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Returns to on-the-job search and wage dispersion[★]

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ABSTRACT

A wide class of models with On-the-Job Search (OJS) predict that workers gradually select into better jobs. We develop a simple method based on the expected number of job offers received that can be used to measure match quality, identify the wage offer distribution and estimate the contribution of OJS to wage dispersion and the increase in wages over the lifecycle. The method uses two sources of identification: (i) time variation in job-finding rates and (ii) individual variation in the time since the last layoff. Applying this method to the NLSY 79, we find that the standard deviation of the wage-offer distribution is 13% and that OJS accounts for 8% of the total wage dispersion and 30% of the wage-increase over the lifecycle.

1. Introduction

In labour-market models with On-the-Job Search (OJS) as developed by Burdett and Mortensen (1998), the ordeal of a job seeker resembles that of the mythological character Sisyphus, who is tasked with rolling a boulder up a hill. Each time he nears the top, the boulder slips from his hands and rolls back down the slope. Sisyphus has no alternative but to start his laborious task all over again. For their part, job seekers climb the hill of rents by selecting into ever better paying jobs drawn from an offer distribution. When their current job is destroyed, they tumble back to the trough of unemployment at the bottom of the hill. As with Sisyphus, all gains from prior selection are lost. They have to restart their task, climbing the same hill from which they have just fallen. This process yields simple predictions regarding the evolution of wages over the course of a worker's career which can be used to identify the shape of the wage-offer distribution, estimate the contribution of OJS to wage dispersion and the overall increase in expected wages over the lifecycle.

The key concept in our approach is the *employment cycle* (see Wolpin (1992)): the interval between two consecutive job-destruction shocks (i.e. two consecutive attempts to roll the boulder up the hill of rents). The selection of ever better offers means that wages increase in the course of each cycle. But when the boulder tumbles back to the bottom of the hill—i.e. when the destruction of a job ends the employ-

ment cycle—average wages return to the level at the beginning of the employment cycle and the process starts again.

Moreover, the speed at which Sisyphus is able to roll the boulder up the hill depends on weather conditions. Job offers arrive more frequently during booms than busts. The higher their arrival rate, the more quickly the job seeker is able to climb the hill of rents. This type of search models yield detailed predictions as to how these differences in job-offer arrival rates affect the profile of wage growth during the employment cycle. Extrapolating from this parable, we can identify two independent sources of variation for the position of Sisyphus on the hill or the position of a worker in the job ladder:

- the timing with which the boulder slips from the worker's hands: the random arrival of job-destruction shocks, which end the employment cycle at different points in time for each job seekers. We use the distinction between quits and layoffs to identify employment cycles;
- 2. differences in weather conditions: the variation in aggregate labourmarket conditions. We distinguish between two concepts of time: *calendar time* and *labour-market time*; the clock of the former runs at a constant pace, that of the latter runs proportional to the joboffer arrival rate (i.e. faster during a boom than a bust). Where the regular return to experience is a function of calendar time, the return

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to search is a function of labour-market time. The difference between these concepts of time enables us to disentangle the two returns.

We show that we can exploit these sources of exogenous variation to create a theoretically consistent measure of match quality. We measure the sum of the job offer arrival rates from the start of the employment cycle until the end of the current job capturing the expected number of job offers paying a lower wage then the current job. We further show how this measure can be used to quantify the contribution of OJS to wage dispersion by means of simple OLS regressions. The specification of these regressions is based on a detailed analysis of the characteristics of the selection process of ever better draws from the offer distribution. We also show that this type of job ladder model yields strong predictions regarding both wages and the distribution of job tenure. The starting date of all jobs (barring the first job) during an employment cycle is predicted to be uniformly distributed over the length of that cycle until the job ends. The intuition for this result is that the current job is the best offer received since the start of the employment cycle. As all draws have equal probability of being the max, its moment of arrival is uniformly distributed over the length of the employment cycle. Furthermore, jobs ending in a quit, rather than a layoff, are expected to have a lower match quality. This implies that the wage is also expected to be lower. The empirical analysis confirms both predictions.

We apply our method to the 1979 cohort of the National Longitudinal Survey of Youth (NLSY). We find that the standard deviation of the distribution of log wage offers is about 13% and that frictions explain around 8% of the total variation in log wages for male workers. This standard deviation is 1.5 times larger for workers with higher than lower education levels. Our estimates imply an average wage loss after layoff of about 13%, consistent with large earnings losses following displacement documented empirically (e.g., Jacobson et al. (1993), Davis and Wachter (2011) and Bertheau et al. (2022)). Our method allows for a decomposition of the overall increase in expected wages over the lifecycle into three components: OJS, experience and tenure. OJS explains about 30% of the overall return, experience about 60%, leaving 10% for the return to tenure.

Our methodology builds on the work of Barlevy (2008) and Hagedorn and Manovskii (2013). Barlevy (2008) shows how record theory can be used to identify the wage offer distribution from a sequence of expected wages in an employment cycle. We show how to use the information on job duration to obtain stronger prediction. Hagedorn and Manovskii (2013) create two measures of match quality from the histories of labour market tightness during the time-interval before the start of the current job and during the current job. We extend this analysis by constructing one unified and theoretically consistent measure of match quality and show how it relates to the wage offer distribution.

Several authors have to attempted to estimate the return to OJS using a variety of methods. Whereas we directly estimate the return to OJS, Hagedorn, Manovskii and Wang (2017) measure frictional wage dispersion as the residual item and Bagger et al. (2014), Tjaden and Wellschmied (2014) and Burdett et al. (2016) use structural approaches. Gautier and Teulings (2015) and Guvenen, Kuruscu, Tanaka and Wiczer (2020) compute a measure of match quality directly from data on worker and job characteristics. Fredriksson, Hensvik and Skans (2018) measure match quality as the similarity of the worker's skills compared to those of co-workers. Vejlin and Veramendi (2020) use wages associated with displacement to provide an upper bound for the contribution of job search for total wage dispersion which is about twice as large as our estimate. Guo (2021) proves that one can identify the offer distribution using the wages of workers with multiple offers and empirically finds very similar results to us in terms of the dispersion of wages.

The next section develops the theoretical concepts employed in this paper, while Section 3 presents our empirical results. Section 4 briefly discusses the implications of our findings.

2. The theoretical argument

2.1. Assumptions

Our model is based on four assumptions:

- 1. workers receive job offers at a rate λ_t that may vary over time, but do not depend on other factors
- 2. the log wage *w* paid in a job is a random draw from a wage-offer distribution *F*(*w*)
- 3. workers accept any job offer that pays a higher wage than their current job
- jobs are destroyed at a job-destruction rate δ_t, which may vary over time but is the same across jobs.

These assumptions imply that the job-offer arrival rate is exogenous and cannot be influenced by job seekers' altering their search effort. This set up is consistent with various wage setting mechanism in the literature such as the wage posting model of Burdett and Mortensen (1998) and extensions such as Bontemps et al. (2000), Gautier et al. (2010) and Burdett et al. (2011) as well as models of bargaining without offer matching such as Gottfries (2021). The implications of violations of these assumptions are discussed in Section 2.5. As is standard in these models, the job-destruction rate δ_t is the same for all jobs, but can vary over time. The wage rate is the only relevant job characteristic (i.e. amenities play no role in mobility decisions) and job mobility is costless. Our method does not require us to specify the dynamic process of λ_t and δ_t .

For the sake of convenience, we assume that unemployed job seekers accept any job offer. When the reservation wage for an unemployed job seeker w^- is the same for all individuals, this assumption is equivalent to a redefinition of the wage-offer distribution and a corresponding redefinition of the job-offer arrival rate:

$$F^{\text{new}}(w) := \left[F^{\text{old}}(w) - F^{\text{old}}(w^{-}) \right] / \left[1 - F^{\text{old}}(w^{-}) \right], \tag{1}$$
$$\lambda_{t}^{\text{new}} := \lambda_{t}^{\text{old}} \left[1 - F^{\text{old}}(w^{-}) \right].$$

In a world where the reservation wage differs between individuals, this redefinition is individual specific. It is therefore equivalent to individual heterogeneity in the arrival rate and the distribution of job offers. We shall show in Section 2.3 that the first order effect of this heterogeneity on our empirical specification is absorbed by the worker fixed effect.

The key concept in our methodology is the *employment cycle*, defined as the time interval that begins at the start of the first job after an unemployment spell and ends when a worker is laid off by a δ_t shock. Without loss of generality, we normalise our measure of calendar time *t* such that it takes the value 0 at the start of the current employment cycle.

It is useful to apply the following definitions:

$$\Lambda_t \equiv \int_0^t \lambda_r dr,$$
$$\Delta_t \equiv \int_0^t \delta_r dr.$$

We refer to Λ_t as the *labour-market time* elapsed since the start of the employment cycle, in contrast to *t*, which is the *calendar time*. While the clock of calendar time runs at a constant pace, the clock of labour-market time runs faster during a boom (when λ_t is high) than during a bust (when λ_t is low). Were $\lambda_t = \lambda$ constant, labour-market time would be proportional to calendar time $\Lambda_t = \lambda t$. We define *a* and *b* as the respective start and end dates of the current job, where a < b. Hence, Λ_a is the labour-market time elapsed since the beginning of the employment cycle up to the starting date of the current job, while Λ_b is the labour-market time elapsed until its termination date. Since a < b, $\Lambda_a < \Lambda_b$.

