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Transforming Private Pensions: An Actuarial Model to Face Long-Term Costs

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Abstract: A common response in public pension systems to population ageing is to link pensions to observed longevity. This creates an automatic stabiliser that arises from the valuation of a private actuarially funded system. However, no private pension plan mechanism has been articulated to adapt to this ageing in relation to the increased costs it entails. Private pension plans focus on saving for retirement; capital is accumulated to pay for it. However, perceptions of health status change over time and, as retirement age approaches, concerns about long-term care (LTC) increase. Moreover, there is not enough time to plan for it sufficiently in advance. This paper proposes to incorporate a mechanism to add an allowance to the financial pension (retirement, disability, rotation) to cover LTC within a private defined benefit pension plan, in the case of a pensioner becoming dependent. Depending on a pensioner's health status, both the expected number of payments and their intensity are transformed. For this purpose, a mechanism is defined (through Markov chains) to adapt the amount of LTC support to a beneficiary's health-related life expectancy. The study's main contribution is that it establishes a private pension plan model that offers to incorporate dependency aid through this mechanism into the economic pensions without increasing the total cost of the plan. It adapts to life expectancy according to a person's state (healthy, disabled, dependent).

Keywords: ageing; dependency; long-term care; private pensions

MSC: 91G05

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1. Introduction

Long-term care (LTC) is defined as those expenditures devoted to the care of older people over a period of time [1]. This care is the direct support for activities of daily living (bathing, dressing, eating, etc.) or through support for instrumental activities (preparing meals, cleaning, managing money, etc.). LTC mainly arises from the loss of autonomy caused by old age [2,3]. The growing number of older people who need healthcare thus increases the financial pressure on healthcare systems [4–6]. At the same time, demand for better access to higher quality services is growing [7]. To realise the importance, according to [8], total LTC spending in the United States in 2012 was 8.7% of total health care expenditure (USD 220 billion). Between one-third and one-half of United States retirees needed nursing and care services, and between 10% and 20% of these required it for more than five years [9]. It is estimated that 43% of the European Union's population will be over 65 by 2025, reaching 129.8 million inhabitants; this will increase health and care expenditure [10]. This is now an area of special relevance in the scientific literature [11].

Health care systems differ widely between countries and taken reforms have changed the approach to LTC. As a result, some authors [12,13] have classified care systems based on integrated dependency on a welfare state and the relationship with other institutions. This allows LTC to be included gradually into both public and private systems.

Although dependency costs can be considered as a natural extension of health insurance, dependency is a contingency that should be considered as important as retirement [14] so that the insured can be protected from the risk of outliving their resources after retirement [15]. In this sense, the literature proposes linking dependency coverage to pensions to extend its effect [16–18].

LTC coverage should be integrated into an individual's particular pension strategy, such as retirement [1], rather than being considered an additional health service [14]. Financial support for LTC is therefore a logical extension of the purpose of pension plans: to provide an adequate complement to meet retirement needs, no matter the individual's health status. Pension plans should therefore finance LTC needs [19]: on the one hand they should provide an income to make up for the lack of a salary and, on the other, should supplement the pension income. In this way, as the individual ages, the needs change; when the baby boom generation reaches an age where LTC is needed, there will be resources to meet them [20].

Reference [21] claims that dependency coverage is already into retirement planning so that the probability of becoming dependent is already included in the planning itself. Other authors [22,23] argue that dependency and mortality are negatively correlated, thus creating a natural demand for each product. Two affecting factors are undoubtedly age and health status, with a degree of uncertainty as to when the insured will become dependent [24], although there is evidence of adverse risk selection or favourable hedging: individuals who are more risk averse and take better care, typically live longer and are more likely to underwrite LTC products [25,26].

LTC coverage and retirement should be planned well in advance [27,28] to avoid the effects of demographic and social changes. Thus, in most countries, the elderly population will increase and family caregiving will decrease [29–34]. It will therefore be necessary to provide resources, products and services that are adapted to people´s needs [35].

The problem lies in the financing of dependency. As a solution, some authors [36,37] propose to finance LTC coverage by paying the premium from the retirement benefit. In contrast to this approach, other authors [38,39] proposed that LTC should be integrated within the coverage itself, which adapts the pension to the dependent situation through an actuarial factor that relates the pension to a pensioners' life expectancy [40,41]. Social security in countries such as Sweden, Norway, Italy, Poland and Latvia employ a mechanism in which individuals receive benefits based on both estimated life expectancy and contributions. This reduces pressure on public resources while tailoring the pension to each generation's characteristics [42,43]. The inclusion of this resource introduces actuarial rationality to the system [44].

This study is novel because, as there is uncertainty as to when a person will become severely or highly dependent, it proposes that beneficiaries prioritise their resources to pay for LTC. Therefore, this paper aims to establish a financial–actuarial model that transforms private economic resources (pensions) by including a supplement to help pay for LTC at a beneficiary's request. The inclusion of an actuarial model results in a social contribution by adapting the private plan to a beneficiary's needs, whether they are a retired or disabled pensioner.

To meet this objective, the following section presents an actuarial model for valuing pensions. This model, under differential mortality rates according to a beneficiary's status, determines the actuarial pension correction factor to be applied, which is based on whether a beneficiary is retired or disabled. The choice of correction factor depends on a beneficiary's life expectancy (healthy, disabled, severely dependent). The third section shows this factor's results when applied to the Spanish mortality experience for independent persons as well as disabled and dependent persons. The final section discusses both the results and the proposed actuarial model, and proposes future research recommendations.

The main results show that expenditure can be adapted to reality. Even while maintaining the original pension, an additional supplement to cover LTC is achieved. As a final result, a dependent person's quality of life improves when LTC expenses are partly covered.

As [45] highlighted, a dependent person may live longer by improving their functional environment.

2. Materials and Methods

2.1. The Actuarial Model

This paper extends the model initially proposed by [46–48] obtaining complemented model to pay LTC needs for higher degrees of dependency. The initial assumption takes a defined benefit (DB) plan and the individual lacks information about his/her future health status. Therefore, the individual's contribution history is independent of the future health status: the plan depends solely on the labour career. Moreover, any additional information about their true health status that emerges over time does not really affect the pension at retirement. In contrast, if the plan were a defined contribution (DC) plan, an individual would progressively acquire information about his/her health status and could therefore underwrite a coverage (insurance, annuity, reverse mortgage, etc.) suited to their LTC needs [49–51].

The classification of degrees of dependency depends on institutional factors in each country. In any case, as the degree of dependency increases, more LTC is needed. In fact, milder degrees of dependency may result in disability pensions as they do not allow a worker to perform their usual work. As derived from the study of populations in the United Kingdom [52,53] and Norway [54], it is also appreciated that mortality for dependent persons is proportional to their age and the level of care needed. It is therefore necessary to focus on the calculation of the probability of a dependent person's death that limits the duration of LTC payments.

