EUSKAL HERRIKO UNIBERTSITATEA /
UNIVERSIDAD DEL PAÍS VASCO

Ensayos sobre valoración de Stock Options

by

Agueda Madoz Mendioroz

A thesis submitted in partial fulfillment for the degree of Finanzas y Economía Cuantitativas / Quantitative Finance and Economics

in the

Facultad de Economía y Empresa (Sarriko)
Departamento de Fundamentos del Análisis Ecónomico II

January, 2019

Preserva tu derecho a pensar, más vale que corras el riesgo de equivocarte que cometas el pecado de no pensar.

- Hipatía

EUSKAL HERRIKO UNIBERTSITATEA / UNIVERSIDAD DEL PAÍS VASCO

Abstract

Facultad de Economía y Empresa (Sarriko)
Departamento de Fundamentos del Análisis Ecónomico II

Finanzas y Economía Cuantitativas / Quantitative Finance and Economics

by Agueda Madoz Mendioroz

According to the main accounting standards, firms should consider possible changes and mean reversions of the volatility when computing the cost of the executive stock options (ESOs). In the first part, we show the effects of the stochastic volatility in both the executive's subjective valuation and shareholder's costs of ESOs. We conduct a sensitivity analysis obtaining relevant pricing differences with respect to the valuations under constant volatility. We analyze the pricing errors when the firm uses a misspecified volatility model. Finally, we check European-style options as approximations to the American ESOs by using the option's expected life.

We study the effects that granting European Executive Stock Options (ESOs) have on the subjective valuation and the incentives of loss-averse executives. The executive's preferences are represented by a kinked utility function. By using the certainty equivalent approach, we show that the executive's discount is comparatively higher than that corresponding to a risk-averse executive. We also find that the executive's incentives to raise the firm's stock price, as measured by the subjective *delta*, and to develop risky investment projects, as measured by the subjective *vega*, are comparatively lower for a loss-averse executive.

Finally, we study the effects of the effort made by a loss averse agent on the subjective valuation of ESOs. We consider that the agent's effort affects, on one hand, the expected negative utility and, on other hand, positively, the value of the company, through the price of it's shares. In the same way, using the the certainty equivalent approach we can compare the changes that occur in the subjective valuation when modifying the parameters that govern the effort.

Acknowledgements

En primer lugar mi más sincero agradecimiento a aquellas personas que han estimulado mi interés en el área de las finanzas y más concretamente en la valoración de opciones, y que me han apoyado y ayudado durante la realización de esta tesis, mis directores Antoni Vaello y Miguel Angel Martinez. También me gustaría agradecer especialmente las aportaciones realizadas por Angel León y Julio Carmona, sin ellos esta tesis no sería posible.

En segundo lugar estoy muy agradecida a todos los catedráticos y profesores del Departamento de Fundamentos del Análisis Económico II de la Facultad de Ciencias Económicas y Empresariales de la UPV por haberme dado la oportunidad de trabajar y presentar esta tesis.

En tercer lugar, gracias a todos los compañeros presentes y pasados que he tenido en este trayecto, muchos de ellos verdaderos amigos a los cuales debo agradecer algo más que el mero apoyo académico. Gracias Paulina.

Por último, agradecer a mi familia, David, Maia y otras veintitres personas tan especiales para mí, que siempre están cuando los necesito.

A María Jesús y Pedro

Contents

List of Figures

List of Tables

1

Introducción

Las Opciones sobre Acciones para Ejecutivos (en adelante, ESO's, por su acrónimo en inglés) tienen, en general, un peso importante en los paquetes de compensación para altos ejecutivos. Hall and Murphy (2002) informan que en 1999, el 94% de las empresas del S&P 500 concedieron ESO's a sus altos ejecutivos, y que estas subvenciones representaron el 47% de la remuneración total de los CEOs del S&P 500. Además Murphy (2013) revela que en 2011 la compensación basada en opciones representa el 21% de la remuneración de las

ESO's. La literatura que cubre las diferentes características de las ESO's ha crecido exponencialmente en los últimos años. Esta situación, junto con el hecho de que, actualmente, las normas contables Financial Accounting Standard Board (2004) exigen a las empresas reconocer el coste total de las ESO's concedidas a los altos directivos y otros empleados, hace que la correcta valoración de las ESO's sea un tema que despierta cada vez mayor interés.

Hall and Murphy (2002) demostraron que los supuestos fundamentales de la teoría de valoración de opciones no son razonables para la valoración de las ESO. Mientras que en el marco tradicional de valoración de opciones los inversores pueden seleccionar libremente la composición de su cartera, los titulares de las ESO's no pueden vender o negociar sus opciones de empresa. Los altos directivos no pueden cubrir sus riesgos vendiendo en corto las acciones de la empresa y, además, la hipótesis de un inversor bien diversificado no se satisface porque su capital humano, sus planes de pensiones, etc.... también dependen del rendimiento de la empresa. Todas estas restricciones pueden conducir a un ejercicio anticipado (subóptimo) de las ESO's.