2.2. Job transition and the growth of wages

Since workers accept any wage offer exceeding the wage in their current job, the log wage $w_t = w_a$ in a job with starting and termination date *a* and *b* (hence: $t \in [a, b)$) is the maximum of all job offers received since the start of the employment cycle at t = 0. Let n_b denote the number of job offers received in the time interval [0, b] since the start of the employment cycle at t = 0. Let n_b denote the start of the employment cycle at t = 0. Let n_b denote the number of job offers received in the time interval [0, b] since the start of the employment cycle until the end of the current job, including the job offers received at t = 0 and at time *b* in the event that the current job ends in a quit. Let $F_{ab} \equiv F(w_a)$ denote the rank of this job. The proposition below specifies the relation between the elapsed labour-market time Λ_b on the one hand and the number of job offers n_b and the expected rank, i.e., match quality, of the current job F_{ab} on the other hand.

Proposition 1 Expected number of offers and rank. Consider the model discussed in Section 2.1.

1. The expected number of job offers n_b satisfies

$$\mathbb{E}[n_b|b, \text{layoff}] = \Lambda_b + 1,$$

$$\mathbb{E}[n_b|b, \text{quit}] = \Lambda_b \frac{\Lambda_b}{\Lambda_b - (1 - e^{-\Lambda_b})}$$

- 2. The expected rank for any job with a > 0, ending either in layoff, $E[F_{ab}|a, b, \text{layoff}]$, or in a quit, $E[F_{ab}|a, b, \text{quit}]$, does not depend on a.
- 3. The expected rank of the current job satisfies

$$E[F_{ab}|b,layoff] = 1 - \Lambda_b^{-1} + \Lambda_b^{-2} (1 - e^{-\Lambda_b}),$$
(2)
$$E[F_{ab}|b,quit] = 1 - 2 \frac{\Lambda_b + (\Lambda_b + 2)e^{-\Lambda_b} - 2}{\Lambda_b + e^{-\Lambda_b} - 1} \Lambda_b^{-1}.$$

The proof is presented in Appendix A. The results can be understood intuitively. The first statement indicates that the expected number of job offers until the moment of separation due to a layoff at time *b* is equal to $\Lambda_b + 1$. The term Λ_b measures the expected number of job offers since the start of the first job of the employment cycle until the moment of separation from the current job; that is, the expected number of offers in the time interval (0, b). This number follows a Poisson distribution with rate $\Lambda_b = \int_0^b \lambda_t dt$. Since the expectation of the Poisson distribution is equal to the rate, the expected number of offers during this time interval is equal to Λ_b . We have to add one for the offer received at the start of the employment cycle at t = 0, which allowed the worker to move from unemployment to employment. Since we assume that unemployed job seekers accept any job offer they receive, a job seeker moving from unemployment to employment is considered to have received exactly one offer at t = 0.

The relation for quits is slightly more complicated. Since the worker received an offer at time b (the offer which made the worker quit), one might presume the expected number of offers to be just one higher than for a layoff due to the additional offer received at time b. Since this offer has been accepted, however, it must have been the highest offer received to date. Hence, relatively few offers must have arrived before, since the more offers the worker has received previously, the higher the expected max of these offers and hence the smaller the probability of receiving an even better offer. The expression in Proposition 1 corrects for this selectivity.

The second statement indicates that the expected rank of any job other than the first job in an employment cycle (by design, a = 0 for that job) depends on its termination date b, but *not* on its starting date a. We shall therefore condition $E[F_{ab}|\cdot]$ not on a, but only on b. The intuition for this result is that, for layoffs, the current job is the max of a set of n_b draws from the wage-offer distribution. Each draw has an equal probability of being the max. Hence, the arrival date a of the max provides no additional information on its expected value over and above the value of n_b . The argument for quits is the same. This statement does not hold for a = 0, since a = 0 implies by definition that the current job is the first job of the employment cycle. This fact is informative about the expected rank, as it implies that none of the offers that the worker

might have received since the start of the employment cycle exceeded the initial offer at t = 0.

The third statement deals with the expected rank of the current job, conditioning on its termination date *b*. Again, this expectation differs between layoffs and quits. For layoffs, the expected rank is equal to the expected maximum of n_b draws from the uniform distribution, which is $1-\mathbb{E}[(n_b+1)^{-1}] \cong 1-\mathbb{E}[n_b]^{-1} = 1-(\Lambda_b+1)^{-1}$. The expression in Proposition 1 corrects for this approximation. For quits, F_{ab} is only the second highest draw from n_b draws; this is because the draw at time *b* must be the highest draw, otherwise the worker would not quit the current job.

An attractive feature of Proposition 1 is that none of its statements depend on the layoff rate δ_t or its integral Δ_t . Our results are thus not affected by a potential covariation of λ_t and δ_t because, intuitively, a layoff only halts the selection process in the current employment cycle but does not interfere with the selection process itself.

2.3. The distribution of job length

The previous section discussed the evolution of the wage rank over the course of an employment cycle. This section discusses the implications for the distribution of job tenures.

Proposition 2 Distribution of job tenures. Consider the model discussed in Section 2.1. For each job other than the first job of an employment cycle, Λ_a/Λ_b is uniformly distributed on the unit interval [0, 1].

The proof is presented in Appendix B. This proposition holds irrespective of the shape of the offer distribution. It only relies on the assumption that job seekers accept any offer that pays a higher wage than their current job; see Assumption 3. The proof requires repeated application of Bayes' rule. The derivation is a little involved, but the intuition is simple. Consider the case where job offers arrive at fixed time intervals rather than by a Poisson process. Hence, the current wage must be the highest of Λ_b offers that have arrived since the start of the employment cycle. Since these offers are independent draws from the offer distribution, all have an equal probability of being the best. The arrival of the highest offer is thus uniformly distributed over the length of the employment cycle. The reasoning is analogous to the rationale for why the expression for the expected rank in equation (2) does not depend on the job's starting date a. The proof shows that this same rationale applies when wage offers arrive at a Poisson rate rather than at fixed intervals.

The proposition implies that, except for the first job of an employment cycle, the expected duration of each job is half the length of the cycle measured in labour-market time:

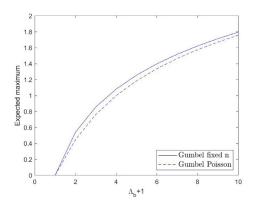
$$\mathbf{E}\left[\frac{\Lambda_a}{\Lambda_b}\right] = \frac{1}{2}$$

2.4. The wage offer distribution

Proposition 1 specifies the law of motion of the expected rank of the job held by a worker over the course of an employment cycle. If the log wage-offer distribution were standard uniform, the rank would be equal to the wage offer. In that case, Proposition 1 would characterise the evolution of the expected log wage over the course of the employment cycle. In that case, the effect of the length of the employment cycle on the wage in a job that ends at time *b* is proportional to Λ_b^{-1} up till a term of order $O(\Lambda_b^{-2})$; it has a finite upper bound consistent with the finite upper support of the uniform distribution. The offer distribution is empirically expected to have an unbounded support, implying that the effect of Λ_b is not bounded.

Here we focus on the special case of the Gumbel distribution. This distribution has some very convenient characteristics for the application at hand. The first two moments of distribution of the maximum of n

A. Gumbel distribution



draws take a convenient form:

$$E[w|n] = \mu + \sigma\gamma + \sigma \ln n,$$

$$Var[w|n] = \sigma^2 \frac{\pi^2}{6},$$
(3)

where $\gamma \cong 0.577$ is Euler's constant and μ and σ are the location and scale parameters, respectively. The expectation increases proportional to $\ln n$ and the variance is independent of n, while the coefficient of proportionality of the increase in the expectation is proportional to the standard deviation of the offer distribution. The higher the standard deviation, the stronger is the effect of n on the expected log wage.

If the number of offers n_b at time *b* were fixed, this formula would allow us to calculate the expected log wage. In our case, n_b follows a Poisson distribution with expectation $\Lambda_b + 1$ for layoffs;¹ for quits, the distribution is somewhat more complicated. For this case, the following proposition applies.

Proposition 3 Expected log wages for a Gumbel offer distribution. Consider the model discussed in Section 2.1. If the offer distribution is *Gumbel, the expected log wages satisfy*

$$E[w_{l}|b, \text{layoff}] = \mu + \sigma\gamma + \sigma \ln \left(\Lambda_{b} + 1\right) + O(\Lambda_{b}^{-1}), \qquad (4)$$
$$E[w_{l}|b, \text{quit}] = \mu + \sigma\gamma + \sigma \ln \left(\Lambda_{b} + 1\right) - \sigma + O(\Lambda_{b}^{-1}).$$

The proof can be found in Appendix C. For large values of Λ_b , the difference between quits and layoffs is only a location shifter, where the size of the shift is equal to σ . The expression is approximate as the actual number of job offers is not $\Lambda_b + 1$, but a Poisson-distributed number with expectation $\Lambda_b + 1$.

Panel A of Figure 1 compares the expected maximum when the actual number of job offers is $\Lambda_b + 1$ with Poisson distributed number of offers. The two lines almost coincide. We shall therefore ignore this difference in the remainder of the paper, except for the variance decomposition in Section 3.5.