Therefore, let X be the random variable "age of death of a new born". F represents the death distribution function

$$F(x) = P(X \leq x) \tag{1}$$

where $x \geq 0$ and $F(0) = 0$.

The complementary is the survival function. For each age x, it gives the probability that a new born will reach that age alive. That is $\forall\ x \geq 0$.

$$s(x) = P(X > x) = 1 - F(x) \tag{2}$$

The derivative function $f(x)$ of the death function $F(x)$ results in

$$f(x) = \frac{dF(x)}{d\ x} = -\frac{ds(x)}{d(x)} = -s'(x) \tag{3}$$

being (μ_x) the instantaneous mortality rate.

$$\mu_x = \frac{f(x)}{1 - F(x)} \tag{4}$$

As $\mu_x \geq 0$ and $f(x) = -s'(x)$, then

$$\mu_x = -\frac{s'(x)}{s(x)} = -\frac{d\ Ln\ (s(x))}{dx} \tag{5}$$

The probability that a person of age x will live t years or more can therefore be defined as

$$_tp_x = e^{-\int_x^{x+t} \mu_x dz} \tag{6}$$

Likewise, v_T is the financial discount factor from the t-th instant to the origin or zero moment, where the financial discount function is defined by the discounting process at the instantaneous rate of interest $\delta(t)$.

$$v_T = e^{-\int_0^T \delta(t)dt} \tag{7}$$

The present value of the compensation at time t-th will be

$$Z_T = b_T \cdot v_T \tag{8}$$

and both magnitudes b_T and v_T depend on the time-to-death.

If the survival function ($s(x)$), the payoff function (b_T) and the financial function (v_T) are known, it is possible to estimate the present value of future pension benefits (assuming a duration from, e.g., retirement age r to maximum life expectancy age w) or actuarial value at age r ($PVFB_r$) as

$$PVFB_r = E(Z_T) = E(b_T \cdot v_T) = \int_r^w b_t \cdot e^{-\int_r^w \mu_t dt} \cdot e^{-\int_r^w \delta(t) dt} \cdot dt \tag{9}$$

This study assumes that when a beneficiary becomes severely or highly dependent at an intermediate age x, between age r and age w, the pension automatically increases by the factor λ_x^d. As a result, the new amount helps to pay for their LTC. This factor, which increases the pension, is counterbalanced by a dependent person's inherent mortality difference. At each age there is a different factor that is affected only by the different mortality probabilities: general mortality and dependent´s mortality.

When an individual becomes dependent, $b_x \cdot \lambda_x^d$ replaces b_x (the pension due at age x). Thus, the pension is automatically increased to provide a higher pension to help to pay for LTC costs.

2.2. The Actuarial Equity Factor

This factor means that there is no additional funding. Therefore, at an age $x > r$, if a beneficiary becomes dependent and decides to transform his/her pension, the present value of future pension benefits has to be equal to the present value of future pension benefits combined with LTC assistance for a dependent person.

$$PVFB_x = PVFLTC_x \tag{10}$$

$PVFB_x$: Actuarial value of future pensions to be received by an independent pensioner valued at age x, such that $x > r$.

$PVFLTC_x$: Actuarial value of a future pension, including the new LTC aid at age x, such that $x > r$.

The result is the conversion factor at age x, such that $x > r$. The factor depends on the differential of mortality tables (general versus dependent's mortality) discounted to the expected return of the pension fund.

$$\lambda_x^d = \frac{\int_x^w e^{-\int_t^{t+1} \mu_t dt} \cdot e^{-\int_r^{t+1} \delta(t) dt} \cdot dt}{\int_x^w e^{-\int_t^{t+1} \mu_t^d dt} \cdot e^{-\int_r^{t+1} \delta(t) dt} \cdot dt} = \frac{\overline{a}_x^m}{{}^d\overline{a}_x^m} \tag{11}$$

$e^{-\int_t^{t+1} \mu_t^d dt}$: Probability that a dependent person of age t will live to age $t + 1$ as a dependent person.

$e^{-\int_t^{t+1} \mu_t dt}$: Probability that a person of age t will live to age $t + 1$, based on a general mortality table.

$\overline{a}_x^m$: Actuarial life annuity of a healthy person at age x. It can be variable or constant, depending on whether it is indexed to an external benchmark.

${}^d\overline{a}_x^m$: Actuarial life annuity of a dependent person at age x. It can be variable or constant, depending on whether it is indexed to an external benchmark.

The resulting LTC complement depends on:

- The age of decision making.
- A cohort's expected mortality.
- A pension plan's performance.

- A dependent person's expected mortality.
- The amount of pension that a beneficiary receives.

With the exception of a dependent person's expected mortality, all other parameters are standard in the development of a private pension plan.

2.3. The Mortality of Dependent Persons

In the literature [55–58], there is consensus that at a given age, the disabled persons' mortality rate (${}^{i}q_x^m$) is higher than the general population mortality rate (q_x^m); there is unanimity that the dependent persons' mortality rate (${}^{d}q_x^m$) is different to and higher than the general mortality rate, as shown in the standard mortality tables used by insurers. It is, of course, significantly higher than the mortality rate of insured persons (${}^{a}q_x^{(m)}$).

$$ {}^{d}q_x^m > {}^{i}q_x^m > q_x^m > {}^{a}q_x^{(m)} \quad (12) $$

This paper initially starts from a simplified type of multi-state transition model based on stochastic Markov processes [59,60] describing the probabilities between various states: active worker to retired (both independent and dependent), active worker to disabled (independent or dependent) and to deceased (Figure 1). For an annual period, it is a multi-state discrete model, where it is assumed that there can be no more than one transition per year and there are no returns to previous states.

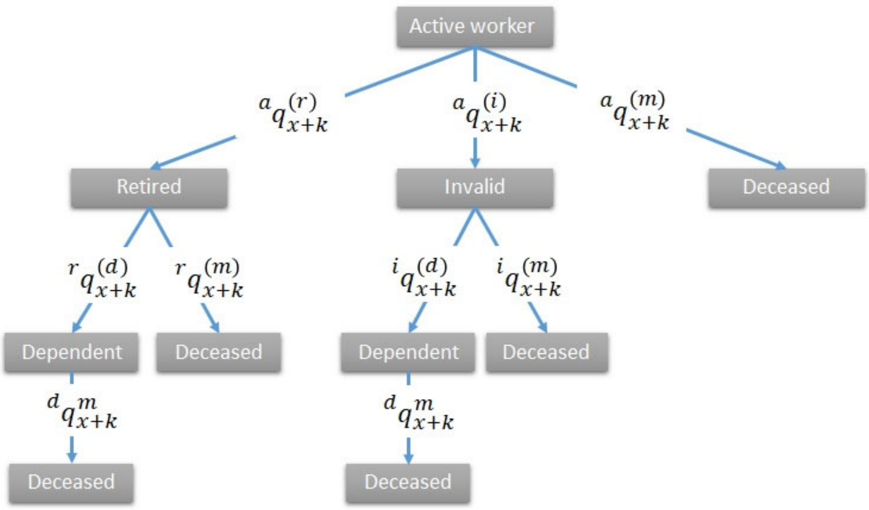


Figure 1. Transition probabilities. Source: Own work.