Por lo tanto, hay tres valoraciones diferentes para los paquetes de ESO's. Siguiendo la terminología de Ingersoll (2006), en primer lugar, está la valoración subjetiva, es decir, el valor de la ESO para un ejecutivo no diversificado y averso al riesgo. Esta valoración es subjetiva ya que depende de las características del empleado: grado de diversificación, aversión al riesgo.... Se basa en el principio de equivalencia cierta: su valor es igual a la cantidad de efectivo entregada en lugar de las ESO en la fecha de concesión que rinda la misma utilidad esperada. En segundo lugar, la valoración objetiva, o coste de la empresa (el llamado valor justo), es la valoración de la opción para un agente neutral en cuanto al riesgo (la empresa) que sabe que el ejercicio de la ESO lo realiza un ejecutivo restringido. El valor subjetivo de la ESO es interesante para analizar los incentivos de las ESO's, mientras que la valoración objetiva es la relevante a efectos contables. En tercer lugar, también podemos calcular la valoración neutral al riesgo para un agente no restringido. Teóricamente, el valor subjetivo de las ESO's debe ser inferior al valor objetivo y éste no debe superar a la valoración neutral al riesgo.

A pesar de la creciente bibliografía sobre la fijación de precios de la ESO, pocos artículos han introducido en la valoración el supuesto de volatilidad estocástica. Sin embargo, como se indica en el párrafo A32 del Financial Accounting Standard Board (2004), uno de los factores a considerar al estimar el precio de las ESO's es exactamente ese, el método de estimación de la volatilidad debe incluir cambios en la volatilidad con la posibilidad de reversión media. En este sentido, entre los trabajos que incluyen este supuesto cabe destacar los de Brown and Szimayer (2008), Tian and Jiang (2008) y León and Vaello-Sebastià (2009).

Tian and Jiang (2008) se centran en el comportamiento de la volatilidad histórica para predecir la volatilidad de la rentabilidad de las acciones a largo plazo como factor relevante para la valoración de la ESO. Concluyen que la volatilidad histórica es un pronóstico pobre y proponen un enfoque alternativo que mejora sustancialmente este resultado. Mientras tanto, Brown and Szimayer (2008) proponen un modelo de valoración de ESO que se centra principalmente en el ejercicio anticipado y en la volatilidad estocástica según Heston (1993). En resumen, ambos estudios demuestran que los valores obtenidos son ligeramente inferiores a los obtenidos con el método Black-Scholes tradicional. León and Vaello-Sebastià (2009) consideran la valoración de las ESO's americanas en el marco de valoración de opciones de tipo GARCH de Duan (1995). No obstante, consideran un modelo de forma reducida en el que el ejercicio de la ESO únicamente depende de una regla exógena. Además, se centran en la valoración objetiva, que sólo es relevante a efectos contables, pero no analizan los efectos de GARCH en la valoración subjetiva (percepción del ejecutivo).

La primera parte de la tesis (capítulo 2) presenta un modelo tanto de la valoracion objetiva como la subjetiva de las ESO's americanas cuando el precio de las acciones de la empresa se rige por el modelo de volatilidad estocástica tipo Heston (1993). Inicialmente se obtiene la valoración subjetiva de la ESO a través del equivalente cierto como en Lambert et al. (1991) y Kulatilaka and Marcus (1994), pero considerando el marco más general de Hall and Murphy (2002), que permite restringir las acciones en la ecuación de riqueza. Mostramos cómo adaptar el algoritmo de Monte Carlo de mínimos cuadrados de Longstaff and Schwartz (2001) para obtener la valoración subjetiva, siguiendo las líneas de León and Vaello-Sebastià (2010). Analizamos el impacto de los parámetros del modelo de Heston (velocidad de reversión media, volatilidad de la varianza y correlación) en la valoración subjetiva de ESO, revelando sesgos significativos. En cuanto a la valoración objetiva, proponemos un nuevo método para estimar la frontera de ejercicio del agente con el fin de calcular el coste de los ESO's. Además, también estudiamos el impacto de utilizar un modelo de volatilidad mal especificado (volatilidad constante) cuando el proceso de varianza real sigue un modelo de volatilidad estocástica. En este caso, el tamaño del sesgo depende de la dinámica (parámetros) del proceso de volatilidad estocástica. Por último, realizamos un análisis de las implicaciones contables utilizando ESO de tipo europeo para aproximar el coste de la ESO, sustituyendo el vencimiento de las ESO por el tiempo de ejercicio previsto.

En la segunda parte (capítulos 3 y 4) de la tesis nos centramos en la valoración subjetiva de las ESO's europeas. Como comentábamos antes, las diferentes características de las ESO han sido ampliamente analizadas en diversos artículos, pero, desde el problema de valoración de las ESO's hasta el diseño óptimo del paquete de compensación del ejecutivo, en todos los casos el modelo estándar asumido es, en gran medida, el de un ejecutivo que maximiza la utilidad esperada de Von Neumann-Morgenstern.