Equation (4) is the workhorse for our empirical inference. We will use this specification as i) it corresponds to the Gumbel distribution and ii) it can be seen as a first order log linearisation for any distribution. A comparison of equation (4) for the Gumbel distribution with

$$\mathbb{E}[w_t|b, layof f] = \mu + \sigma\gamma + \sum_{n=0}^{\infty} (n!)^{-1} \Lambda_b^n e^{-\Lambda_b} \sigma \ln (n+1).$$

Fig. 1. Expectation log wage (layoffs) Notes: The parameters of the Gumbel distribution are $\sigma = 6^{1/2}/\pi$ and $\mu = -\gamma\sigma$ such that the mean is zero and the variance is 1. Similarly the parameters of the Normal distribution is $\sigma_N = 1$ and $\mu_N = 0$ such that the mean and variance is the same as for the Gumbel distribution. The Gumbel rescaled plots $b_n \log(\Lambda_b + 1)$.

equation (2) in Proposition 1 for the uniform distribution supports the previous intuition. Ignoring the terms $O(\Lambda_b^{-1})$, $E[w_a|b, layoff]$ evolves proportional to $\ln (\Lambda_b + 1)$ for the Gumbel distribution and proportional to $1 - \Lambda_b^{-1}$ for the uniform distribution. Since the right tail of the Gumbel distribution is fatter, the increase in the expected log wage plateaus more slowly.

B. Normal distribution

Gumbel fixed n (rescalled)

Normal

 $\Lambda_b + 1$

Normal Poisson

1.8

1.6

14

0.6

0.4

0.2

Expected maximum

Panel B of Figure 1 presents the expected maximum of the log wage assuming that log wages follow the Normal distribution with parameters such that the mean and variance are the same as for the Gumbel distribution in Panel A. A few things are apparent. First, the increase in log wages is initially similar under both distributions but for larger values of the Λ_b , the increase is slower under the Normal distribution (as was anticipated by the earlier discussion). Second, the expected maximum under the Normal distribution with a Poisson distributed number of offers can be approximated quite well with the first order log linear approximation (as can be seen by comparing the red dashed line with the solid black line). In this sense, our simple approach is useful irrespective of the exact empirical distribution.²

The intuition from the previous section is that the evolution of $E[w_a|b, layoff]$ is informative with respect to the right tail of the offer distribution. For example, for a uniform distribution (like *F* in Proposition 1), the upper support of the offer distribution is bounded. Hence, the slope of $E[w_a|b, layoff]$ with respect to *b* will converge to zero, as the actual log wage approaches the upper support and there is no room left for further improvement. The fatter the right tail, the slower the slope of $E[w_t|a, b, layoff]$ will decline. The following proposition shows how information on the evolution of the expected log wage can more generally be used to identify the whole wage offer distribution.^{3,4}

Proposition 4 Non-parametric identification. Consider the model discussed in Section 2.1. The function $E[w_a|b, \text{layoff}]$ identifies the distribution F(w) non-parametrically.

The proof can be found in Appendix D. To avoid the selectivity issues for these jobs (see Proposition 1), we have not proven this theorem for jobs ending in a quit, but we suspect that it would be possible to do so.

¹ The exact formula applies the expectation

² The working paper version of this paper approximates the expected maximum for other offer distributions by means of a polynomial in $log(\Lambda_b + 1)$ using Proposition 1.

³ This argument seems to suggest that only the right tail of the distribution is identified. Our proof shows that full identification is obtained for an infinite number of observations. However, the actual number of observations required for reliable identification is obviously larger for the left than the right tail.

⁴ We make the further assumption that the expectation w exists and is finite for any number of draws n. This assumption is violated for e.g., the Cauchy distribution.

2.5. Discussion of assumptions

This section discusses the implications of some violations of the Assumption 1-4 in Section 2.1. First, there could be heterogeneity in the job offer arrival rates. Equation (4) allows for an evaluation of the implications of individual heterogeneity in the job-offer arrival rate. Let λ_{it} be the job-offer arrival rate for worker *i* at time *t*; we assume $\lambda_{it} = \bar{\lambda}_i \lambda_t$ where $E[\bar{\lambda}_i] = 1$; $\bar{\lambda}_i$ measures individual heterogeneity, while λ_t measures aggregate fluctuations; hence $E[\lambda_{it}|t] = \lambda_t$ and $\Lambda_{ib} = \bar{\lambda}_i \Lambda_b$. A Taylor expansion of $\sigma \ln (\Lambda_{ib} + 1)$ yields

 $\sigma \ln \left(\Lambda_{ib} + 1 \right) = \sigma \ln \bar{\lambda}_i + \sigma \ln \left(\Lambda_b + 1 \right) + O(\Lambda_b^{-1}).$

In a wage regression based on equation (4), heterogeneity in the joboffer arrival rate would therefore be reflected in the individual fixed effect in a first-order approximation.

Second, consider the consequences of individual heterogeneity in the standard deviation σ_i with $\mathbb{E}[\sigma_i] = \sigma$. Since the individual variation in σ_i and the aggregate variation in Λ_b are independent, the coefficient σ is consistently estimated, while the term $\mathbb{E}[(\sigma_i - \sigma) \ln \lambda_i]$ is again absorbed by the worker fixed effect. Up to a first-order approximation, heterogeneity in the workers' wage-offer distribution is, therefore, absorbed by the individual fixed effect. Individual heterogeneity in the reservation wage, e.g., due to differences in flow income in unemployment or assets, will impact both the wage offer distribution as well as the probability of acceptance, but by the previous argument, these effects are absorbed in the fixed effect to a first order.

Assumption 1-4 are consistent with many models in the literature, most prominently, the canonical wage posting model developed by Burdett and Mortensen (1998) and extensions such as Bontemps et al. (2000), Gautier et al. (2010) and Burdett et al. (2011), the model of bargaining with renegotiation but without offer matching in Gottfries (2021) as well the model of bargaining with decreasing returns to scale in Elsby and Gottfries (2022).⁵ There are, however, some classes of search models that do not fit our assumptions. We discuss three classes of models.

First, if either the on-the-job search intensity is endogenous, as in Christensen et al. (2005), or firms differ in their separation rates, as in Pinheiro and Visschers (2015) and Jarosch (2021), there are additional selection on top of that coming from the selected acceptance emphasised in this paper. Assume, for example, that the exogenous separation rate is lower in high paying firms then Λ_b reflects the lower acceptance rate, less endogenous search and lower exogenous separation rate in higher paying firms. Each mechanism implies a positive relationship between wages and Λ_b . In this case, the estimates are still informative about the total amount of selection, but the precise interpretation of the coefficients depends on how these additional channels are introduced.

Second, our assumptions are inconsistent with models of offer matching (Cahuc et al., 2006; Dey and Flinn, 2005; Postel-Vinay and Robin, 2002), when focussing on wages. However, since these models feature transitions based only on match quality where any offer with a higher match quality is accepted, Λ_b can be used as a measure of match quality. Hence, Proposition 1 hold for the match quality rank (rather than wage rank). Postel-Vinay and Robin (2002)'s model implies that initial wages for workers coming from unemployment are decreasing in match quality, since workers can better use outside offers to bargain higher wages in these jobs. This allows a novel test of the sequential auctions model which we implement in the next section.

Third, our assumptions are not consistent with the wage setting in Burdett and Coles (2003) since turnover depends on tenure in that model. This deviation is discussed in greater detail in the next section.

3. Empirical analysis

3.1. Empirical specification of the wage regression

Let w_{it} be the log wage of worker *i* at time *t*, let $\Lambda_b(i, t)$ be the integral of the job offer arrival rate λ_t from the beginning of the employment cycle to the end of the job held at time *t*, and let D(i, t) be a dummy variable for jobs that end by a layoff. We apply the following specification:

$$w_{it} = \beta_i + \beta' X_{it} + \beta_u u_t + \beta_\Lambda \ln\left(\Lambda_b + 1\right) + \beta_D D + \varepsilon_{it},\tag{5}$$

where we drop the arguments (i, t) for notational convenience. Equation (4) implies $\beta_{\Lambda} = \beta_D = \sigma$. We allow w_{it} to vary with timevarying personal characteristics such as education and experience, summarised in the vector X_{it} , and with the business cycle as approximated by the unemployment rate u_t . It is critical that the time varying component in X_{it} is independent of the draw from the wage offer distribution at the start of the current job. If not, a wage offer is not a sufficient statistic for a job seeker to decide whether or not to accept it, since he must also consider the differences in the expected future wage growth between his current job and the new offer. The inclusion of a tenure profile would potentially violate this requirement. Finally, ϵ_{it} is the error term, measuring both the deviation of the current wage offer from its expectation $\beta_{\Lambda} \ln (\Lambda_b + 1)$ and all other random components, such as measurement error in wages and unobserved personal characteristics. Note that the model implies that these other components are independent of the max of all offers received during the current employment cycle. Hence, the coefficient β_Λ on its expectation $\ln(\Lambda_b+1)$ is estimated consistently.

Estimation of equation (5) requires data on the job-offer arrival rate λ_i , which cannot be inferred directly from the data as we observe accepted offers only, not offers that have been turned down. Because unemployed job seekers accept any job offer, their job-offer arrival rate λ_{ul} can be directly observed. We assume that the arrival rates of both groups of job seekers vary proportionally:

$$\lambda_t = \psi \lambda_{ut},\tag{6}$$

consistent with the empirical evidence, e.g., Nagypál (2008), which shows that UE and EE flows are both strongly procyclical. Moreover, this assumption is in line with the theoretical literature on the business cycle; see Moscarini and Postel-Vinay (2013), Coles and Mortensen (2016) and Lise and Robin (2017).

