Viana, V., Nagy, D., Xavier, J., Neiva, A., Ginoulhiac, M., Mateus, L. & Varela, P. (Eds.). (2022). Symmetry: Art and Science | 12th SIS-Symmetry Congress [Special Issue]. *Symmetry: Art and Science*. International Society for the Interdisciplinary Study of Symmetry. 138-145. https://doi.org/10.24840/1447-607X/2022/12-17-138

APPLICATION OF APERIODIC TILING TO LATTICE SPATIAL STRUCTURES: A FAMILY OF APERIODIC PATTERNS ON SPHERICAL CAPS.

JAMES DURN and S. ALIREZA BEHNEJAD

Name: James Durn (b. Brighton, England, 1999)
Profession: MEng student (Civil and Environmental Engineering)
Fields of interest: Civil engineering
Affiliation: University of Surrey
E-mail: james.e.durn@gmail.com
Homepage: www.linkedin.com/in/james-durn-60a730195/
Major publications and/or exhibitions:
Durn, J. & Behnejad, S. A. (2022). *A New Family of Lattice Domes: Spherical Caps of Aperiodic Tiling*. University of Surrey.

Name: S. Alireza Behnejad (b. Tehran, Iran, 1972)
Profession: Educator (Senior Teaching Fellow in Civil Engineering), Researcher (Geometry of Spatial Structures)
Fields of interest: Configuration processing, Design and construction of spatial structures, Engineering education, Formex algebra, Structural performance of bamboo (also human relations and Persian poetry)
Affiliation: University of Surrey
E-mail: a.behnejad@surrey.ac.uk; a.behnejad@gmail.com
Homepage: www.surrey.ac.uk/people/s-alireza-behnejad
Major publications and/or exhibitions:
Behnejad, S. A. and Zahedi, M. (2010) 'Symmetry in Traditional Persian Poetry', *Symmetry: Art and Science*, 8, 16-19.
Behnejad, S. A., Parke, G. A. R. and Nooshin, H. (2013) 'Sphere Packing for Regularisation of Lattice Domes', *Beyond the Limits of Man*, p.231.
Behnejad, S. A. (2018) 'Engineering' In *Exploring Pedagogic Frailty and Resilience,* Leiden, The Netherlands: Brill | Sense.
Behnejad, S. A. (2018). Geometrical data for lattice spatial structures: regularity, historical background and education (Doctoral thesis, University of Surrey).
Behnejad, S. A., Parke, G. A. R. and Samavati, O. A. (2021). Inspiring the Next Generation. Proceedings of the 7th International Conference on Spatial Structures and the Annual Symposium of the International Association for Shell and Spatial Structures IASS 2020-21, University of Surrey, pp. 3590.

Abstract: *The aim of this report is to expand on the existing available configurations for lattice spatial structures and in turn widen the range of design options that can be presented to architects and/or clients. This is achieved by developing a new family of configurations in which the aperiodic tiling pattern 'Penrose Kite and Dart' tiling is applied to spherical caps using formex configuration processing techniques. To further expand this new family, the pattern modification of triangulation is suggested along with alternative projection methods of changing the shape of the initial tiling patch as well as using pellevation. To determine a degree of feasibility, the regularity of components is studied and compared to configurations of existing, periodic patterns such as ribbed and diamatic domes. As a result, the spherical caps under two decompositions of aperiodic tiling were found to have similar regularity indicators to the periodic configurations and are therefore considered feasible options. Each proposed modification was also found to be low impacting on feasibility by introducing a low number of new member lengths. However, the use of aperiodic tiling under three or more decompositions is not recommended due to the significant increase in new member lengths.*

Keywords: Aperiodic Patterns; Penrose Tiling; Lattice Spatial Structures; Configuration Processing; Regularity

INTRODUCTION

A wider range of available configurations for lattice spatial structures provides a greater array of interesting design options open for consideration. With most configurations consisting of periodic patterns, the expansion of available families of configurations can be achieved by projecting aperiodic tilings, such as Penrose tiling, onto spherical surfaces to produce a family of aperiodic patterns on spherical caps.

A tiling is periodic if a translation exists that leaves the tiling invariant. Among the symmetries of the tiling, it should be possible to outline a region of tiling and tile the plane by translating copies of that region without rotation or reflection. If this type of translation is not possible, the tiling is deemed as non-periodic. Aperiodic tiling is considered as tiling with a set of prototiles that admit infinitely many tilings of the plane, yet no such tiling is periodic. Aperiodic tiling can be generated by various means such as decomposition (Grünbaum & Shephard, 2016).

