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Estimating Price Impact via Deep Reinforcement Learning

Yi Cao* Jia Zhai†

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Abstract

Price impact is the adverse change of the asset price against trader's action. As a crucial part of indirect trading cost, price impact has attracted increasing attention in both econometric and data science literature. In this paper, we draw upon both strands of the literature and develop a deep neural network enhanced recursive (DeRecv) model to estimate temporary and permanent price impact of an order or trade. The temporary price impact is calculated as the sum of the expected immediate impact at each time point after taking action in an ad-hoc market condition. The permanent price impact is defined as a new permanent level at which the information of the incoming order is entirely absorbed by the market. Through the experimental evaluation based on data from 10 stocks at NASDAQ and Shanghai Stock Exchange, we show that the proposed DeRecv model is better than the reinforcement learning model and the traditional vector autoregressive model.

JEL code: C63, F47

Keywords: Finance; Price Impact; Reinforcement Learning; Deep Neural Network.

*Management Science and Business Economics Group, Business School, University of Edinburgh, 29 Buccleuch Place, Edinburgh EH8 9JS, UK. Email: jason.caoyi@gmail.com

†Business School, Xi'an Jiaotong-Liverpool University, Suzhou, Jiangsu Province, China 215123, jia.zhai@xjtlu.edu.cn

1 Introduction

It is widely known that the trading action conversely influences the asset prices. A buy (or sell) action increases (or decreases) the asset price (see Hasbrouck (1988, 1991); Parlour and Seppi (2008); Hautsch and Huang (2012)). The change of the price due to the trading action is usually called price impact or market impact. The action includes both the execution of a market order and the placement of a limit order. The theoretical studies suggest a monotonic positive relation between the price and volume of the trading action and the magnitude of the impact (see the work of Hasbrouck (1988, 1991)). A trading action with a larger size or a more aggressive price incurs a bigger impact. However, the magnitude of the impact can not be obtained directly from the market. Therefore a number of studies investigate the methods for estimating the price impact of a trading action.

Early studies, such as the work of Jones et al. (1994) show that the market order direction (buy or sell) has significant explanatory power in estimating permanent price impact. Furthermore, Huberman and Stanzl (2004) demonstrate that the magnitude of permanent price impact is linear to the size of a market order (trade). Hasbrouck (1991) shows that a linear relationship between the impact and the order size is not robust and may lead to a 'misspecified model'. Therefore, Hasbrouck (1991) proposes a polynomial term in the vector autoregressive (VAR) model for capturing the non-linear relation between the permanent price impact and the size of the market order (trade). Meanwhile, Parlour and Seppi (2008) and Hautsch and Huang (2012) show that the submission of a limit order also exerts price impact even if the order is not immediately executed. Thus, Hautsch and Huang (2012) propose a VAR based model to estimate the temporary and permanent price impact of a limit order. Philip (2019) reconfirms the existence of a non-linear relation between the price impact and trade size and shows that the traditional VAR model is not applicable in capturing the impact in today's trading environment. To solve this, Philip (2019) proposes a recursive model by modifying a reinforcement learning (RL) framework. Due to its Markovian assumption, the RL framework is highly flexible in modelling the non-linear relation, and therefore, has been applied in modelling limit order book for years, particularly in optimal execution via a sequence of trade (see Bertsimas and Lo (1998); Nevmyvaka et al. (2006); Hendricks and Wilcox (2014); Ning et al. (2018)). Philip (2019) removes the optimisation of a trader's objective and revises the RL framework as a recursive model—the permanent price impact of a trade is the immediate price impact plus the permanent impact of all subsequent trades caused by the initial action. By using the imbalance of the depth between the bid and ask side, Philip (2019), on one hand, simplifies the market condition as a binary variable—either a positive or a negative imbalance, on the other hand, leaves two research gaps. The first is the distinction of the impacts under different market conditions that provide the same imbalance value. For example, the depth imbalance of one (depth of bid has one more share than the depth of ask) and the depth imbalance of 10,000 (depth of bid has 10,000 more shares than the depth of ask) are considered as identical market states '+DI' in the model of Philip (2019). However, the two market conditions apparently exert significantly different impacts than illustrated in previous studies (e.g. Hasbrouck (1988, 1991); Parlour and Seppi (2008); Hautsch and Huang (2012)). The second research gap is the estimation of the temporary price impact of an order.

1.1 Research gap and motivation

Price impact has been considered as one important component of the indirect cost of a trading action and is even higher than the transaction cost due to the study of the Deutsche bank Ferraris (2008). An accurate estimation of the price impact is a key determinant of investment performance. This work is motivated by the reinforcement learning (RL) model in Philip (2019). We summarise our motivations as follows:

- The RL model can be used to model the non-linear relation between a market order and its permanent impact on the market. A research gap that has not been solved yet lies in modelling the market not by a simplified binary variable of market depth imbalance, but by the limit order book, which reflects an ad-hoc market condition. This gap needs to be addressed with further studies.

- The probability of the market reactions to an action is estimated by a binary variable Philip (2019). Any changes of markets or order formats lead to massive expansions of subsequent cases. This leaves estimating the transition probability as an addressable gap.

- The optimal execution of a large position is often performed over a short period Ning et al. (2018). Accurate forecasts of execution costs are crucial in optimal execution strategy. The temporary price impact of an order over a short trading horizon contributes to a large part of the execution cost. This leaves another gap in the literature. Our study is motivated by and represents a step towards addressing these gaps.

1.2 Main contribution

In this work, we aim to address the discussed three research gaps and propose a Deep neural network enhanced Recursive (DeRecv) model to estimate the temporary and permanent price impact of a trade or order. Our main contribution in this study falls into the following three points.

1. We model the limit order book by a vector of its price and depth (volume) at both bid and ask sides, and update the order book by the order-matching rule (SEC (2000)) via a recursive framework. Thus, our method models the order book as an ad-hoc state of the market. The estimated impact shows a continuous, monotonic, and smooth relation with the trading price, volume, and the market condition as well. This overcomes the work of previous studies that the estimated impact may remain identical when the market condition changes.

2. On the financial market, a trading action may incur unlimited types of following actions. We model the incurred actions as a classification problem by categorising them into eight primary types. We then forecast the probability of each type using a deep neural network with a Softmax activation function at output layer. By this, we consider a non-linear relation between a trading action and its incurred actions. Therefore, it considers the probability forecasting under more complex ad-hoc market conditions.

3. We also contribute to the literature by offering comprehensive empirical evidence that our DeRecv model outperforms the benchmark models of Philip (2019) and Hautsch and Huang (2012). Previous studies usually compare the impact of different trading actions under the same market condition. In addition to this, we further compare the impact of the same trading action under different market conditions. This empirical study highlights the importance of the limit order book model in estimating the trading cost and clearly illustrates the advantage of our proposed model.

1.3 Findings

We document several findings. First, we re-confirm that non-linearity exists in the relation between the permanent price impact and the size of either a trade or a limit order. The relation is across 10 different stocks in both developed and emerging markets.

Second, we show that the DeRecv model generates smooth and monotonic permanent impact of a limit order or a trade under different market conditions. In contrast, VAR Hautsch and Huang

(2012) and RL Philip (2019) model estimate identical impacts across the change of the market due to the linearity of the model and the oversimplified market variable respectively.

Third, we find that the DeRecv model captures the temporary price impact across time with a monotonic relation, whereas the VAR model fails to generate an impact that maintains stable and consistent monotonicity to the size of a trade or an order across time.

The remainder of this paper is organised as follows. Section 2 develops the DeRecv model. Section 3 describes the data used in the evaluation and compares the DeRecv, RL, and VAR models in price impact estimation. Finally, section 4 summarizes and concludes the study.

2 Model Construction

In this section, we first briefly outline the recursive framework proposed in Philip (2019). We then develop our model and show that our model addresses the existing problems and fills the gaps discussed in Section 1.1.

2.1 Recursive framework

This work follows the recursive framework proposed in Philip (2019) for estimating the price impact: the permanent price impact $Q^*(s,a)$ of a trading action a on the market s can be modelled as

$$Q^*(s,a) = R(s,a) + \gamma \sum_{s' \in S} \sum_{\bar{a} \in A} P(s,a,\langle s',\bar{a}\rangle) Q^*(s',\bar{a}) \tag{1}$$

where the $R(s,a)$ represents the immediate impact after taking action a in s; the γ is the discount factor; the s' is the subsequent market after taking the action a in s; the $\bar{a}$ is the following action induced by a after the market has been transited from s to s'; and the market transition probability can be represented as $P(s,a,\langle s',\bar{a}\rangle)$. The permanent price impact $Q^*(s,a)$ of taking action a in s equals the immediate response of this action plus the sum of the expected permanent impact of all subsequent actions induced by the first action a. The A defines all possible actions that can be immediately induced by the first action a. The framework in equation 1 is originally defined in the work of Philip (2019). We follow this framework but revise the definition of all variables, i.e., a, s, $R(s,a)$, and $P(s,a,\langle s',\bar{a}\rangle)$.