Allais (1953) and Ellsberg (1961) fueron de los primeros autores en cuestionar las predicciones de la teoría de la utilidad esperada. Desde entonces, muchos estudios experimentales influyentes han cuestionado su capacidad para explicar muchos de los fenómenos observados, respaldando la necesidad de nuevos desarrollos teóricos. El más influyente entre todos los modelos alternativos de elección bajo incertidumbre es la teoría prospectiva ("prospect theory") de Kahneman and Tversky (1979). Aquí, las preferencias del agente se definen de diferente manera sobre las ganancias que sobre las pérdidas separándolas por algún punto de referencia, de tal manera que el agente es más sensible a las pérdidas que a las ganancias. De este modo, la función de utilidad que representa estas preferencias se define a trozos, con una pendiente más pronunciada en la región de pérdidas que en la región de ganancias. Esto es lo que se conoce como aversión a las pérdidas, que es un elemento esencial de la teoría prospectiva. Los méritos relativos para explicar lo que parecen ser anomalías para la teoría de la utilidad esperada están documentados en Camerer (2000) y Subrahmanyam (2007) que proporciona un estudio de la literatura de las finanzas del comportamiento.El artículo de de Meza and Webb (2007) analiza la compensación basada en opciones bajo la aversión a las pérdidas encontrando que habrá situaciones en las que el beneficio no es sensible al rendimiento de la empresa. En esta línea, Dittmann et al. (2010) calibra un modelo de agente-principal donde el agente (ejecutivo) es averso a las pérdidas y demuestran que su modelo puede explicar algunos hechos, como, por ejemplo, las posesión de opciones y los altos salarios base.

En el capítulo 3, nos centraremos en la valoración subjetiva para ejecutivos con aversión a las pérdidas y en los incentivos. Para obtener el valor subjetivo del ejecutivo seguimos la literatura que comienza con Lambert et al. (1991), y luego continúa con Hall and Murphy (2002), Tian (2004) y Cai and Vijh (2005) entre otros, que utiliza el enfoque de equivalente cierto para obtenerlo. Nuestros resultados sugieren que la asignación de activos del ejecutivo y la valoración subjetiva muestran un patrón diferente con respecto al caso de aversión al riesgo. Por lo tanto, las sensibilidades subjetivas (*delta, vega*), comúnmente utilizadas para medir los incentivos, también son diferentes.

Esta valoración subjetiva, por otro lado, se ve afectada por la selección de activos que el ejecutivo tiene a su disposición para mantener su patrimonio. En este sentido, consideraremos un activo libre de riesgo y un índice de mercado como sus alternativas para mantener la riqueza. Esto conduce naturalmente a un examen de la decisión de asignación de riqueza del ejecutivo como parte de nuestro examen de la valoración subjetiva de la ESO, una cuestión que también examinamos. Una vez obtenida esta valoración subjetiva, podemos discutir los incentivos que ofrecen las ESO's a los ejecutivos con aversión a las pérdidas.

Por último, en el capítulo 4, analizamos el problema cuando el objetivo del ejecutivo es maximizar la utilidad esperada de su riqueza final considerando que, además de ser un agente averso a las pérdidas, el esfuerzo que realiza para aumentar su riqueza provoca, por un lado, alguna utilidad negativa y, por otro, un aumento de la empresa de valor. Nuestro modelo se caracteriza por una función de utilidad definida a trozos, donde la curvatura de las pérdidas es mayor que la de las ganancias y en la busqueda del esfuerzo óptimo que debe realizar el agente para maximizar su riqueza final.

Siguiendo Desmettre S. (2010) el modelo propuesto corresponde a un ejecutivo que tiene la posibilidad de influir en el precio de las acciones de la empresa gracias al esfuerzo realizado con su trabajo, es decir, consideramos que si el agente trabaja bien (hace un mayor esfuerzo), la empresa mejorará sus resultados y con ello el precio de las acciones de la empresa aumentará. Pero, por otro lado, un mayor esfuerzo conduce a su vez a una utilidad negativa, es decir, a mayor esfuerzo en el trabajo, obtiene menor utilidad.

En ambos casos, se realiza una presentación detallada del modelo, se describe el entorno del ejecutivo dado por el valor de su patrimonio inicial, el paquete retributivo pagado por la empresa y los activos disponibles para la asignación de la parte de su patrimonio que no se posee en forma de ESO's. A continuación, presentamos la función de utilidad que describe las preferencias del ejecutivo representativo averso pérdidas, prestando especial atención a su nivel de referencia, introducimos el enfoque para medir la valoración subjetiva de los ESO's, que será ampliamente utilizado para obtener nuestros resultados y realizamos un análisis de sensibilidad de cómo varias variables de interés se ven afectadas por la selección de parámetros del modelo y el número de ESO's en sus paquetes de retribución.

2

Option Grants, Fair Value and Stochastic Volatility

2.1 Introduction

In recent decades, Executive Stock Options (henceforth, ESO) have popularized as part of compensation packages that companies benefit their top executives.

This fact along with that, currently, accounting standards Financial Accounting Standard Board (2004) require companies to recognize the full cost of the ESO granted to top executives and other employees, makes the proper ESO valuation a topic of interest.

