T]he regress in the series of appearances, as a determination of the magnitude of the world, proceeds *in indefinitum*.... This is equivalent to saying that, although the sensible world has no absolute magnitude, the empirical regress (through which alone it can be given on the side of its conditions) has its own rule, namely, that it must always advance from every member of the series, as conditioned, to one still more remote; doing so by means either of our own experience, or of the guiding-thread of history, or of the chain of effects and causes.... [T]he regress does not proceed to the infinite, as if the infinite could be given, but only indeterminately far, in order [by means of the regress] to give that empirical magnitude which first becomes actual in and through this very regress. (Kemp Smith 1933, A 521 B 549-A 523 B 551.)

So whether it is supposed to end *qua* finite or supposed *not* to end *qua* infinite, it is supposed that the whole series exists as a whole.

He denies that these apparent alternatives oppose one another since they share the assumption that there is such a thing as the whole series. His contrary view is that there is *no* such thing as the whole series as far as the human mind is concerned and thus he hopes to avoid the rationalist *and* the empiricist stance towards the series of conditions.

¹⁵ Kant is careful to distinguish between the various kinds of regresses there are but, for the purposes of my argument, the only sort of regress that we need concern ourselves with is the logical regress to which all of the regresses, in the final analysis, reduce.

“Since the world does not exist in itself, independently of the regressive series of my representations, it exists in itself neither as an infinite *whole* nor as a finite *whole*.” (Kemp Smith 1933, A 504, B 533) (my italics), he says.

Two things must be noted immediately. One is that, whether the series be deemed ‘infinite’ or ‘indefinite’, the fact remains that it is something that can’t be completed—an indefinite series possesses no greater degree of completeness than an infinite or a transfinite one.¹⁶ The other is that Kant identifies the incompleteness of the series with the world as it *appears* to us—with the-world-as-we-know-it—and the *completeness* of the series with the world as it is in itself. In particular he argues that our lack of knowledge of the world as it is in itself lies at the root of our inability to complete the series. In this way he is himself working in a fundamental, and fairly explicit way, with the complete/incomplete opposition that is the central theme of this thesis. It turns out that Kant has succeeded in freeing himself from the irrevocably complete/irrevocably incomplete opposition *only if the category of the irrevocably complete is to be regarded as actual rather than potential*; only if—*pace* Aristotle—all irrevocably complete things are what Cantor calls transfinite. Given that the irrevocably complete/irrevocably incomplete

distinction *cuts through* the indefinite/transfinite distinction, it follows that Kant's retreat from the infinite and the finite to the *not*-infinite but *not*-finite does nothing to mitigate the tension there is between the irrevocably complete and the irrevocably incomplete; rather it reaffirms the reality of this opposition in terms of the distinction between the indefinite and the *not-indefinite*.

2.3. Rationalism and Empiricism—Similarities and Differences

Kant's and Leibniz's philosophies of the infinite are not so very different. One difference is that Leibniz recognises, and Kant does not, that the series of conditions as a whole is necessarily infinite rather than finite, but infinite in the complete rather than the incomplete sense of this term. Kant feels he has to consider the rationalist proposal that the series comes to an end to be the proposal that it is finite but this is because he fails to see—as Leibniz does—that the existence of something unconditioned is incompatible with that of the series in the first place, that the unconditioned must be something “without negation”, something singular and solitary. By dint of his view that the world of appearances is always indefinite, Kant is

¹⁶ Leibniz, for example, is torn between thinking that the-world-as-we-know-it is

forced to regard the world as it is in itself as complete. He couldn't—even if he wanted—say either that it is complete in a finite or an infinite way. Leibniz reminds that the finite is incomplete and thus that this world as a whole is something infinite. The other—and very significant—difference is that for Leibniz the irrevocably complete is the infinity of fullness rather than emptiness, of largeness rather than smallness. This is the source of the paradox as this affects Leibniz. If this concept is as full as he says it is—and he says that it has the “*maximum*” amount of “positive reality”—then there should be nothing else besides this concept; there should be no *negative* reality, no nothingness, and so nothing other than God. Let me briefly illustrate Leibniz's problem. In the *Dialogue on Human Freedom and the Origin of Evil* (Leibniz 1989, pp.113–44) Leibniz draws a distinction between God and his creatures in terms of the pure being of God and the being *and* nothingness of creatures. Leibniz's mouthpiece in the dialogue wants to account for evil by appeal to “another infinite cause capable of counterbalancing the influence of divine goodness.” In response to the charge that he would therefore be a Manichean, Leibniz's mouthpiece appeals to the principle of nothingness and says of nothingness that it “is infinite, eternal... It includes an infinity of things, for all things that do not

indefinite and thinking that it is transfinite but he does not imagine that he can do

exist are included in nothingness, and all things that are no longer have returned to nothingness.” Given the purity of God, given that God is not one of the parts of the world but something infinitely whole, it follows that Leibniz cannot distinguish between the purity of being and purity of nothingness. If he could then there can be no doubt that God—even before the advent of the creation—is himself an admixture of being and nothingness and therefore not fundamentally different from a created monad in respect of his “original imperfection”.

For Kant, on the other hand, the concept of the world as a whole is empty; the irrevocably complete is the infinity of emptiness, of smallness. This is the source of the following well-known criticism of Kant: strictly speaking he cannot even speak of the world as it is in itself since, from the human point of view, there is nothing to be spoken of. Perhaps Kant believes that this objection can be countered by saying, as he does in the preface to *The Critique of Pure Reason*, that “... though we cannot *know* these objects as things in themselves, we must yet be in position at least to *think* them as things in themselves; otherwise we should be landed in the absurd conclusion that there can be appearance without anything that appears.” i.e. by distinguishing between perception and *conception* (Kemp

without something irrevocably complete in which to ground this world.