3.2. Data

We use the 1979 Cohort of the National Longitudinal Survey of Youth (NLSY 79) for the years 1979 to 2012. Since childbearing interrupts the careers of many women, a phenomenon not accounted for in our theoretical model, we use data from men only. Similarly, because our model applies to primary jobs, the sample is restricted to the primary jobs of men over the age of 18 who are not enrolled in full-time education.⁶ We exclude job spells of fewer than 15 hours per week, shorter than four weeks, and starting before 1979. When there are multiple jobs, the primary job is defined as the job with the highest number of hours. Jobs with inconsistencies in their start or end date are adjusted or removed.⁷ If schooling is not reported for a given month, we assign the maximum

⁵ The assumptions are consistent with the wage setting in Elsby and Gottfries (2022) and their special case without idiosyncratic shocks but not their full model.

⁶ Enrolment in education is not recorded for some periods of 2008, 2010 and 2012, but at this point in the sample the respondents were in their 40s and few had been enrolled in the previous periods.

⁷ Observations missing information on the month or year when the job started or ended are removed. If the day is unknown, we set it to 15. If the day reported is greater than the number of days in the month (e.g., 31st of February), we set it to the last day of the month. If during the interview the worker reported that the job ended after the interview date, we set the end date to the interview date. Jobs where the start date is reported as being after either the interview date or the end date are removed.

from the previous months; if it is less than previously reported, we use the max previously reported.

To construct the variable Λ_b , we categorise job terminations into either quits (belonging to the same employment cycle) or layoffs (starting a new cycle). To this end we follow Barlevy (2008), who classifies a separation as a quit when the new job starts within eight weeks of the termination of the previous job and the stated reason for separation was voluntary (where a non-response is treated as voluntary). If two jobs overlap, we consider the transition to be voluntary if the last job was the primary job during the overlapping period. Jobs that begin as non-primary and subsequently become primary jobs are removed, as are all following jobs in the employment cycle. This allows us to determine whether or not two consecutive jobs belong to the same employment cycle.

Having defined the concept of employment cycles, we have to decide which jobs to include in our analysis. We exclude jobs that have not yet ended. Jobs are deemed to have ended when the worker reports no longer working in the job, the job becomes a secondary job, or the worker at an interview during the subsequent year does not report having worked for the firm during the past year. Jobs where the worker reports being self-employed or working for a family business, where the hourly wage is below \$1 or above \$500, or where covariates are missing are excluded. Wages are deflated using seasonally adjusted national CPI (CPIAUCSL).

We calculate the job arrival rate λ_{ut} for job seekers who are unemployed for fewer than five weeks and employed in the subsequent month using monthly CPS data.⁸ We restrict our analysis to a sample of males aged 25 to 54.⁹ We use the non-seasonally adjusted unemployment rate for men aged 25 to 54 (LNU04000061).

The parameter ψ in equation (6) can be estimated from the difference in the value of Λ_b between the first job and all subsequent jobs in an employment cycle (see Appendix E). In particular, if ψ is high, job offers arrive frequently and the duration of the first job should be short compared to subsequent jobs: the high arrival rate of offers allows for rapid improvement in match quality and hence a decline in the acceptance rate later on in the employment cycle. We obtain $\psi = 0.2$ and average values for λ_t , λ_{ut} and δ_t of 0.08, 0.4 and 0.02 per month, respectively.

Table 1 provides summary statistics for the variables of interest.

3.3. Wage regressions

Table 2 presents the estimation results for equation (5). Standard errors are clustered at the job level. We run separate regressions for lower and for higher educated workers, respectively. The controls used in the subsequent regressions are a linear time trend, years of education (quadratically), and experience and tenure up to a third-order polynomial with interactions, as well as dummies for region, marriage and urban versus rural location.

As discussed in Section 3.1, controls for tenure are inconsistent with our model. As a specification test, we present results both with and without tenure controls. In both specifications, the variable $\ln (\Lambda_b + 1)$ is highly significant with the expected sign. However, the controls for tenure are also highly significant, contradicting the assumption that match quality has a constant effect on wage differentials between jobs over the duration of job spells (see equation (5)). We return to this issue when discussing the distribution of job tenures and the decomposition

Table 1	
Summary	Statistics

	First Job	Second Job	Subsequent Jobs	Total
	1 1150 000	becond bob	bubbequent bobb	Total
urban	0.798	0.799	0.791	0.789
	(0.416)	(0.410)	(0.424)	(0.426)
Fraction high Edu.	0.315	0.394	0.443	0.359
0	(0.465)	(0.489)	(0.497)	(0.480)
In real hourly wage	-2.856	-2.676	-2.545	-2.681
, ,	(0.532)	(0.558)	(0.578)	(0.558)
$\ln(\Lambda_b + 1)$	0.709	1.300	1.797	1.345
	(0.598)	(0.690)	(0.668)	(0.879)
$\ln(\Lambda_{Th} + 1)$	1.569	1.952	2.190	1.968
	(0.837)	(0.666)	(0.582)	(0.803)
Individuals	2572	1470	607	2582
Jobs	12623	2254	991	15868
Observations				33386

Notes: The columns for first, second and subsequent jobs refer to the sequence in an employment cycle. For these columns, only the first observation for each job is used, whereas the total column includes all observations. Standard deviations are in parentheses. High edu. refers to individuals with more than 12 years of education. T_b refers to the sum of the length of all previous employment cycles (i.e. experience) up until the end of the current job. Individuals refers to the number of people included with no missing values.

of the overall expected wage growth in the effects of experience, tenure and OJS. Controls for tenure are included in all subsequent regressions.

The coefficient on u_t is in accordance with the literature and stable across all subsequent specifications. Hence, we do not report it in subsequent tables.

Table 3 provides the results of several specification tests. Panel A splits the variable $\ln (\Lambda_b + 1)$ for different employment cycles over the individual's lifecycle. Its coefficient is highly stable between subsequent employment cycles, which provides support for the model: after losing his grip on the boulder, Sisyphus indeed has to climb the same hill of rents yet again.

Panel B tests the impact of omitting the business cycle as source of variation in $\ln (\Lambda_b + 1)$; the remaining variation is due to individual heterogeneity in the arrival of layoff shocks, which allows us to distinguish between the effects of general experience and the length of the ongoing employment cycle. To this end, we replace $\ln (\Lambda_b + 1)$ with $\ln (\lambda b + 1)$, where λ is the mean value of λ_t over the observations. This is equivalent to letting calendar and labour-market time run at the same pace. Thus, individual heterogeneity in the starting dates of new employment cycles is the only source of variation in $\ln (\lambda b + 1)$. The estimated coefficients are virtually identical to those reported in Table 2. The variation in the pace of labour-market time is therefore not essential for the estimation.

Panel C reports what happens when both sources of variation in $\ln (\Lambda_b + 1)$ are entered simultaneously. As one would expect, the variable $\ln (\Lambda_b + 1)$, which allows for both sources of variation, drives out the variable $\ln (\lambda b + 1)$, which ignores the business cycle variation in λ_i . Surprisingly, the coefficient on $\ln (\Lambda_b + 1)$ is twice as large as in Table 2, while $\ln (\lambda b + 1)$ has a negative sign, though this coefficient is only significant when combining the data on lower and higher educated workers since $\ln (\Lambda_b + 1)$ and $\ln (\lambda b + 1)$ are highly correlated. This suggests that business cycle fluctuations disproportionately affect the job transition rates for workers close to the top of Sisyphus' hill. This result is consistent with the notion that quits rates are highly pro-cyclical, a finding which is consistent with Beaudry and DiNardo (1991) warranting further research.

Panel D tests whether a layoff genuinely resets the clock of job selection from the offer distribution to zero or whether some gains from selection are carried over to the next employment cycle. We enter $\ln(\Lambda_{Tb} + 1)$ as a regressor, where Tb denotes the total work experience since the start

⁸ Due to changes to the CPS classification, the monthly files cannot be matched for several months (07/1985, 10/1985, 01/1994, 06/1995, 07/1995, 08/1995 and 09/1995). For these months we use the predicted values from a regression of the transition rate on a linear trend, a monthly fixed effect and the current unemployment rate.

⁹ We match the monthly CPS data using the codes provided by Shimer (2012) and the variables suggested by Drew et al. (2014). In addition, we use race and age as extra controls.

Table 2

Estimates with and without tenure controls

	No tenure		With tenure			
	All	Low-Edu.	High-Edu.	All	Low-Edu.	High-Edu.
u _t	-0.011***	-0.013***	-0.008***	-0.012***	-0.014***	-0.008***
	(0.002)	(0.002)	(0.003)	(0.002)	(0.002)	(0.003)
$\ln(\Lambda_b + 1)$	0.124***	0.109***	0.152***	0.105***	0.087***	0.134***
0	(0.005)	(0.006)	(0.009)	(0.005)	(0.006)	(0.010)
Observations	33386	20885	11722	33386	20885	11722
R^2	0.643	0.563	0.674	0.644	0.564	0.675

Standard errors in parentheses * p < 0.10, ** p < 0.05, *** p < 0.01 Notes: Regressions of the logarithm of the real wage. The controls are described in the main text.