Being

${}^{a}p_{x+k}^{(a)}$: Probability that an active worker aged $x + k$ lives one year more as active worker.

${}^{a}q_{x+k}^{(i)}$: Probability that an active worker aged $x + k$ becomes disabled in less than one year, exposed to other causes of exit (death and retirement).

${}^{a}q_{x+k}^{(m)}$: Probability that an active worker aged $x + k$ dies in less than one year, exposed to other causes of exit (disability and retirement).

${}^{a}q_{x+k}^{(r)}$: Probability that an active worker aged $x + k$ retires in less than one year, exposed to other causes of exit (death and disability).

If $x + k$ is previous to the retirement age ($x + k < x < r$), the following equivalence is obtained

$$ {}^{a}p_{x+k}^{(a)} + {}^{a}q_{x+k}^{(i)} + {}^{a}q_{x+k}^{(m)} + {}^{a}q_{x+k}^{(r)} = 1 \quad (13) $$

This is true for the whole period of activity.

Once an active worker becomes disabled and is receiving a disability pension, for an age $x + k$, the following is fulfilled

$$ {}^{i}p_{x+k}^{(i)} + {}^{i}q_{x+k}^{(m)} + {}^{i}q_{x+k}^{(d)} = 1 \tag{14} $$

where:

${}^{i}p_{x+k}^{(i)}$: Probability that a disabled person aged $x + k$ lives one year more as disabled.

${}^{i}q_{x+k}^{(m)}$: Probability that a disabled person aged $x + k$ dies in less than one year, exposed to other causes of exit (dependency).

${}^{i}q_{x+k}^{(d)}$: Probability that a disabled person aged $x + k$ becomes dependent in less than one year, exposed to other causes of exit (death). This is a binomial value (0; 1). Zero value when a beneficiary decides to continue receiving his/her pension and one value when the beneficiary decides to transform the pension.

As in the case of disability, from the age of retirement ($x > r$), a retirement pension is paid if the beneficiary is alive. The beneficiary can apply for a supplement in case of severe dependency. Therefore

$$ {}^{r}p_{x+k}^{(r)} + {}^{r}q_{x+k}^{(m)} + {}^{r}q_{x+k}^{(d)} = 1 \tag{15} $$

${}^{r}p_{x+k}^{(r)}$: Probability that a retirement pension beneficiary aged $x + k$ lives one year more as retired.

${}^{r}q_{x+k}^{(m)}$: Probability that a retirement pension beneficiary aged $x + k$ dies in less than one year, exposed to other causes of exit (dependency).

${}^{r}q_{x+k}^{(d)}$: Probability that a retirement pension beneficiary aged $x + k$ becomes dependent in less than one year, exposed to other causes of exit (death). This is a binomial value (0; 1). Zero value when a beneficiary decides to continue receiving his/her retirement pension and a value of one when the beneficiary decides to transform the pension.

Finally, to determine dependent persons:

${}^{d}p_{x+k}^{d}$: Probability that a retirement pension beneficiary who is dependent aged $x + k$ lives one year more as a dependent person.

${}^{d}q_{x+k}^{m}$: Probability that a retirement pension beneficiary who is dependent aged $x + k$ dies in less than one year.

Evidently, the sum is unity at age $x + k$.

$$ {}^{d}p_{x+k}^{d} + {}^{d}q_{x+k}^{m} = 1 \tag{16} $$

If the model applies a factor (λ_x^d) to a pension benefit when becoming a dependent person, only the probability of death as a dependent should be determined.

3. The Spanish Experience Results

3.1. Dependency Degrees and Mortality Tables

Institutional LTC systems organise dependency according to the degree of severity: from the mildest to the most severe dependence and depending on the number and type of activities of daily living that an individual can perform. The classification has a direct impact on the public aid received; both the classification and amount of aid differs for each country.

In Spain, the coverage for the highest levels of dependency is offered [61] either through insurance products or through pension plans. In fact, dependency coverage [62] was added as a contingency in a private pension plan and was exclusively for degrees of severe or high dependency [48,63].

The PERM/F 2000 mortality tables for the general population were chosen—specifically for the year 2008 [64]. There is one life table for men and one for women.

For the disabled persons group, the Spanish social security actuarial tables for pensioners receiving a life annuity for disability [65] provide information on the entire population with permanent disability, whether or not they are dependent. All ages from 16 to 108 years are included. The data regarding gender were obtained from the 2008 Spanish population census [66].

3.2. Mortality of Severely and Highly Dependent Persons

In Spain, only the national survey on disability, personal autonomy and dependency [67] provides data for this group of people. It is impossible to determine degrees of dependency from this survey because it focuses solely on the habits and care of dependent persons. However, there are studies [46] that have established the life expectancy of an individual suffering from the most severe stages of dependency. In these works, based on an overall mortality, a dependent individual has an excess mortality expressed by a multiplicative correction (θ)

$$ {}^{d}q_x^m = \theta \cdot q_x^m \tag{17} $$

This correction can be variable at each age, although [68] indicated that a fixed correction adjusts the mortality of older dependents better than other types of approximations. However, the multiplicative correction tends to overestimate the mortality of a dependent individual at lower ages and underestimate it at higher ages. It is therefore more correct to make an additive adjustment (ε) to the overall mortality by considering age as an independent variable in a functional form [69]

$$ {}^{d}q_x^m = q_x^m + \varepsilon \tag{18} $$

where $\varepsilon = f(x)$.

Thus, mortality rates are lower at younger ages and are increasing with the level of dependency. For lower dependency levels, no excess mortality applies [70].

Reference [71] determined the probability of death of severely and highly dependent persons; they used general mortality tables and adjusted them to HID 98-01 statistics for France.

They found that excess mortality differentials with respect to overall mortality decreased from age 96 onwards. To capture this effect, they included a variation of the Rickayzen and Walsh formula, which is based on a mixed correction for overall mortality, to model dependent mortality. In this mixed correction, this study's authors considered an additive modification under the Rickayzen and Walsh expression and a multiplicative correction on the overall mortality numbers that reflects the decline in absolute mortality differentials in older ages. The function is

$$ {}^{d}q_x^m = \begin{cases} q_x^m + \frac{\delta}{1+\gamma^{x_i - x}} \; \forall x_i < 95 \\ q_x^m \cdot (1+\beta) + \frac{\delta}{1+\gamma^{x_i - x}} \; \forall x_i \geq 95 \end{cases} \tag{19} $$

δ: Maximum value to be incorporated according to the age at which it asymptotically converges.