Smith 1933, Bxxvi-Bxxvii). But this response is neglectful of the fact that our failure to know the world as it is in itself is not, for Kant, merely a perceptual failure but a *conceptual* one. This is because the world as it is in itself is the world as it is *prior* to its having been brought under the categories, *prior* to its having been so much as conceived. Kant's predicament here can be compared to that of Wittgenstein in the *Tractatus*. Wittgenstein argues that the world as it is independently of expression in language is inexpressible and concludes this work with the suggestion that this claim is itself an inexpressible one. Given the 'nature' of the world as it is in itself, Kant ought—if he is honest with himself—to draw the same conclusion as Wittgenstein. However, he dares not do this.

The relevance of this anti-Kantian commonplace to the issue at hand by can be established by pointing out, firstly, that the unspeakability of the whole must apply also to the parts that make it up (at least in so far as they are to be referenced—as they are by Kant—to the whole). We can therefore go on to say, secondly, there is more than an semantic/epistemic problem here since the question arises as to what *ontological* status the parts can hold in the circumstance that the whole is to all intents and purposes pure nothingness: if the whole is nothing then how can it be that the parts are something? If Leibniz's problem is to explain how, given the pure being of

God, there is any nothingness, Kant's problem is to explain how, given the pure nothingness of the thing in itself, there is any being.

There are two apparently different ways of encountering the irrevocably complete/ incomplete contradiction. One is the Leibniz's way where you affirm the irrevocably complete and have—as a human—to deal with the contradictory shadow of the irrevocably incomplete; the other is Kant's way where you—as a human—*affirm* the irrevocably incomplete and have to deal with the contradictory shadow of the irrevocably complete; in the one case, the irrevocably complete is in the foreground and, in the other, it is in the background; in the one case it is conspicuous by its presence and in the other by its absence; Leibniz takes the first way, Kant the second; neither fully faces up to the contradiction but neither fully shies away from it either. The difference between Leibniz's positive attitude to the irrevocably complete and Kant's negative attitude isn't what at first it seemed to be. It isn't that for Leibniz the whole is infinite and for Kant it is not but that for Leibniz it is an infinitely large whereas, for Kant, it is infinitely small. But the supposed presence of things that are neither infinitely large, nor infinitely small, directs Leibniz to Kant, and Kant to Leibniz.

Let me further develop the point I am trying to make by reference to the consideration that the back and forth that we have discovered to go on between the irrevocably complete and the irrevocably incomplete is that between rationalism and empiricism. Consider James's response to the question of what these terms mean (posed in the first lecture in *A Pluralistic Universe*): "Reduced to their most pregnant difference", he says, "*empiricism means the habit of explaining wholes by parts, and rationalism means the habit of explaining parts by wholes.*" (James 1977, p.9). What makes it the case that rationalists explain things in terms of wholes and empiricists in terms of parts? Their shared assumption of the limited—the *partial*—nature of the human knowledge and human being combined with their disagreement as to how the existence of such knowledge and being is to be explained. As is suggested by the case of Leibniz, rationalists explain it on a basis of the consideration that this partiality participates in something whole, God. Empiricists, on the other hand, argue that this partialness means that we must content ourselves with knowing in part and therefore cannot say anything positive about the whole. James who describes himself as a "radical empiricist" sums up the empiricist attitude to the whole in this way:

No philosophy can ever be anything but a summary sketch, a picture of the world in abridgement, a foreshortened bird's eye view of the perspective of events... All philosophers ... have conceived of the whole world after

the analogy of some particular feature of it which has particularly captivated their attention. All follow one analogy or another; and all the analogies are with some one or other of the universes sub-divisions. Everyone is nonetheless prone to claim that his conclusions are the only logical ones, that they are necessities of universal reason, they being all the while, at bottom, accidents more or less of personal vision which had far better be avowed as such. (James 1977, pp.9–10.)

The whole is for James nothing in itself but only becomes something on account of an imaginary extrapolation from the parts. For the rationalist, the whole is purely positive. It is pure being, pure knowledge, pure goodness... for, otherwise, the partial being, knowledge, and goodness of man cannot be explained. These parts are to be explained on account of the fact that they are imperfect forms of God's *whole* being, knowledge, and goodness.¹⁷ For the empiricist, however, the whole is to be seen as purely negative. It is not pure being so much as it is pure nothingness. (If rationalism and theism go hand in hand, then so do empiricism and atheism). When those who take empiricism seriously want also to recognize their commitments to the whole they make of it a negative thing, something that we must, as Wittgenstein says in the *Tractatus*, "pass over in silence"(Wittgenstein 1961, 7). Hegel says rightly of Kant's thing-in-itself that it is "utter abstraction, total

¹⁷ The biblical doctrine that the human is a fallen form of the divine is therefore perhaps the classical piece of rationalism.

emptiness ... an ‘other-world’ the *negative* of every image, feeling, and definite thought” (my italics) (Wallace 1973, Section 44).

The Critical philosophy has one great negative merit. It has brought home the conviction that the categories of understanding are finite in their range, and that any cognitive process confined within their pale falls short of the truth. But Kant had only a sight of half the truth. He explained the finite nature of the categories to mean that they were subjective only, valid only for our thought, from which the thing-in-itself was divided by an impassable gulf... This stage of ‘appearance’ however—the phenomenal world—is not the terminus of thought: there is another and a higher region. But that region was to the Kantian philosophy an inaccessible ‘other world’. (Wallace, 1973, Section 60)

