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Abstract. The inherent modelling of the operational wind turbines and rotating machines do not agree in general with the assumptions of the operational modal analysis (OMA) methods developed for civil engineering, where time invariant systems are considered. Current OMA methods for rotating machines introduce data-pre-processing to adapt classical identification methods. However, they show strong limitations and rely on strong assumptions, such as the isotropy of the rotor, making them hardly applicable in practice. To overcome these limitations, this paper proposes to employ the Floquet theory of periodic system to approximate rotating systems as time invariant systems. Thus, classical identification methods can be used to retrieve the parametric signature of the periodic systems. This Floquet-based approximation gives a physical meaning to the identified eigenmodes. The proposed approach is validated on both a small numerical model and an aero-servo-elastic numerical model of a rotating 10MW wind turbine, with isotropic and anisotropic rotors, using the stochastic subspace identification to retrieve the modes and their uncertainty.

Keywords: Operational Modal Analysis · Wind Turbine · Subspace Methods · Vibration and Modal Analysis

1 Introduction

Given the prospects for growth in the number of wind farms in the coming years and the continuing quest to reduce maintenance costs, it is important to implement reliable monitoring methods based on data collected during operation of the wind turbines. Operational modal analysis (OMA) methods are powerful output-only identification methods to monitor structural properties. Many OMA methods have been developed for civil engineering applications, where the monitored structure can be accurately approximated by linear time invariant (LTI) systems [4]. However, the rotation of the rotor of a wind turbine breaks the LTI hypothesis, and these systems should rather be modelled as linear time periodic (LTP) systems. As they no longer meet the basic OMA assumptions, these LTI-based OMA methods must be extended to LTP-systems.

In this context, the first method developed for OMA on wind turbine is the multi blade coordinate (MBC) method [2]. It is based on a change of variables to transform an LTP-system into an LTI-system, making classical OMA methods applicable. However, the MBC is valid only if the rotor is isotropic, i.e. when all the blades have the same properties, which is not verified with experimentation [16]. A second OMA method for wind turbines is based on the so-called harmonic transfer function (HTF) [1] to create a map between inputs and outputs of an LTP-system. The approach was originally defined for control purposes [18]. It pre-processes the measurements to increase the dimension of the outputs by defining the exponentially modulated periodic (EMP) signal space. The theoretical number of modulation is a priori unknown and must be tuned by the user. This method has been extended to subspace identification with the same pre-processing [15]. Finally, the last OMA method for wind turbines is based on classical subspace methods adapted to LTP-systems, as the SSI-LPTV method [10]. The latter identifies the eigenmodes of an LTP-system and does not require a pre-processing of the measurements or isotropic properties of the rotor. However, the identified modes converge to their theoretical value with a rate proportional to rotational speed, which makes it adapted for systems with high rotational speed, as helicopters, but poorly suitable for systems with low rotational speed as wind turbines.

This paper proposes to define an identification method for LTP-systems valid even for lower rotational speeds. The approach exploits the Floquet decomposition of such systems. Indeed, an LTP-system can theoretically be approximated by a sum of eigenmodes of periodic amplitude. These periodic amplitudes are projected on a Fourier series, where each Fourier coefficient corresponds to the harmonic of a Floquet mode. It has been shown in previous works [5] that this sum can be truncated to retain only the few harmonics that contribute to the signal. The associated participation factors can be used for this selection [3]. The truncated sum leads to an expression of the data almost identical to the expression corresponding to an LTI-system, making then possible the use of identification methods on this new modelling. The main contribution of the paper is to prove that LTP-systems can be approximated as LTI-systems thanks to the Floquet-Fourier formulation. By taking advantage of this, classical OMA methods can be used to identify this new approximation and monitor such systems.

The organisation of the paper is as follows. Section 2 presents the dynamical behavior of an LTP-system and the associated approximation as an LTI-system. Section 3 validates the approximation using a frequency analysis with an example of a simple wind turbine model. Finally, Section 4 demonstrates that classical OMA methods can be used to identify the modes of the approximation, with examples on data computed with an aero-servo-elastic wind turbine model.