In Philip (2019), market s is defined by the depth imbalance only (i.e. +DI and -DI) and the action a is defined by large or small trade. In this study, we modify the definition of the market s as a four dimension vector. As action a includes limit or market order, and is occurred asynchronously, we use the order time rather than clock time in modelling and estimating the price impact following Hautsch and Huang (2012).

$$s \equiv [p_t^a, p_t^b, d_t^a, d_t^b]^\intercal \tag{2}$$

where the p_t^a and p_t^b represent the best ask and bid price at order time t respectively, and d_t^a and d_t^b represent the depth at the best ask and bid level at order time t, respectively. As a result of this modification, in our model, the market represents the true condition of the limit order book at order time t. Table 1 shows examples of limit order book of stock Google, each of which represents a market at t.

We also modify the definition of the action and distinguish a from $\bar{a}$ — a represents the participant's action in market s and $\bar{a}$ denotes the market's reaction (induced following actions) observed by the participant. We define the action a as a real limit or market order with any price or size. We define it by the price p_t and volume $\pm v_t$ of the order

$$a \equiv [p_t, \pm v_t]^\intercal \tag{3}$$

where $+$ and $-$ indicate buy and sell orders, respectively and $[p_t = 0, \pm v_t]^\intercal$ represents the market order.

When an order a is placed at the market s, the market transits to a new one s'. As we use the order time t to represent the market and order, we can also represent the market as $s_t \equiv [p_t^a, p_t^b, d_t^a, d_t^b]^\intercal$ and the order as $a_t \equiv [p_t, \pm v_t]^\intercal$. The s_t represents the market status immediately before the order a_t at t. Therefore the new market s' equals to s_{t+1}. The s_{t+1} is calculated by the order book matching rule in O'Hara (1997); SEC (2000).

The permanent impact is defined as a new permanent level at which the information of the incoming order is entirely absorbed by the market (Hasbrouck (1991)). The temporary price impact at time t_n is defined as the sum of expected immediate impacts n order time after placing an order a_t at t, where $n = 1, ...N - 1$, and $t_N = t + N$ is the order time at which the price impact reaches the permanent level. A placed order a generates a sequence of temporary price impacts and a final permanent impact. The temporary impacts show how the market responds to the placed order across time and how those responses converge to the final permanent level.

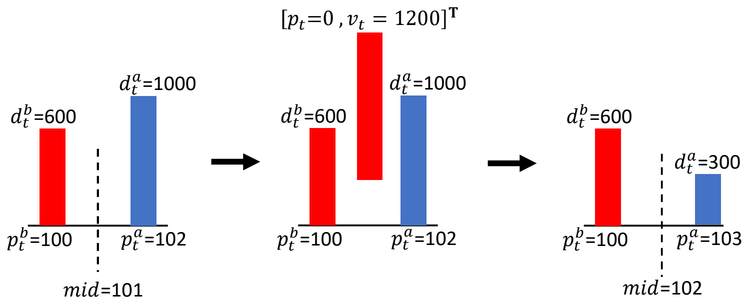
Once an order is placed in the market, other participants might be induced to submit subsequent orders as a reaction. The reacting order contains arbitrary price and size and takes infinite possible formats, which follow a nonstationary stochastic process (Hautsch and Huang (2012); O'Hara (2015)). Therefore, estimating how likely an ad-hoc order is induced at a specific market s generates an unsound result. Instead, the reacting order can be categorised to represent several types. The probabilities of those types being induced lead to an overall trend of how participants react to market actions. Accordingly, we group all orders that are possibly induced by a market action into eight primary classes and estimate the probability of each class. Because we measure an expected overall impact of a specific order, an estimation of how likely certain classes of an order may be induced after the market s has been transited to s' by an action a provides a straightforward and measurable outcome. Accordingly, we define the market response action $\bar{a}^{(i)}$, i=1,...,8 as eight primary classes/types:

1. $\bar{a}^{(1)}$: buy limit order at current bid price p_t^b;
2. $\bar{a}^{(2)}$: buy limit order with price $p_t \in (p_t^b, p_t^a]$;
3. $\bar{a}^{(3)}$: sell limit order at current ask price p_t^a;
4. $\bar{a}^{(4)}$: sell limit order with price $p_t \in [p_t^b, p_t^a)$;
5. $\bar{a}^{(5)}$: large buy market order;
6. $\bar{a}^{(6)}$: small buy market order;
7. $\bar{a}^{(7)}$: large sell market order; and
8. $\bar{a}^{(8)}$: small sell market order.

As shown by Hautsch and Huang (2012), the aggressiveness of the price significantly determines the impact of a limit order. We group the limit orders as classes #1-4 mainly by their prices. Following Philip (2019), the market orders are grouped as classes #5-8 mainly by their sizes. As a result of this modification, in our model, the market participant submits order $a = [p_t, \pm v_t]^\intercal$ in market $s = [p_t^a, p_t^b, d_t^a, d_t^b]^\intercal$, which transits the market to state s', where the participant primarily observes eight classes of action $\bar{a}$ with probability $P(s, a, \langle s', \bar{a}\rangle)$. In our model, the size of a particular type of order is measured by the average daily volume (ADV) of this order type.

We use a simple example to show the process of estimating the immediate price impact by the modified RL framework. The initial market is $s_t = [p_t^a = 102, p_t^b = 100, d_t^a = 1000, d_t^b = 600]^\intercal$ and a participant's action is a market buy order with size 1200 $a_t = [p_t = 0, v_t = 1200]^\intercal$. As the buy order consumes all depth at the best ask level, the best ask price moves up to the second best ask level. We assume the new market status can be calculated as $s_{t+1} = [p^a_{t+1} = 103, p^b_{t+1} = 100, d^a_{t+1} = 300, d^b_{t+1} = 600]^\intercal$. Thus, the mid-point price increases from 101 to 102, generating a 99 bps immediate impact. Figure 1 illustrates the transition from market s_t to s_{t+1} (equivalent to s' in equation (1)) by taking action a_t.

Figure 1: This figure shows an example of a participant taking an action of market buy order $a_t = [p_t = 0, v_t = 1200]^\intercal$ in the market $s_t = [p^a_t = 102, p^b_t = 100, d^a_t = 1000, d^b_t = 600]^\intercal$



Later, eight types of subsequent actions might be induced with different probabilities as illustrated in Figure 2. Thus, the permanent impact of a market buy order a_t is the sum of the immediate impact of a_t and the permanent impact of the eight classes of subsequent orders.

Figure 2: This figure shows that eight types/classes of subsequent reactions are induced after the market s_t has been transitioned to s_{t+1} by action a_t. LO: Limit Order; MO: Market Order