γ: Slope factor

x_i: Age at the inflection point where the curve changes shape from convex to concave.

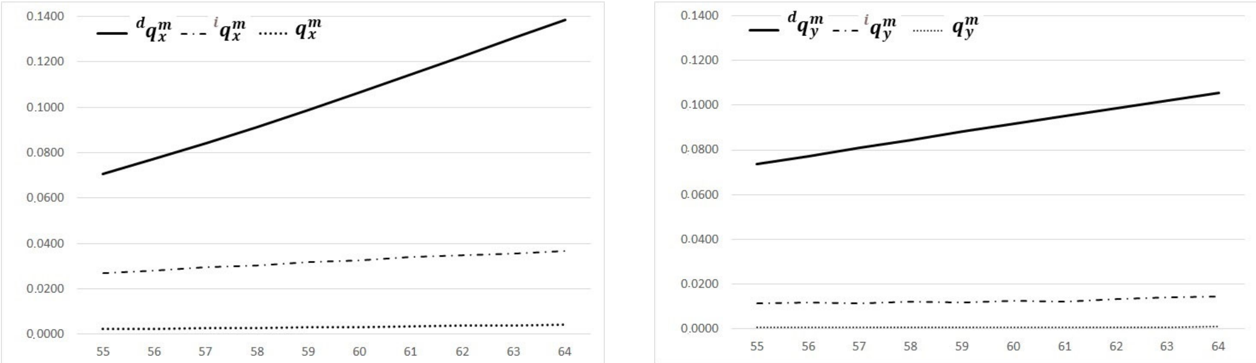
β: Multiplicative factor on overall mortality.

The values obtained with an ordinary least square procedure with respect to the crude values of severe dependency estimated for Spain are given in Table 1.

Table 1. Dependent excess mortality factors for the level of severe and high dependency in Spain. Source: [71].

Factors	Men	Women
δ	0.245	0.165
γ	1.135	1.09
x_i	62.50	58.61
β	0.1142	0.0962

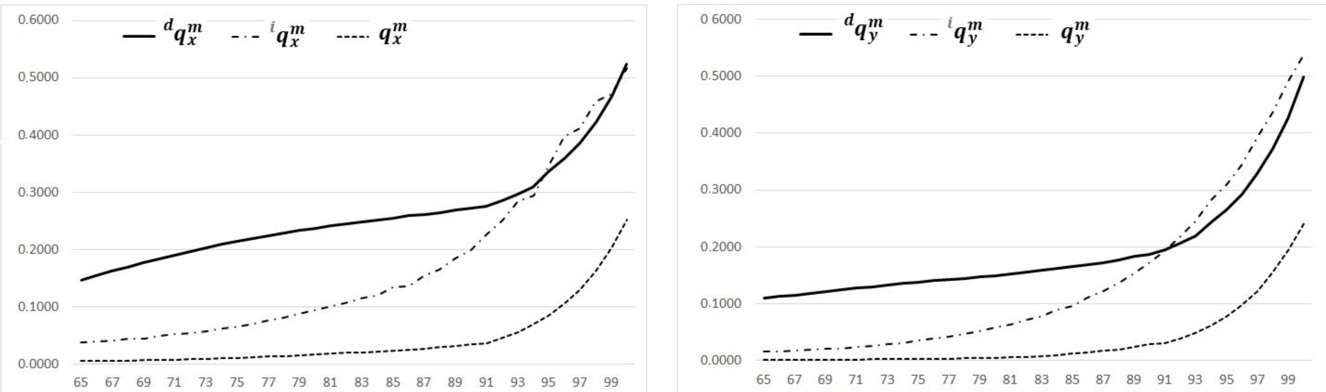
Mortality rates for severely and highly dependent persons are higher than the overall mortality for all ages (Figure 2). Therefore, expression (12) is fulfilled in the Spanish mortality experience. Figure 2 shows the different mortality rates for different periods and genders for the 10 years prior to retirement for both men (a) and women (b). In all cases, male mortality values are higher than female mortality values.



(**a**) (**b**)

Figure 2. Mortality differential by status and gender in the last 10 years of work activity. (**a**) Men; (**b**) women. Source: Own work. Database: PERM/F 2000 mortality tables for the general population [64]; Spanish social security actuarial tables for disability [65]; severe and high dependency mortality tables in Spain [71].

This inequality can also be seen in the period after retirement (Figure 3) for both men (a) and women (b). In this case, disabled persons tend to reach a mortality rate in their later years, which is close to that of severely and highly dependent persons. Logically, the degeneration of the human body makes the mortality status of both coincide (Figure 3).



(**a**) (**b**)

Figure 3. Mortality differential by status and gender after retirement age at 65. (**a**) Men; (**b**) women. Source: Own work. Database: PERM/F 2000 mortality tables for the general population [64]; Spanish social security actuarial tables for disability [65]; severe and high dependency mortality tables in Spain [71].

There is a difference in mortality by gender, in all age groups and for the different states. Thus, it can be seen in Figure 4 that the percentage of excess mortality in men is much higher. In the 10-year age brackets, a gradual decrease in the excess mortality differential can be seen in all states. In the last years of estimated life, the mortality rate values are almost equal so that excess mortality tends to decrease (d).

