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Impact of LBL Calibration on the Accuracy of Underwater Localization

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Abstract: The use of Long Baseline (LBL) systems is quite consolidated in the underwater domain, especially within applications where it is important to precisely localize submerged devices close to the sea bottom. Indeed with a LBL acoustic array the nominal positioning accuracy for seabed applications results to be not dependent on the depth and almost constant at any point inside the area delimited by the transponders. Despite the above advantages, the achievable accuracy of LBL systems is actually affected by different factors, mainly related with technical limits of the used instruments and with the level of knowledge of the physical characteristics of the acoustic medium. Another important element, possibly reducing the precision, concerns the way the LBL system is operated, and is related with the calibration of the acoustic array after its deployment on the sea bottom. Indeed if the positions of all the transponders are not perfectly known, errors in the localization procedure unavoidably arise. The paper specifically focuses on this last aspect and investigates the linkage existing between the error on the position of transponders and the resulting error in the localization procedure. A detailed theoretical analysis of the problem is proposed and for some basic transponders geometries a closed form relationship is obtained. Some simulations are finally reported to support the achieved results.

Keywords: Error analysis, Positioning systems, Position accuracy, Calibration, Transponders

1. INTRODUCTION

A Long Baseline (LBL) system is a positioning system constituted by an array of transponders, in fixed known positions, that can exchange signals with the object to be localized. The estimation of the object position is obtained through the measurements of the Time-of-Flights (ToFs) of the exchanged signals.

The use of ToF measurements for localization is not a new concept. Several applications can be found in literature, as for example Lange and Seitz (2001), Chandrasekhar et al. (2006), Lanzisera et al. (2006). Existing techniques are typically based on the following dual-step procedure: first, each ToF is simply transformed into the corresponding distance (or range), by taking into account the speed of propagation of the signals in the considered medium; then, if a sufficient number of range estimations is available, the localization problem is solved by applying a standard trilateration-based algorithm as in Manolakis (1996). Within terrestrial applications, the use of such a strategy is well consolidated and is normally employed, for example, by GPS receivers, processing the ToFs of radio signals received by a proper set of satellites.

Conversely, in the underwater domain, the application of the above methodology is not as straightforward because of the impossibility of exploiting electromagnetic signals in water as they suffer from strong absorption effects which almost totally prevent their employment. Therefore the use of acoustic signals is the only viable solution. However different issues are known to affect the underwater acoustic channel, as indicated, among the others, by Milne (1983) and Jensen et al. (1994). The main source of problems within underwater localization is the anisotropy of the acoustic medium, which makes the speed of sound (SoS) in water a not constant parameter, as it is actually dependent on the physical characteristics of the considered zone. As a consequence, ray bending phenomena arise and multi-path effects become more crucial.

The above problems certainly represent the more serious limiting factors in the achievable accuracy of any ToF-based localization technique, as indicated by several works, among which Chandrasekhar and al. (2006), Caiti et al. (2005) and Kussat and al. (2005). However, together with the limitations induced by the physics of the underwater acoustic medium, another factor, impacting on the accuracy of LBL-based localization, is represented by the calibration of the acoustic array after its deployment. Indeed if the positions of all the transponders are not perfectly known, errors in the localization procedure unavoidably arise.

To avoid such a risk, within the off-shore industry, for long-term seabed applications, where a high level of accuracy in positioning submerged devices is requested, sophisticated and costly instruments are normally used together with consolidated calibration procedures.

Unfortunately the same cannot be stated for shallow water shorter–term operations, like the survey-oriented missions often performed by low-cost autonomous underwater vehicles (AUVs), like the FOLAGA available at the authors’ research group and described in Alvarez et al. (2009). For such a kind of applications, very frequently the GPS position of the supporting boat, registered at the point where the transponder is dropped down, is considered the transponder position. Sometimes the localization errors occurred during the survey can be mitigated by post-processing the acquired mission data. But in any case, very often, calibration errors are considered negligible by definition. Despite such a consideration can be reasonable in various situations, quantitative indications about the real impact of a non precise calibration process on the resulting localization errors are, at the best of authors' knowledge, still lacking in literature.