Several famous examples of aperiodic tiling were produced by Sir Roger Penrose (1979) following his pursuit to tile a plane with pentagons with no gaps or overlapping. The Penrose tiling under consideration in this report is Penrose Kite and Dart tiling, otherwise known as *P2 tiling*, see Figure 1. This tiling is remarkable in that it consists of just two prototiles while still tiling aperiodically (Grünbaum & Shephard, 2016).

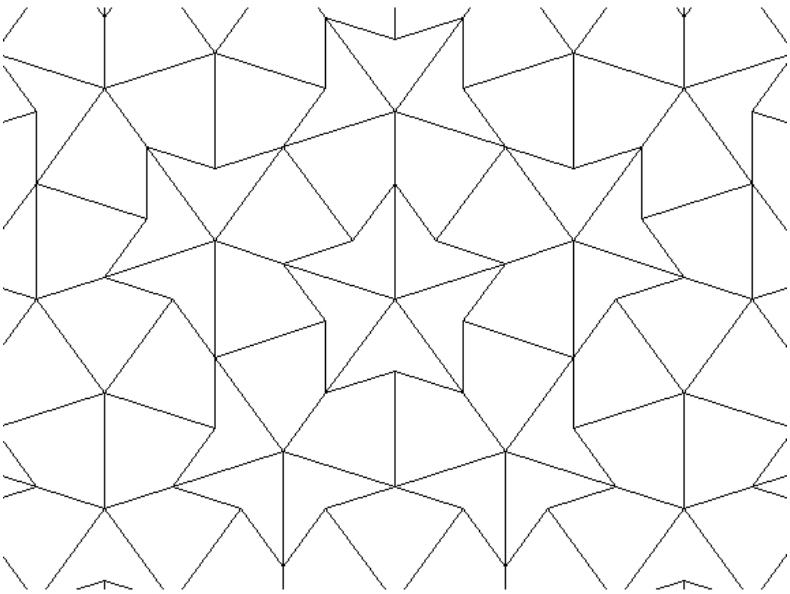


Figure 1 Penrose Kite and Dart tiling patch (adapted from Penrose (1979)).

The regularity of a structure can be viewed in various ways such as the visual regularity, regular arrangements, and the regularity of components (Behnejad, 2018). The two formers are subjective terms based on interpretation by the viewer. Whereas the latter can be discussed quantitatively using regularity indicators including the number of members (n), the number of member lengths (b), or ‘bundles’, the length ratio (r), and the length deviation (d) – all of which can be presented in a

Length Profile (LEP) chart as discussed by Nooshin, *et al*. (1997). Regularity indicators are determined using geometrical data given in the formulation process.

The comparison of regularity of components should be performed between configurations consisting of a similar number of members, or 'crowdedness' (Durn & Behnejad, 2021), with the number of bundles being the governing indicator of regularity. With a lower variation in structural components, a structure with fewer bundles provides cost and time savings during construction as well as the manufacturing of members (Behnejad, *et al*., 2013).

The length ratio and deviation can be used to further determine a level of feasibility by gaining an understanding of how similar the lengths of each bundle are. Although a structure is defined as more regular as the values for the length ratio and deviation become closer to one and zero, respectively (Nooshin, *et al.*, 1997), it can be argued that greater values for these parameters can be beneficial. Bundles with greater differences in length are more identifiable, and thus reduce the probability of human error during the labelling, sorting, and assembly processes.

METHODOLOGY

A spherical cap is formed by projecting a pattern onto a spherical surface. This can be achieved using formex configuration processing techniques (Nooshin, 2017). To reduce warping of the pattern during projection, the author suggests the use of a pattern with an isosceles triangle perimeter with an acute top angle. Figure 2 shows the generation of P2 tiling up to three decompositions starting with half of a dart prototile.