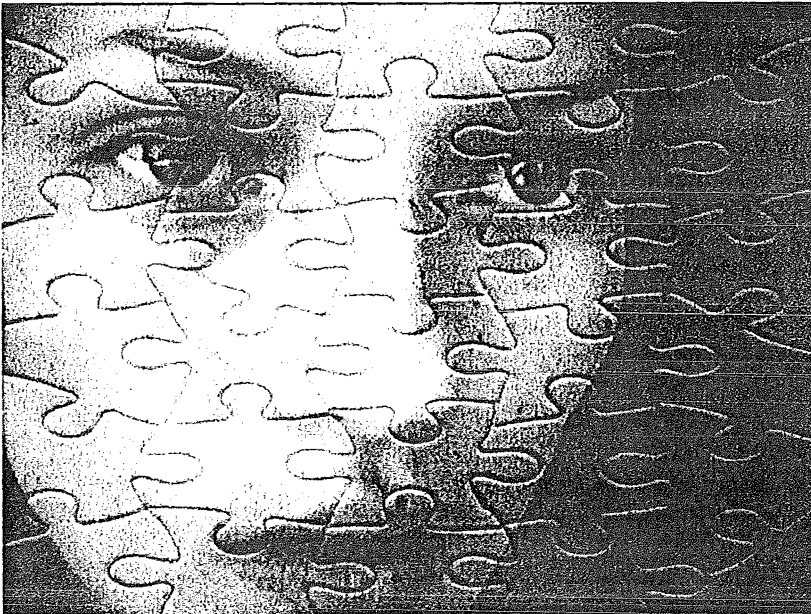
Ironically rationalism and empiricism imply each other. The parts in a rationalist scheme of things owe their positive characteristics to the fact that they are parts of a purely positive whole, it is maintained, but what about their *negative* characteristics? A purely positive whole *has* no negative characteristics and so the mere existence of parts and the associated need for the negative means that there is no whole in the first place. Rationalism turns into empiricism, but the matter is not settled. If the bugbear of rationalism is negation and the incompatibility of negation with a pure real whole, the bugbear of empiricism is *the positive* and its incompatibility with a purely unreal whole. Empiricists maintain that certain parts of the world have reality. For the same reason that rationalists admit negation into their scheme

of things—the limitation of human mind and being—empiricists must admit the positive into their scheme of things. But this admission spoils the purity of the negativity of the whole in just the same way that the rationalist admission of negativity spoiled the positivity of the whole. Empiricism then turns into rationalism.

This situation can be clearly seen by reference to the geometrical conception of the world that the dispute turns on in the first place. Imagine that the world is the jigsaw puzzle in figure one.¹⁸ For the rationalist, the puzzle as a whole represents the reality with a capital G. Each of the pieces of the puzzle represents something that is real in so far as it is a part of the whole puzzle but unreal in so far as it is not. In Leibniz's case, for

¹⁸ Yes this analogy is inadequate—as we have seen, nothing but the whole of the world will suffice to model the whole of the world—but it will have to do!

Figure One: *The Rationalist World-Puzzle*



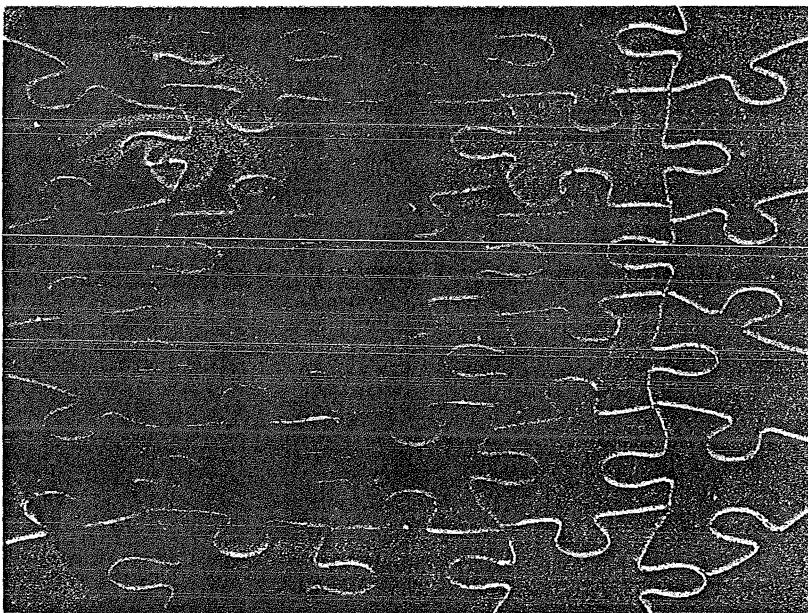
example, these pieces may be analogically regarded as having all the positive characteristics that Leibniz ascribes to monads—being, simplicity, mindfulness, activity—but they can also be regarded as having all the *negative* characteristics he ascribes to monads—nothingness, complexity, physicality, and passivity. The positive characteristics are the ones the pieces have in the proportion to which they make the whole puzzle whereas the negative characteristics are the ones that they have in the proportion to which they *fail* to make up the whole puzzle. Any one piece is one thirtieth of the whole puzzle and twenty-nine thirtieths shy of it. A piece is real, simple, mindful and active to the degree that it makes up the whole puzzle

and unreal, complex, physical, and passive, to the degree to which it *fails* to make up the whole puzzle.¹⁹ As far as the rationalist—as far as someone for whom the parts are to be explained only by reference to their place in the whole—is concerned, the Platonic trio of truth, beauty and goodness together—all the purely positive things in life—do not ever quite reside in the parts of the world. Every one of these parts is negative to a degree proportional to its inequality to the whole. This then is the paradox of rationalism: *the whole is not purely positive after all*. For example God is, according to Leibniz, pure being and is nothingness to no degree at all. Created monads, on the other hand, are partly being and yet nothingness to *some* degree or another. But, by looking at the figure, it should become clear that any nothingness is no less involved in the puzzle—and therefore the world—as a whole than is being. The difference between any of the pieces of the puzzle and the whole puzzle is after all the *other* pieces. In so far as there is any nothingness it is right there in the whole. The same is bound to go for *any* negative attribution by rationalists to the parts as goes for nothingness. It seems then that the whole, if there is one, is either purely positive or merely another part. If rationalists admit—as they do—the

¹⁹ There is another obvious and related problem with my analogy due to its finite nature. There are a finite number of pieces... but imagine, if you can, a jig-saw puzzle with infinite pieces of differing sizes.

limitation of human being then they must admit that the whole is not after all purely positive and so the picture we have before us must be reversed. There is *no* whole; the whole is purely negative; the parts are *positive* so far as they differ from the whole and that they are *negative* in so far as they go to make up the whole. In so far as it admits limitation, rationalism apparently collapses into empiricism.