Hall and Murphy (2002) showed that the fundamental assumptions of the modern theory of option pricing are not reasonable to ESO valuation. While in the traditional option pricing framework investors can select freely their portfolios, ESO holders cannot sell or trade their options. Moreover, the executives cannot hedge their risks by short-selling the firm's stocks and in addition, the assumption of a well-diversified investor is not satisfied because his human capital, pension plans, etc... also depend on the firm performance. All these restrictions can lead to early exercise (suboptimal) of the ESOs

Hence, there are three different valuations for the ESO packages. Following the terminology of Ingersoll (2006), first, there is the subjective valuation, that is, the ESO value to a non diversified risk-averse executive. This valuation is subjective since it depends on the employee's characteristics: diversification, risk-aversion... It is based on the certainty equivalent principle: the amount of cash delivered in place of the ESOs at the grant date yielding the same expected utility. Secondly, the objective valuation, or firm cost (the so called fair value), is the option valuation for a risk neutral agent (the firm) who knows that ESO exercise is performed by a restricted employee. The ESO subjective value is interesting to analyze the incentives of ESO plans, while the objective valuation is the relevant one for accounting purposes. Third, we could also calculate the risk-neutral valuation for an unconstrained agent. The ESO subjective value should be less than the objective value and this one must not exceed the risk neutral valuation.

Despite the growing literature on ESO pricing, few articles have introduced the time-varying volatility assumption in the valuation. However, as stated in the Financial Accounting Standard Board (2004), paragraph A32, one of the factors to consider when estimating the price of the ESO is exactly that, the method of estimation of volatility must include changes in the volatility with the possibility of mean reversion. In this sense, among the papers that include this assumption it is worth highlighting those of Brown and Szimayer (2008), Tian and Jiang (2008) and León and Vaello-Sebastià (2009).

Tian and Jiang (2008) focus on the performance of historical volatility Financial Accounting Standard Board (recognized by 2004) to forecast the long-term stock return volatility as the relevant input for the ESO valuation. They conclude that historical volatility is a poor forecast and propose an alternative approach that improving substantially this result. Meanwhile, Brown and Szimayer (2008) propose an ESO valuation model that is primarily focused on three aspects: performance hurdles, early exercise and stochastic volatility according to Heston (1993). In short, both studies demonstrate that the obtained values are slightly lower than the traditional Black-Scholes ones. Further, considering performance hurdles increases the ESO's incentives and it could also mitigate the dilution effect. León and Vaello-Sebastià (2009) consider the valuation of American ESOs in the GARCH option pricing framework of Duan (1995). Nonetheless, they consider a reduced-form model where the ESO exercise depends on an exogenous rule. Moreover, they focus on the objective valuation, which is only relevant for accounting purposes, but they do not analyze the GARCH effects in the subjective valuation (executive's perception).

This paper aims to model both objective and subjective American ESO valuations when the company's stock price is driven by the Heston's stochastic volatility model. We initially obtain the subjective valuation of the ESO through the certainty equivalent as in Lambert et al. (1991) and Kulatilaka and Marcus (1994), but considering the more general framework of Hall and Murphy (2002), allowing for restricted stocks in the wealth equation. We show how to adapt the Least-Squares Monte Carlo algorithm of Longstaff and Schwartz (2001) to obtain the subjective valuation, following the lines of León and Vaello-Sebastià (2010). We analyze the impact of the Heston's model parameters (speed of mean reversion, volatility of variance and correlation) on the ESO subjective valuation revealing significant biases. Regarding the objective valuation, we propose a new method to estimate the employee's exercise frontier in order to calculate the firm cost of ESOs. Moreover, we also study the impact of using a misspecified volatility model (constant volatility) when the true variance process faces stochastic volatility. In this case, the size of the bias depends on the dynamics (parameters) of the stochastic volatility process. Finally, we carry out an analysis of the accounting implications using European-style ESOs to approximate the ESO cost replacing the maturity of the ESOs with their expected exercise time.

The rest of the paper is organized as follows. Section 2.2 describes the model and the solving algorithm. The impact of the different parameters of the volatility dynamics on the subjective valuation is addressed in Section 2.3. Several aspects of the objective value are analyzed in Section 2.4. Finally, Section 2.5 summarizes our results.

2.2 The model

First, we start with the executive's ESO valuation, that is, the amount of cash CE that makes the executive indifferent between being compensated with ESOs or CE. To do this, we assume the framework of Hall and Murphy (2002), and afterward we provide some technical details regarding the simulations.

2.2.1 The executive's framework

We assume that the executive has an initial wealth, W_0. The amount W_0 is not immediately available to the executive. A proportion of the initial wealth, ω, is invested in restricted stocks of the firm, which cannot be sold until the ESO maturity, T. Thus, given the stock price at grant date, S_0, the number of restricted stocks in the compensation package are

$$N_S = \omega \cdot W_0 / S_0. \tag{2.1}$$

Secondly, a proportion α of the initial wealth are ESOs, which have been valued by the firm using the Black-Scholes-Merton model[1]. The granted ESOs are American-style call options with exercise price K that mature in T years and have an initial vesting period of τ years. Then the number of ESOs granted to the executive are

$$N_{op} = \alpha \cdot W_0 / BS. \tag{2.2}$$

[1]Although BS valuation is a biased methodology to account for ESO costs, in this case it is simply used to establish the number of ESOs granted to the executive.