2 Modal analysis and approximation of LTP-system

2.1 Dynamic model

The motion of a constant rotating wind turbine can be expressed as an LTP-system,

$$\mathcal{M}(t)\ddot{\xi}(t) + \mathcal{C}(t)\dot{\xi}(t) + \mathcal{K}(t)\xi(t) = v(t), \quad (1)$$

where $\xi(t) \in \mathbb{R}^m$ represents the displacements of the structure at the degrees of freedom (DOF) of the system, and $\mathcal{M}(t+T) = \mathcal{M}(t)$, $\mathcal{C}(t+T) = \mathcal{C}(t)$, $\mathcal{K}(t+T) = \mathcal{K}(t)$, respectively the mass, damping and stiffness matrices. T represents the rotational period. The unknown input $v(t)$ is assumed to be a Gaussian white noise. In the following, the mechanical system is expressed in a state-space form, from the definition of the state vector $x(t) \in \mathbb{R}^n$ where $n = 2m$ and the observation $y(t) \in \mathbb{R}^r$.

$$x(t) = \begin{bmatrix} \xi(t) \\ \dot{\xi}(t \end{bmatrix} \quad \text{and} \quad y(t) = C_a\ddot{\xi}(t) + C_v\dot{\xi}(t) + C_d\xi(t), \quad (2)$$

where C_a, C_v and C_d are selection matrices. A noise $w(t)$ can be added to the observation. $w(t)$ is assumed to be a Gaussian white noise. This leads to the following state-space expression:

$$\begin{cases} \dot{x}(t) = A_c(t)x(t) + B_c(t)v(t) \\ y(t) = C(t)x(t) + D(t)v(t) + w(t) \end{cases}, \quad (3)$$

with all the matrices defined in [5]. Using the Floquet theory [9], the homogeneous part of the state vector can be expressed as a sum of n eigenmodes with a periodic amplitude, namely the Floquet modes

$$x_h(t) = \sum_{j=1}^{n} X_j(t) \exp(\mu_j t)\, q_j(t_0), \quad (4)$$

with μ_j the j–th characteristic exponent, $q_j(t_0)$ depending of the initial conditions and $X_j(t)$ the associated T-periodic amplitude.

2.2 Approximation of the Floquet modes

Once the Floquet mode decomposition is applied, the observation vector is expressed as a finite sum of eigenmodes to obtain the description of a time invariant system.

Similarly, the Floquet mode decomposition of the observation $y_h(t)$ is

$$y_h(t) = \sum_{j=1}^{n} Y_j(t) \exp(\mu_j t)\, q_j(t_0), \tag{5}$$

where $Y_j(t) = C(t)X_j(t)$ is both the amplitude of the j–th Floquet mode of the observation and a periodic vector of period $T = \frac{2\pi}{\Omega}$. Using Equation (5) and expanding into a Fourier series the amplitudes of the Floquet modes, it comes

$$y_h(t) = \sum_{j=1}^{n} \sum_{l=-\infty}^{\infty} Y_{j,l} \exp\left((\mu_j + il\Omega)\, t\right) q_j(t_0). \tag{6}$$

with $Y_{j,l}$ the Fourier coefficients. The contributions of the harmonics in the expansion of $y_h(t)$ are determined by the participation factor [3]

$$\phi^y_{j,l} = \frac{\|Y_{j,l}\|}{\sum_{l=-\infty}^{\infty} \|Y_{j,l}\|}. \tag{7}$$

A minimum participation factor (ϕ^y_{min}) is applied to select harmonics with the highest contribution. Then, an approximation of the observation is constructed as $\hat{y}(t)$ by a finite sum of eigenmodes,

$$\hat{y}_h(t) = \sum_{(j,l),\phi^y_{j,l} \geq \phi^y_{min}} Y_{j,l} \exp\left((\mu_j + il\Omega)\, t\right) q_j(t_0), \tag{8}$$

$\hat{y}(t)$ can then be expressed as a sum of $\tilde{n}$ eigenmodes

$$\hat{y}_h(t) = \sum_{p=1}^{\tilde{n}} Y_p \exp(\mu_p t)\, q_p(t_0), \tag{9}$$

where each index p corresponds to a pair (j, l) and $\mu_p = \mu_j + il\Omega$.

Finally the Floquet modes of an LTP-system have been approximated by a finite number of eigenmodes identical to those of an LTI-system. Consequently the dynamical behavior of an LTP-system can theoretically be approximated as the behavior of an LTI-system. The next section is dedicated to the validation of this approximation with a small model of a wind turbine.