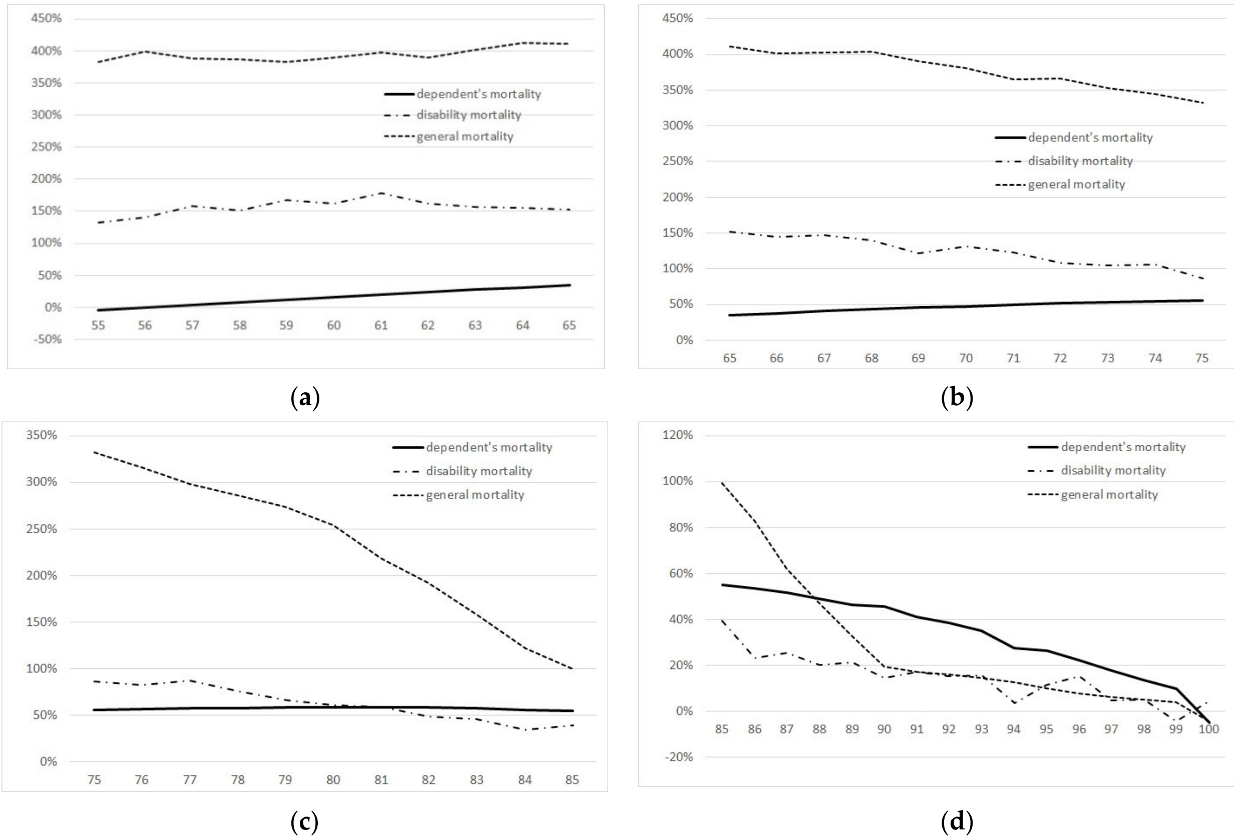


Figure 4. Existing excess mortality in each state by gender and age (**a**) 55 to 65 years; (**b**) 65 to 75 years; (**c**) 75 to 85 years; (**d**) 85 to 100 years. Source: Own work. Database: PERM/F 2000 mortality tables for the general population [64]; Spanish social security actuarial tables for disability [65]; severe and high dependency mortality tables in Spain [71].

3.3. LTC in Retirement

A beneficiary receives their retirement pension according to the temporary payment expectations based on the mortality rate. However, a change in status will not affect any accumulated capital, but it will affect the type of pension received; the priority will not be to replace wages but to help with LTC expenses. Therefore, the expected payment under the new circumstances is affected, which leads to a reduction in the number of payments in line with the increased probability of death as a dependent.

To help to clarity, Table 2 refers to retirement outcomes for people who were born in the 1960s. It not only shows the different mortality rate values according to status (general versus dependent), but it also shows the various factors to be applied when an individual becomes severely dependent, which provides the value of LTC aid and an analysis by gender. Although a common denominator is the increase with age of the overall mortality rate for both men (q_x^m) and women (q_y^m), in a dependent situation the same is true for the number of men (${}^d q_x^m$) compared to those of women (${}^d q_y^m$). However, the mortality gap decreases with increasing age in both sexes ($\Delta q_x^m / {}^d q_x^m$).

Analysing the LTC factor, as the individual becomes dependent in younger ages, we obtain higher amounts per euro of retirement pension, for both men (λ_x^d) and for women (λ_y^d). For men, the values are up to three times higher than the pension they receive in the first years of retirement.

Table 2. Retirement outcomes for the 1960s generation. Source: Own work. Software used: Microsoft Excel.

	Male Mortality Rates			Female Mortality Rates			LTC Factors		
Age	q_x^m		$\Delta q_x^m/{}^dq_x^m$	q_y^m	${}^dq_y^m$	$\Delta q_y^m/{}^dq_y^m$	λ_x^d	λ_y^d	$\Delta\lambda_x^d/\lambda_y^d$
65	0.01050	0.15069	1335.03%	0.00292	0.10727	3576.65%	2.981	2.352	26.76%
66	0.01119	0.15874	1318.54%	0.00316	0.11075	3404.81%	3.000	2.328	28.86%
67	0.01212	0.16679	1275.75%	0.00341	0.11415	3247.12%	3.009	2.299	30.87%
68	0.01318	0.17469	1225.33%	0.00369	0.11748	3082.76%	3.008	2.266	32.77%
69	0.01416	0.18223	1186.87%	0.00407	0.12080	2868.18%	2.998	2.228	34.57%
70	0.01508	0.18940	1156.12%	0.00441	0.12398	2713.09%	2.978	2.186	36.26%
71	0.01633	0.19649	1103.52%	0.00492	0.12720	2484.29%	2.949	2.139	37.87%
72	0.01798	0.20355	1031.96%	0.00540	0.13028	2313.58%	2.912	2.089	39.40%
73	0.01973	0.21031	966.07%	0.00609	0.13343	2091.59%	2.867	2.035	40.86%
74	0.02188	0.21684	891.19%	0.00703	0.13648	1841.54%	2.814	1.978	42.26%
75	0.02416	0.22304	823.27%	0.00816	0.13951	1609.91%	2.755	1.918	43.60%
76	0.02648	0.22886	764.30%	0.00953	0.14254	1395.12%	2.689	1.856	44.88%
77	0.02899	0.23441	708.66%	0.01119	0.14559	1200.89%	2.617	1.791	46.12%
78	0.03182	0.23981	653.71%	0.01302	0.14854	1040.54%	2.541	1.725	47.32%
79	0.03515	0.24521	597.52%	0.01528	0.15157	892.00%	2.460	1.657	48.47%
80	0.03859	0.25029	548.62%	0.01825	0.15483	748.26%	2.375	1.588	49.58%
81	0.04277	0.25556	497.49%	0.02144	0.15852	639.20%	2.286	1.518	50.62%
82	0.04721	0.26065	452.06%	0.02456	0.16203	559.60%	2.195	1.448	51.57%
83	0.05136	0.26516	416.26%	0.02871	0.16609	478.53%	2.100	1.378	52.38%
84	0.05449	0.26913	393.88%	0.03402	0.17079	402.07%	2.003	1.309	53.04%
85	0.05921	0.27401	362.78%	0.03969	0.17559	342.44%	1.904	1.240	53.56%
86	0.06471	0.27923	331.53%	0.04554	0.18038	296.08%	1.803	1.171	53.93%
87	0.06918	0.28353	309.85%	0.05289	0.18600	251.64%	1.701	1.104	54.06%
88	0.07473	0.28840	285.95%	0.06072	0.19178	215.85%	1.599	1.039	53.98%
89	0.08129	0.29382	261.43%	0.06695	0.19856	196.56%	1.498	0.975	53.65%
90	0.08830	0.29938	239.07%	0.07771	0.20630	165.49%	1.398	0.913	53.09%
91	0.09546	0.30490	219.41%	0.09013	0.21205	135.26%	1.302	0.854	52.40%
92	0.10916	0.31679	190.20%	0.10434	0.22296	113.70%	1.213	0.800	51.64%
93	0.12427	0.33014	165.67%	0.12050	0.23555	95.48%	1.132	0.750	50.79%
94	0.14172	0.34579	143.99%	0.14021	0.25100	79.02%	1.061	0.708	49.88%
95	0.16034	0.37696	135.10%	0.16276	0.27970	71.85%	1.006	0.675	49.06%
96	0.18176	0.39947	119.78%	0.18852	0.30277	60.60%	0.933	0.631	47.76%
97	0.20646	0.42607	106.37%	0.21794	0.32980	51.33%	0.866	0.592	46.24%
98	0.23498	0.45760	94.74%	0.25149	0.36153	43.75%	0.807	0.558	44.52%
99	0.26853	0.49565	84.58%	0.29037	0.39930	37.52%	0.756	0.530	42.72%
100	0.30749	0.54117	75.99%	0.31217	0.44103	41.28%	0.718	0.508	41.23%