Figure Two: *The Empiricist World-Puzzle*



Empiricism however fares no better. For the empiricist, the parts of the puzzle represent what is real. Each of the pieces of the puzzle represents something that is real in so far as it is real in its own right rather than something whose reality resides in its place in the whole puzzle. Let us first examine the example of Kant. Kant grasps hold of a fact that the rationalists

and the empiricist know very well, namely, that the drama of humanity is played out at the level of the pieces rather than at that of the whole puzzle. Kant therefore works his model of the world around the idea—encountered earlier—that the concept/object distinction itself guarantees both that there is a negative thing that is the world as it is independently of any conception of it and that this world is unknowable except as a limit to knowledge. Wittgenstein in the *Tractatus* makes more or less the same case in semantic rather than epistemic terms: Kant stresses that the whole of the puzzle is something that cannot be known, Wittgenstein that it is something that cannot be spoken of. While Kant wants to have it both that the whole is unknowable but that there ‘is’ a whole, Nietzsche promotes the ‘death’ of the whole, i.e. he seeks to forswear the idea that there is any whole whatsoever. This leads Nietzsche to a view of the pieces of the puzzle as things that must be created rather than things that are derived from their participation in the whole. “Truth”, Nietzsche says, “is... not something that might be found or discovered—but something that must be created” (Kauffmann and Hollingdale 1968, sec 552). The model of the world that starts with Kant, and continues in Nietzsche, is most clearly taken to its logical extreme by Sartre. In *Being and Nothingness* Sartre draws a distinction between *being-in-itself* (*en-soi*) and *being-for-itself* (*pour-soi*)

(Barnes 1957). The first sort of being is that possessed by things, the second by people. His radically empiricist move turns on his claim that *being-for-itself* is nothingness, that it has no essence. Sartre casts himself in the role of the discoverer of the absolute nature of human freedom; he compares and contrasts his anarchistic conception of human possibility with a psychological and philosophical tradition that limits human nature by appealing to “opaque” drives and goals and insisting on their universality. Sartre is clearly in Nietzsche’s camp but he goes further than Nietzsche and denies the influence of *everything* that might limit human freedom, including Nietzsche’s will to power. *Being-for-itself*, he says, is *pure* nothingness. For Western Philosophy this is apparently an incredibly radical stance and it prompts William Barrett in his classic text on existentialism to declare that

Never in the thought of the West has the self been so pervaded by negation. One would have to go to the east, to the Buddhist philosopher Nagarjuna... to meet the awesome list of negations Sartre draws up.
(Barrett 1960, p.220)

But Sartre’s rationalist hens come home to roost in the form of *being-in-itself*. In contrast to being-for-itself, being-*in*-itself *is* something. Sartre says that *being-for-itself* is doomed to forever strive, and yet to forever fail, to achieve the status of being-in-itself. *Beings-in-themselves* then are those

pieces of Sartre' version of the jig saw puzzle of the world that are being rather than nothingness, those pieces that serve to sustain the distinction between being and nothingness and stop the puzzle from collapsing into undifferentiated nothingness. (For the rationalist this role is played by nothingness and in this their role is to sustain the distinction between being and nothingness by stopping the puzzle from collapsing into undifferentiated being.) But, in exactly the same way that the rationalist model according to which the whole is pure being forces the realization that rationalism is false and that empiricism is true, the empiricist model according to which the whole is pure nothingness forces the realization that rationalism is true. The argument is the reverse of the previous: *if there is being in the parts then there is being in the whole and if there is any being in something having this degree of purity then it must exist.* The rationalist refutes the empiricist by means of an ontological argument to pure being: the empiricist admits to the being of the parts and thus must admit to the being of the whole. But then the empiricist refutes the rationalist by appeal to the ontological argument in reverse: there is *nothingness* in the parts, the rationalist will admit, and, in that case, there is nothingness in the whole. Therefore the whole *cannot* exist. Figure Three illustrates the thought that the empiricist model of the world is just the rationalist model in reverse.

Given that the whole/parts relationship is the idea on which empiricism—as well as rationalism—rests, it emerges that what is at issue between champions and the critics of completeness is not completeness *per se* but rather that of the positive or negative character of that completeness. Cantor's term 'the increasable infinite' —for example— is deceptive in that it encourages us to imagine an infinite regress in positive rather than negative terms, in terms of largeness rather than smallness. We are inclined to conclude perhaps that the empiricist is a champion of the irrevocably incomplete, that he has no time for the idea of an infinite that admits of completeness. This however is a mistaken conclusion: all that is needed to reveal the lurking 'holism' of the empiricist is the notion of the irrevocably small rather than that of the irrevocably large. *This* conception of infinity is based on smallness rather than largeness; it is infinity, not of being, but of nothingness, and yet it is no more amenable to alteration or manipulation—it is no less complete in its own negative way—than the positive infinity of the rationalist.²⁰ For the rationalist, the limitation of the parts creates the problem of increasable infinity, the problem how to bridge that never-narrowing gap that divides the world, as we know it, from the

²⁰ For the rationalist, the finite reaches out for the infinite of which it is a pale imitation; the empiricist, by contrast, *recoils* from the infinite; for the empiricist, the infinite is no more than a pale imitation of the finite.

world of the ultimately real. Since the reality of the parts is for the empiricist dependent on their difference *from*, rather than their similarity *to*, the whole, these must aspire to be as small rather than as large as possible and yet, in the same way that the parts of the rationalist whole can never achieve their desired minimum size, the parts of the empiricist whole can never disappear altogether.²¹