Moving from such a consideration, the authors propose a detailed analysis of the errors induced by a wrongly calibrated acoustic array. All the other possible sources of localization errors are here purposely neglected, in order to isolate the addressed phenomenon. The effects induced by the way acoustic signals propagate underwater were the subject of other dedicated works, as in Turetta et al. (2011) and Casalino et al. (2010), and are here not taken into account at all. For the same motivations, all the electric, human-induced and environmental possible sources of noise are them also not considered hereafter. And the same is true also for the quantization errors induced by the signal processing in charge of detecting the presence of an incoming acoustic pulse and measuring its ToF.

In conclusions, in the following analysis, acoustic signals are assumed to move at a constant and perfectly known SoS, along straight path and in an ideal, noiseless, environment. Further, the used instruments are them also considered as ideally perfect, as they are capable of measuring all the ToFs with infinite resolution and precision. Moving from such premises, the work starts by introducing the algorithm used for the localization process in the absence of transponder positioning errors. Successively the transponder errors are introduced and the closed form for the resulting localization error is obtained. Then the different parameters related with the localization error are analyzed in details. Simulative results validate the theoretical analysis carried out and finally some conclusions on the performed activities are drawn.

2. SOLUTION IN THE ABSENCE OF ERROR SOURCES

The most used algorithm for localizing an AUV through a LBL system is the spherical-based one. According to that, first the measured ToF from the i-th transponder (t_i) is translated by the AUV into an estimation of its distance from the same transponder (d_i), by considering the SoS (v).

$$d_i \hat{=} t_i \cdot v \quad ; \quad i = 1,..,n \tag{1}$$

being n the number of transponders.
Then by defining (x,y,z) the coordinates of the AUV w.r.t. to a given Cartesian frame and (x_i, y_i, z_i) the coordinates of the i-th transponder, the following relationships can be formulated:

$$(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2 = d_i^{\,2} \; ; \; i = 1,\ldots,n \tag{2}$$

each one of them clearly stating that the AUV is located on the surface of a sphere centred in the corresponding transponder position and with radius equal to d_i.
Now, by considering known the AUV z coordinate (as it is given from an onboard sensor like a pressure gauge or an altimeter) and by assuming that (as it often happens) all the transponders are deployed at the same depth ($z_1=z_2=\ldots=z_n$), the original 3D problem can be reduced to a 2D one (for the unknown x and y), consisting in finding out the intersecting point of the set of circumferences obtained by intersecting the above spheres with the horizontal plane at the known depth of the AUV:

$$(x - x_i)^2 + (y - y_i)^2 = r_i^{\,2} \quad ; \quad i = 1,\ldots,n \tag{3}$$

where the term

$$r_i^{\,2} \hat{=} d_i^{\,2} - (z - z_i)^2 \quad ; \quad i = 1,\ldots,n \tag{4}$$

is obtained by the measured ToF and the known vertical distance between the AUV and the transponders.
By now manipulating terms of (3) the following linear relationship can be obtained (in the case of n=4 transponders) for the 2D vector $X = [x,y]^T$:

$$A \; X \; = \; R - D \tag{5}$$

where:

$$A \hat{=} \begin{bmatrix} x_1 - x_2 & y_1 - y_2 \\ x_2 - x_3 & y_2 - y_3 \\ x_3 - x_4 & y_3 - y_4 \\ x_4 - x_1 & y_4 - y_1 \end{bmatrix} \; ; \; R \hat{=} \frac{1}{2}\begin{bmatrix} r_2^{\,2} - r_1^{\,2} \\ r_3^{\,2} - r_2^{\,2} \\ r_4^{\,2} - r_3^{\,2} \\ r_1^{\,2} - r_4^{\,2} \end{bmatrix} \; ;$$
$$D \hat{=} -\frac{1}{2}\begin{bmatrix} (x_2^{\,2} + y_2^{\,2}) - (x_1^{\,2} + y_1^{\,2}) \\ (x_3^{\,2} + y_3^{\,2}) - (x_2^{\,2} + y_2^{\,2}) \\ (x_4^{\,2} + y_4^{\,2}) - (x_3^{\,2} + y_3^{\,2}) \\ (x_1^{\,2} + y_1^{\,2}) - (x_4^{\,2} + y_4^{\,2}) \end{bmatrix} \tag{6}$$

the above equation can then be solved with a least squares method, giving the following estimated position $\hat{X} = [\hat{x}, \hat{y}]^T$:

$$\hat{X} \; = \; (A^{\mathrm{T}} A)^{-1} A^{\mathrm{T}} (R - D) = A^{\#}(R - D) \tag{7}$$

where it is worth noticing that both A (and consequently $A^{\#}$) and D are constant terms that can be computed just once offline.