3 Validation of the approximation

The previously defined approximation is first tested on a small example, using a frequency analysis. A model of wind turbine with five DOF is used [14], with the

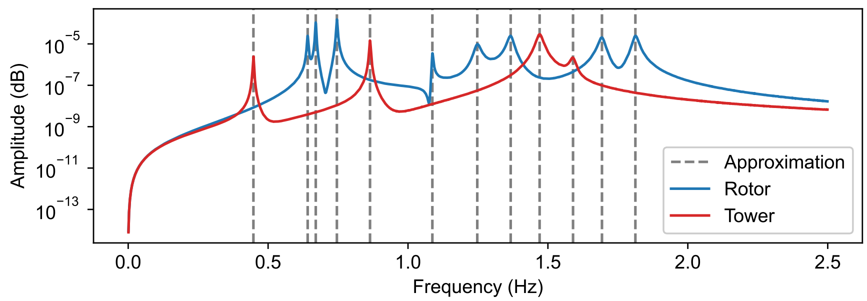

Fig. 1: Comparison of the frequencies of the approximation and the PSD of a free decay of the small model of wind turbine for two DOF

same structural parameters as described therein and a rotational speed of 1.4 rad/s. For this example the approximation is defined with a minimum participation factor of 1%. The objective is to compare the identified frequency peaks obtained by a Power Spectral Density (PSD) analysis using a decay test with the theoretical frequencies of the approximation modes ($\frac{\mu_p}{2\pi}$). An illustration of their superposition, for the accelerations of one rotor DOF and one tower DOF is shown in Figure 1.

As seen in Figure 1, each peak in the PSD spectrum corresponds to one frequency of the approximation modes. And vice versa, all theoretical approximation modes are observable in the PSD. Moreover, a good agreement between the frequency peaks of the approximation and the reference is observable. As a conclusion, approximating an LTP-system as an LTI-system is plausible and such approximation can describe the dynamical behavior of such system. The approximation has been illustrated on a simple model of wind turbine. The next section is dedicated to the identification of the approximation modes from simulated data of an operating wind turbine.

4 Identification: Application to an aero-servo-elastic wind turbine model

To confirm the proposed approach, an identification of the modes related to an aero-servo-elastic wind turbine model is performed. The considered model is precisely a finite element model of the DTU 10MW wind turbine [8] with an isotropic rotor, computed with the software OpenFAST [11]. The simulation is performed with turbulent wind and a rotational speed controlled by an external controller (Figure 2). The objective is to assess that the identified modes obtained with the current method are those computed with the theoretical approximation. In the simulation, the rotor has a mean rotational speed of 6 rpm, so a constant rotational speed of 6 rpm is assumed in the approximation. To compute the Floquet modes of the model, the

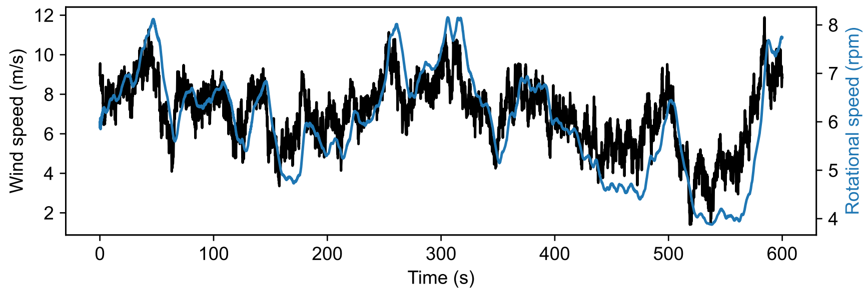

Fig. 2: Evolution of the wind speed and the rotational speed during the simulation

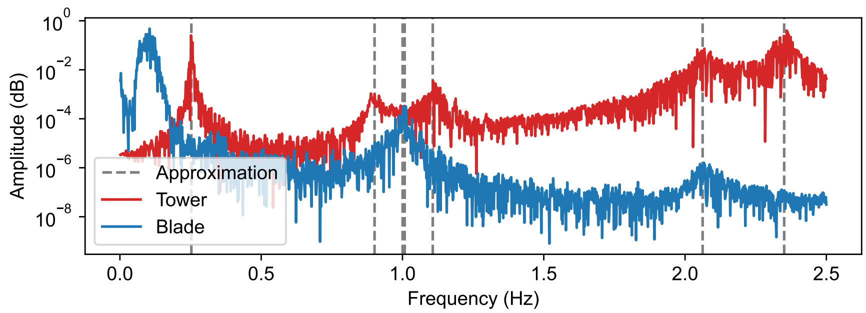

Fig. 3: Normalized PSD of different DOF, with the comparison with the frequency of specific approximation modes

MBC transform method is used on the system matrices, using the method described in [2]. A minimum participation factor of 5% is applied to derive the approximation from the Floquet modes. The data used in the identification are the Fore-Aft (FA) and Side-Side (SS) accelerations at mid- and top-tower, as well as the flap and edge blade root bending moments. The data are sampled at 50 Hz during 600s.