At risk here is the very distinction between the irrevocably complete and the irrevocably incomplete. Although Kant fails to solve the paradox, something of the utmost importance immerses from ‘his’ attempt to do so. I refer to his proposal that there is no ‘analytical’ difference—no logical difference—between one idea of the world as a whole and another. The thing about both the infinity of the irrevocably complete and the infinity of the irrevocably incomplete therefore is their irrevocability, i.e. the fact that they concern the world as a whole. What the failure of the Kantian solution suggests is that, regardless of whether preference is given to the whole (as it is by the rationalist) or to the parts (as it is by the empiricist), the whole must be treated either as maximally large or something maximally small and, in either case it is bound to be something to which nothing can be added or

²¹ It is surely not a coincidence that empiricism seems enamored of ‘disappearance’ theories.

subtracted, something irrevocable. It turns out that, as Leibniz fears, there is nothing *to* an irrevocably incomplete world; it's incompleteness is not as we might have imagined the incompleteness of something, like Schubert's Eighth Symphony, that exists but cannot be finalized; rather it is the incompleteness of 'something', like Schubert's Tenth Symphony, that is incomplete because it doesn't exist in the first place.²² There is no simply logical difference between the irrevocably complete and irrevocably incomplete here because complete being and complete nothingness are equally undifferentiated, meaning that logic can distinguish them if, and only if, it cannot do so.²³ The question arises then as to whether there is *any* difference whatever between them. What differences are there, you might want to ask, when there are no logical differences? Sartre quotes Hegel as saying that "Being and Nothingness are empty abstractions, and the one is as empty as the other." (Barnes 1957, p.15). On Hegel's argument here is no difference whatever between rationalism and empiricism as I have characterized them but, as Sartre responds, Hegel "forgets that emptiness is

²² The bug-bear of Aristotle's view of the past as infinite and the infinite as the irrevocably incomplete is the question of how there can be such a thing as the present under these circumstances. The same objection can be leveled in logical rather than cosmological terms: how can there be such a thing as the logically present in the circumstance that the logical past of any statement or thought is irrevocably incomplete?

emptiness *of* something. Being is empty of all other determination than identity with itself, but non-being is empty of *being*. In a word, we must recall here against Hegel that being *is* and nothingness *is not*" (Barnes 1957, p.15). Logic is sensitive to any difference there is here iff it is not yet, if there is *no* difference— if, as Hegel says, being and nothingness are indeed “empty abstractions”—then there is no such thing as *difference*. That the world is differentiated is the most fundamental presupposition of logic, the most fundamental condition of the possibility of logic and, accordingly, any reason there is for this view lies inside logic iff it does not: logic cannot prove, it can only assume, that the world is differentiated. But—and this is the point—the world is differentiated only if there *is* a difference between being and nothingness, only if somehow there *is* some sort of interaction of these, and only if this interaction somehow results in difference and thus in finitude.²⁴ Otherwise the parts collapse into the whole and, whether the

²³ The parent idea that, although infinite being could not be diminished in any way by nothingness, infinite nothingness could somehow receive an injection of being is no doubt that of creation *ex nihilo*.

²⁴ According to G. Macdonald Ross, Leibniz thought that the world is a binary construction out of being and nothingness, out of the arithmetical ideas of zero and one. Apparently “he was so proud of this idea that he planned to commemorate it with a medal bearing the legends: THE MODEL OF CREATION DISCOVERED BY G.W.L., and ONE IS ENOUGH FOR DERIVING EVERYTHING FROM NOTHING” (Macdonald Ross, 1984, p102). Macdonald Ross’s claim fits well with Leibniz’s view—expressed in the *Theodicy* and later in the *Monadology*—that God is a logically singular thing, and thus it seems that Leibniz has an even more radical understanding of the infinite than I have so far given him credit for. However, despite the fact that he saw that the-world-as-we-know-it as something derivable from pure being and pure nothingness, it seems unlikely that he would have been able to appreciate the thought that there is no *logical*

whole is supposed to be being or nothingness, it is something undifferentiated. Given that the world is differentiated then, despite any similarity they may possess when seen from a logical point of view, being and nothingness are not the same.

2.4. An Ethical Post Script

These considerations allow us to conclude with Leibniz and Kant that, for the sake of logic, the irrevocably complete and the irrevocably incomplete must go together. This is however scant justification for their togetherness since it is merely to say that otherwise there can be no such thing as logical difference, and thus no logic. The *real* reason behind my insistence on their togetherness is ethical: at this level of the inquiry, the connection between logic and ethics can be seen to be an intimate one: the paradox of rationalism and empiricism (let's call it) must not be resolved from the human point of view for, if ever it is, then that is because the world is complete being and therefore makes *complete* ethical demands on us or because it is complete nothingness and therefore makes *no* ethical demands on us. Ethically speaking, these extreme positions are equivalent because,

difference between pure being and pure nothingness, that it is the non-logical interaction of these that makes logical difference possible. In the first place this proposal has a heretical air to it and, in the second place, Kant was the first Western thinker to seriously entertain the idea that the world as whole was a wholly negative thing.

either way, you can do whatever you please to do. In the first case this is because you could do no other; in the second case any alternative courses of action are only limited by your imagination but, since any objection to a chosen course of action can only be one that you pose yourself, you are no less free to overturn it as you are to allow it to stand.

The basic problem with rationalism without empiricism, or empiricism without rationalism, is not so much that logic depends on their union but that '*religion*' does. The religious attitude, says James, involves two fundamentals: (1) the idea that there is a problem, that all is not right with the world and (2) that there is a solution to this problem or a means by which this problem can be mitigated. (James 1961) James calls this attitude religious but it might as well be called ethical, or scientific, so long as it is agreed that common to all of these is the assumption that life poses a problem and that there are at least limited solutions to this problem. The difference, if there is one, is that science doesn't propose a complete solution in the way that some religions do, but it is based on the possibility of *improvement*. Any philosophy of life that is not to be promoting of either the blindly optimistic belief that everything is perfect just as it is or the stoically pessimistic belief that, although things are imperfect, there is nothing to be done about it must accept (1) *and* (2). To deny (1), or to affirm (1) and deny