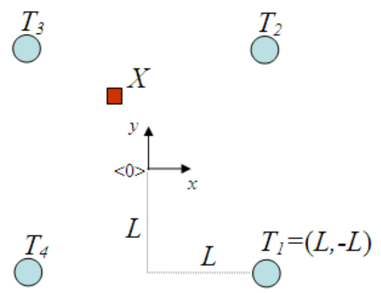


Fig. 1 – squared configuration of transponders

Further, if the transponders are located in a rectangular configuration and if the coordinate system is selected with the origin in the center of the rectangle, it is easy to see that D vector is always null. Moreover, if the configuration is square, A matrix assumes a very simplified form. Indeed by numbering transponders as indicated in figure 1, it turns out that:

$$A = \begin{bmatrix} 0 & -2L \\ 2L & 0 \\ 0 & 2L \\ -2L & 0 \end{bmatrix} \quad (8)$$

and consequently that:

$$A^{\#} = \begin{bmatrix} 0 & 1/4L & 0 & -1/4L \\ -1/4L & 0 & 1/4L & 0 \end{bmatrix} \quad (9)$$

In conclusion, in the above assumption of a squared configuration the solution of the localization procedure with the spherical based algorithm can be formulated in a closed form as follows:

$$\begin{aligned} \hat{x} &= \frac{+R_2 - R_4}{4L} \\ \hat{y} &= \frac{-R_1 + R_3}{4L} \end{aligned} \quad (10)$$

being R_i the i-th term of vector R.

Before concluding this section it is worth noticing that under the considered assumptions, the above solution results to be absolutely exact, as it can be verified by substituting into it the expressions of R_i:

$$\begin{aligned} \hat{x} &= \frac{\left[(x-x_3)^2+(y-y_3)^2\right]-\left[(x-x_2)^2+(y-y_2)^2\right]}{8L} + \\ &\quad + \frac{\left[(x-x_4)^2+(y-y_4)^2\right]-\left[(x-x_1)^2+(y-y_1)^2\right]}{8L} \\ \hat{y} &= \frac{\left[(x-x_4)^2+(y-y_4)^2\right]-\left[(x-x_3)^2+(y-y_3)^2\right]}{8L} + \\ &\quad + \frac{\left[(x-x_1)^2+(y-y_1)^2\right]-\left[(x-x_2)^2+(y-y_2)^2\right]}{8L} \end{aligned} \quad (11)$$

that, after some math, leads to the expressions:

$$\begin{aligned} \hat{x} &= x \\ \hat{y} &= y \end{aligned} \quad (12)$$

clearly showing how the estimated position is actually errorless.

3. SOLUTION IN THE PRESENCE OF ERRORS IN THE TRANSPONDERS POSITIONING

If the transponders are not deployed in their nominal position, errors in the localization procedure may arise. The problem is not originated by the presence of displacements between the desired and actual positions; whenever such displacements can be measured, it is obviously possible to compensate for them. Matrix A would not have the simplified form (8) and vector D would not be the null one; however, in the knowledge of the displacements, the same computation seen before, despite not anymore in its simplified form, can be still applied with no resulting errors.

Problems rather occur if displacements exist but their existence is unknown. Indeed in such a scenario the localization procedure runs by assuming that transponders are in their *nominal* positions. However measured ToFs (and hence computed distances) are instead dependent on the *actual* positions of transponders. From this inconsistency, localization errors arise.

In order to better characterize such errors, be $dX_i=[dx_i, dy_i]^T$ the vector representing the difference between the nominal and the actual position of the i-th transponder.