4.1 Frequency spectrum analysis

The normalized PSD of the blade root edge bending moment and a mid-tower side-side acceleration are displayed in Figure 3. The frequencies of some theoretical eigenmodes of the approximation are also added, namely the two first modes of Side-Side bending and the harmonics of the three different Floquet modes of the edge bending mode of the blades (backward, forward and collective). One can see that frequency peaks of the PSD are well characterized by the approximation. This illustrates once again the capacity of the approximate LTI modelling to describe accurately the dynamic of the turbine even tough the considered system is more complex.

A frequency spectrum analysis is done to identify the modes of the approximation. To retrieve the full modal information, a subspace identification with uncertainty quantification is now performed to compare the identified modes with the theoretical approximation modes.

4.2 Subspace identification method

Before performing the subspace identification, the state-space representation of the approximation is needed. From the definition of an LTP-system and its associated approximation, it is possible to define the following discrete state-space

$$\begin{cases} z_{k+1} = \mathbf{A} z_k + \mathbf{B}_k v_k \\ y_k = \mathbf{C} z_k + \mathbf{D}_k v_k + \tilde{w}_k \end{cases}, \tag{10}$$

with z_k the new state vector based on the approximation of the outputs, $\mathbf{A}$ and $\mathbf{C}$ two constant matrices and $\mathbf{B}_k$ and $\mathbf{D}_k$ two periodic matrices of period $T_d = \frac{T}{\Delta t}$ (Δt the data time step). The complete definition of the state-space of the approximation and the demonstration of the applicability of the Stochastic Subspace Identification (SSI) method is detailed in [5]. Here, an efficient version [6] of the SSI method [17] is used. The SSI [17] aims to identify the eigenmodes of the system through the sample correlations. The SSI covariance-driven is presented in the following. The first step is to construct a Hankel matrix filled by correlations, directly constructed from matrices gathering the observation data

$$\hat{H} = \mathcal{Y}^+ \left(\mathcal{Y}^-\right)^T. \tag{11}$$

where $\mathcal{Y}^+ \in \mathbb{R}^{(p+1)r \times N}$ and $\mathcal{Y}^- \in \mathbb{R}^{qr \times N}$ are defined in [17]. $\hat{H}$ can be seen as the Hankel matrix filled with the correlations $\hat{R}_i$, the estimate of the correlation $R_i = \mathbb{E}\left(y_k y_{k-i}^T\right) = \mathbf{C}\mathbf{A}^{i-1} G$, where $G = \mathbb{E}\left(z_{k+1} y_k^T\right)$. From the estimation of the Hankel matrix, by using a singular value decomposition it is possible to identify the system matrices $\mathbf{A}$ and $\mathbf{C}$. Then, the eigenmodes can be computed with the eigenvalue decomposition of $\mathbf{A}$

$$\mathbf{A} = \Psi \left[\mu_i\right] \Psi^{-1}. \tag{12}$$

The continuous time eigenvalues λ_i are found from the discrete time eigenvalues μ_i by $\lambda_i = \frac{\log(\mu_i)}{\Delta t}$. Then the frequency ($f_i$) and the damping ($\zeta_i$) of the associated mode are defined such that $f_i = \frac{|\lambda_i|}{2\pi}$ and $\zeta_i = -100.\frac{Re(\lambda_i)}{|\lambda_i|}$. Finally, the mode shape matrix is found from $\Phi = \mathbf{C}\Psi$.

Following the identification, an uncertainty computation method is used [7]. Defined in [13], the method estimates the covariance matrices of the identified modes from the same data used for the identification. From the covariance matrices it is possible to express the standard deviations σ_f and σ_ζ of the frequency and damping, respectively.

4.3 Identification

In this last section, the identification of the DTU 10 MW wind turbine modal properties is done based on the SSI method recalled previously. The identification is performed using the blade root bending moments and tower accelerations together. These two quantities do not have the same order of magnitude, so data are normalized first. Once the identification is performed the modes shapes are re-normalized to relate to physical quantities.