The degree of executive's diversification depends on ω and α. The less diversified the executive, or the larger $\omega + \alpha$, more linked the executive's wealth dynamics to the performance of the firm. The remainder of the executive's wealth, or free-wealth, $(1 - \omega - \alpha) \cdot W_0$, is invested in the risk-free asset.[2]

The pricing methodology works backwards on a set of simulated paths of stock prices and variances.

The terminal wealth of the executive, conditional on non previous ESO exercise, is:

$$W_{T|T} = W_0 \, (1 - \omega - \alpha) \, e^{rT} + N_{op} \, (S_T - K)^+ + N_S \, S_T \, e^{dT}, \tag{2.3}$$

where $W_{i|j}$ denotes the executive's wealth at time i conditional on ESO exercise at time $j \leq i$, r is the annual risk-free rate, and S_t denotes the stock price at time t. The expression $(b)^+$ represents the maximum between b and zero. We assume that the restricted stock dividends are reinvested in more restricted stocks, that is, the executive does not receive any payoff from the restricted stocks until T.

If the executive decides to exercise the ESO package at any time before maturity, $t \leq T$, he will invest the ESO payoffs in the risk-free asset from that moment until maturity. In this case, his terminal wealth becomes:

$$W_{T|t} = W_0 \, (1 - \omega - \alpha) \, e^{rT} + N_{op} \, (S_t - K)^+ \, e^{r(T-t)} + N_S \, S_T \, e^{dT}, \tag{2.4}$$

[2]Other works, such as Tian (2004) and Cai and Vijh (2005), consider that the non-restricted executive's wealth can be invested between the risk-free asset and the market portfolio, but under a constant volatility framework. However, Tian (2004) only considers European-style ESOs, which simplifies the problem of the optimal exercise of the ESOs, and the grid in Cai and Vijh (2005) is not well suited to include a third state-variable (stochastic volatility), since the number of nodes growths exponentially.

Finally, we establish the utility function for the risk averse executive. We assume the standard power utility function respect to her wealth,

$$U(W) = \frac{W^{1-\gamma}}{1-\gamma}\,\mathbb{1}_{\{\gamma\neq 1\}} + \ln(W)\,\mathbb{1}_{\{\gamma=1\}} \tag{2.5}$$

where $\mathbb{1}_{\{\ldots\}}$ is an indicator function that takes a value equal to one if the expression inside the brackets is true, and zero otherwise. The parameter $\gamma > 0$ is the relative risk aversion coefficient, thus the larger γ the higher the executive's risk aversion.

2.2.2 The subjective valuation

The executive maximizes the expected utility of his terminal wealth conditioned to the initial time, or grant date, by selecting his optimal ESO exercise time, t^*. Therefore, the executive's problem can be expressed as:

$$\underset{t}{max}\ \mathbb{E}_0\left[U\left(W_{T|t}\right)\right]. \tag{2.6}$$

We will use the certain equivalent principle for the executive's ESO valuation. This means to calculate the amount of money, CE, that the executive is willing to receive instead of the option to get the same expected utility. If this cash amount is delivered in place of an ESO at the grant date, and it is invested in the risk free asset, the executive's total wealth at maturity becomes

$$W_T^{CE} = \left[W_0\,(1-\omega-\alpha) + N_{op}\cdot CE\right] e^{rT} + N_S\,S_T\,e^{dT}. \tag{2.7}$$

Obtaining the ESO subjective value leads to search for the amount CE that solves the next equation:

$$\mathbb{E}_0\left[U\left(W_T^{CE}\right)\right]=\mathbb{E}_0\left[U\left(W_{T|t^*}\right)\right]. \tag{2.8}$$

Once we have computed the optimal time of exercising the option, t^*, and the corresponding (maximum) expected utility, $\mathbb{E}_0\left(U\left(W_{T|t^*}\right)\right)$, the amount CE from the above equation is obtained by a quadratic distance minimization:

$$\underset{CE}{min}\left[\mathbb{E}_0\left(U\left(W_T^{CE}\right)\right)-\mathbb{E}_0\left(U\left(W_{T|t^*}\right)\right)\right]^2. \tag{2.9}$$

Similarly, we can obtain the subjective valuation of the restricted stocks. The subjective valuation of the restricted stocks is the certain amount of money CE' granted in substitution of every restricted stock. In this case, the executive's terminal wealth will be:

$$W_T^{CE'}=\left[W_0\left(1-\omega-\alpha\right)+N_S\cdot CE'\right]e^{rT}+N_{op}\left(S_{t'}-K\right)^+e^{r(T-t')}, \tag{2.10}$$

Notice that the composition of executive's wealth has changed (certain cash in place of restricted stocks), thus the optimal ESO exercise rule will be different than in the standard case. Then the optimal exercise time is here denoted by t' in contrast to equations (2.8) and (2.9) where we used t^*.

2.2.3 The algorithm

Solving equation (2.8) requires the previous step of finding the optimal exercise time, t^*. We use a similar approach to the Least Squares Monte Carlo (LSMC) procedure by Longstaff and Schwartz (2001) to estimate the investor's expected utility at the initial moment or grant date, denoted as time 0. This approach is based on simulations and it aims to get the expected utility if the ESOs are early exercised and compare this value, at the same time, with the expected utility of the terminal wealth if the ESOs are maintained for at least one period. More details of the algorithm are provided in the appendix.