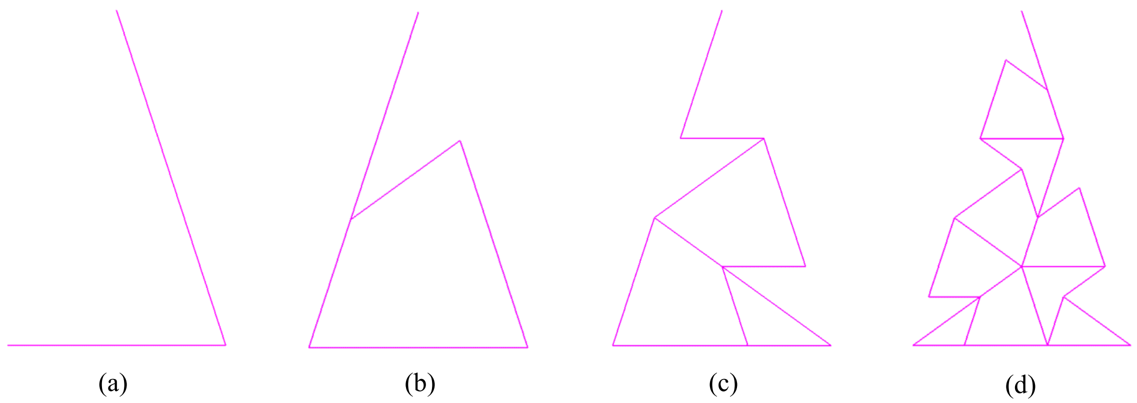


Figure 2 Initial patch shape of half a dart prototile (a) under one (b), two (c) and three (d) decompositions of P2 tiling.

When projecting the pattern, a suitable scale factor should be applied to the circumferential direction so that the pattern covers a certain angle about the origin based on the pattern frequency. The angle covered by a single section should allow the section to be repeated about the origin a number of times to form a complete spherical cap, see Figure 3 for an example. The flat bottom edge of the tiling patch gives the spherical cap a level, circular base post projection.

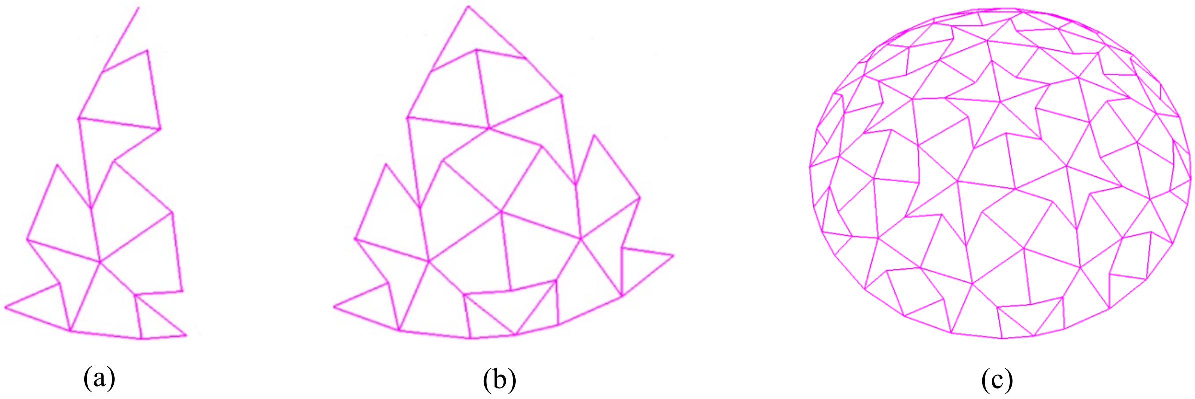


Figure 3 Section of spherical cap covering 36° about the origin in the circumferential direction (a). Reflected section for symmetrical tiling (b). Full spherical cap of aperiodic tiling after five repetitions of the section of symmetrical tiling (c).

DISCUSSION

In addition to varying the number of tiling decompositions, the family of aperiodic patterns on spherical caps can be extended by producing variations of the original configuration through various methods. These can include pattern modification, such as triangulation, and alternative projection methods, such as changing the tiling patch shape and the use of pellevation, a configuration