Та	ble 3	
~		

Specification tests

	All		Low-edu.		High-edu.	
	coeff.	std.error	coeff.	std.error	coeff.	std.error
Panel A						
$\overline{\ln (\Lambda_b + 1)}_{\text{cycle } 1}$	0.119***	(0.011)	0.093***	(0.013)	0.150***	(0.018)
$\ln (\Lambda_b + 1)_{\text{cycle 2-4}}$	0.118***	(0.006)	0.088***	(0.007)	0.149***	(0.010)
$\ln (\Lambda_b + 1)_{\text{cycle 5-7}}$	0.103***	(0.007)	0.095***	(0.009)	0.115***	(0.014)
$\ln (\Lambda_b + 1)_{\text{cycle 8-}\infty}$	0.066***	(0.009)	0.071***	(0.010)	0.072***	(0.019)
D _{layoff}	0.042***	(0.007)	0.056***	(0.008)	0.014	(0.013)
Panel B						
$\overline{\ln\left(\lambda b+1 ight)}$	0.104***	(0.005)	0.086***	(0.006)	0.134***	(0.010)
Panel C						
$\ln (\Lambda_b + 1)$	0.265***	(0.092)	0.203*	(0.111)	0.254	(0.172)
$\ln(\lambda b + 1)$	-0.161*	(0.092)	-0.117	(0.111)	-0.120	(0.173)
Panel D						
$\ln(\Lambda_b + 1)$	0.094***	(0.008)	0.065***	(0.010)	0.144***	(0.015)
$\ln (\Lambda_{Tb} + 1)$	0.024*	(0.013)	0.050***	(0.016)	-0.020	(0.023)
Panel E						
$\ln (\Lambda_b + 1)$	0.116***	(0.012)	0.106***	(0.015)	0.123***	(0.021)
$\ln \Lambda_a$	0.002	(0.007)	-0.012	(0.008)	0.020*	(0.011)
$\ln (\Lambda_b + 1)_{\text{first job}}$	-0.023^{*}	(0.012)	-0.024	(0.015)	-0.005	(0.021)
D _{first job}	0.016	(0.019)	0.023	(0.023)	-0.015	(0.034)
Panel F						
$\ln (\Lambda_b + 1)_{\text{first job}}$	0.086***	(0.007)	0.081***	(0.009)	0.101***	(0.013)
$\ln (\Lambda_b + 1)$ first job&tenure<6m						
	-0.002	(0.008)	-0.014	(0.010)	0.018	(0.013)

Notes: Regressions of the logarithm of the real wage. The controls are described in the main text and each Panel describes remaining RHS variables. The column describes the population which the regression is run for.

of the first employment cycle. We find evidence that gains are carried over from previous cycles for workers with lower but not higher education levels.

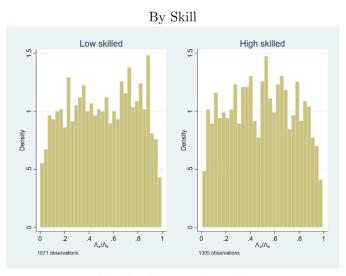
Panel E tests whether $\ln \Lambda_a$ has a significant impact on wages. Since information on *a* might be relevant for the first job of an employment cycle (see the discussion related to Proposition), we add a dummy and a separate slope coefficient for these jobs. According to point 2 of Proposition, it should not. The estimation results confirm this for both education levels. In a similar model, Hagedorn and Manovskii (2013) use both $\ln \Lambda_a$ and $\ln (\Lambda_b - \Lambda_a)$ to control for the expected log wage offer.¹⁰ However, we have shown that the theoretically consistent measure of match quality to be $\ln (\Lambda_b + 1)$. This object cannot be written as a linear combination of $\ln \Lambda_a$ and $\ln (\Lambda_b - \Lambda_a)$. When entering the theoretically correct variable $\ln (\Lambda_b + 1)$, $\ln \Lambda_a$ turns out to be insignificant.

Panel F runs the regression for first jobs and estimate the coefficient on $\ln(\Lambda_b + 1)$ separately for workers with low tenure. The interaction coefficient is small in magnitude and insignificant suggesting that there is a stable return to a good match supporting the model of Burdett and Mortensen (1998). This is in line with the results in Panel E which presented a regression coefficients for first job which is similar in magnitude to that for subsequent jobs. If the offer matching model of Postel-Vinay and Robin (2002) were to hold, the wage of a worker coming from unemployment would be negatively related to match quality, since a higher match quality yields better prospects for future wage increases when the worker receives outside offers. This extra option value of a high quality job is offset by a lower initial wage. In contrast, our results are in line with the reduced form work in Di Addario et al. (2022) and in favour of models where the return to match quality is stable such as the wage posting model or models with bargaining without offer matching.

3.4. The distribution of job tenure

This section tests the prediction of our statistical model regarding the distribution of job tenures. Figure 2 presents the histogram of Λ_a/Λ_b , separately for lower and higher educated workers. Proposition implies that this statistic should be distributed uniformly on the unit interval.

¹⁰ In fact, Hagedorn and Manovskii (2013) use $\Theta_s \equiv \int_0^s \theta_r dr$ rather than $\Lambda_s \equiv \int_0^s \lambda_r dr$, where θ_t is labour market tightness (vacancies over unemployment) rather than the job offer arrival rate λ_t . Selection is proportional to λ_t . Under a standard Cobb Douglas matching function with CRS, $\lambda_t = A\theta_t^{1-\alpha}$, where α is the share of unemployment in the matching function. Hence, its integral Λ_t is not proportional to Θ_t .



By Employment Length

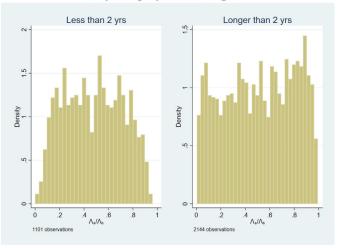


Fig. 2. Test of the arrival rate of the maximum Notes: The histograms present the distribution of Λ_a/Λ_b by skill and by employment length.

The actual distributions fit the uniform distribution remarkably well, except for the last decile of the distribution. The latter deviation might be due to the fact that we ignore jobs lasting less than four weeks; e.g., if the employment cycle at the termination date of the current job has lasted for two years, Λ_a/Λ_b can never be above 25/26 = 0.96. To assess this more formally, we run the test separately for b < 2 years and $b \ge 2$; for the latter, this effect should be smaller since the feasible maximum of the empirical value of Λ_a/Λ_b is closer to its theoretical value of 1. The results in Figure 2 show that the sample for $b \ge 2$ exhibits a much smaller decline in the density function near its upper support, and this decline starts at a higher point in the distribution. A Kolmogorov-Smirnov test for the null hypothesis on all data rejects the null of a uniform distribution at the 1% level. If we restrict the sample to $\Lambda_a/\Lambda_b \in [0.05, 0.95]$ to avoid this censoring issue at the extremes of the support, the null hypothesis is not rejected at the 10% level.

These results are puzzling. On the one hand, they provide surprisingly strong confirmation of our model without a return to tenure for job durations. On the other hand, they are hard to square with the estimation results for wages, which clearly show the presence of a "true" return to job tenure. The inconsistency is most apparent when extending equation (5) with a linear tenure profile where t - a is the tenure in the job (where *a* is the starting date of current job) and where the parameter β_T measures the return to tenure. Clearly, the worker's optimal strategy is to accept any outside offer w_{it}^o coming in at time *t* that satisfies $w_{it}^o > w_{ia}^o + \beta_T(t - a)$, i.e. it is acceptable only if it is so much better than the current job that it offsets the accumulated return to tenure $\beta_T(t - a)$ in the current job.

In this model, a job seeker therefore does not accept every offer w_{it}^o that exceeds the value w_{ia}^o of the current job. The selection process can still be described as the max over a number of draws from an offer distribution, but the offer distribution is non-stationary: it gradually deteriorates relative to the current job by β_T per unit of calendar time. Hence, labour-market time accrued early in the employment cycle is more valuable from the perspective of the selection process of receiving better draws from the offer distribution, because early draws allow the worker to accumulate a higher return to tenure than later draws. This implies that workers will change jobs early in the employment cycle more often than is predicted by Proposition. A distribution of Λ_a/Λ_b that is skewed to the left relative to the uniform distribution is therefore consistent with a return to tenure. This combination of on the one hand a return to tenure and on the other hand a job-tenure distribution consistent with a model without a return to tenure is curious. We return to this issue in Section 3.5, where we decompose the overall wage increase over the lifecycle into the effects of general human capital, OJS and the return to tenure.

3.5. A variance decomposition

We can use these estimates of the standard deviation of the wageoffer distribution to analyse the contribution of job search to wage dispersion. For this decomposition, we ignore the difference between lower and higher educated workers and focus on the average effect of search frictions for all workers.

It is convenient to decompose the error term ϵ_{it} into two components: the random effect of search frictions $w_{ia}^o - \mathbb{E}[w_{ia}^o|\Lambda_{ib}] = \beta_{\Lambda} \ln (\Lambda_{ib} + 1)$ plus a constant), and all other random factors $\overline{\epsilon}_{it}$, in particular measurement error in wages and unobserved personal characteristics. The variance of log wages over the lifecycle can be decomposed into three components: (i) $\beta_i + \beta' X_{it}$, (ii) w_{ia}^o and (iii) random shocks $\overline{\epsilon}_{it}$:

$$\operatorname{Var}\left[w_{it}\right] = \operatorname{Var}\left[\beta_{i} + \beta' X_{it} + w_{ia}^{o} + \overline{\varepsilon}_{it}\right]$$

The model implies that $w_{ia}^o - \mathbb{E}[w_{ia}^o|\Lambda_{ib}]$ and $\overline{\epsilon}_{it}$ are independent, since the random effect of variations in the quality of job offers, $w_{ia}^o - \mathbb{E}[w_{ia}^o|\Lambda_{ib}]$, is beyond the worker's control.