(2), is—without the benefit of the paradox—to provide oneself with an excuse for anything one pleases. Extreme rationalism however must deny (1) because, in the context of complete being, there is no possibility of a problem or of anything being other than exactly what it should be. Extreme empiricism must at least deny (2) because, in the context of complete nothingness, nothing can be changed. The paradox problematises any premise according to which the world is perfect or that it is unalterable. So long as such questions can be raised in the first place, the paradox exists and it can do its work.²⁵

²⁵ It is important to note that it can and should be argued, on a basis of the same premises, that the paradox helps draw our attention to the artificiality of dualisms, and therefore to what de Bono disparagingly calls 'rock logic'. (This is the logic behind the sort of ethical thinking that Fanon calls and 'Manichean', the thinking behind prejudice and hatred.) . At first sight any such argument might seem to be drawing quite the opposite moral from the paradox from that drawn here, but not when it is considered that—paradoxically—dualism *implies* monism. It does so because unified things are needed in the first place in order to oppose one thing to another. Before dualism comes monism and thus the paradox poses no less of a problem for the former than it does for the latter.

A CONCLUSION

Leibniz is determined to put the irrevocably complete and irrevocably incomplete together. His conception of completeness—of God—*always* goes hand in hand with his conception of the *incompleteness* of creation. Consider his predicate-in-subject principle for example. The basis of this principle is God's omniscience, God's ability to see that their subjects contain the predicates of true propositions. We creatures cannot possibly see this, Leibniz believes. Is omniscience really the basis of this principle? Suppose that *nobody* could see that their subjects contain the predicates of all true propositions. Would it nonetheless be possible to uphold the principle? No, because the principle takes knowledge to consist in the grasping of concepts, and to grasp the concept of the whole world perfectly is, as we have seen, to grasp only this concept itself. Thus Leibniz's theory of truth is actually a correspondence theory in disguise; it turns out that the matching of our imperfect concepts to the perfect concept of God is the same thing as matching them to the world. To deny that *anyone* sees that the predicates of all true propositions are in their subjects then is to deny that there *is* a world. Leibniz's view of human knowledge is that it is a substandard version of divine knowledge and that thus it is dependent on the

reality of divine knowledge. Given the nature of omniscience, it follows that God is for Leibniz irrevocably complete, that the-world-as-we-know-it is irrevocably incomplete, and so that Leibniz is making knowledge of the irrevocably complete a condition of *any* knowledge.

Consider also his Cosmological Argument for the existence of God. Again the complete and the incomplete are allied by means of the same style of logic. Human *existence* in this case is the incomplete version of divine existence. Everything in the human world is limited, says Leibniz, and therefore points to something outside itself in the way that any limit inevitably does. If this regress of limits is to go on *ad infinitum* then there is no possibility that the world is complete and thus, alongside this infinite series of limits, Leibniz insists that there be something *limitless*. Why should there be, ask Hume and Russell and others in the empiricist tradition? Leibniz's answer is that otherwise there is no complete world, i.e. no *world* at all:

I believe that where there are only beings by aggregation, there aren't any real beings... every being by aggregation presupposes things endowed with real unity, because every being derives its reality only from the reality of those beings of which it is composed, so it will not have any reality at all if each being of which it is composed is itself a being by aggregation, a being for which we must still seek further grounds for its reality, grounds which can never be found in this way, if

we must always continue to seek for them. (Ariew and Garber, 1989, p. 85.) (My italics.)

This quest for unity *cannot* end with a plurality of things, however, because this plurality is itself only an aggregate in Leibniz's terms—an aggregate of all created monads. If it can be ended it must end with something *supremely* unified. To think with Hume and Russell that the regress could go on forever is in Leibniz's view to think that *there is no real being*. In this case Leibniz is making the existence of something irrevocably complete a condition of the existence of anything else. In both cases he is making the incomplete world of human experience an inadequate version of the complete celestial world. Although the true nature of this latter world is hidden from us, it is no less amenable to rational scrutiny than the former.

These notions of infinity seem to contradict one another, and we are inclined therefore to dispense with one or the other. However, it seems that this is more easily said than done. Suppose, firstly, that we take the world to be irrevocably complete. But from the perspective of someone who can see the world as complete, the question of limits—the question of what is *not*—cannot conceivably arise. This is because to conceive of the world as an omniscient mind conceives it is to conceive only of this conception itself. God's conception of the world, this is to say, involves the collapse of the

concept/object distinction. Otherwise, there are things unknown to God—things as they are independently of God’s conception of them, Kant’s noumena, or the thing-in-itself. Moreover, to conceive the world as an omniscient mind is to conceive only a single thing. God’s conception of the world, *this* is to say, means the collapse of *all* distinctions. Otherwise the all-embracing concept under which the mind of God is to bring a plurality of things is unable both to subsume all things under, or within, itself *and* function just as its own object. There is, in this case, an inevitable ontological divide between this concept and the things falling under it, and so there is, again, a domain of the unconceived to which God is not privy. This is the consideration on which rests the neglected but vitally significant Leibnizian doctrine that there is from God’s perspective no negation, no limitation, no nothingness. This means that these ideas cannot so much as *exist* from this perspective and so the question as to the existence of limits cannot from this perspective so much as arise. This is no good we accordingly conclude; obviously the world is incomplete since obviously the question of whether the world is complete *does* arise for us. We can therefore conclude with equanimity that, firstly, the world is incomplete and secondly that it cannot be completed for, *pace* Hegel, any world that involves some degree of limitation is never going to *grow* into a complete

world but will always remain incomplete. But what am I talking about when I say that the world is not irrevocably complete? How can I say this when, as we have agreed, this world is an undifferentiated one, a world whose concept is its object? In so far as I have a concept of the irrevocably complete, the world is indeed complete ... but it can't be since, as we have agreed, I can raise the question of whether it is complete. That I *speak of* lacking the perfect conception seems to involve me in a contradiction just because the very idea of the perfect conception is the same thing as this perfect conception itself. Thus I do *not* lack the perfect conception since I speak of it. But I also speak of *lacking* the perfect conception, a consideration that cannot arise from the perspective of the perfect conception... and so it goes ad infinitum, back and forth... back and forth ad infinitum. It only stops when I grow tired of thinking about it.