If LTC benefits are differentiated, the LTC benefits' amount for men is higher than for women ($\Delta\lambda_x^d/\lambda_y^d$), reaching 50% higher at ages close to 90. This is because men's mortality rates are higher than those of women.

Figure 5 generalises the above by analysing the value of the actuarial equity factor by age and gender according to a beneficiary's birth year.

An analysis of the results shows that:

1. For both men (a) and women (b), once retired and in the event of requesting conversion for severe dependency or high dependency, an LTC supplement is generated, which doubles or even triples their original pension's value.
2. Regardless of gender, the older the age, the lower the actuarial equity factor tends to be; this leads to a lower LTC supplement.
3. Older generations show smaller increases in LTC allowances. This shows that the effect of the mortality in severe and high dependence individuals will be significantly higher in younger generations.
4. Men experience larger increases in LTC supplements than women in the 75–85 age bracket (Figure 5c). At higher ages the actuarial equity factor tends to equalise.

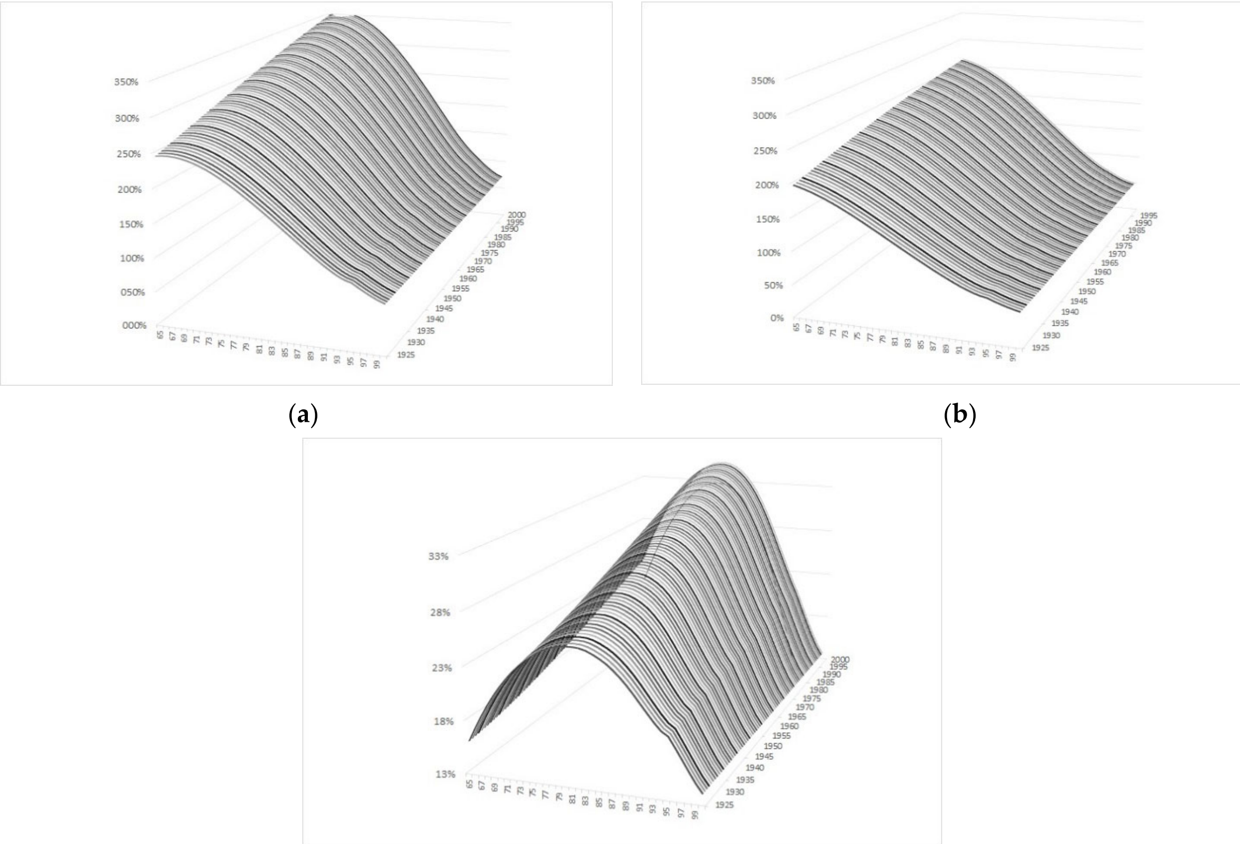


Figure 5. Actuarial retirement equity factor. Percentage of LTC supplement by age and gender. (**a**) Men; (**b**) women; (**c**) effect of gender on the percentage of LTC by age. Source: Own work. Software used: Microsoft Excel.

If it is taken into account that the highest percentage of individuals suffering from dependency are those over 65 years of age, the pension calculation should also take into account that it is probable that an individual will be both independent and dependent. If this calculation was not performed, the elderly's life expectancy would be overestimated. A reduction in the number of payments causes the total pension (retirement plus LTC) to increase threefold depending on the age at which an individual becomes severely dependent. Therefore, the adoption of the actuarial factor takes into account both the expected number and intensity of payments.