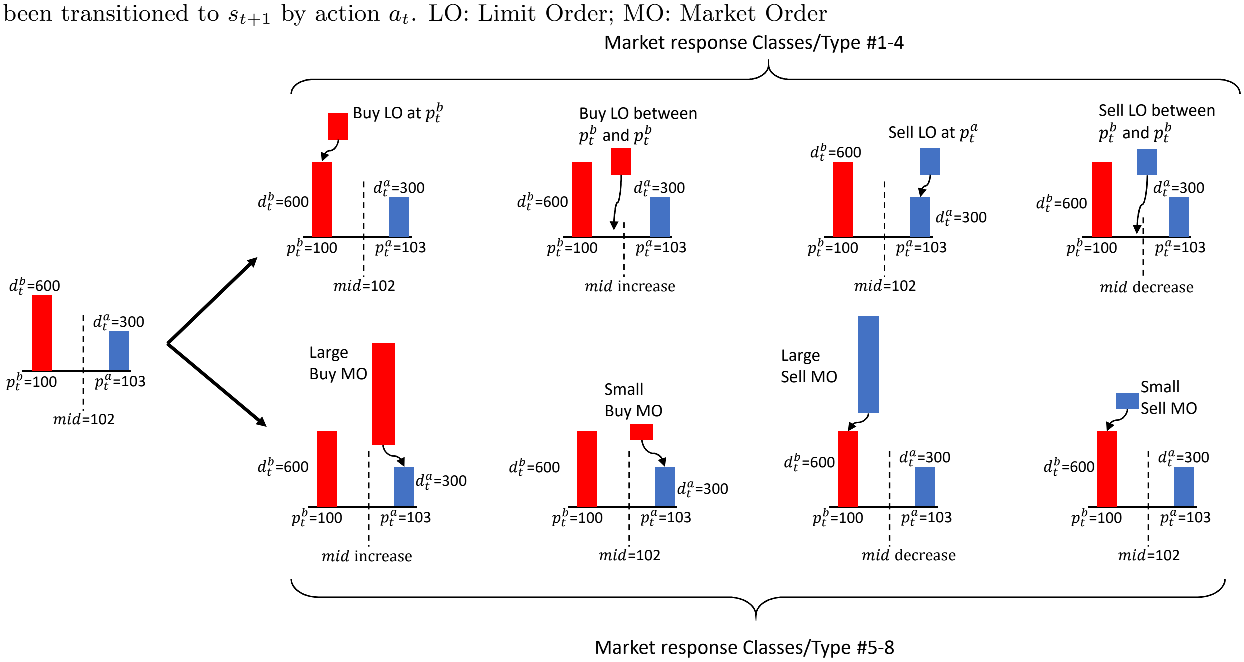
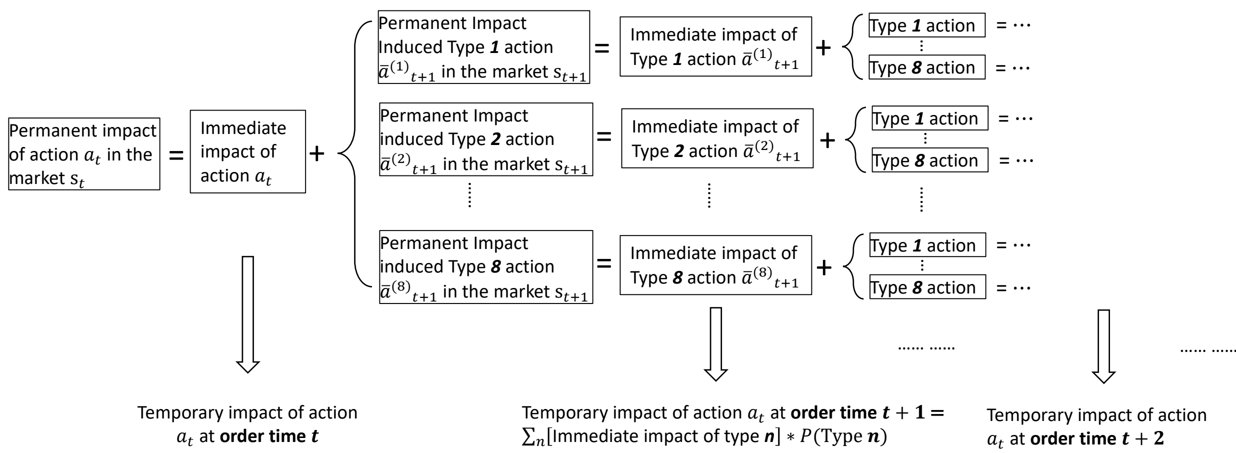


Figure 3 depicts the recursive process of calculating both the permanent impact and temporary impact. The temporary impact at order time t is equal to the immediate impact of the action

a_t. Later, eight types of subsequent orders are induced in the market s_{t+1}. Thus, the temporary impact at order time $t+1$ is the sum of the immediate impacts of the eight types of actions multiplied by their probability of occurrence. The temporary impact at order time $t+2$ is the sum of the immediate impacts of 8×8 types of actions multiplied by their probability of occurrences, respectively.

Figure 3: This figure shows the calculation process of the permanent and temporary impacts of action a_t. The permanent impact of a_t is the immediate price impact induced by a_t plus the permanent impact of the eight induced subsequent actions multiplied by their probability of occurrences. The temporary impact is the sum of expected immediate impacts n order time after placing an order, where $n = 1, ...N-1$, and N is the order time at which the price impact reaches the permanent level.



2.2 Modelling transition probability

After defining the market s, actions a_t and $\bar{a}$, and the computation process of permanent and temporary price impacts, the remaining parameter of equation (1) to be estimated is the transition probability $P(s, a, \langle s', \bar{a}\rangle)$. As we define eight primary market responses, the transition probability can be rewritten as $P(s_t, a_t, \langle s_{t+1}, \bar{a}^{(i)}_{t+1}\rangle)$, where i=1,...,8. As the state s_{t+1} can be directly calculated by adding action a_t to s_t as illustrated in Figure 1, the probability of transiting from tuple $\langle s_t, a_t\rangle$ to tuple $\langle s_{t+1}, \bar{a}^{(i)}_{t+1}\rangle$, $P(s_t, a_t, \langle s_{t+1}, \bar{a}^{(i)}_{t+1}\rangle)$, can be further simplified as the probability of inducing type i subsequent action $\bar{a}^{(i)}_{t+1}$ after taking action a_t in market s_t, $P(s_t, a_t, \bar{a}^{(i)}_{t+1})$. Inspired by the application of deep reinforcement learning in optimal execution (Ning et al. (2018)), we model the probability $P(s_t, a_t, \bar{a}^{(i)}_{t+1})$ by a deep neural network.

As $P(s_t, a_t, \bar{a}^{(i)}_{t+1})$ represents the probability of induced subsequent action type i, it can be formulated as a probabilistic interpretation of a classification problem following the early study of Bridle (1990). We consider eight primary responses at order time $t+1$ as eight output classes of our model. Given the input of current market s_t and action a_t, the model outputs the likelihood of each reaction $a_{t+1}^{(i)}$, i=1,...,8. Instead of generating a single class label as the traditional classification problem, our DNN model produces the probabilities of eight classes being induced after taking action a_t in market s_t. Therefore, we term it as probabilistic DNN (pDNN). For example, in Figure 1, if one participant takes market buy order $a_t = [p_t = 0, v_t = 1200]^\intercal$ in the market $s_t = [p_t^a = 102, p_t^b = 100, d_t^a = 1000, d_t^b = 600]^\intercal$, other participants may follow the trend of a_t; thus, the probability of following buy (or sell) actions might be increased (or reduced). Therefore, the output of our model might be given as $y = [0.1499, 0.3499, 0.0001, 0.0001, 0.3499, 0.1499, 0.0001, 0.0001]^\intercal$, which indicates that the probability of class 2 and 5 (aggressive buy action) is 0.3499, the probability of class 1 and 6 (normal buy action) is 0.1499, and the probabilities of sell orders are all 0.0001.

2.2.1 Network architecture

Following the network architecture in Ning et al. (2018), we use a fully connected feed-forward deep neural network with four hidden layers with 20 neurons in each of the first three hidden layers and 8 neurons in the last hidden layer, which connects the output softmax layer. Each neuron in the hidden layer contains a ReLU activation function to prevent vanishing gradients and provide sparsity. The network takes the market $s_t = [p_t^a, p_t^b, d_t^a, d_t^b]^\intercal$ and action $a_t = [p_t, v_t]^\intercal$ at t as input and outputs the probabilities of induced eight classes of actions at $t+1$ via the Softmax layer.

The softmax function in each neuron in the output layer is defined as

$$p(y = j|\mathbf{x}) = \frac{e^{(\mathbf{w}_j^T\mathbf{x}+b_j)}}{\sum_{k=1}^{8} e^{(\mathbf{w}_k^T\mathbf{x}+b_k)}}. \tag{4}$$

The softmax layer has the same length as the connected hidden layer, takes all neurons k=1 to 8 from the previous hidden layer as input, and calculates the probability of the output being one of the j classes, j=1 to 8. With the softmax function at the output layer, we follow the study of Bridle (1990) to use the cross-entropy function as the cost function for optimizing the whole network. The cross-entropy function is defined as

$$H(y, \hat{y}) = -\sum_{k=1}^{8} y_k \log \hat{y}_k \tag{5}$$

where y_k and $\hat{y}_k$ are the actual and predicted probabilities of each output. The cross-entropy cost function is the sum of the products of all actual probabilities with negative log of the predicted probabilities. In multi-class classification, the cross-entropy function outperforms the traditional mean squared error function with softmax activation function at the output layer (Bridle (1990)).

2.2.2 Training the network

To train the pDNN, we implement the model using Keras (Chollet (2016)). Following the works of Mnih et al. (2013); Chollet (2016); Ning et al. (2018), we use RMSprop optimizer for training the model. We apply stochastic dropout regularisation following Gal and Ghahramani (2016) with a low dropout value of 0.1. Next, we randomly drop 10% of the input units in each iteration during the training process for reducing the overfitting risk and better regularisation. Furthermore, we split the training data into two parts: 80% as the training part and 20% as the validation part. The first part is to train the network and tune the parameters for minimising the loss function. After passing across all samples in the first part (one epoch), the trained network predicts the unknown samples from the validation part. Once the validation loss does not decrease for a predefined period (patience period), the training is stopped, and the model is saved. We select a maximum training duration of 1000 epochs and early stopping patience of 10 following the studies of Fischer and Krauss (2018).