The term $\operatorname{Var}[w_{ia}^o]$ can be decomposed into three orthogonal terms:

- (i) the variance in the log expected number of job offers $\ln(\Lambda_{ib} + 1)$ due to the random arrival of layoff shocks that induce the start of a new employment cycle;
- (ii) the variance in the actual number of job offers conditional on the expected number of offers due to the Poisson arrival rate of these offers;
- (iii) the variation in log wages conditional on the actual number of job offers, due to the variance in the wage-offer distribution.

For the Gumbel distribution, we obtain a particularly simple formula for the approximation to this decomposition:¹¹

$$w_{it} = \beta_i + \beta' X_{it} + \beta_T (t-a) + \beta_u u_t + \beta_\Lambda \ln\left(\Lambda_b + 1\right) + \beta_D D + \varepsilon_{it},\tag{7}$$

¹¹ The simple approximation ignores the difference between quits and layoffs since the coefficient on quits is small in equation (5) but the outcome of the approximation is close to the theoretical variance of wages of 0.024 based on the distribution of outstanding matches $G(F) = F/(1 + \lambda/\delta(1 - F))$ and the values of λ , δ and σ .

$$\operatorname{Var} \begin{bmatrix} w_{it}^{o} \end{bmatrix} = \operatorname{Var} \left[\ln \left(\Lambda_{b} + 1 \right) \right] \sigma^{2} + \operatorname{E} \left[\operatorname{Var} \left(\ln n_{ib} | \Lambda_{ib} \right) \right] \sigma^{2} + \operatorname{E} \left[\operatorname{Var} \left(w_{a}^{o} | n_{ib} \right) \right]$$
$$\cong \left(\operatorname{Var} \left[\ln \left(\Lambda_{b} + 1 \right) \right] + \operatorname{E} \left[\frac{\Lambda_{b}}{\left(\Lambda_{b} + 1 \right)^{2}} \right] + \frac{\pi^{2}}{6} \right] \sigma^{2} \cong 0.0258,$$
$$\begin{array}{c} 0.16 \\ 0.16 \\ 0.16 \end{array} \right) \sigma^{2} \cong 0.0258,$$
$$(8)$$

where we apply the first-order approximation for the variance of the Poisson distribution.¹² The variance of the max of n_{ib} draws from the Gumbel distribution $\operatorname{Var}\left[w_a^o|n_{ib}\right]$ is independent of n_{ib} (see equation (3)). The variance of $\ln(\Lambda_b + 1)$ is taken from Table 1.

The main source of variation is the variance of the maximum conditional on the actual number of offers, $\operatorname{Var}\left[w_{a}^{o}|n_{ib}\right] = \pi^{2}/6\sigma^{2}$. Our estimate $\sigma = 0.10$ is taken from Table 2 (the coefficient for all observation including a return to tenure). This corresponds to a standard deviation of the wage offer distribution of $\sigma \frac{\pi}{\sqrt{6}} = 0.13$. Guo (2021) estimates a dispersion of the wage offer distribution of 0.14, in line with our results. Since the total variance in wages is about 0.3, search frictions account for around 8% of the overall wage dispersion. Tjaden and Wellschmied (2014) and Vejlin and Veramendi (2020) estimate that search frictions account for 14% and 20%, respectively. Our paper requires weaker assumptions on measurement error. In particular, the scale parameter is estimated by means of a regression on the mean number of offers which are (given the model) orthogonal to measurement error in wages and unobserved worker heterogeneity. Furthermore, our estimate is in line with the share of variance explained by firm fixed effects once limited mobility bias has been accounted for in two-way fixed effects models as reported by Bonhomme et al. (2020).

The model can also be applied to calculate the expected log wage loss following a layoff by comparing the log wage at the moment of layoff to the expected wage in the first job of the next employment cycle; the expectation of this loss is $\sigma \ln (\Lambda_{ib} + 1)$. Using the average value of δ_t and λ_t , the expected loss in log wages can be calculated as

$$\mathbf{E}\left[w_{t}^{o}|\Lambda_{ib}\right] - \mathbf{E}\left[w_{t}^{o}|\Lambda_{ib}=0\right] \\
= \int_{0}^{\infty} \Pr\left(t\right)\beta_{\Lambda}\ln\left(\lambda t+1\right)dt = -\beta_{\Lambda}\exp\left(\delta/\lambda\right)\operatorname{Ei}(-\delta/\lambda) \tag{9}$$

using $Pr(t) = \delta e^{-\delta t}$. The expected loss in log wages depends on the standard deviation of the wage-offer distribution and on the ratio of the job-destruction to the job-offer arrival rate, δ/λ . Taking $\sigma = 10\%$ and our estimate of $\delta/\lambda = 0.25$, the average wage loss of about 13%. This estimate is lower than the empirical estimates based on mass displacement (e.g., Jacobson et al. (1993) and Davis and Wachter (2011)), but these studies restrict the analysis to high-tenured workers with a high average match quality. The model mechanism is also consistent with recent evidence that highlights the loss of firm premium in explaining earnings losses after displacement (Bertheau et al., 2022; Gulyas and Pytka, 2020; Schmieder et al., 2022).

The increase in workers' wages over the lifecycle can be decomposed into three components: (i) the accumulation of general human capital; (ii) the tenure profile in wages; and (iii) the selection into better matches due to OJS. To make this decomposition, we first obtain the total increase in workers' wages over the lifecycle by running a log wage regression with a fourth-order polynomial in experience and the same controls as in our previous regressions, but omitting $\ln(\Lambda_{ib} + 1)$ and the polynomial in tenure.

$$\operatorname{Var}\left[\ln n_{b}|\Lambda_{b}\right] \cong \left(\frac{d\ln \operatorname{E}\left[n_{b}\right]}{d\operatorname{E}\left[n_{b}\right]}\right)^{2} \operatorname{Var}\left[n_{b}|\Lambda_{b}\right] = \frac{\Lambda_{b}}{(\Lambda_{b}+1)^{2}},$$

since $\operatorname{Var}\left[n_{b}|\Lambda_{b}+1\right] = \Lambda_{b}$ and $\operatorname{E}\left[n_{b}\right] = \Lambda_{b}+1.$

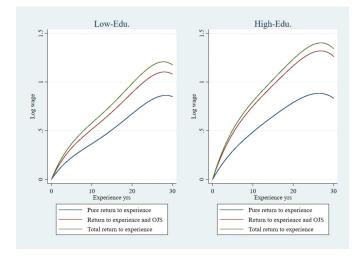


Fig. 3. Experience Profile with and without controlling for OJS Notes: The Figure presents a decomposition of the total return to experience, the green line, into components coming from returns to 1) tenure, the difference between the green and the red line, 2) climbing the job ladder, difference between the red and blue line, and 3) pure returns to experience, blue line. The method is described in the main text.

Next, we derive an estimate of the tenure profile, using the same regression but adding these omitted variables. The inclusion of $\ln(\Lambda_{ib} + 1)$ corrects the estimated return to tenure for the bias introduced by the effect of heterogeneity in match quality ('good jobs survive'). We use these coefficients to eliminate the effect of tenure from our individual data on log wages. Then, we regress this tenure-corrected log wage on a fourth-order polynomial in experience. This experience profile includes the returns to experience and OJS, but excludes the return to tenure. The gap between this experience profile and the total increase in workers' log wages is the contribution of the return to tenure to the increase in workers' wages over the lifecycle.

Finally, we obtain the return to experience by regressing log wages on fourth-order polynomials in experience and tenure, including the polynomial in $\ln(\Lambda_{ib} + 1)$.¹³ The coefficients associated with experience measure the pure experience effect.

The results are shown in Figure 3. The return to OJS explains 30% of the total increase in workers' wages over the lifecycle. Correcting for this return yields a much flatter experience profile. The contribution of tenure to the total return to experience is small at less than 10%, which is good news for the model (see the discussion in Section): a high return to tenure is hard to square with our theoretical model. While it is hard to make a direct comparison due to different modelling strategies and non-additive decompositions, the contribution of the job ladder is larger than the estimates in Altonji et al. (2013) and Vejlin and Veramendi (2020) while smaller than those presented in Bagger et al. (2014).

We perform a similar decomposition for the tenure profile. First, we obtain the total return to tenure by running a wage regression with the standard controls and fourth-order polynomials in experience and tenure but omitting $\ln(\Lambda_{ib} + 1)$. Next, we run the same regression including $\ln(\Lambda_{ib} + 1)$. The tenure profiles derived from these regressions appear in Figure 4. The results suggest that most of the raw tenure profile can be attributed to survival bias, in particular for workers with high education levels. This result explains why we find a uniform distribution for Λ_a/Λ_b . The pure tenure profile is relatively small.

¹² The approximation reads

¹³ Changing the order of the decomposition in the three components does not affect our results.

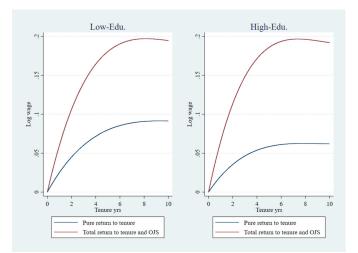


Fig. 4. Tenure Profile with and without controlling for OJS Notes: The Figure presents a decomposition of the total return to tenure, the red line, into components coming from pure return to tenure, the blue line, and one that comes from selection from OJS, difference between the red and blue line. The method is described in the main text.

4. Conclusion

No single structural model will be able to fully explain the empirical pattern of job-to-job transitions and wage-dynamics. Too many mechanisms play a role for all of them to be captured in a single structural framework. From this perspective, this paper shows how the wage offer distribution in the baseline OJS model characterised by efficient job-tojob transitions and rank-preserving cyclical fluctuations in wage setting can be estimated by a simple OLS regression. The paper also develops a measure of match quality that is also consistent with alternative wage setting mechanism, such as sequential auctions, that preserve efficient transitions.