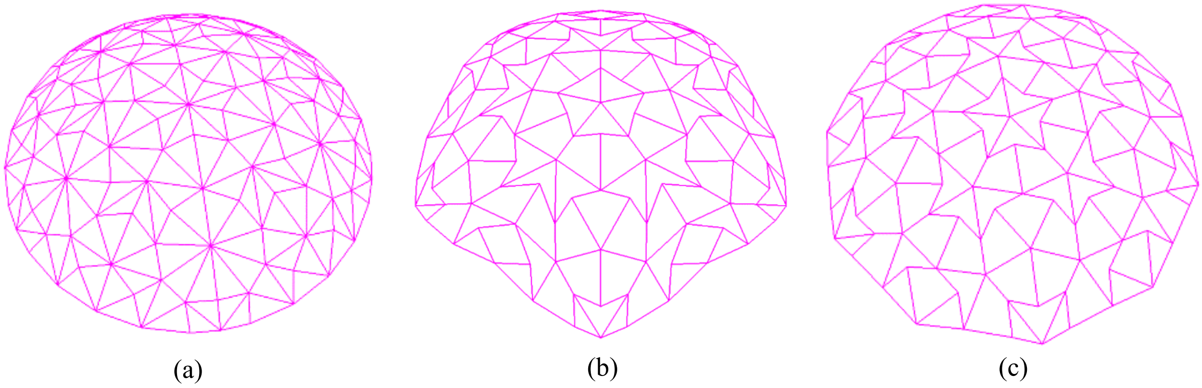
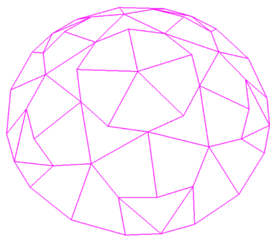
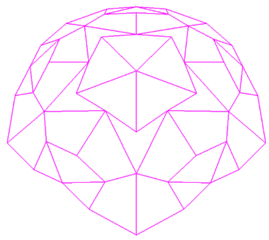
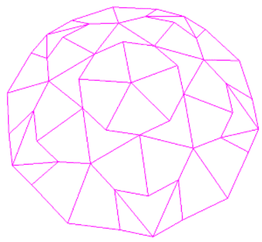
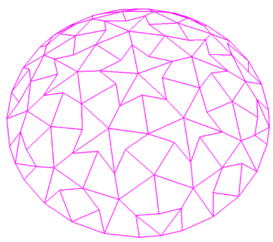
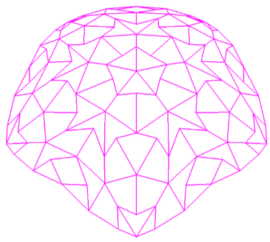
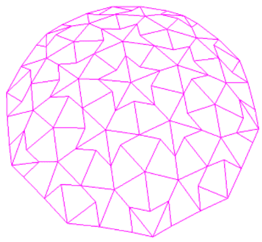
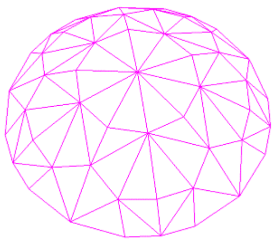
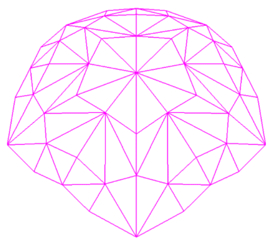
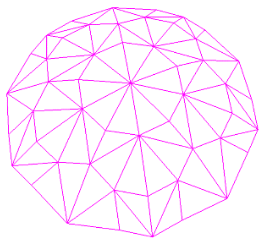
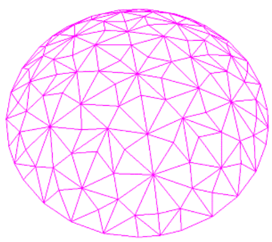
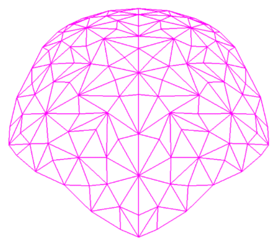
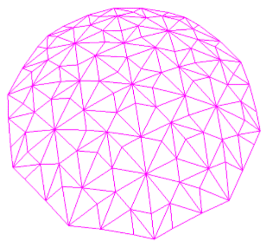


Figure 4 P2 spherical caps under triangulation (a), a change of tiling patch shape (b), and circular cap pellevation (c).

processing technique developed by Hofmann (1999). Triangulation can be applied to tilings with non-triangular prototiles, such as P2 tiling, by simply introducing a new member acting vertically through the centre of each prototile. The result of such pattern modification can be seen in Figure 4a. Triangulation has the potential in improving the structural stability and performance of the lattice spatial structure.