To prepare the training dataset, we combine the data of the market s_t and the action a_t at order time t as an input vector $[p_t^a, p_t^b, d_t^a, d_t^b, p_t, \pm v_t]^\intercal$ for pDNN. We bucket the actions that are immediately taken after a_t to eight classes as the output targets of pDNN. Thus, the pDNN is defined as a multi-class classification model to predict the probabilities of eight types of immediate responses after the action a_t at market s_t. Table 1 shows an example of organising market action and limit order book data as input features and output targets of pDNN model. In this example, we use high-frequency data of Google stock after the market opening on 21 June 2012. The column 'Time' shows the seconds after the market opening (9:30 am). The columns 'p_t^a' and 'p_t^b' depict the best ask and bid prices, respectively and the columns 'd_t^a' and 'd_t^b' are the depth at the best ask and bid prices. The columns 'p_t' and '$\pm v_t$' represent the price and shares of the action a_t. A positive (or negative) 'v_t' indicates a sell (or buy) order. A zero 'p_t' represents a market order. The 'Input Features' represents the input vector $[p_t^a, p_t^b, d_t^a, d_t^b, p_t, \pm v_t]^\intercal$. The 'Output Class' shows the type of immediate response after taking the current action. For example, at time t=0.0599 (0.0599 seconds after the market opening), the action a_t is a sell order with price 579.4 and 300 shares, which consumes 300 shares of depth at the best bid price level. The action a_{t+1} immediately after a_t is at the time 0.1132: a sell order with price 579.40 and 167 shares, which is a class 4 action:

sell limit order with price $p_{t+1} \in [p_t^b = 579.40, p_t^a = 580.23)$. Similarly, the action a_{t+2} immediately after a_{t+1} is at the time 0.1542: a buy order with price 579.40 and 100 shares, which is a class 1 action: buy limit order at current bid price $p_{t+1}^b = 579.40$. Hence, we construct the market s_t and action a_t at each order time t as the input feature, and the class of the response action a_{t+1} at each order time $t+1$ as the output target.

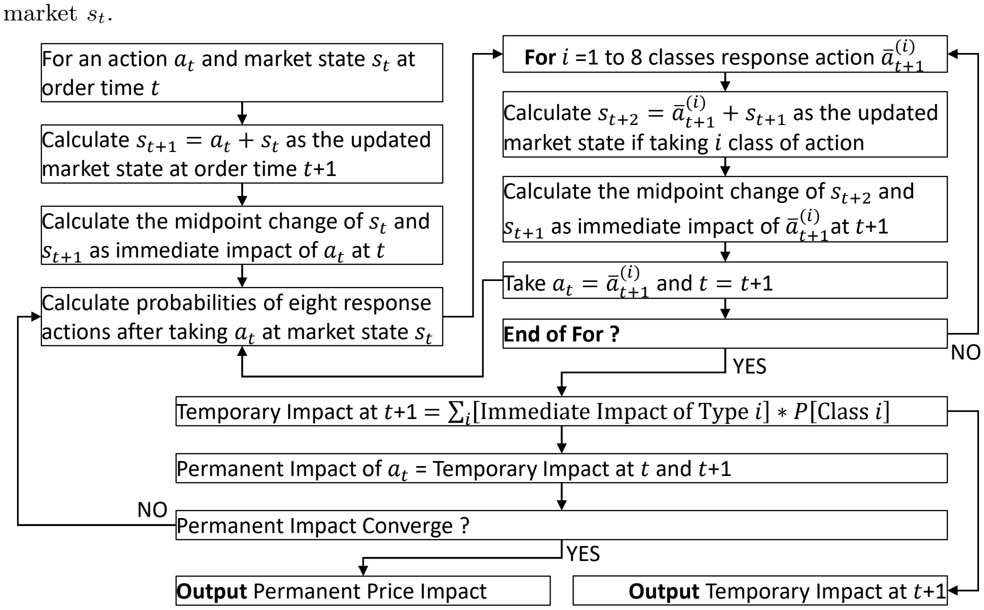
Table 1: This table shows examples of market s_t (via limit order book) and action a_t after market opening on 21 June 2012. Column 'Time' shows the seconds after market opening (9:30 am). The 'Output Class' is determined by the order class at next order time.

	market s_t					Action a_t		Input Features	Output Class
ID	Time	p_t^a	p_t^b	d_t^a	d_t^b	p_t	$\pm v_t$	$[p_t^a, p_t^b, d_t^a, d_t^b, p_t, \pm v_t]^\intercal$	$\{1, ..., 8\}$
1	0.0599	580.23	579.40	100	496	579.40	-300	[580.23,579.4,100,496,579.4,-300]	4
2	0.1132	580.23	579.40	100	196	579.40	-167	[580.23,579.4,100,196,579.4,-167]	1
3	0.1542	580.23	579.40	100	29	579.40	100	[580.23,579.4,100,29,579.4,100]	2
4	0.2023	580.23	579.40	100	129	579.89	25	[580.23,579.4,100,129,579.89,25]	2
5	0.2023	579.89	579.40	25	129	579.95	50	[579.89,579.4,25,129,579.95,50]	3
6	0.2023	579.89	579.40	25	129	579.89	-50	[579.89,579.4,25,129,579.89,-50]	3
7	0.2023	579.89	579.40	25	129	579.89	-60	-	-

2.2.3 Summarised procedures and an illustrative example

We illustrate the procedure of estimating the permanent and temporary price impacts in Figure 4 and summarise the transition probability estimation via a pseudo code.

Figure 4: This figure shows procedures of estimating the permanent and temporary price impacts of action a_t at market s_t.



Pseudo code of modelling transition probability
Model Training
1. Given full limit order book (LOB): Construct Input Feature and Output Class in Table 1 as the training dataset
2. Train the pDNN model by the constructed training dataset
3. Calculate Average Daily Volume (ADV) of class i of responses, i=1,. . . ,8
Probability Estimation
1. For a new action a_t and market s_t, construct Input Feature
2. Feed Input Feature to trained pDNN model and generate probabilities of responses of the eight classes

We show an example using the full order book data of Google stock (symbol 'GOOG') on 21 June 2012 to illustrate the estimation procedure. The data contains 112,673 records from 9:30 am to 4:00 pm, including every order submission, execution, and deletion. We organise the raw data as the examples in Table 1 and randomly select 90,140 records (80%) to construct the dataset to train the pDNN model. The pDNN model is implemented using Keras (Chollet (2016)) and is trained on dual-core Xeon e5 CPU with 128GB RAM. To construct the RL framework in equation (1) and Figure 3, we calculate the ADV of each class of order based on the training set of Google stock, as shown in Table 2.

Table 2: This table shows the ADV of eight classes of orders based on the full order book of Google stock on 21 June 2012. The definition of eight classes is in Section 2.1

	class 1	class 2	class 3	class 4	class 5	class 6	class 7	class 8
ADV	89.4913	70.2024	72.1074	67.2667	78.8095	79.4844	77.7972	72.7208

After training the pDNN model, we use the following example to illustrate the procedure of estimating the permanent and temporary price impacts. In this example, the input feature of the pDNN model is $[p^a_t, p^b_t, d^a_t, d^b_t, p_t, \pm v_t]^\intercal$=$[569.91, 569.75, 30, 1000, 569.91, -20]^\intercal$, where a sell limit order is placed at the best ask price. Therefore, the updated market is s_{t+1}=$[569.91, 569.75, 50, 1000]^\intercal$. The mid-point changes when order time t is 0. We feed the input feature to the trained pDNN and receive the probabilities of eight classes of market responses, as shown in the Table 4.

Table 3: This table shows a random example of the market and an action: a sell limit order with 20 shares and price 569.91 submitted to the limit order book at 10961.6729 seconds after market opening at 9.30 am. The current best bid and ask prices are 569.75 and 569.91, respectively. We use this illustrative example to show the procedure of estimating the price impact.

market s_t					Action a_t		Input Features
Time	p^a_t	p^b_t	d^a_t	d^b_t	p_t	$\pm v_t$	$[p^a_t, p^b_t, d^a_t, d^b_t, p_t, \pm v_t]^\intercal$
10961.6729	569.91	569.75	30	1000	569.91	-20	[569.91,569.75,30,1000,569.91,-20]

We can observe the intuition from the generated probabilities: after a sell limit order a_t, the probabilities of a further sell order (class 3, 4, 7, and 8) are higher than a buy order (class 1, 2,

Table 4: This table shows the probabilities of eight classes of responses generated by the pDNN model based on the input feature $[569.91, 569.75, 30, 1000, 569.91, -20]^\intercal$. The model is trained by the full limit order book of Google stock on 21 June 2012.

	class 1	class 2	class 3	class 4	class 5	class 6	class 7	class 8
Probability	0.1034	0.1057	0.1908	0.1626	0.0807	0.0840	0.1362	0.1366

5, and 6). Using the market update rule by Figure 1, we can calculate the immediate impact of taking each of the eight actions following the order matching rule in O'Hara (1997); SEC (2000). For example, if taking class 2 action (buy limit order with price $p_t \in (p_t^b, p_t^a]$), the new action a_{t+1} is constructed as $[569.91, 70.2024]^\intercal$. The updated market at $t+2$ is as:

$$\begin{aligned}
s_{t+2} &= s_{t+1} + a_{t+1} \\
&= [569.91, 569.75, 50, 1000]^\intercal + [569.91, 70.2024]^\intercal \\
&= [569.91, 569.75, 70.2024 - 50, 1000]^\intercal \\
&= [569.91 + 0.060236, 569.75, 20.2024, 1000]^\intercal \\
&= [569.97, 569.75, 20.2024, 1000]^\intercal
\end{aligned}$$