The model has a number of testable implications for both wages and job durations: (i) a layoff restarts the selection process, (ii) log wages in an ongoing job depend on its termination date, not its starting date, (iii) other things being equal, jobs ending in a quit rather than a layoff pay lower wages, and (iv) the starting date of the current job is uniformly distributed over the length of the current employment cycle until the end of this job.

These implications have largely been confirmed by the data. The finding that the job duration is uniformly distributed over the current employment cycle (implication (iv)) came as something of a surprise. One might have expected the an asymmetric distribution due to worker's acquisition of firm-specific human capital implying that the current job is more likely to have started in the beginning rather than at the end of the current employment cycle. Our data do not support this conclusion.

Using the simple methodology, we find that (i) the standard deviation of the log offer distribution is about 13%, and search frictions account for around 8% of wage dispersion; (ii) search frictions account for about 30% of the total increase in workers' wages over the lifecycle, while tenure accounts for only 10%; (iii) the standard deviation of the offer distribution is about 50% higher for more highly educated workers.

We see several potentials for future research building on the basic insights of this paper. Most promisingly, the methodology creates a proxy Λ_b for the worker ranking of jobs but is silent on the extent to which the value of a job is match or firm specific. With matched employer-employee data one could additionally use the Bagger and Lentz (2019) poaching rankings of firms to create an average firm ranking. The extent to which our worker ranking is correlated with this ranking is then informative on the relative importance of the firm compared to the match. Both these measures have the value of relying only on the structure of transitions. If one additionally incorporating wages via a two-way fixed effects regression, one could potentially tease out the relative value of amenities compared to wages. Thus, combining this firm ranking and with our worker ranking of jobs offers the potential to estimate directly, and transparently, many aspects of search models.

Appendix A. Proof of Proposition 1

All probabilities and expectations here condition on b. For the proofs for quits, there must be a job offer at b (the new job allowing the worker to quit). For the sake of notational convenience, we omit these conditions in the equations below.

1. Expected number of job offers n_b :

The worker receives one offer at time 0 for the transition from unemployment to employment and n_b^o offers in the time interval (0, b). Since n_b^o follows a Poisson distribution with parameter Λ_b (and hence expectation Λ_b), we have

$$E[n_b^o] = \Lambda_b$$

(a) Jobs ending in a layoff:

Since the arrival of a layoff shock is independent of the previous arrival of job offers, we have

$$\Pr(n_b^o | \text{layoff}) = \Pr(n_b^o).$$

Taking the expectation of n_b^o and adding 1 for the offer received at time 0 yields

$$\mathbf{E}[n_b] = 1 + \Lambda_b.$$

(b) Jobs ending in a quit:

The worker receives an offer at time *b* that allows him or her to quit. Hence, the number of offers received in the time interval [0, b] is $n_b = n_b^o + 2$. For an offer received at time *b* to lead to a quit, it must be the highest of n_b offers. Since all i.i.d. draws are equally likely to be the maximum, this implies

 $\Pr(\operatorname{quit}|n_b^o) = \left(n_b^o + 2\right)^{-1}.$

Since n_b^o follows a Poisson distribution with parameter Λ_b , the probability of a quit conditional on an offer is

$$Pr(quit) = \sum_{n_b^o=0}^{\infty} Pr(n_b^o) Pr(quit|n_b^o) = e^{-\Lambda_b} \sum_{k=0}^{\infty} \frac{\Lambda_b^k}{k!(k+2)}$$
(A.1)
$$= e^{-\Lambda_b} \left(\Lambda_b^{-1} \sum_{k=0}^{\infty} \frac{\Lambda_b^{k+1}}{(k+1)!} - \Lambda_b^{-2} \sum_{k=0}^{\infty} \frac{\Lambda_b^{k+2}}{(k+2)!} \right)$$
$$= \Lambda_b^{-2} (\Lambda_b + e^{-\Lambda_b} - 1),$$

using $\sum_{k=0}^{\infty} \frac{\Lambda_b^o}{k!} = e^{\Lambda_b}$. Finally, $\Pr(n_b^o)$ follows from the Poisson distribution

$$\Pr\left(n_{b}^{o}\right) = \frac{\Lambda_{b}^{n_{b}^{o}}}{n_{b}^{o}!}e^{-\Lambda_{b}}$$

Combining these expressions yields

$$\Pr(n_b^o|\text{quit}) = \frac{\Pr(n_b^o)\Pr(\text{quit}|n_b^o)}{\Pr(\text{quit})} = \frac{\Lambda_b^{n_b^o}e^{-\Lambda_b}}{n_b^o!(n_b^o+2)} \frac{1}{\Lambda_b^{-2}(\Lambda_b+e^{-\Lambda_b}-1)}$$

Hence, using that $n_b = n_b^o + 2$ gives

$$\mathbf{E}[n_b|\mathbf{quit}] = \sum_{n_b^o=0}^{\infty} \Pr(n_b^o|\mathbf{quit})(n_b^o+2) = \frac{\Lambda_b^2}{\Lambda_b + e^{-\Lambda_b} - 1}$$

2. The expected rank for any job with a > 0, ending either in a layoff, $E[F_{ab}|a, b, layoff]$, or a quit, $E[F_{ab}|a, b, quit]$, does not depend on a. The probability of a, F_{ab} and layoff occurring jointly at b is given by

$$\Pr(a, b, \text{layoff}, F_{ab}) = \lambda_a F_{ab} \exp\left[-\Lambda_b \left(1 - F_{ab}\right) - \Delta_b\right] \delta_b$$

$$\Pr(a, b, \operatorname{quit}, F_{ab}) = \lambda_a F_{ab} \exp\left[-\Lambda_b \left(1 - F_{ab}\right) - \Delta_b\right] \lambda_b \left(1 - F_{ab}\right)$$

These expressions are derived in the proof of Proposition 2. The distribution of F_{ab} conditional on *a* and *b* is therefore independent of *a*

$$\begin{aligned} &\Pr(F_{ab}|a, b, \text{layoff}) = C_{b, \text{layoff}} F_{ab} \exp\left[-\Lambda_b \left(1 - F_{ab}\right)\right], \\ &\Pr(F_{ab}|a, b, \text{quit}) = C_{b, \text{quit}} F_{ab} \exp\left[-\Lambda_b \left(1 - F_{ab}\right)\right] \left(1 - F_{ab}\right). \end{aligned}$$

3. Expected rank for jobs starting at *a* and ending at *b*:

(a) Jobs ending in a layoff:

$$\Pr\left(F_{ab}|\text{layoff}\right) = \Pr\left(F_{ab}\right),$$

where the probability of a layoff at *b* is independent of the value of F_{ab} . The cumulative distribution function of F_{ab} conditional on the initial offer at time 0 and Λ_b is

$$\Pr\left(F_{ab} \le F\right) = F e^{-\Lambda_b (1-F)}$$

Differentiating this equation with respect to F yields the density function of F_{ab} :

$$\Pr\left(F_{ab} = F\right) = \left(\Lambda_b F + 1\right) e^{-\Lambda_b (1-F)}.\tag{A.2}$$

The expectation of F_{ab} can therefore be written as

$$E[F_{ab}] = \int_0^1 F(\Lambda_b F + 1) e^{-\Lambda_b(1-F)} dF$$

= 1 - $\Lambda_b^{-1} + \Lambda_b^{-2} (1 - e^{-\Lambda_b}).$

(b) Jobs ending in a quit:

Using the above equations yields

$$\Pr(F_{ab} = F | quit) = \frac{\Pr(F_{ab} = F) \Pr(quit|F_{ab} = F)}{\Pr(quit)}$$

$$= \Lambda_b \frac{(1 - F)(\Lambda_b F + 1)e^{-\Lambda_b(1 - F)}}{1 - \Lambda_b^{-1} + \Lambda_b^{-1}e^{-\Lambda_b}}.$$
expected match quality is similarly.

The expected match quality is similarly

$$\begin{split} \mathsf{E}(F_{ab}|\mathsf{quit}) &= \int_{0}^{1} \Lambda_{b} \frac{F(1-F) (\Lambda_{b}F+1) \mathrm{e}^{-\Lambda_{b}(1-F)}}{1-\Lambda_{b}^{-1} (1-\mathrm{e}^{-\Lambda_{b}})} dF \\ &= 1-2 \frac{\Lambda_{b} + (\Lambda_{b}+2) \mathrm{e}^{-\Lambda_{b}}-2}{\Lambda_{b} + \mathrm{e}^{-\Lambda_{b}}-1} \Lambda_{b}^{-1}. \end{split}$$

Appendix B. Proof of Proposition 2

Let F_{ab} be the realisation of the rank F of the worker's current job. The probability that the length b of a worker's employment cycle exceeds t and that the last job has a rank $F < F_{ab}$ satisfies

$$\Pr\left(F < F_{ab}, b > t\right) = F_{ab} \exp\left[-\Lambda_t \left(1 - F_{ab}\right) - \Delta_t\right]. \tag{B.1}$$

The first factor F_{ab} is the probability that the first offer of the employment cycle at time 0 is less than F_{ab} , while the second factor is the probability that no offer higher than F_{ab} has been received during the time interval (0, t). Similarly, the probability that the worker's employment cycle lasts beyond the starting date *a* of the current job and that he/she has not previously received a better offer than the value F_{ab} in his current job is

$$\Pr\left(F < F_{ab}, b > t\right) = F_{ab} \exp\left[-\Lambda_a \left(1 - F_{ab}\right) - \Delta_a\right].$$

Conditional on the starting date of the current job *a* and its rank F_{ab} , the density function of its end date *b* satisfies

$$\Pr(b|F_{ab}, a) = \left[\lambda_b \left(1 - F_{ab}\right) + \delta_b\right] \times$$

$$\exp\left[-\left(\Lambda_b - \Lambda_a\right) \left(1 - F_{ab}\right) - \left(\Delta_b - \Delta_a\right)\right].$$
(B.2)

Now let F_{ab} be stochastic. The joint density $Pr(a, b, F_{ab})$ is the product of four probabilities:

(i) the joint probability $Pr(F < F_{ab}, a < t)$ that the length of the employment cycle exceeds *a* and that there has been no prior offer greater than F_{ab} ;

(ii) the probability λ_a that there is an offer at *a*;

(iii) the probability that this offer is of rank *F*; this is unity since *F* is uniformly distributed;

(iv) the probability $\Pr(b|F_{ab},a)$ that this job ends at b conditional on F_{ab} and a.