Now by referring with $T_i=[x_i, y_i]^T$ the nominal position of the i-th transponder, the actual position T_{ai} becomes:

$$T_{ai} = T_i + dX_i \quad ; \quad i = 1,\ldots,n \quad (13)$$

As a consequence the circumferences (3) now become:

$$[x-(x_i+dx_i)]^2 + [y-(y_i+dy_i)]^2 = r_i^2 \;\; ; \; i=1,\ldots,n \quad (14)$$

Now as, by assumption, the existence of transponder displacements is unknown, A matrix (depending only on the nominal position of transponders), remains as in (6) as well as vector D.

As a consequence (again under the assumption of a nominal squared configuration) the form of the solution is still (10) but now, by substituting into it the expressions of the new r_i terms (14), it turns out that the estimated position becomes after some maths:

$$\begin{aligned} \hat{x} &= x + a_x + b_x + c_x \\ \hat{y} &= y + a_y + b_y + c_y \end{aligned} \quad (15)$$

where it has been posed

$$\begin{aligned} a_x &\hat{=} \frac{(dx_3^2+dy_3^2)-(dx_2^2+dy_2^2)+(dx_4^2+dy_4^2)-(dx_1^2+dy_1^2)}{8L} \\ a_y &\hat{=} \frac{(dx_4^2+dy_4^2)-(dx_3^2+dy_3^2)+(dx_1^2+dy_1^2)-(dx_2^2+dy_2^2)}{8L} \end{aligned} \quad (16)$$

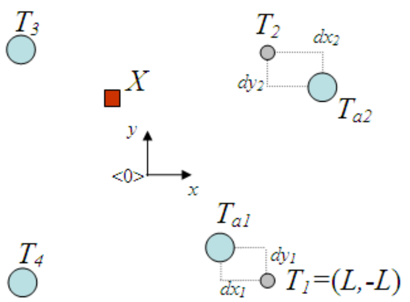


Fig. 2 –Nominal positions (grey) and actual positions (cyan) of transponders in the presence of displacements

$$b_x \triangleq \frac{(dy_3 - dx_3) + (-dx_2 - dy_2) + (-dx_4 - dy_4) + (dy_1 - dx_1)}{4}$$
$$b_y \triangleq \frac{(-dx_4 - dy_4) + (dx_3 - dy_3) + (dx_1 - dy_1) + (-dx_2 - dy_2)}{4} \tag{17}$$

$$c_x \triangleq \frac{-(x \cdot dx_3 + y \cdot dy_3) + (x \cdot dx_2 + y \cdot dy_2)}{4L} + \frac{-(x \cdot dx_4 + y \cdot dy_4) + (x \cdot dx_1 + y \cdot dy_1)}{4L}$$
$$c_y \triangleq \frac{-(x \cdot dx_4 + y \cdot dy_4) + (x \cdot dx_3 + y \cdot dy_3)}{4L} + \frac{-(x \cdot dx_1 + y \cdot dy_1) + (x \cdot dx_2 + y \cdot dy_2)}{4L} \tag{18}$$

From (15) it is clear that in this case (as expected) an estimation error vector actually exists, as follows:

$$e \triangleq \begin{bmatrix} \hat{x} - x \\ \hat{y} - y \end{bmatrix} = \begin{bmatrix} a_x + b_x + c_x \\ a_y + b_y + c_y \end{bmatrix} \tag{19}$$

In order to make easier the analysis of the characteristics of e, it is convenient introducing the unit versors describing the directions from the origin to the nominal positions of transponders (see Fig.3):

$$u_i \triangleq \frac{X_i}{\|X_i\|} \quad ; \quad i = 1, \ldots, n \tag{20}$$

By exploiting (21), the position estimation error (20) can be expressed in the following more compact form:

$$e = \frac{1}{4} \sum_i u_i \left[-\frac{\|dX_i\|^2}{2L} - dX_i \cdot u_i + \frac{dX_i \cdot X}{L} \right] \triangleq \frac{1}{4} \sum_i u_i \lambda_i \tag{21}$$

from which it emerges how the error vector e is actually the combination of four vectors, each one aligned with a unit versors u_i. and with a coefficient λ_i. For understanding the direction and the norm of the resulting error vector e, it is therefore necessary analyzing the nature of coefficients λ_i. Each one of them is actually composed by three terms:

$$a_i \triangleq -\frac{\|dX_i\|^2}{2L} \ ; \ b_i \triangleq -dX_i \cdot u_i \ ; \ c_i \triangleq \frac{dX_i \cdot X}{L} ; 1, \ldots, n \tag{22}$$

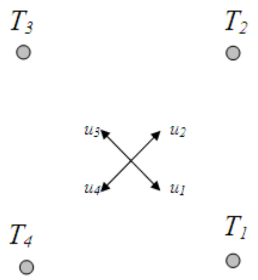


Fig. 3 – Unit versors u_i.