The mid-point change is 0.060236/2=0.030118. Similarly, we can calculate the mid-point change for all eight classes. Hence, we update the Temporary Impact (TI) in equation (1) and Figure 3 as

$$\begin{aligned}
TI(s_t, a_t) = 0 + 0.99(&P(class1)Q(s_{t+1}, class1) + P(class2)Q(s_{t+1}, class2) \\
&+ P(class3)Q(s_{t+1}, class3) + P(class4)Q(s_{t+1}, class4) \\
&+ P(class5)Q(s_{t+1}, class5) + P(class6)Q(s_{t+1}, class6) \\
&+ P(class7)Q(s_{t+1}, class7) + P(class8)Q(s_{t+1}, class8)) \\
&= 0 + 0.99(0.1034 \times 0 + 0.1057 \times 0.030118 + 0.1908 \times 0 + 0.1626 \times 0 \\
&+ 0.0807 \times 0 + 0.0840 \times 0 + 0.1362 \times (-0.02854) + 0.1366 \times 0) \\
&= -0.000697
\end{aligned}$$

The first iteration provides a 0.000697 decrease of the bid-ask mid-point. It shows that although a small sell order brings zero immediate impact on the mid-point, after one order time, the following actions from other participants push down the mid-point by a tiny magnitude. The second iteration is based on eight market actions. We feed eight input features $[s_{t+1}$, action class i$]^\intercal$ to pDNN model and generate the probability of 8×8=64 market responses. We show the iterations in Table 5. After iteration 53, the permanent price impact estimation does not change to a precision of 6 decimal places for further iterations. Therefore, the permanent price of a 20-share sell limit order placed at the best ask price 569.91 generates a -0.001408 impact on the bid-ask mid-point. The result is consistent with our expectation, that is, a small sell limit order placed at the ask price generates a small negative price impact.

Table 5: Estimation of the permanent price impact after each iteration.

Iteration 1	Iteration 2	Iteration 3	Iteration 4	Iteration 5	……	Iteration 50	Iteration 51	Iteration 52	Iteration 53
-0.000697	-0.000711	-0.000746	-0.000801	-0.000837		-0.001407	-0.001408	-0.001408	-0.001408

3 Model evaluation

We compare the performance of estimating temporary and permanent price impacts using our proposed DeRecv model with RL model in Philip (2019) and traditional VAR model in Hautsch and Huang (2012).

3.1 Data

We use high frequency full order book data from the NASDAQ exchange market, and the Shanghai Stock exchange (SSE) market for the 21 June 2012 to 18 July 2012 time period. From NASDAQ, we select the stocks of the five most popular companies: Google, Microsoft, Apple, Amazon, and Intel. From SSE, we select the five largest stocks based on market value. Table 6 shows the summary statistics for the 10 stocks. The data contains the submission and trade of every order. The stock selection is based on two reasons. First, NASDAQ is a mature exchange market, and the five selected stocks are highly liquid. Second, SSE, though relatively new, is ranked fourth worldwide by market capitalisation and monthly trading volume. Jiang et al. (2019) show that the price impact on SSE market is more significant than well-developed markets. An evaluation based on high frequency data in both the well-developed market and emerging market brings a thorough performance comparison of the proposed model across distinctive market mechanism and conditions.

We compare the performance of our proposed DeRecv model with the RL model of Philip (2019) and the VAR model of Hautsch and Huang (2012) using two groups of experiments. The first group compares the performance of estimating permanent price impacts via DeRecv and RL model. The

Table 6: This table shows the summary statistics for the ten sample stocks. The sample contains 20 trading days from 21 June 2012 to 18 July 2012. 'No. Obs.' is the number of order and trades. 'Price' is the averaged executed price in dollar and RMB. 'Spread' is the average difference between the bid and ask. Size is the number of shares of the order and trade. The 'Avg. size.', 'Med. size', and 'Max size' are the average, median, and maximum share, respectively.

	Stock	No. Obs.	Price	Spread	Avg. size.	Med. size	Max size
NASDAQ	GOOG	2563365	570.84	0.30011	62.88	100	2300
	AAPL	2206348	583.27	0.15650	88.37	100	15000
	AMZN	2779675	222.82	0.13483	84.72	100	33570
	INTC	2905150	27.05	0.01246	477.30	300	82396
	MSFT	2979000	30.55	0.01253	544.69	300	200000
SSE	PetroChina (601857)	1883970	43.53	0.02802	269.76	295	7130938
	Kweichow Moutai (600519)	1895554	88.26	0.11928	557.07	250	60234
	SAIC Motor (600104)	1555686	8.54	0.01204	579.44	407	796157
	Yangtze Power (600900)	1624788	10.08	0.01208	880.74	693	519050
	Daqin Railway (601006)	1793484	8.51	0.01151	1432.43	981	1781252

second group compares the performance of estimating temporary price impacts via DeRecv and VAR model.

3.2 Comparative Evaluation: Permanent Price Impact

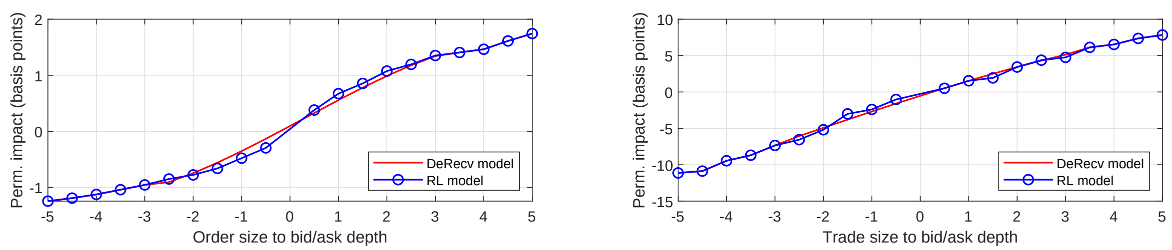
3.2.1 Different Orders on Same Market

We first estimate the permanent price impact of limit orders with different sizes on the same market. The evaluation is based on our proposed DeRecv model and RL model of Philip (2019) on the selected stocks. We compare the estimated permanent price impact and illustrate the results in Figure 5. In Panel A, the permanent price impact of buy and sell limit order placed at the best bid and ask level is estimated by DeRecv and RL models. The order size is represented as a ratio of shares to the depth at the best bid and ask level, respectively. For example, if the depth at the best bid is 100 shares, the size of a buy order placed at the best bid price with 200 shares can be represented as $200/100 = 2$. The permanent impact of such a buy order is 1 basis point. The sell order is represented with a negative sign. Similarly, a sell order with 200 shares can be represented as $-200/100 = -2$ and generates -0.7 basis point permanent impact. It is clear from Figure 5 Panel A that the DeRecv and RL models generate a similar relation between the order size and the permanent price impact. The relation curve generated by the DeRecv model is slightly smoother than the one by the RL model. In Figure 5 Panel B, the relation between the trade size and permanent price impact is closer to a linear relationship. In this example, the DeRecv model captures a smoother linear curve while the RL model shows a relation with tiny noise.

Figure 5 Panel B shows another example of permanent price impact estimation via DeRecv and RL models using the data of Petro China stock from the SSE. Its permanent impact shows a different pattern to Google in NASDAQ. The buy (or sell) limit order of Petro China stock generates weaker (or stronger) impact than the ones of the Google stock. However, the buy (or sell) trade of Petro China stock induces stronger (or weaker) impact than the ones of the Google stock. This result conforms to the previous studies of price impact on emerging markets (e.g. Jiang et al. (2019)). Similar to the result in Panel A, a monotonic relation between trade size and the magnitude of the permanent price impact is captured by the DeRecv and RL models. We can observe from Panel B that the DeRecv model shows a smoother relation compared to the curve estimated by the RL model, which generates a more discrete and non-smooth relationship. The right column in Panel B shows a clearer comparison. The curve estimated by the RL model appears not as smooth as the one by the DeRecv model. We assume the reason behind it is that an oversimplified discrete market in RL model brings a piece-wise discrete curve. To investigate the reason, we zoom-in Figure 5 and conduct a further experiment by order (or trade) size with small intervals.

Figure 5: This figure shows the permanent price impact of limit order (left column) and trade (right column) with different sizes using the DeRecv and RL models for the data of Google (Panel A) and Petro China (Panel B) stocks from 21 June 2012 to 18 July 2012.