As previously mentioned, the use of a triangular tiling patch with a horizontal bottom edge produces spherical caps with level, circular bases. By manipulating the shape of the patch, the shape of the base ring can be altered. Reflecting the tiling patch along one of its long edges to form a dart shaped tiling patch produces spherical caps with “zig-zagged” base rings, see Figure 4b. Furthermore, the use of pellevation (Hofmann, 1999) can create levic domes following the method presented by Behnejad and Rognoni (2019), thus creating another variation in base-ring shape, see Figure 4c. The resulting change in geometry from these projection methods may be suitable for projects where the load on the structure is required to be directed into point loads around the perimeter of the dome. Following the modification techniques discussed, the author has used formex configuration processing techniques to produce a new family of lattice domes based on P2 tiling applied to spherical caps, see Table 1. Within this family, the spherical caps can be classified based on the number of tiling decompositions, pattern modifications and base shapes, the latter being the result of different projection methods.

Table 1: Family of P2 spherical caps.

P2 Spherical Caps				
		Circular Base	Zig-Zagged Base	Levic
Original Pattern	2D	 n = 95, b = 9	 n=95, b = 10	 n = 95, b = 11
	3D	 n = 235, b = 21	 n = 235, b = 24	 n = 235, b = 25
Triangulated Pattern	2D	 n = 130, b = 11	 n = 130, b = 14	 n = 130, b = 14
	3D	 n = 330, b = 26	 n = 330, b = 32	 n = 330, b = 35

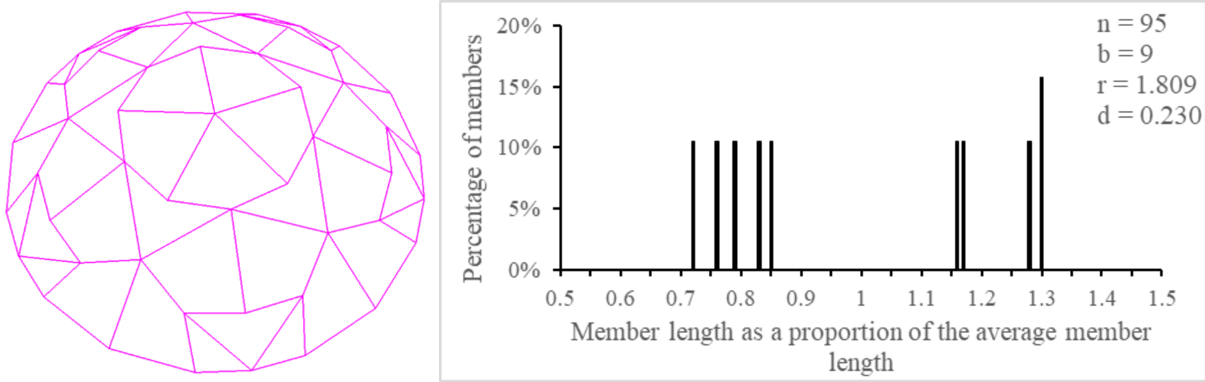
To avoid repetitive language, the author suggests a naming convention to easily identify the number of decompositions a tiling has undergone. For a spherical cap of P2 tiling under two decompositions, the configuration is referred to as the "P2-2D" spherical cap in which the 2D indicates that the tiling has undergone two iterations of decomposition.

Through comparison of the number of bundles, the P2-2D spherical cap with a circular base has the highest degree of regularity, consisting of the lowest number of bundles (b = 9), see Figure 5a. In addition, a comparison performed by Durn & Behnejad (2021) concluded that the number of bundles in the P2-2D spherical cap with a circular base sits in the upper range presented by a group of existing periodic configurations of four to 11 bundles. These configurations included diamatic, geodesic, ribbed, lamella, and Schwedler domes formulated to be of equal size and similar crowdedness to that of the aperiodic configuration.