The impact of each term in the resulting error vector e can be appreciated by considering the orders of magnitudes of different quantities within a typical application: normally L is at least hundreds of meters, while $\|dX_i\|$ is expected to be at most few meters.

Analysis of the errors due to a_i terms

In the light of the last consideration, a_i terms clearly play a very limited role, as their order of magnitude is the centimeter. Further, as their values are never negative and they combine along four orthogonal directions, it follows that what is relevant for the final error vector e is just the difference (if any) between the norms of dX_i vectors. Indeed if the norms of the four transponder displacements are equal each other (despite the four dX_i vectors might have different directions), the impact of a_i terms on e is null. Actually for having a null contribution of a_i terms, it is sufficient that:

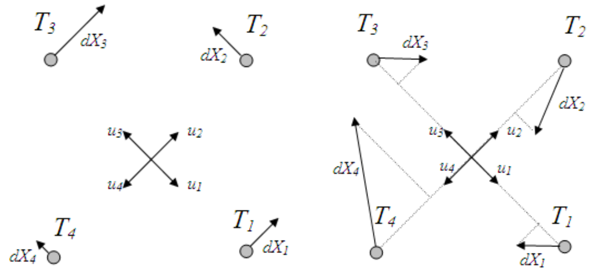
$$\begin{cases} \|dX_1\| = \|dX_3\| \\ \|dX_2\| = \|dX_4\| \end{cases} \Rightarrow \sum_i a_i u_i = 0 \tag{23}$$

Analysis of the errors due to b_i terms

As it emerges from (22), b_i terms are the only ones not dependent on the baseline. Indeed each one of them depends on the inner product between the corresponding dX_i vector and the u_i versor. Therefore also the directions of displacement vectors are relevant. In this light, the luckiest scenario (where every b_i terms is null) is depicted in Figure 4a. Further, analogously to what previously said for a_i terms, it is also possible to have non null b_i terms, providing a null contribution to e as depicted in Figure 5b. Indeed:

$$\begin{cases} \|dX_1\| \cos(\delta_1) = \|dX_3\| \cos(\delta_3) \\ \|dX_2\| \cos(\delta_2) = \|dX_4\| \cos(\delta_4) \end{cases} \Rightarrow \sum_i b_i u_i = 0 \tag{24}$$

For given norms of displacement vectors dX_i, the worst-case scenario (i.e. when the error contribution is bigger) is instead depicted in Fig. 5. It occurs when dX_1 and dX_3 lie along the direction of u_1 (with the same orientation) and dX_2 and dX_4 lie along the direction of u_2 (with the same orientation). In such a case (under the same assumptions than before) order of magnitude of the contribution of b_i terms to e is the meter.



(a) (b)

Fig. 4 – Scenarios of null contribution provided by b_i terms. *a*) every b_i term is null, as every dX_i vector is orthogonal to its corresponding u_i; *b*) b_i terms are not null but their resulting contribution to error vector e is null

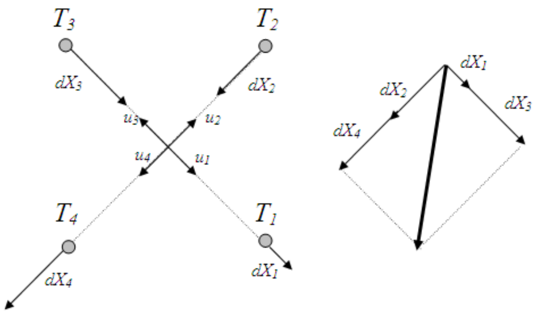


Fig. 5 – Scenario of non null contribution provided by b_i terms. The resulting vector is in bold.