The dynamics for the firm's stock price, S_t, with an instantaneous variance for the stock return, V_t, being stochastic is driven by Heston's model. Under the physical measure, this process is given by the following two diffusion equations:

$$dS_t = (\mu_s - d)S_t dt + S_t \sqrt{V_t} dZ_t^s, \tag{2.11}$$

$$dV_t = \kappa \left(\theta - V_t\right) dt + \sigma_v \sqrt{V_t} dZ_t^v, \tag{2.12}$$

$$\rho_{sv} dt = dZ_t^s dZ_t^v, \tag{2.13}$$

where μ_s and d denote, respectively, the annual expected return and the dividend yield of the firm's stock price. Note that the dynamics for V_t is driven by a mean-reverting square-root stochastic volatility process. The parameters implied in this equation are a total of three. The parameter θ is the constant long-term mean of the variance, κ represents the rate (speed) at which the variance reverts to θ and the volatility for the variance process is denoted by σ_v. In the last equation, the parameter ρ_{sv} represents the correlation coefficient between the two standard Brownian processes Z_t^s and Z_t^v.

First, we simulate a large number for paths of the stock price, S_t, as indicated by equations (2.11) to (2.13). We use an Euler discretization to simulate S_t. When discretizing the variance process, V_t, using the Euler's scheme a drawback arises. Specifically, the probability of a negative (discrete) variance is greater than zero. Thus, as a solution, we implement the "full truncation scheme" proposed by R. Lord and van Dijk (2010). The scheme is as follows:

$$\ln(S_{t+\Delta t}) = \ln(S_t) + \left(\mu_s - d - \frac{V_t}{2}\right)\Delta t + \sqrt{V_t}\Delta Z_t^s, \tag{2.14}$$

$$\widetilde{V}_{t+\Delta t} = \widetilde{V}_t + \kappa\left(\theta - (\widetilde{V}_t)^+\right)\Delta t + \sigma_v\sqrt{(\widetilde{V}_t)^+}\Delta Z_t^v, \tag{2.15}$$

$$V_{t+\Delta t} = (\widetilde{V}_{t+\Delta t})^+, \tag{2.16}$$

$$\Delta Z_t^s = \rho_{sv}\Delta Z_t^v + (1-\rho_{sv}^2)\Delta Z_t^i, \tag{2.17}$$

where Z_t^i and Z_t^v are independent standard normal variables.

We have selected this discretization since it performs other Euler schemes and, unlike other methods such as L.Andersen (2008), it is easier to implement. Moreover, for pricing path-dependent options, like our case, the exact scheme of Broadie and Kaya (2006) is not suitable. For European-style contracts, this approach seems very interesting. Nonetheless, for long-term (ten years) American options, where we do not only need the terminal stock price, but also the complete stock price path, the scheme of Broadie and Kaya (2006) becomes unfeasible due to the huge computational cost.

2.3 Numerical Results

First, we show a numerical study for the subjective ESO valuation and second, a sensitivity analysis on the valuation when changing the value of the parameters in the stochastic volatility process.

In all numerical studies we use a general parameter setting, but changing only one parameter in each analysis to validate the model. Suppose that the executive has an initial wealth of 5 million dollars to by shared out between cash and the firm's stock. We also consider that the executive is granted with 5 year stock options issued at the money with an exercise price of $30. The risk free interest rate is $r = 0.03$, the time of maturity of the ESO's is 5 years and the vesting period, τ, extends for the first two years. We consider eight cases, given different values for the degrees of diversification, α and ω, and the relative risk aversion γ.

We consider two cases of risk aversion, $\gamma = 2, 3$, as risk aversion increases we can see a general fall in all cases of subjective ESO valuation, as expected. Regarding the degree of diversification, two possible cases are also studied, $(\alpha, \omega) = (1/3, 1/3)$ and $(\alpha, \omega) = (1/4, 1/4)$. Likewise, the study is repeated when the presence of restricted stock is testimonial, $\omega = 0.05$, maintaining the level of stock options.

The number of simulated paths for the stock price and stochastic volatility is 50,000 (including antithetic), and the ESO valuation is obtained as the average of 100 previous estimations using different seeds. We approximate the price of the American-style ESO with the price of a Bermudan ESO with monthly exercise frequency, nonetheless we use a daily frequency, 252 time steps per year, to simulate the two state-variables, S_t and V_t.

Table 2.1 displays the subjective valuation, ESO_{sub}, for our hypothetical ESO under both constant and stochastic volatility models, CV and SV columns respectively, for alternative degrees of the executive's diversification, (α, ω), and risk-aversion, γ. In this case, the parameters of the variance process are $\kappa = 1,\ \theta = 0.09,\ \sigma_v = 0.1$, and $\rho_{sv} = -0.50$. The valuation under CV assumes a constant volatility equal to the square root of the long-term variance, θ.