Panel A



Panel B

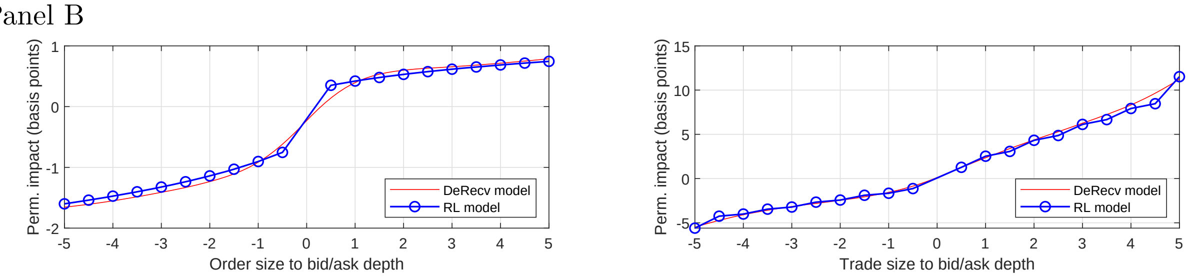
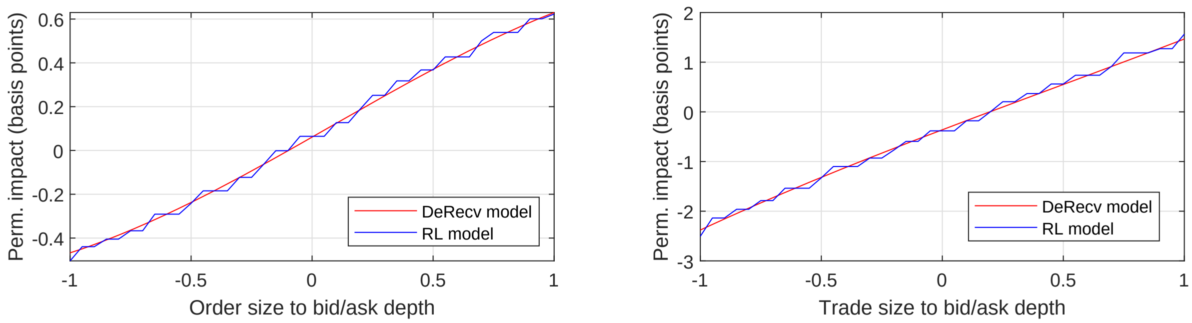


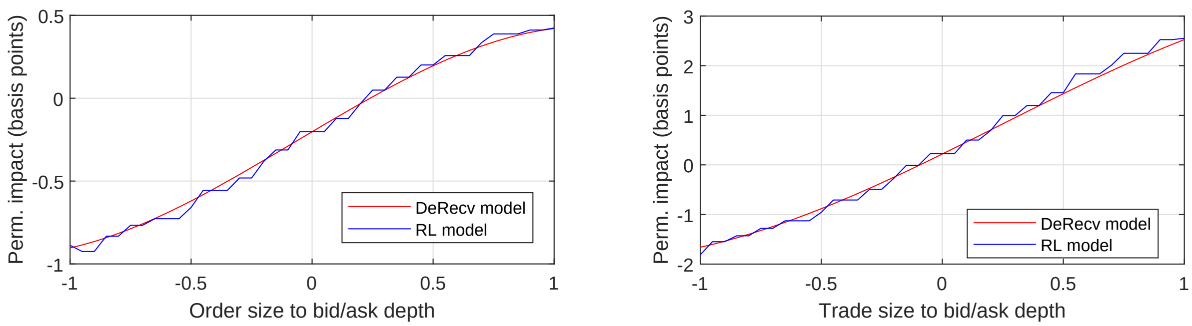
Figure 6 shows a ‘zoomed-in’ investigation of the permanent price impact estimation by the DeRecv and RL models. The x-axis is the ratio of the shares of the buy (or sell) order or trade to the depth at the best bid (or ask) level. We use 0.05 as the ratio increment in the experiments. It is evident that the DeRecv model captures a smoother and accurate relationship between the size and the permanent impact. The RL model, however, generates noisy and step-wise estimations for the two example stocks. This further shows that in the RL model, a discrete market state by depth imbalance only oversimplifies the market condition and yields a close but not smooth and non-accurate step-wise relation between the size and impact magnitude.

Figure 6: This figure shows a 'zoomed-in' illustration of the permanent price impact of limit order (left column) and trade (right column) with different sizes using the DeRecv and RL models for the data of Google (Panel A) and Petro China (Panel B) stocks from 21 June 2012 to 18 July 2012.

Panel A



Panel B



3.2.2 Same Order on Different Market

We then compare the price impact of the same order on different markets. When the market depth at bid and ask side is different, one order (or trade) generates significantly different price impact (see the work of Hautsch and Huang (2012)).

We first compare the price impact of a buy action with size 500 shares submitted to a market, where the depth of the best ask is 500, and the depth of the best bid is changing from 50 to 1000. The Figure 7 compares the price impacts of Google and Petro China stocks generated by our proposed DeRecv, RL (Philip (2019)), and VAR model (Hautsch and Huang (2012)). In theory, when the depth of the best bid increases from 50 to 1000, the permanent impact of a buy limit order decreases. In Panel A, at the bid depth size of 50 and 100, the VAR model generates impact values of 2.53 and 1.44, which shows a negative linear relation. When the bid depth size is from 150 to 500, the impact value by the VAR model remains the same of 0.61. When the bid depth size increases to 600 to 1000, the impact by VAR model remains at 0.44 as well. It is clear that when the bid depth size changes within a large range from 150 to 1000, the impact value by VAR model shows only two steps of 0.61 and 0.44. It indicates that VAR model expects the same price impact under the market change. It is apparently against the theoretical studies of Hasbrouck (1988, 1991); Parlour and Seppi (2008); Hautsch and Huang (2012). It is also a dangerous result as the price impact by VAR model is remarkably under estimated when the bid depth size is from 150 to 400. An under-estimated impact may mislead the trading strategy by over estimating the potential profit. The RL model by Philip (2019) provides even worse results as it generates only two impact values when the bid depth increases from 50 to 1000. Among three models, only the DeRecv model (red curve in Figure 7) provides a smooth monotonic relation of the impact and bid depth size. In Panel B, we observe the same pattern that the RL and VAR models generate a piece-wise impact value of a trade under market conditions with different bid depth sizes. The DeRecv model provides a monotonic and smooth estimation of the price impact corresponding to the change of the market.

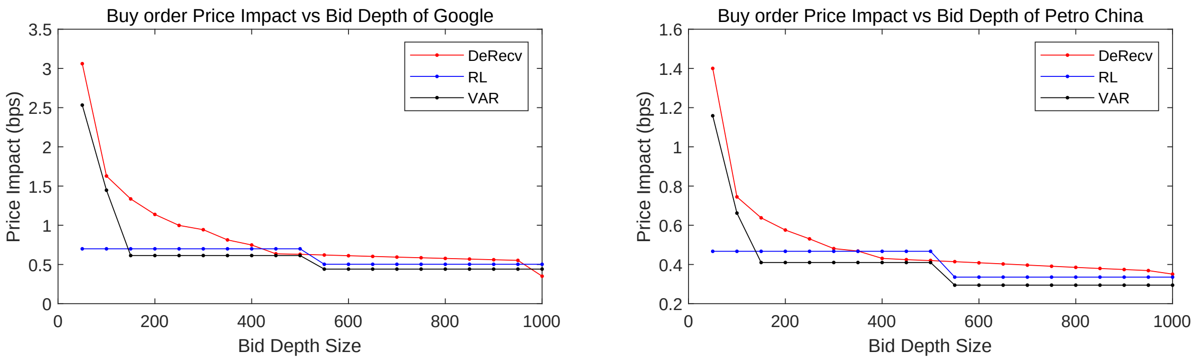
Second, we compare the price impact of a sell action with size 500 shares submitted to a market, where the depth of the best bid is 500, and the depth of the best ask is changing from 50 to 1000. In theory, a sell action pushes down the price. We observe a general relation between the impact and the market: when the ask depth size increases, the impact of a sell action decreases. However, the pattern in Figure 8 is the same as the pattern in Figure 7: the RL and VAR models estimate a piece-wise impact values across different depth sizes at ask side. Only the DeRecv model estimates a monotonic and smooth price impact according to the market change. From the Panel A and B, we can clearly observe that when the ask side depth size is between 150 to 400, the RL and VAR models under-estimate the impact of a sell action (either a sell limit order or a sell trade). The under-estimation might misdirect a trading strategy by over-estimating its potential profit.