Kant provides an argument for the view that this is not a real contradiction at all. The fundamental premise both of this argument and of the critical philosophy as a whole is the undoubtedly limited nature of human knowledge. Both the rationalist and the empiricist alike accept the truth of this premise and yet, Kant argues, this acceptance commits them to the conclusion that what is the most fundamental dispute between them is only the *illusion* of a dispute. In my terms, the dispute turns on the question

of whether the world is irrevocably complete or irrevocably incomplete, whether the “conditioned” things we meet with in experience have at their root, or alongside somehow, something “*unconditioned*” by anything other than itself, something complete in itself. The rationalist answer to this question is and always has been ‘yes’ while the empiricist answer is ‘no’. Kant argues that this ancient contest is a vain one since, given the limitation of the human perspective—given the perspective-bound nature of the relationship of the human mind to the world—it follows that all concepts of a whole are indistinguishable. There is therefore no genuine opposition between the fundamental theses of rationalism and empiricism. The *apparent* opposition between these is, Kant says, “dialectical” rather than “analytic”:

I beg permission to entitle this kind of opposition dialectical, and that of contradictories analytical. Thus of two dialectically opposed judgments both may be false; for the one is not a mere contradictory of the other, but says something more than is required for a simple contradiction. If we regard the two propositions, that the world is infinite in magnitude and that it is finite in magnitude, as contradictory opposites, we are assuming that the world, the complete series of appearances, is a thing in itself that remains even if I suspend the infinite or the finite regress in the series of its appearances. If, however, I reject this assumption, or rather this accompanying transcendental illusion, and deny that the world is a thing in itself, the contradictory opposition of the two

assertions is converted into a merely dialectical opposition. (Kant, 1933, A 504, B 532.)

The profundity of the point Kant is making here can hardly be overestimated but it nonetheless has the defect that it cannot be so much as stated unless it is false. The simple objection is just that the concept of the whole is the same as its object and thus that, to employ it at all, Kant has to affirm the thing he aims to deny, but there is a much more subtle way of looking at this situation. What, it can be asked of Kant, is *he* opposing to the completed world in virtue of which it might be the case that rationalism and empiricism can be seen to be in agreement here? The world as it is *depends* on our concepts of it, the phenomenal world, the apparent world. In particular, Kant says of this world that it is 'indefinite' rather than either finite or infinite, and his thought is that an indefinite world is unlike a finite or an infinite one, not required to be complete.

Kant supposedly diffused the rationalism/empiricism conflict (as this concerns the regress of conditions) by identifying the finite world with the infinite but in doing so he deprived the infinite world of its incompleteness, of the thought that an infinite regress never ends. There is *still* a regress that never ends but it is to be called indefinite rather than infinite because an infinite regress involves the complete series in a way that an indefinite

regress does not. That Kant has not merely renamed the conflict that exists between rationalism and empiricism and thus that he should proceed to smooth it over by means of considerations drawn from his own philosophy is something we can be sure of only if there is a clear-cut distinction between the indefinite and the infinite, that is only if the an indefinite series is *not* a complete series. Given however that the thing in itself is complete—albeit that its completeness is a negative rather than a positive thing—it follows that anything else must be subsumed by, or *consumed* by it, or be complete in its own right. Kant's philosophy of the infinite involves just the same conflict as Leibniz's philosophy of the infinite, the difference being that the world as it is in itself is regarded by Leibniz as the largest possible thing whereas for Kant it is regarded as the smallest; for the one it is infinite fullness, for the other infinite emptiness. Moreover the similarities and differences there are between Leibniz and Kant serve as a blueprint of the similarities and differences there are between rationalism and empiricism. Both parties understand knowledge and being on analogy with the whole/parts relationship; and both parties admit the human predicament is that of something with partial knowledge and partial being; and both parties admit—albeit implicitly—to an irrevocably complete whole. Where they differ is on the question of nature of the whole of which they are part:

rationalists see this whole in terms of something in possession of maximal knowledge and maximal reality whereas empiricists see it in terms of something possessing *minimal* knowledge and *minimal* reality.²⁶ The admitted presence of things that are neither a minimum nor a maximum means, however, that both end up having, subconsciously, admitted the “opposite”: if they are ‘maximalists’ their challenge is to acknowledge the minimal and, if ‘minimalists’, their challenge is to acknowledge the maximal. But, as Kant’s work suggests, there is no simply logical difference between these apparent extremes, between the irrevocably complete and the irrevocably incomplete. Any such difference is prior to logic since, without the combination of these modes of infinity, there could be no such thing as difference and thus no such thing as logic.

Pascal distinguishes two kinds of infinity, the infinity of the smallest possible object and the infinity of the largest possible object, an infinity of complete being and an infinity of complete nothingness. “For what is man in nature?” he asks. “A nothing compared with the Infinite; an all compared with nothing; a mean between nothing and everything...” (My italics)

²⁶ Rationalists, it might be said, is on the side of the Hindus while Empiricists are on that of the Buddhists: these Eastern traditions are in agreement in so far as each has it that ultimate reality is singular, undivided; they disagree as to whether this reality is something or nothing, an ultimate fullness (the Brahman ‘Self’) or an ultimate emptiness (the Buddhist ‘Void’).