Analysis of the errors due to c_i terms

Differing from what happen for a_i and b_i terms, the values of c_i terms depend always on the AUV position. with the sole exception occurring in the unlikely case that dX_1=dX_3 and dX_2=dX_4. In such an extreme case (see Figure 6) it is easy to see that the overall contribution of c_i terms is always zero, notwithstanding the AUV position. In all the other displacement vectors configurations, the contribution of c_i terms to the overall error depends on the position of the AUV and might be null or not according to where the AUV is. The worst-case scenario (for given norms of displacement vectors dX_i and given AUV distance from the origin X) occurs when all the five vectors are aligned each other and with directions as depicted in Figure 7: i.e. dX_1 opposite to dX_3 and dX_2 opposite to dX_4. However it is worth noticing how, for the same configuration of dX_i vectors, the contributions of c_i terms to the overall error e assumes different values, depending on the position of the AUV in the plane (also if it maintains a constant distance from the origin). Indeed in the case of Figure 8 (obtained with the same dX_i vectors than in Figure 7) as X vector is orthogonal to all dX_i, the the resulting contribution to error e is null. Analogously, in all the other possible AUV positions, contributions intermediate values are clearly obtained.

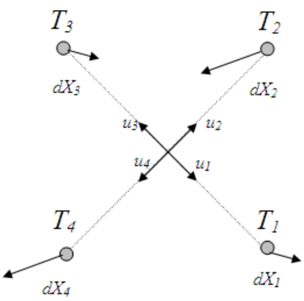


Fig. 6 – Null resulting contribution to the error e provided by c_i terms

As a final remark, note how the error contribution provided by c_i terms, (for a given AUV direction) is linearly dependent with the ratio between the AUV distance from the center of the area and the baseline. Such a property provides an important operative indication: in order to minimize the localization errors induced by the uncertainties on the transponder positioning, transponders should be deployed as far each other as possible (compatibly with technological limitations of the device at hand), notwithstanding the size of the operative area of the AUV.

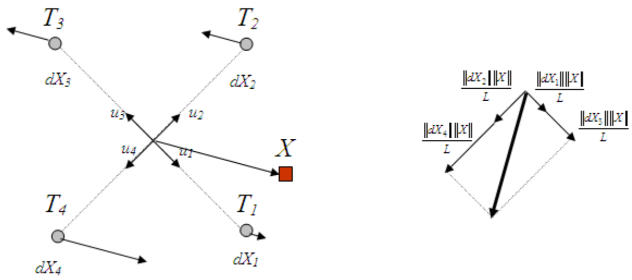


Fig. 7 – Worst-case scenario for the error contribution provided by c_i terms. The resulting vector is in bold..

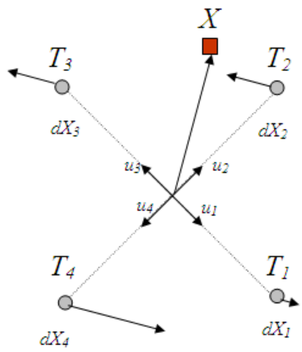


Fig. 8 – Same displacement vectors as in Figure 7 but null error contribution provided by c_i terms

4. SIMULATIVE RESULTS

In order to validate the above analysis, some simulative tests have been carried out with Matlab Simulink®.
In all of them the AUV has been moved on a horizontal plan, making it follow a spiralled path, starting from the origin and moving outwards and reaching a distance from it higher than one kilometre, as represented in Figure 9.

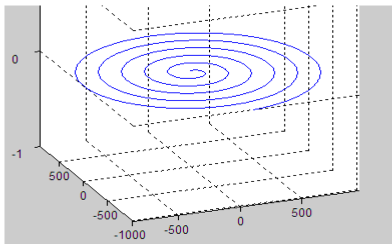


Fig. 9 –AUV simulated motion (meters)

Four transponders are located 50 meters deeper than the AUV in a squared nominal configuration. For every test two baselines have been considered, namely 400 and 800 meters (i.e. L=200, L=400 respectively). The quantization effects related with the ToF measurements have been neglected, by assuming onboard the AUV the presence of an ideal signal processing functionality with an infinite resolution. Finally the assumed speed of sound in all the tests was 1500 m/s.
In the first set the transponders displacements error vectors have been selected as: dX_1=[1m, -1m]^T; dX_2=[1m, -1m]^T; dX_3=[-1m, 1m]^T; dX_4 =[-1m, 1m]^T. In this way, as the left

sides of relationships (23) and (24) are both satisfied, the contributions of a_i terms and b_i terms are null. As a consequence in this configuration the whole error e is originated by c_i terms only. The variation of the norm of the error vector as dependent on the norm of the AUV vector X can be appreciated in Figure 10.