3.2.3 Temporary Price Impact

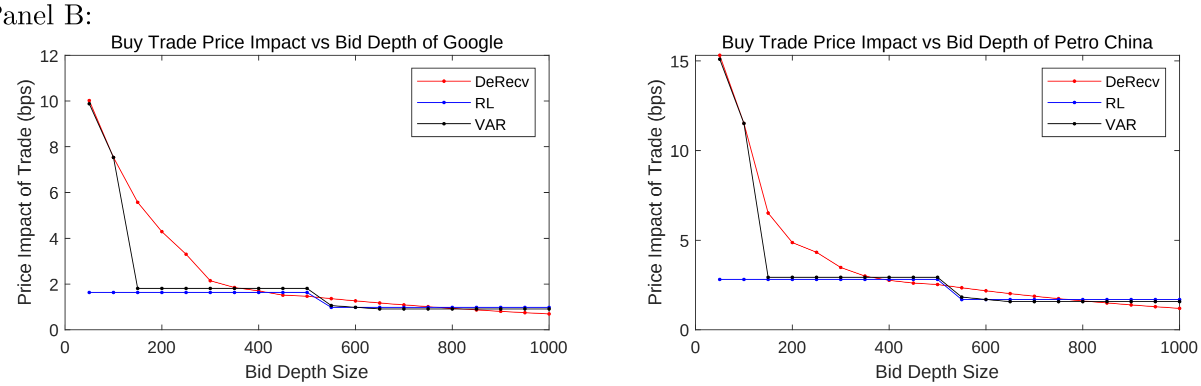
We estimate the temporary price impact using our proposed DeRecv model and the VAR model of Hautsch and Huang (2012) on the selected stocks. We show the estimated temporary price impact using the data of Google stock in Figure 9. The left column of Figure 9 shows the temporary price

Figure 7: This figure shows the price impacts of a limit buy order (Panel A) and a trade (Panel B) with 500 shares of Google (left) and Petro China (right) stocks. The impacts are estimated by DeRecv, RL and VAR models under different market conditions, where the depth change at the best bid is from 50 to 1000. The x-axis shows the bid depth size and the y-axis shows the impact in basis points. The depth size at the best ask is 500 shares.

Panel A:



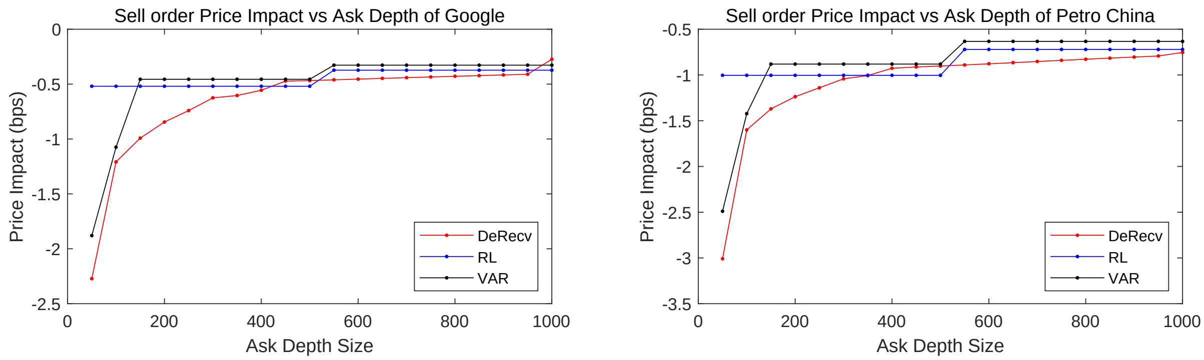
Panel B:



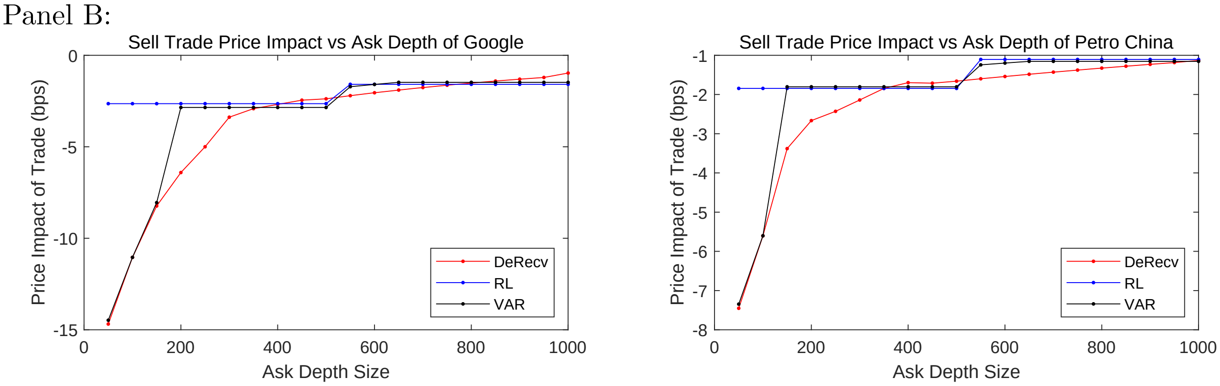
impact exerted by buy (or sell) limit order with shares equal to the depth at the best bid (or ask) level via DeRecv and VAR models, respectively. The y-axis represents the price impact in basis points. Specifically, the upper two curves show that the arrival of a buy limit order exerts positive temporary price impacts across event times (x-axis) from 0 to 50 and converges to the permanent impact of 0.685 at around the event times 45 to 50. The first difference between the DeRecv and VAR models is the smoothness of the temporary impact curve. The DeRecv generates a much smoother curve (red line) compared to the one using the VAR model (black dot-line). From an economic perspective, the temporary impact increases monotonically across time and converges to the permanent impact (Dufour and Engle (2000); Engle and Patton (2004); Hautsch and Huang (2012); Jiang et al. (2019)). Although the DeRecv model captures the monotonic relation for all event times, the temporary impact estimated by VAR model is noisy and shows some unexpected non-monotonic values from event times 8 to 15. The lower two curves show that an incoming sell limit order exerts a negative price impact and converges to the permanent impact of -0.436 at around the event times 45 to 50. Like the upper curves, the temporary impact generated via the

Figure 8: This figure shows the price impacts of a limit Sell order (Panel A) and a Sell trade (Panel B) with 500 shares of Google (left) and Petro China (right) stocks. The impacts are estimated by DeRecv, RL and VAR models under different market conditions, where the depth change at the best ask is from 50 to 1000. The x-axis shows the ask depth size and the y-axis shows the impact in basis points. The depth size at the best bid is 500 shares.

Panel A:



Panel B:

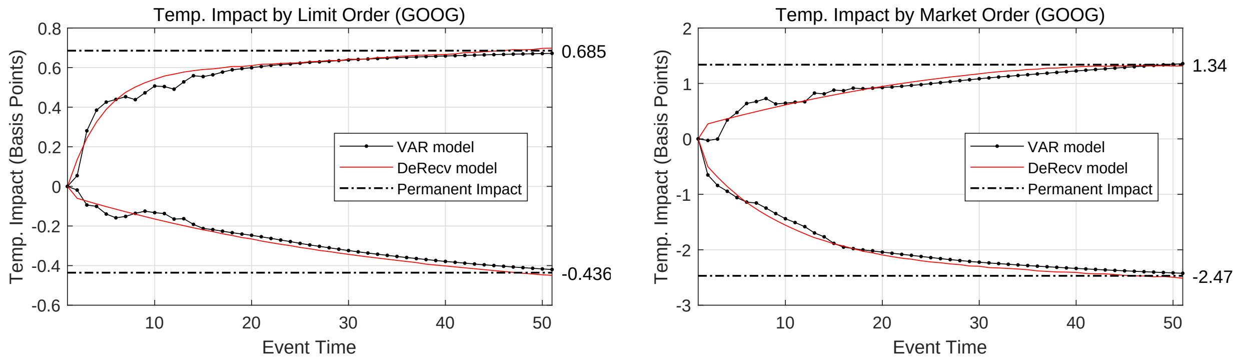


DeRecv model is unwrinkled and decreases monotonically with event time. In contrast, the curve generated by the VAR model shows some uncertain noises from event time 0 to 15 and does not recover to a smooth monotonic change across time.

The right column of Figure 9 shows the estimation of temporary impact exerted by market orders (trades). The upper two curves illustrate how the temporary price impact changes after the arrival of a market buy order with shares equal to the depth at the best bid level. Similar to the left column, the red line—generated by the DeRecv model—shows a smoother and monotonic increase of the temporary impact across the event time. However, the black dotted-line—generated by the VAR model—shows some random fluctuations in the estimated temporary price impact from event times 0 to 10. The lower two curves show identical patterns from event times 0 to 50. The two curves indicate that a sell market order exerts an immediate negative impact, and the magnitude of the impact increases across time monotonically.