(Hazelton 1974, p.79). This idea of Pascal's nicely introduces my final conclusion. As Pascal says, we find ourselves caught between the infinity of being and that of nothingness—"extremes elude us" he says—and yet, as philosophers we are involved in a quest for infinity, the most extreme thing imaginable. In order to complete this quest, however, we would have literally to become either nothing or everything in which case it would be impossible for us to recognise the existence of anything else. The recognition of the existence of others though is the basis not merely of logic but of ethics. As the case of Spinoza²⁷ indicates, radical rationalists have a tendency to think that they *are* everything, as the case of Sartre indicates, radical empiricists that they are nothing and both these thoughts are promoting—theoretically at least—of an anything-goes philosophy of action. This consideration is for me perhaps the deepest reason why I go along with Leibniz and insist that, at very bottom, the world is irrevocably complete *and* irrevocably incomplete. This is a contradiction but a very special one, one to be encountered only at this most basic level of inquiry: given that both these modes of infinity are undifferentiated but must be

²⁷ It was Spinoza's view, for example, that we should not blame people for wrong-doing since it was beyond their power to do otherwise. The objection that we should not, by the same token, blame people for blaming people for wrongdoing this exempts people from any ethical responsibility. As Kant says, "ought implies can".

differentiated in order for us to speak of them, this is a contradiction according to which the world is, and is not, contradictory....

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APPENDIX 1

Completeness and Self-Reference

One of my main tasks I set myself in the present work is to show that the concept of the irrevocably complete is a self-referential one. If I can show this then I am free to maintain—as I do—that in so far as there is such a concept there is an object corresponding to it, namely itself. Then I can, and do, go on to maintain that, in the light of the degree of completeness possessed by this concept/object, it is a singular, solitary thing, that there is nothing else besides this thing... and so on until the conclusion that there is, from the human perspective, an inescapable paradox affecting any attempt to conceive of ‘Reality’, that Leibniz understandably sees this as irrevocably complete *and* irrevocably incomplete... Although I haven’t developed this it in any explicit way here, it is my view that the connection between the irrevocably complete and self-reference is more intimate still, that not only does the one imply the other but also that the two are equivalent. Recall the map of the empire that is identical with this empire itself. The reason that this map needs to be so large is that a smaller map would inevitably be a sketch of; an abridgement of—a falsification of—of the empire and this is something the highly ambitious cartographer cannot tolerate. In particular,

any such cartographer must aim to include the map of their empire within itself. Given, however, that nothing can be anything *other* than itself, it follows that any map that is supposed to include itself within itself can include nothing else besides; it cannot include anything other. Otherwise this self-referential map must be shrunk in order to make room for these other things

Figure Three: *A Map that is Inclusive of Itself?*



The problem is that a self-referential map cannot be reproduced. If it could be reproduced then it would be possible to make a further map featuring both the original *and* the reproduction... A map that was indeed a map of itself would already include every aspect of itself and, like the ability to be other than itself, the ability to

reproduce itself can form no part of a thing.) By a twist on this same reasoning it can be concluded that the any self-referential map must be irrevocably complete. Otherwise it would be possible to create a larger map that included this self-referential map *and* the things it does not include. The possibility of a larger map—and therefore of a reproduction of the original map— though serves to undermine the assumption that this map is self-referential... Thus there is nothing that a self-referential map does not include and thus a self-referential map is irrevocably complete. Moving from the case of the self-referential map to that of the self-referential thing generally: self-reference implies irrevocable completeness.

Let's put these considerations in their appropriate context, that of the liar paradox. One of the first things that ever struck me about the sentence 'This sentence is false.' was its apparent difference from the sentence 'The cat is on the mat.' In the case of the first sentence, but not the second, I had a strong sense that every time I wrote the words 'This sentence is false.' I was referring to different thing. At first I was satisfied by the thought that you can refer to the sentence from afar by the use of 'that' or 'the sentence embedded in the second sentence of this paragraph' or whatever other 'namely rider' takes

your fancy, but I was not satisfied for long. The received wisdom is that self-referential sentences can be re-referred to—consider where Gödel's Theorem and the 'Theorems' of Self-Reference be without it—takes it that there is a difference between the form and the content of a self-referential sentence, so that the content of the liar sentence can be reproduced at will (at very least, in the mind). My contrary wisdom is that if there really is such difference then supposedly self-referential sentences don't really refer to themselves after all. A sentence is its form *and* its content and so if you want to refer to it you have to refer to both of these; exactly like the irrevocably complete, the self-referential sentence involves the collapse of the form/content distinction.

The liar paradox does not arise so much from the ambition of the liar sentence to say of itself that it is false, but from its ambition to be a self-referential thing in an incomplete world. Any self-referential sentence threatens to say of itself that it is untrue because, in the circumstance that it is not irrevocably complete, its lack of truth is a condition of its truth; or—rather—its lack of existence is a condition of its existence. What am I saying? If the sentence is self-referential then it is irrevocably complete and nothing can exist *alongside*

something irrevocably complete. All the sentences we know of though are built on the premise that they are parts of a world where anything that exists does so alongside other things. All the self-referential sentences we know then presuppose their own non-existence. All self-referential *sentiments* (sentences, statements, thoughts etc.), it can be said, are forms of the liar paradox: these sentiments do not merely declare that they are not true; they declare that they are *not*. It is therefore no coincidence that *all* of the historical ‘solutions’ to the liar paradox involve—in one-way or another—a hostile attitude towards self-reference/irrevocable completeness, for self-reference/irrevocable completeness comes at the expense the-world-as-we-know-it. (Needless to say, this hostility is quite unhelpful since, far from pointing to any solution to the paradox that inspires it, it is a form of this paradox.)

The fact that irrevocable completeness implies self-reference is one I have appealed to throughout this thesis in order to support the conclusion that the idea of the infinity of completeness is paradoxical rather than merely self-refuting. And, of course, I have used this

conclusion to justify the further conclusion that Leibniz was right to insist on the rationality of this mode of infinity etc.. The fact that self-reference implies irrevocable completeness is one to which I have made no appeal. This is because there was no direct need, in this context, to justify the conclusion that the paradoxes of self-reference have the same form as the paradox of irrevocable completeness. (The thesis after all was not directly intended as a diagnosis of the paradoxes of self-reference.) Since, however, this is something that lies just beneath the surface of the thesis throughout, and since it is something that everyone who understands me at all will at least wonder about, I thought I'd better say something about the intimate nature of the connection I find to exist between the two (irrevocable completeness and self-reference.). I thought also that this would be better of outside the main body of the text.