Moreover, the last two columns exhibit the subjective value of the restricted stock. Below each valuation, in parenthesis, we show the price standard deviation computed over the one hundred estimations. At the top on the left-hand side of the table, we show the corresponding benchmark risk-neutral valuation or Black-Scholes price, BS hereafter, for comparison. Note that $BS = 9.5964$, is rather the same value as the corresponding one for the stochastic volatility framework, namely the European Heston price equal to 9.5825 (for $\lambda = 0$).

We observe some results. First, for European ESOs the executive's discount respect to BS is very high. The intuition is that the executive cannot obtain any payoff from the ESOs until maturity, thus they provide very low utility. For American-style ESOs, the subjective valuation increases significantly. Moreover, in both cases we can appreciate that the executive's discount is higher when the executive is further from the risk neutral situation: lower diversification and higher risk aversion (higher (α, ω) and γ). Moreover, the differences between CV and SV are higher when the executive is further from the risk neutral situation. The last two columns show that the subjective valuation of the restricted stocks behaves quite similar to the case for ESOs. Note that restricted stocks may be understood as European-style ESOs with an exercise price of $K = 0$. In several cases, the value of restricted stock using SV is less than the CV case.

Here, we analyze the valuation effects for different parameters of equations (2.12) and (2.13). We show the behavior of ESO_{sub} when changing separately κ, σ_v and ρ_{sv} corresponding to the left-hand side graphics of Figure 2.1.[3] We also study the corresponding pricing errors when valuing ESOs under CV, denoted as ESO_{sub}^{cv}, but the true process is SV, denoted as ESO_{sub}^{sv}, through the relative bias.

[3]The effects for the long-term variance, θ, will be studied later in Section 2.4.

Table 2.1: Subjective ESO valuation under stochastic volatility

BS=9.5964			European ESO		American ESO		Restricted Stocks	
γ	α	ω	*CV*	*SV*	*CV*	*SV*	*CV*	*SV*
2	0.333	0.333	3.3545	3.4801	4.5504	4.6358	22.4651	22.2946
			(0.0041)	(0.0063)	(0.0217)	(0.0228)	(0.0597)	(0.0625)
		0.05	6.0808	6.3101	6.8550	7.0384	25.0781	25.1191
			(0.0147)	(0.0157)	(0.0178)	(0.0175)	(0.3533)	(0.3231)
	0.25	0.25	4.8747	5.0649	5.8824	6.0077	24.7852	24.8180
			(0.0069)	(0.0087)	(0.0237)	(0.0234)	(0.0873)	(0.0657)
		0.05	7.1430	7.3927	7.7114	7.8727	26.4727	26.4844
			(0.0206)	(0.0214)	(0.0200)	(0.0204)	(0.0824)	(0.0001)
3	0.333	0.333	1.6619	1.7076	2.9608	2.9544	19.4865	19.0440
			(0.0065)	(0.0076)	(0.0201)	(0.0210)	(0.0599)	(0.0633)
		0.05	4.2823	4.4592	5.4348	5.5985	22.5996	22.3477
			(0.0053)	(0.0070)	(0.0152)	(0.0173)	(0.2378)	(0.1124)
	0.25	0.25	2.9584	3.0729	4.3154	4.3895	22.1080	21.9246
			(0.0050)	(0.0068)	(0.0214)	(0.0216)	(0.0551)	(0.0631)
		0.05	5.3352	5.5474	6.3400	6.5106	24.2227	24.1611
			(0.0098)	(0.0110)	(0.0181)	(0.0175)	(0.1721)	(0.1122)

This table shows the subjective valuation for a hypothetical ESO and one restricted firm share for different degrees of diversification, α, and risk-aversion, γ. *CV* and *SV* columns exhibit the valuations under constant and stochastic volatility, respectively. Below each price, in parenthesis, we show the corresponding price standard deviations. The relevant parameters are: $S_0 = 30$, $r = 0.03$, $\mu = 0.07$, $T = 5$, $\tau = 2$, $\theta = 0.09$, $\kappa = 1$, $\rho_{sv} = -0.5$ and $\sigma_v = 0.1$. The ESO is issued at-the-money.

This measure is computed as $\left(ESO_{sub}^{cv} - ESO_{sub}^{sv}\right)/ESO_{sub}^{sv}$ and it is displayed in the right-hand graphics of Figure 2.1.

Figure 2.1a analyzes the effects of κ, that is, the speed of reversion of the variance. We can observe that ESO_{sub} initially decreases with κ but it remains quite stable for values of κ larger than 2. In consequence, the relative biases decrease in absolute value for lower values of κ and keep rather constant for higher values. Note that these biases are positive in the case of less diversified executives $(\gamma, \alpha, \omega) = (3, 1/3, 1/3)$, whereas in all other cases it is negative and tend to zero when κ increases. Moreover, for a less diversified executive, $(\alpha, \omega) = (1/3, 1/3)$, a higher risk aversion degree leads to a higher positive size of bias. Thus, an executive's overvaluation.

Figure 2.1b exhibits the same study as before, but respecting σ_v. In this case, biases can be higher than 10% in absolute value and they are almost always negative except in the case $(\gamma, \alpha, \omega) = (3, 1/3, 1/3)$ and small values of σ_v, suggesting an executive's overvaluation when using CV. For more diversified positions, biases aproximates to zero for smaller values of σ_v while they are negative for larger values.