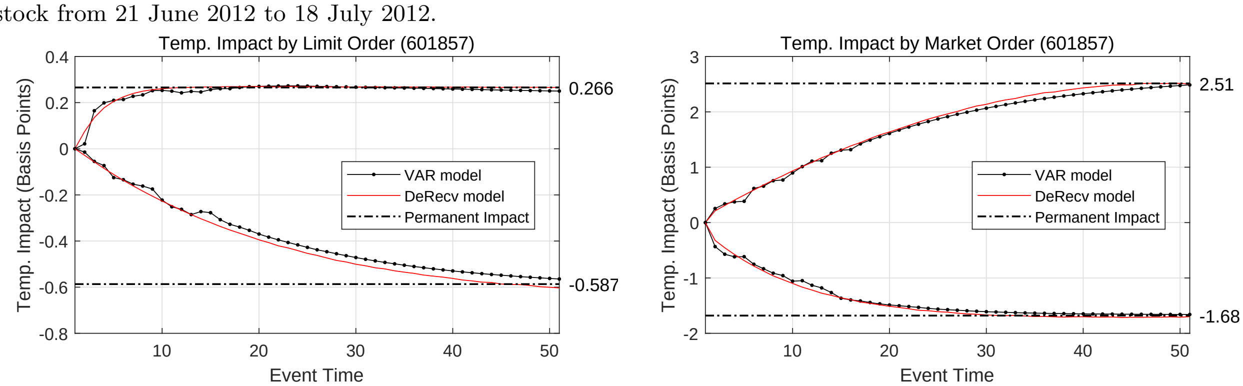
The estimation of temporary price impact using the data of Petro China stock is shown in Figure 10. The pattern in Figure 10 is similar to the example in Figure 9. The DeRecv model provides a

Figure 9: This figure illustrates the estimation of temporary price impact of buy (or sell) limit / market order with size equal to the depth at the best bid (or ask) level using the DeRecv and VAR models for the data of Google stock from 21 June 2012 to 18 July 2012.



smoother and accurate estimation of temporary price impact across time and performs consistently better relative to the VAR model. The VAR model, in contrast, provides a noisy impact, which is close but not as accurate as of the one estimated by the DeRecv model.

Figure 10: This figure illustrates the estimation of temporary price impact of buy (or sell) limit / market order with size equal to the depth at the best bid (or ask) level using DeRecv and VAR models for the data of Petro China stock from 21 June 2012 to 18 July 2012.



From the comparison, we note that the DeRecv model generates a smoother and monotonic relation of the permanent price impact to the size of the order or trade and performs better than the one using the RL model of Philip (2019). In addition, the DeRecv model produces smoother and monotonic estimation for the temporary price impact across event times and is better than the one using the VAR model of Hautsch and Huang (2012). Therefore, our proposed DeRecv model captures the non-linear relation of both the permanent price impact on the size of order or trade, and the temporary price impact across time.

We conduct a similar evaluation by using ten stocks, as illustrated in Table 6. All results confirm the same findings as examples shown in Figure 10 and 9. To conserve the space of the main body of the paper, other results are shown in the Appendix.

4 Conclusion

The evidence that the price impact has a non-linear relation to the size of the order or trade has crucial implication for estimating the temporary and permanent price impacts. In this study, we modify and develop a DeRecv model in which the permanent price impact is defined as a new permanent level at which the information of the incoming order is entirely absorbed by the market. The temporary price impact is defined as the sum of expected immediate impact n order time after placing an order, where $n = 1, ...N - 1$, and N is the order time at which the price impact reaches the permanent level. To estimate the expected immediate impact, the probabilities of the primary types of reactions are estimated using a pDNN model. The pDNN model takes the raw limit order book and the initial action as the input features and estimates the probability of certain types of reactions. Specifically, the pDNN model considers the probability forecasting as a classification problem by answering the question 'what types of reaction an order or trade may exert in an ad-hoc market condition; and what are the probabilities of those types of reaction'. We compare the performance of price impact estimation of the proposed DeRecv model, the RL model of Philip (2019), and the VAR model of Hautsch and Huang (2012). The high-frequency limit order book data is selected from the NASDAQ and SSE market for the 21 June 2012 to 18 July 2012 period. In the permanent price impact estimation, the DeRecv model captures a smoother and accurate relationship of the size and the permanent impact. The RL model, however, generates noisy and step-wise estimations. Particularly, the DeRecv model generates a monotonic, smooth, and continuous impact of the same order when the market condition changes. This is because of the continous model of the limit order book in DeRecv. In contrast, the RL and VAR model estimate discrete and step-wise impacts under different market conditions. In the temporary price impact estimation, the DeRecv model provides a smooth and accurate estimation of temporary price impact across time and is better relative to the VAR model in all experiments. The VAR model, in contrast, provides a noisy impact, which is close but not as accurate as of the one estimated by the DeRecv model. Consequently, our proposed DeRecv model solves the existing problems in price impact estimation and addresses the gaps in extant literature.

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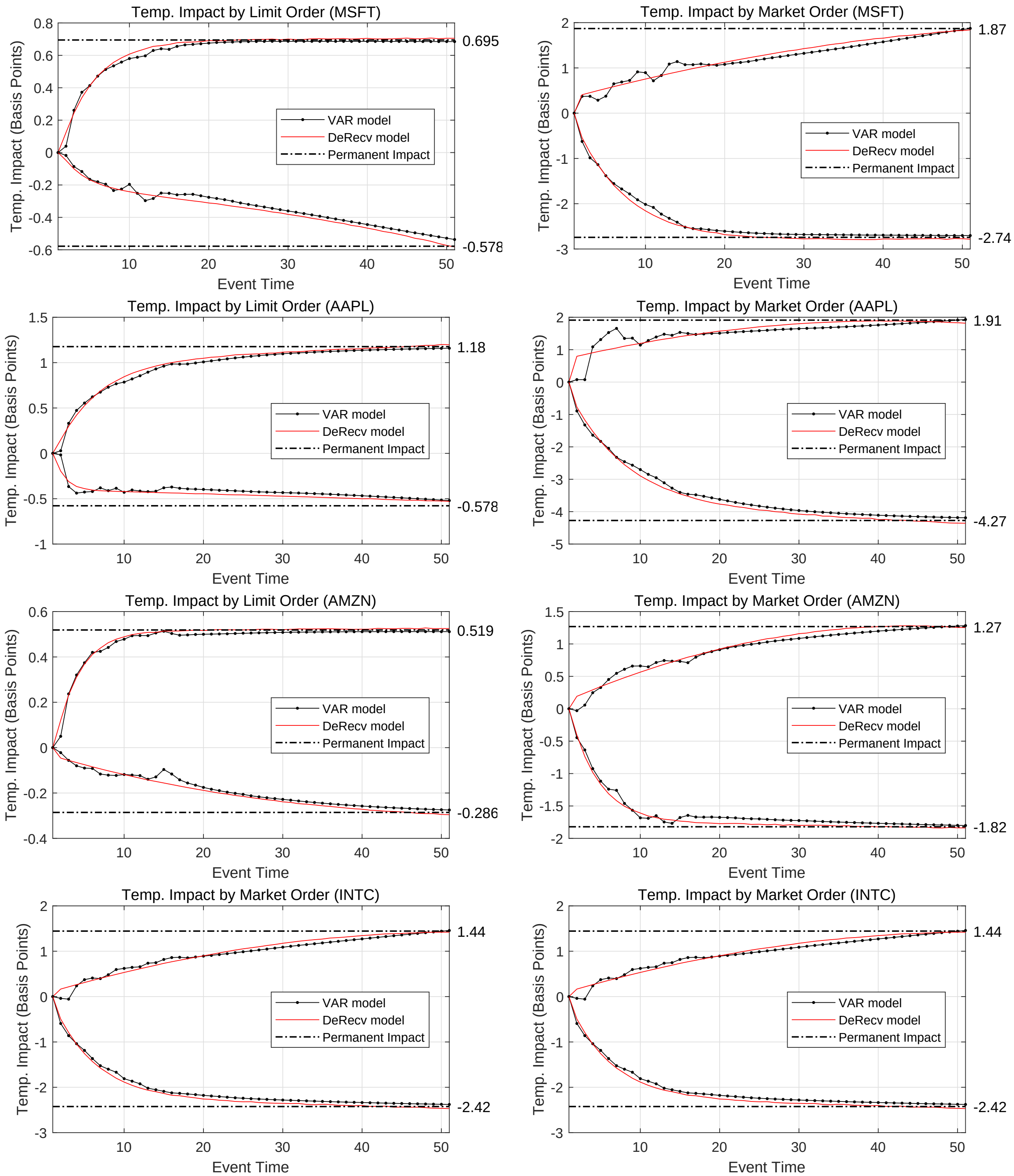
5 Appendix

The appendix contains the results of the experiments on the estimation of temporary price impact using the DeRecv and VAR models. The experiments cover eight stocks in total, four from NASDAQ and four from the SSE market. The estimation of the temporary price impact using the DeRecv model is consistently monotonic and smooth across time. However, the impact estimated by the VAR model is not smooth and contains a non-monotonic relation.

Appendix-Figure 1

This figure illustrates the estimation of temporary price impact of buy (or sell) limit order and trade with size equal to the depth at the best bid (or ask) level using the DeRecv and VAR models for the data of stocks from NASDAQ between 21 June 2012 and 18 July 2012 period. The stocks include Microsoft, Apple, Amazon, and Intel



Appendix-Figure 2

This figure illustrates the estimation of temporary price impact of buy (or sell) limit order and trade with size equal to the depth at the best bid (or ask) level using the DeRecv and VAR models for the data of stocks from SSE between 21 June 2012 and 18 July 2012 period. The stocks include 600104 (SAIC Motor), 600519 (Kweichow Moutai), 600900 (Yangtze Power), and 601006 (Daqin Railway)