The identified modes are displayed in Figure 4 using a stabilization diagram, comparing the frequencies of the identified modes at different increasing orders and the reference (theoretical) frequencies of the approximation. It can be seen in Figure 4, that the modes obtained by the frequency spectrum analysis are properly identified. Once the modes are identified and selected, they are matched with the approximation modes using the Modal Assurance Criterion (MAC) [12], with a minimum matching value of 0.7. The modes are matched twice, once using only the tower DOF, once using only the rotor DOF. Finally, eight identified modes are matched with the approximation modes. They are reported in Table 1, where $(\cdot)_{ap}$ denotes a quantity of the approximation modes and $(\cdot)_{iden}$ a quantity of the identified modes. Some approximation modes, like the mode 2-FA and the flap bending modes, are not identified due to their high damping ratios (more than 30% for the flap bending modes and around 13% for the mode 2-FA).

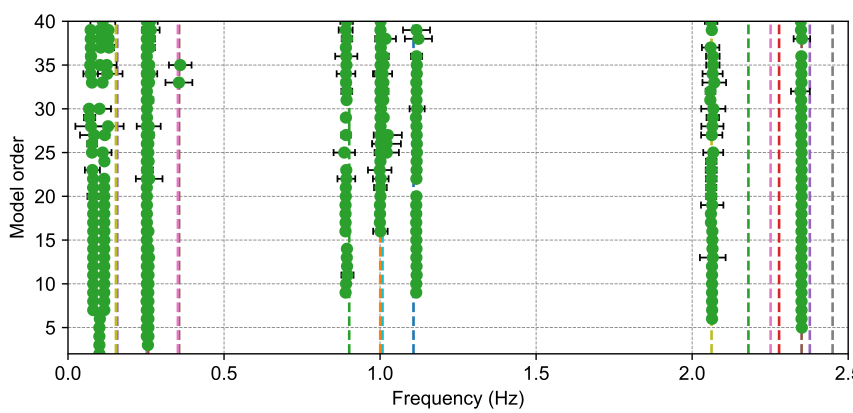

Fig. 4: Stabilization diagram from an identification with all the data – Identified modes (●) – Frequencies of the approximation modes (- - -) and related standard deviations (error bars) – Only modes with $\sigma_f < 0.05$ Hz are displayed

Table 1: Summary of the identification results, with the comparison between the identified modes and the approximation modes

	f_{ap} (Hz)	f_{iden} (Hz)	σ_f (Hz)	ζ_{ap} (%)	ζ_{iden} (%)	σ_ζ (%)	MAC	Names
Tower	2.351	2.351	0.003	0.869	0.987	0.124	1.000	2-SS
	2.062	2.064	0.006	1.778	1.732	0.280	1.000	Col. edge
	1.107	1.116	0.004	2.066	2.715	0.436	0.907	Fw. edge tower
	0.901	0.892	0.006	1.908	2.252	0.274	0.975	Bw. edge tower
	0.257	0.258	0.004	7.776	7.502	1.204	0.998	1-FA
	0.253	0.253	0.001	0.275	0.352	0.277	0.744	1-SS
Rotor	2.062	2.064	0.006	1.778	1.732	0.280	0.969	Col. edge
	1.008	1.010	0.011	2.269	1.771	1.184	0.759	Fw. edge rotor
	1.000	1.001	0.005	1.719	1.792	0.417	0.915	Bw. edge rotor
	0.257	0.258	0.004	7.776	7.502	1.204	0.983	1-FA

The relative gap between the identified modes from the outputs and the approximation modes computed from the model is lower than 1% for all the identified modes. It can be concluded that the identified modes correspond to the approximation modes and that the formulation proposed in the paper is relevant.

In a second step, the model is now complexified and modified with the introduction of anisotropy in the rotor. The same parameters are used, except that one blade has a stiffness of 95% compared to the others. In this case, the identification results obtained are nearly identical to those obtained from the isotropic wind turbine, with the same identified modes, equivalent uncertainties with a mean frequency standard deviation of 5 10^{-3} Hz for the isotropic case and 5.6 10^{-3} Hz for the anisotropic case. The gaps to the reference are also equivalent, with a mean MAC criterion of 0.920 for the isotropic case and 0.963 for the anisotropic case. It can be concluded that the approximation is valid for a wind turbine with an anisotropic rotor and that a wind turbine can be monitored no matter its rotor configuration.

5 Conclusion

It has been shown that the dynamical behavior of an LTP-system can be described by a reduced LTI modelling resulting from an approximation of the LTP Floquet modes. A direct frequency spectrum analysis of a numerical model of wind turbine has demonstrated the validity of such approximation. The major interest of such approximation is that classical identification methods, as the SSI, can be employed. Finally, the interest and efficiency of the method is demonstrated on an aero-servo-elastic model of a 10 MW wind turbine, where the LTI identification of the modes is done. More generally, this work demonstrates the interest of OMA of wind turbines.

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