Hence, the joint probability of a, b and F_{ab} reads

$$\Pr(a, b, F_{ab}) = \lambda_a F_{ab} \exp\left[-\Lambda_b \left(1 - F_{ab}\right) - \Delta_b\right] \left[\lambda_b \left(1 - F_{ab}\right) + \delta_b\right].$$

This density function takes account of a factor λ_a depending on *a* and a second factor depending on *b* and F_{ab} . Integrating over F_{ab} yields the joint probability Pr(a, b). Based on the multiplicative structure of $Pr(a, b, F_{ab})$, this can be written as

$$\Pr(a, b) = \lambda_a c(b),$$

where c(b) is some function of b. Applying Bayes' rule yields

$$\Pr(a|b) = \frac{\Pr(a,b)}{\Pr(b)} = \lambda_a C(b),$$

where C(b) is again some function of *b*. Since $a \in [0, b]$, $C(b) = \int_0^b \lambda_a da = \Lambda_b$. The density of *a* is therefore

 $\Pr(a|b) = \frac{\lambda_a}{\Lambda_b}$

 Λ_a/Λ_b is thus uniformly distributed.

Appendix C. Proof of Proposition 3

1. Based on Proposition 1, $E[n_b|layoff] = \Lambda_b + 1$. Since w_a is the maximum of n_b offers and since the maximum of n_b identical offers from a Gumbel distribution with parameters $\{\mu, \sigma\}$ is $E[w_a|n_b] = \mu + \gamma \sigma + \sigma \ln n_b$

 $E[w_a|\text{layoff}] = \mu + \gamma \sigma + \sigma E[\ln n_b|\text{layoff}]$

$$= \mu + \gamma \sigma + \sigma \ln \mathbb{E}[n_b | \text{layoff}] + O(\Lambda_b^{-1})$$
$$= \mu + \gamma \sigma + \sigma \ln (\Lambda_b + 1) + O(\Lambda_b^{-1}).$$

The equation uses $\mathbb{E}[n_b^k] - \mathbb{E}[n_b]^k = o(\Lambda_b^{k-1})$ for $k \in \mathbb{N}_+$.¹⁴

 For a quit, w_b is the highest offer, while w_a is the second highest offer. The formula for the expectation of the second highest offer implies

$$\begin{split} \mathbf{E}[w_a|n_b, \mathrm{quit}] &= \mathbf{E}[w_a|n_b, \mathrm{layoff}] - n_b \big(\mathbf{E}[w_a|n_b, \mathrm{layoff}] \\ &- \mathbf{E}[w_a|n_b - 1, \mathrm{layoff}] \big). \end{split}$$

Using part 1, this implies

$$\begin{split} \mathbf{E}[w_a|\text{quit}] &= \mu + \gamma \sigma + \sigma \ln\left(\Lambda_b + 1\right) - \sigma \mathbf{E}\left[n_b\left(\ln\left(\Lambda_b + 1\right)\right) \\ &- \ln\Lambda_b\right)\right] + O(\Lambda_b^{-1}) \\ &= \mu + \gamma \sigma + \sigma \ln\left(\Lambda_b + 1\right) \\ &- \sigma\left(\Lambda_b + 1\right)\ln\left(1 + \Lambda_b^{-1}\right) + O(\Lambda_b^{-1}) \\ &= \mu + \gamma \sigma + \sigma \ln\left(\Lambda_b + 1\right) - \sigma + O(\Lambda_b^{-1}). \end{split}$$

Appendix D. Proof of Proposition 4

The distribution of the number of offers conditional on a layoff is

$$\Pr(n_b|\Lambda_b) = \frac{\Lambda_b^{n_b-1}e^{-\Lambda_b}}{(n_b-1)!}.$$

Hence, the expectation of w_a conditional on Λ_b satisfies

$$\mathbb{E}\left[w_a|\Lambda_b\right] = \sum_{n=1}^{\infty} \frac{\Lambda_b^{n-1} e^{-\Lambda_b}}{(n-1)!} w_n$$

¹⁴ This holds as the moment-generating function for the Poisson distribution with parameter Λ_b is $M(t) = e^{\Lambda_b(e^t-1)}$, which implies that $M^k(0) = \Lambda_b^k + O(\Lambda_b^{k-1})$.

where w_n denotes the expected maximum from *n* draws. First, we prove that the function $\mathbb{E}[w_a|\Lambda_b]$ characterises the sequence w_n . Since the above equation holds for all Λ_b , we can take the derivative with respect to Λ_b . Multiplying both sides by e^{Λ_b} and taking the k^{th} derivative yields

$$e^{\Lambda_b}\left[\sum_{i=0}^k \frac{k!}{(k-i)!i!} \frac{\partial^i \mathbb{E}[w_a|\Lambda_b]}{\partial \Lambda_b^i}\right] = \sum_{n=k+1}^\infty \frac{\Lambda_b^{n-k-1}}{(n-k-1)!} w_n,$$

where we adopt the convention that the $(i = 0)^{th}$ derivative of a function is the function itself. This expression holds for all Λ_b . Evaluating it for $\Lambda_b = 0$ yields

$$w_{k+1} = \sum_{i=0}^{k+1} \frac{k!}{(k-i)!i!} \frac{\partial^i \mathbb{E}\left[w_a | \Lambda_b\right]}{\partial \Lambda_b^i} |_{\Lambda_b = 0}.$$

The function $\mathbb{E}[w_a|\Lambda_b]$ therefore characterises the sequence w_n . The sequence of expected maxima w_n in turn identifies any distribution (see Theorem 6.3.1 in Arnold, Balakrishnan and Nagaraja (2008)).

Appendix E. Identification of ψ

For the estimation of ψ , we rely on a steady-state argument whereby labour-market time runs at a constant pace. Hence, we drop the suffix *t* of λ_t and δ_t . First, we derive the expected duration of the first job of an employment cycle. The duration of a job of rank *F* follows an exponential distribution with parameter $\delta + \lambda(1 - F)$. Hence, the expected duration of a job conditional on its rank is $[\delta + \lambda(1 - F)]^{-1}$. Since the rank of the first job is a random draw from the uniform distribution, its expected duration satisfies

$$\mathbb{E}[b|1^{st} \text{ job in emp.cycle}] = \int_0^1 [\delta + \lambda(1-F)]^{-1} dF = \lambda^{-1} \ln(1 + \lambda/\delta).$$
(E.1)

Next, we derive the expected termination date *b* of all subsequent jobs. First, we calculate the joint density among all jobs of the rank *F* of the current job, its start date *a* and its termination date *b*. This density is composed of three parts: (i) the fraction $F \exp[-(\delta + \lambda(1 - F))a]$ of workers remaining at *a* with a rank less than *F*, (ii) the arrival rate λ of an offer at *a*, and (iii) the probability $[\delta + \lambda(1 - F)] \exp[-(\delta + \lambda(1 - F))(b - a)]$ that a match ends at *b* conditional on its having started at *a*. Hence, this density is proportional to

$$Pr(F, a, b) \propto F \exp \left[-(\delta + \lambda(1 - F))a\right] \times \lambda \times [\delta + \lambda(1 - F)]$$
$$\times \exp \left[-(\delta + \lambda(1 - F))(b - a)\right]$$
$$\propto F \exp \left[-(\delta + \lambda(1 - F))b\right][\delta + \lambda(1 - F)].$$

In the second line, *a* is dropped. We can ignore the job-offer arrival rate λ , since it does not depend on *F*, *a* or *b*. We integrate this density over the possible start dates $a \in (0, b)$ to arrive at the joint density of match quality *F* and end date *b*:

$$\Pr(F, b) = \frac{F \exp\left[-(\delta + \lambda(1 - F))b\right][\delta + \lambda(1 - F)]b}{\frac{\delta + \lambda}{\lambda^2} \ln\left(\frac{\delta + \lambda}{\delta}\right) - \lambda^{-1}}.$$

Hence:

$$E[b|subseq.jobs] = \int_0^1 \int_0^\infty b \Pr(F, b) \, db \, dF$$

= $\frac{2}{\lambda} \frac{\lambda/\delta - \ln(1 + \lambda/\delta)}{(1 + \delta/\lambda) \ln(1 + \lambda/\delta) - 1}.$ (E.2)

We can derive information on $E[b|1^{st}$ job in emp.cycle] and E[b|subseq.jobs] from the data. This yields a system of two equations, which can be solved for δ and λ . The ratio of λ to λ_u provides an estimate for ψ .

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