3.4. LTC in Disability

A disability pension is paid to a beneficiary in accordance with the temporary payment expectations based on a disabled person's mortality rate. However, a worsening of their condition will not change the accumulated capital but will change the type of pension they receive because the priority will not be to replace the disability but rather to increase it with an LTC allowance, as in the case of retirement. The expected number of payments under the new circumstances is therefore, affected with a reduction in line with the increased mortality of a dependent person.

Table 3 shows the mortality rate values for disabled men (${}^{i}q_{x}^{m}$)) and women (${}^{i}q_{y}^{m}$) who were born in 1960; the ages range from 45 to 99, in 2-year periods.

Table 3. Disability outcomes for the generation born in 1960. Source: Own work. Software used: Microsoft Excel.

	Male Mortality Rates			Female Mortality Rates			LTC Factors		
Age	${}^{i}q_x^m$	${}^{d}q_x^m$	$\Delta {}^{i}q_x^m/{}^{d}q_x^m$	${}^{i}q_y^m$	${}^{d}q_y^m$	$\Delta {}^{i}q_y^m/{}^{d}q_y^m$	λ_x^d	λ_y^d	$\Delta\lambda_x^d/\lambda_y^d$
45	0.02011	0.02600	29.28%	0.00973	0.03966	307.60%	0.858	1.563	−34.43%
47	0.02094	0.03232	54.38%	0.01026	0.04507	339.36%	0.943	1.611	−28.29%
49	0.02220	0.04009	80.53%	0.01051	0.05101	385.19%	1.034	1.654	−21.42%
51	0.02384	0.04943	107.32%	0.01059	0.05743	442.19%	1.127	1.688	−13.93%
53	0.02595	0.06051	133.19%	0.01047	0.06417	512.61%	1.222	1.710	−5.70%
55	0.02690	0.07269	170.17%	0.01154	0.07111	516.13%	1.314	1.718	2.21%
57	0.02947	0.08638	193.11%	0.01142	0.07828	585.50%	1.400	1.712	10.34%
59	0.03174	0.10128	219.13%	0.01185	0.08558	622.01%	1.478	1.687	18.34%
61	0.03427	0.11748	242.77%	0.01229	0.09294	656.08%	1.542	1.643	26.40%
63	0.03573	0.13382	274.59%	0.01390	0.10014	620.29%	1.585	1.580	33.64%
65	0.03777	0.15069	298.98%	0.01497	0.10727	616.61%	1.599	1.500	40.47%
67	0.04133	0.16679	303.57%	0.01674	0.11415	581.83%	1.580	1.403	46.11%
69	0.04478	0.18223	306.91%	0.02023	0.12080	497.24%	1.534	1.291	50.85%
71	0.05131	0.19649	282.91%	0.02302	0.12720	452.62%	1.461	1.170	54.47%
73	0.05733	0.21031	266.85%	0.02801	0.13343	376.38%	1.368	1.041	57.61%
75	0.06467	0.22304	244.88%	0.03477	0.13951	301.21%	1.257	0.906	59.88%
77	0.07691	0.23441	204.80%	0.04113	0.14559	254.01%	1.129	0.774	61.01%
79	0.08741	0.24521	180.52%	0.05244	0.15157	189.06%	1.004	0.641	61.78%
81	0.10113	0.25556	152.71%	0.06377	0.15852	148.56%	0.874	0.518	61.22%
83	0.11453	0.26516	131.53%	0.07840	0.16609	111.85%	0.738	0.399	59.65%
85	0.13387	0.27401	104.68%	0.09598	0.17559	82.94%	0.595	0.286	56.05%
87	0.15308	0.28353	85.22%	0.12191	0.18600	52.58%	0.452	0.179	52.43%
89	0.18499	0.29382	58.83%	0.15231	0.19856	30.36%	0.314	0.084	47.98%
91	0.22578	0.30490	35.04%	0.19244	0.21205	10.19%	0.193	−0.000	43.79%
93	0.28344	0.33014	16.48%	0.24508	0.23555	−3.89%	0.111	−0.063	40.15%
95	0.34617	0.37696	8.89%	0.31026	0.27970	−9.85%	0.052	−0.095	34.77%
97	0.41222	0.42607	3.36%	0.39398	0.32980	−16.29%	0.035	−0.133	29.19%
99	0.47065	0.49565	5.31%	0.49107	0.39930	−18.69%	0.051	−0.151	24.13%

The values obtained for disability pension recipients who become dependent and opt for LTC support are up to 1.5 times the original disability pension, for both men and women (higher for men) in the years around age 65 because of the higher probability of death as a dependent than as a disabled person. However, as a recipient ages, the value of their benefit decreases sharply. This reflects the fact that disabled mortality is very close to dependent mortality. In fact, with the tables used, women's disability pensions would be reduced if they were to become dependent from the age of 95. This anomaly is due to the different sources from which the disability mortality tables are obtained (an insured population) as opposed to a general population for dependent mortality.

The analysis for other generations does not differ much from what was obtained. Thus, Figure 6 shows the value of the actuarial equity factor according to age and gender in a disability situation.

An analysis of the results shows that:

1. For both men (a) and women (b), if they apply for an LTC supplement when they become dependent, the actuarial equity factor takes a positive value and can reach higher amounts than the disability pension they were receiving (dependency age before 80 years).
2. Regardless of gender, the older an individual becomes, the lower the actuarial equity factor tends to be, although men have higher bonuses if they become dependent in the last years before retirement age. This is a direct mortality effect.
3. With the exception of the pre-retirement period, from age 55 onwards, there is a positive differential in dependent mortality in favour of the actuarial equity factor for

men. The gap increases until around age 90, when it stabilises at around 25% higher (Figure 6c).