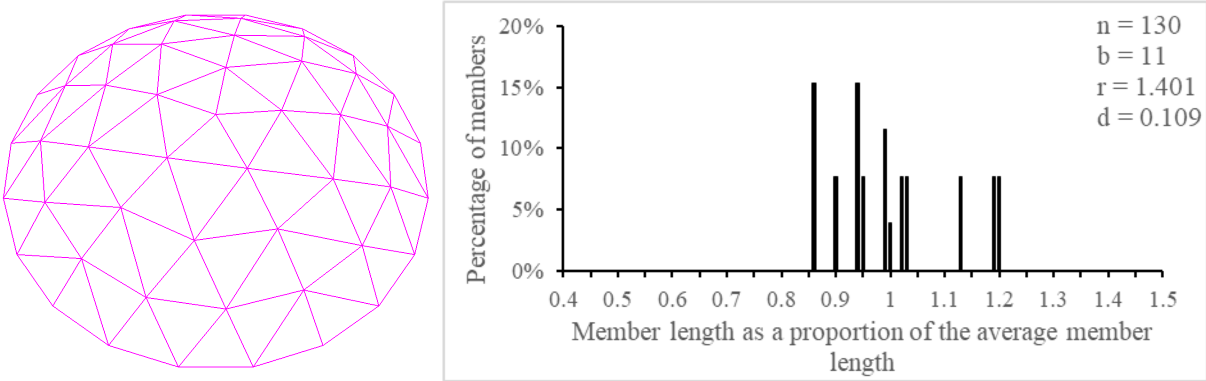
Regarding the length ratio and length deviation, the P2-2D spherical cap with a circular base again presents values that are within the range given by the periodic configurations. The regularity indicators therefore suggest that the time and cost savings, as well as probability of human error when handling members, is similar to that of the periodic configurations. However, the two clusters of bundles presented on the LEP chart in Figure 5a present potential difficulty in identifying members belonging to each bundle due to the proximity in member length within each cluster of bundles. Greater care should therefore be taken during the labelling, sorting, and assembly processes. This challenge is also presented by the diamatic dome in Figure 5b but avoided by the more simplistic ribbed dome in Figure 5c where the member lengths are more evenly spread.

As expected, the pattern modification of triangulation increases the number of bundles within each configuration, as do the alternative projection methods. However, the number of new bundles introduced by these variations are low compared to the original number of bundles and therefore do not significantly impact the configurations regularity and feasibility.

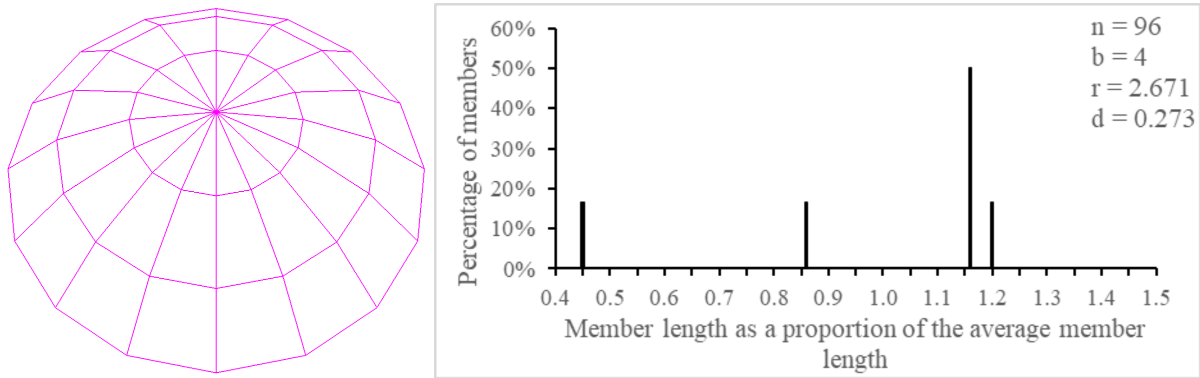
With a similar degree of regularity in components to other existing periodic configurations, the P2-2D spherical caps present themselves as viable, alternative design options. However, under a greater number of pattern decompositions, the configurations become more crowded, and significantly more bundles are introduced. As a result, the P2-3D solutions are considered less feasible than the configurations of low crowdedness under two decompositions.



(a)



(b)



(c)

Figure 5 P2-2D (a), diamatic (b) and ribbed (c) spherical caps with corresponding LEP charts and regularity indicators.

CONCLUSION

Through the projection of aperiodic tiling patterns onto spherical surfaces using formex configuration processing techniques, a new family of lattice domes consisting of aperiodic spherical caps has been produced. This family is expanded through the application of pattern modifications, such as varying the number of tiling decompositions and triangulation and altering the projection method by manipulating the shape of the tiling patches before projection as well as using pellevation. Through the study of regularity, the more feasible design solutions have been identified as those under two pattern decompositions, opposed to those of three, while the proposed design modifications have no significant impact on regularity and feasibility.

As a result, new design options for lattice spatial structures have been produced which can be presented to architects as a range of interesting solutions to select from. The application of aperiodic tiling has also opened the possibility for various other families of lattice spatial structure designs by applying different aperiodic tilings and to various other structural forms.