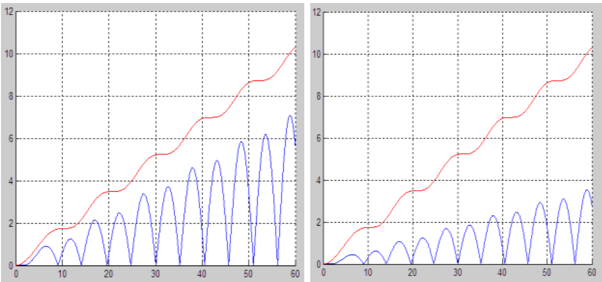


Fig. 10 – Time behaviour of the norm of *e* in meters (blue) and the norm of the *X* in hundreds of meters (red)

Once emphasized the role of c_i terms, in order to show the effects of b_i terms, the transponders displacements error vectors have been selected as dX_1 =[1m, -1m]^T; dX_2 =[1m, -1m]^T; dX_3 =[1m, -1m]^T; dX_4 =[1m, -1m]^T which allow to have zero contribution from both a_i and c_i terms. As expected (see Figure 11) the norm of the error vector is constant and not dependent on the AUV position, nor on the baseline.

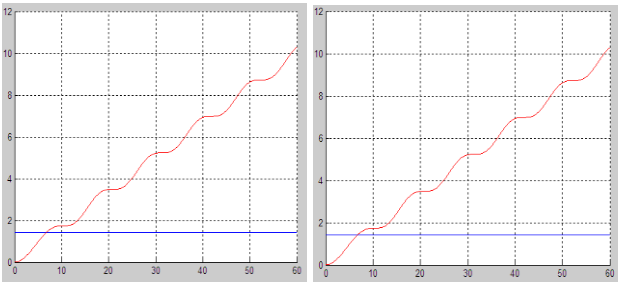


Fig. 11 – Time behaviour of the norm of *e* in meters (blue) and the norm of *X* in hundreds of meters (red)

As the final considered scenario, the following intermediate case was selected dX_1 =[1m, -1m]^T; dX_2 =[1m, -1m]^T; dX_3 =[1m, -1m]^T; dX_4 =[-1m, 1m]^T where both contribution of b_i and c_i terms are present. Results are clearly consistent with expectations: there is a variation in the error norm depending on the AUV distance combined with a constant offset. It is also interesting noticing how the increment of the baseline reduces just the oscillations but not the offset, as it is given to b_i terms which are not dependent on the baseline.

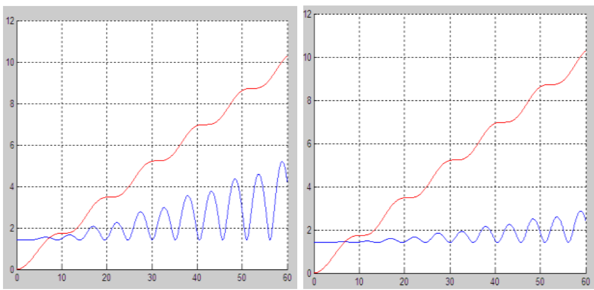


Fig. 14 – Time behaviour of the norm of *e* in meters (blue) and the norm of *X* in hundreds of meters (red)

5. CONCLUSIONS

The effects of a non precise knowledge of the transponders positions on the resulting accuracy of a LBL system have been investigated and analyzed in details. Other than clearly depending on the existing displacements between the nominal and actual position of transponders, localization errors came out to be linearly dependent also on the ratio between the size of the baseline and the distance of the object to be localized from the center of the operative area. A fact this last that provides an important operative indication, especially useful when the LBL system is employed for localizing low-cost AUVs within short-term missions: transponders should be always deployed as far each other as possible (compatibly with technological limitations of the device at hand), notwithstanding the size of the operative area of the AUV.

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