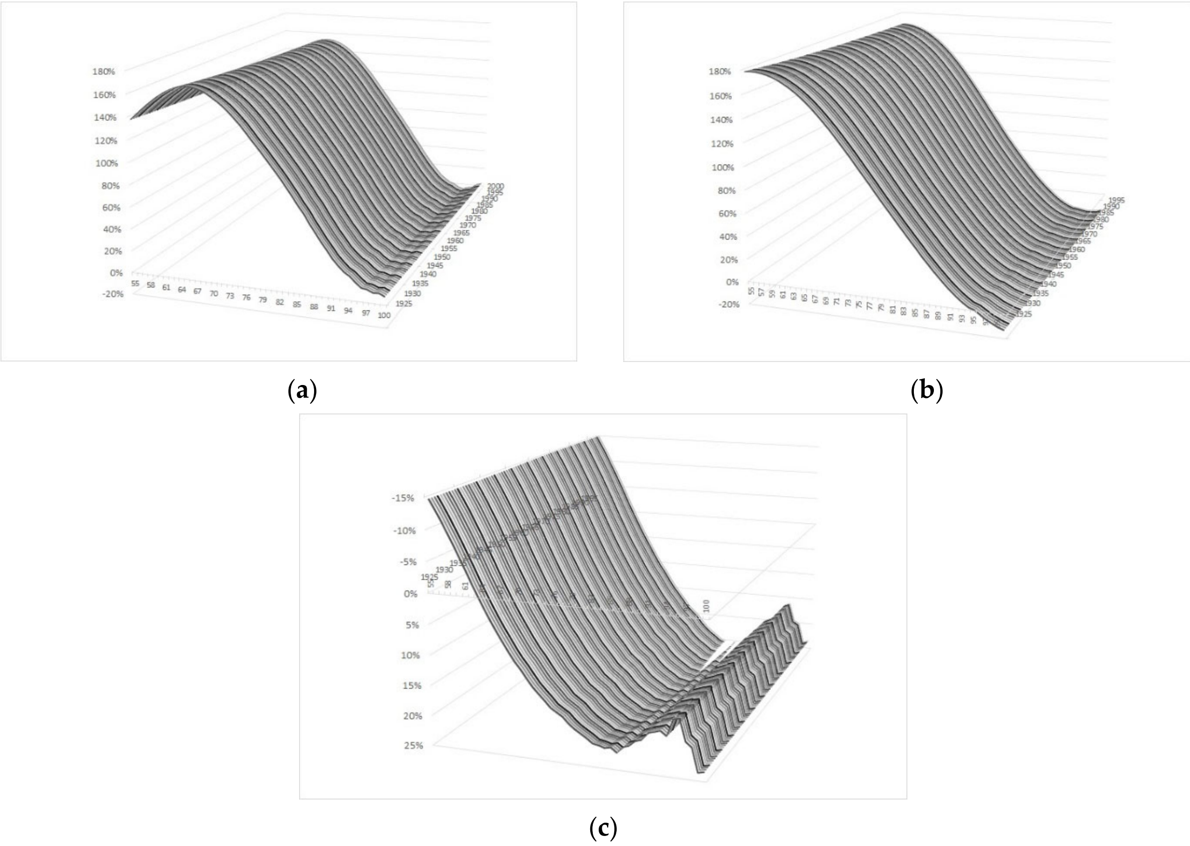


Figure 6. Actuarial equity factor for disability. Percentage of LTC supplement by age and gender. (**a**) Men; (**b**) women; (**c**) effect of gender on the LTC percentage by age. Source: Own work. Software used: Microsoft Excel.

4. Discussion

This study has presented an actuarial model that allows for the incorporation of an LTC payment to a DB pension beneficiary. This assistance enhances a beneficiary's benefit. It does not replace it but transforms it, according to the expected duration of the new dependent status. It is precisely this new status that enables the LTC benefit to exist. This approach is unlike others because it does not finance LTC with contributions that are deducted from a beneficiary's pension while they are independent. This model proposes adjusting the pension payment according to the specific mortality rates. In this way, once a beneficiary is severely dependent, the individual will receive both the pension and LTC support for a shorter period of time. The purpose is not to provide a solution for severely or highly dependent persons but rather to make their pension consistent and adapt it to their life expectancy: they receive assistance when they need it.

A dependent individual seeks to reduce the burden of expenditure due to their status. Therefore, once the individual is a dependent person, he/she has LTC expenses. Through the actuarial equity factor, the dependent individual will then have more resources available to meet these new expenses.

- These resources may be targeted to reimburse part of the LTC expenditure rather than being paid directly to a beneficiary.
- In the case of surplus, it would increase the retirement or disabled benefit.

Although the actuarial factor has been included in this DB private pension model, it requires a request from the beneficiary and is therefore voluntary. That is, it depends on an individual's own will and financial need to meet the LTCs.

From a private pension plan point of view, the actuarial equity factor reflects an internal redistribution of costs and benefits in a pension plan and changes how risk is shared by each generation, altering the way longevity risk is pooled. A healthy individual receives the pension entitled, and on becoming a dependant, they will receive the initial pension together with the LTC aid, proportional to the shorter duration of life.

This concept should help to redefine DB pension plans when it comes to the pension promise made to younger workers. It represents a paradigm shift in the responsibility for old-age income, not only to maintain living standards after retirement but also to have the resources to meet LTC.

This actuarial model has been designed to be implemented without much difficulty and at no cost in private DB plans. These plans currently assume a general mortality rate for the insured but not for severely or highly dependent persons. For this reason, the inclusion of the actuarial factor as a conversion factor with the dependent persons' mortality tables leads to an LTC benefit without additional costs or contributions. The payment expectation is adjusted, which increases the total pension (pension plus assistance). Future work should include the design of a DC plan that includes LTC coverage, although it would be in competition with other insurance products. In a DC plan, an individual is aware of their situation from year to year and, unlike a DB plan, can supplement their coverage with dependency insurance products. In any case, in DC plans or DB plans, we are not focused on the assets side. Our hypothesis has been the transformation of the benefit under the constraints of the same amount of funds. For larger costs, it would be necessary to increase the coverage with additional products.

This paper only uses the second pillar. However, it can easily be extended to the other pillars using accumulated assets (financial or family home) to fund LTC requirements to complement the LTC cost. In this sense, several countries use public provision if an individual is not exceeding a minimum level. If they are, it is required to contribute to the cost of LTC provision by wholly or partially selling some assets, e.g., via partial reverse mortgage. This point of view has strong links to third pillar insurance products as life care annuities [18] and life care tontines [72]. An exploration of how to establish a comprehensive coverage plan to address LTC (second and third pilar) is a direction for future research.

The above model could be included automatically in a pension plan and not at a beneficiary's request, but this would require:

- To have enough data on the new number of dependent individuals by age, degree of dependency and sex. Actuarial science needs large data with which to obtain meaningful probabilities. Therefore, it would automatically incorporate the transition probability of becoming a dependent person so that, at each age, the actuarial equity factor that allows a zero-cost conversion would be obtained.
- Knowing the above transition probability, it would also be possible to design a pension plan independent and additional to retirement and disability pensions. In this case, obviously, the final cost would be higher but it would provide an opportunity to finance it progressively during the period of labour activity.
- If the optimal coverage for severe dependence levels is found, it would also mean that the average cost could be standardised at each LTC age.

In the latter case, the automatic design with a separate benefit makes it possible to differentiate (within an occupational DB pension plan) that there is a part of the pension that is designed to pay for LTC costs, and another part to replace the worker's wages.

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A.G. and I.D.M.; supervision, J.I.D.L.P.; validation, M.C.F.-R., A.G. and I.D.M.; visualization, M.C.F.-R., A.G. and I.D.M.; writing—original draft, J.I.D.L.P., M.C.F.-R., A.G. and I.D.M.; writing—review and editing, J.I.D.L.P., M.C.F.-R., A.G. and I.D.M. All authors have read and agreed to the published version of the manuscript.

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