REFERENCES

Behnejad, S. A. & Rognoni, T. I. (2019). 'Structural Application of Non-Periodic Patterns: Gridshell Configurations Inspired by Penrose Tiling', Japan: *Symmetry Art and Science, 2019 – 11th Congress and Exhibition of SIS*. [Accessed 19 February 2021].

Behnejad, S. A. (2018). *Geometrical Data for Lattice Spatial Structures: Regularity, Historical Background and Education*. Doctoral Thesis. Department of Civil and Environmental Engineering, University of Surrey. DOI: https://doi.org/10.15126/thesis.00849532. [Accessed 01 December 2020].

Behnejad, S. A., Parke, G. & Nooshin, H. (2013). 'Member Length Regularity of Lattice Domes', *ISIS Congress-Festival Symmetry: Art and Science*, Symposium: Bali, Crete, 9-15 September 2013.

Durn, J. & Behnejad, S. A. (2021). *A New Family of Lattice Domes: Spherical Caps with Aperiodic Tiling*, University of Surrey. Available at: https://www.researchgate.net/publication/359746043_A_New_Family_of_Lattice_Domes_Spherical_Caps_of_Aperiodic_Tiling.

Grünbaum, B. & Shephard, G. C. (2016). 'Aperiodic Tilings', in *Tilings and Patterns*. New York: Dover Publications, pp.519-582. [Accessed 01 April 2021].

Hofmann, I. S. (1999). *The Concept of Pellevation for Shaping Structural Forms*. Doctoral Thesis. University of Surrey. Available at: https://epubs.surrey.ac.uk/855512/. [Accessed 18 February 2021].

Nooshin, H. (2017). 'Formex Configuration Processing: A Young Branch of Knowledge', *International Journal of Space Structures*, Vol. 32 (3-4), pp.136-148. Available at: https://epubs.surrey.ac.uk/844887/. [Accessed 05 November 2020].

Nooshin, H., Ishikawa, K., Disney, P. L. & Butterworth, J. W. (1997). 'The Traviation Process', *Journal of the International Association of Shell and Spatial Structures*, Vol. 38, No. 3 December, pp.165-175. Available at: https://www.ingentaconnect.com/content/iass/jiass/1997/00000038/00000003/art00005.

Penrose, R. (1979). 'Pentaplexity A Class of Non-Periodic Tilings of the Plane', *The Mathematical Intelligencer*, 2, pp.32-37. Available at: https:/link.springer.com/article/10.1007/BF03024384. [Accessed 25 April 2021].

James DURN

James Durn is a final year MEng student in the early stages of a career in civil engineering. After growing up in Brighton, James began a master's degree in Civil and Environmental Engineering at the University of Surrey in 2017. At the start of his studies, James was awarded the ICE Scholarship Scheme and partnered with Arcadis, a global design and consultancy firm. Now nearing towards the end of his degree, he has maintained grades at a first-class level while also gaining 17 months of professional experience with Arcadis across two summer placements in 2018 and 2021 and a year in industry across 2019/20. On top of this, James was recently nominated for the IStructE Surrey Region Student Award for his dissertation on aperiodic lattice domes. Post-graduation, James looks to progress his career by hopefully joining Arcadis as a graduate engineer and to continue his pursuit to become a Chartered Engineer. Outside of engineering, James has achieved Grade 6 in playing the drums and has been a part of the University of Surrey Powerlifting Club for over 3 years.

Alireza BEHNEJAD

Alireza Behnejad is the Director of the Spatial Structures Research Centre of the University of Surrey, UK, with thirteen years of experience as an Architect, prior to joining the Centre. Alireza is a Senior Teaching Fellow (Associate Professor) at the Department of Civil and Environmental Engineering, and he was awarded the Teacher of the Year in 2021. He achieved his first master's degree in architectural engineering, followed by a second MSc in Structural Engineering, a PhD in Geometry of Spatial Structures, along with a Graduate Certificate in Learning and Teaching. Alireza is Editor in Chief of the International Journal of Space Structures, founding editor of the Spatial Structures; Movers and Shakers, member of the Executive Council of the International Association for Shell and Spatial Structures (IASS) and the founding chair of the working group on Teaching of Shell and Spatial Structures (IASS-WG20). His research interests include the tactile strategies in engineering education, regularity of structural configurations and the application of biobased structural materials (e.g bamboo) in construction.