The effects of correlation between the two Brownian processes are studied in Figure 2.1c. It is shown that changing ρ_{sv} affects more under a less diversified position with biases reaching positive values. In all cases, an increase in the correlation means an increase in the relative bias.

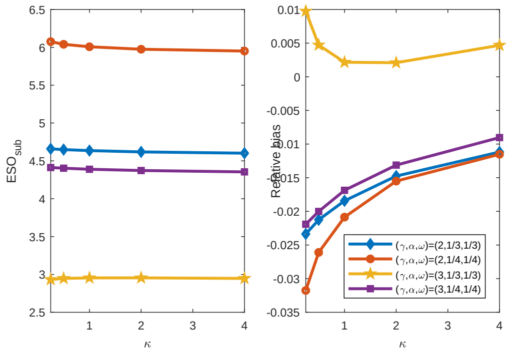
In general, like in Table 2.1, the more different biases are obtained for $(\gamma, \alpha, \omega) = (3, 1/3, 1/3)$, that is, the greatest differences between CV and SV models occur when the executive is further from a risk-neutral position: lower diversification and greater risk aversion.

2.4 Objective ESO valuation

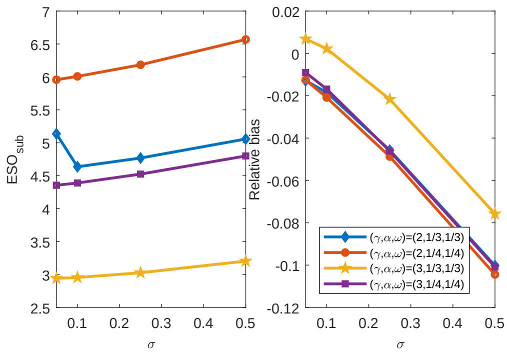
The objective valuation is made by a risk-neutral agent but considering that the ESO exercise is made by a risk averse and undiversified agent. This

FIGURE 2.1: Subjective ESO valuation and Heston's parameters

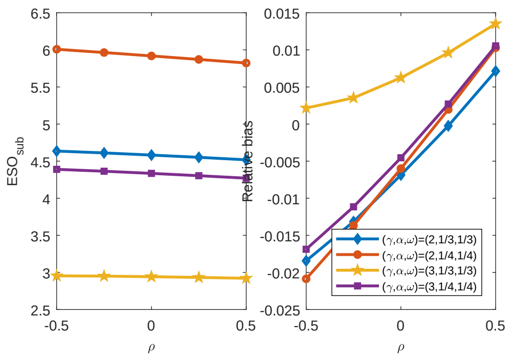
(A) Sensitivity to κ



(B) Sensitivity to σ



(C) Sensitivity to ρ_{sv}



This figure shows the subjective valuation for our hypothetical ESO under Heston's model for different values of κ, σ and ρ_{sv} (figures (a), (b) and (c), respectively). The left-hand side graphics exhibit the above values and the right-hand side graphics the relative biases, respecting the constant volatility model, when changing separately the parameters κ, σ and ρ_{sv}.

valuation, ESO_{obj}, corresponds to the cost of the firm (fair value) of issuing ESOs. As we have mentioned in the section of introduction, new accounting standards require firms to recognize the full value of the ESO granted to executives. We provide some numerical results for ESO_{obj} and the sensitivity analysis for some parameters, specifically the volatility risk premium, λ, and the long-term variance, θ. Second, we will study the effects of misspecification for ESO_{obj} when valuing under constant volatility, instead of the true process of stochastic volatility. This analysis becomes relevant because we are simulating stock returns for the ESO valuation at a daily frequency which supports more, according to the financial empirical evidence, a time-varying volatility rather than a constant volatility framework.

2.4.1 Fair value

To obtain the fair value, we need to build the exercise price threshold, which is the frontier holding the executive such that he becomes indifferent between exercising or continuing with the ESOs. This exercise frontier is contingent to the volatility level, thus we will obtain a different threshold price for every volatility level. In order such an exercise surface, we proceed as follow. When running the LSMU algorithm for the subjective valuation, we separate the exercised paths by volatility deciles. Then, for every decile, we record de minimum price of all exercised paths belonging to that decile. In this way we obtain ten exercise thresholds. Every exercise threshold corresponds to the a different volatility decile. Later, we take the average of the 100 different frontiers obtained with the 100 seeds. Thus, we simulate both the price and variance processes under the risk-neutral measure, driven by equations (2.18) to (2.20), and take the price threshold to determine the exercise. The first time the stock price hits the threshold, the ESOs are exercised and their payoffs are discounted at the risk-free rate.

The risk-neutral measure for the Heston's model is driven by the following equations:

$$dS_t = (r - d)S_t dt + S_t \sqrt{V_t} dZ_t^{s*}, \tag{2.18}$$

$$dV_t = \kappa^* \left(\theta^* - V_t\right) dt + \sigma_v \sqrt{V_t} dZ_t^{v*}, \tag{2.19}$$

$$\rho_{sv} dt = dZ_t^{s*} dZ_t^{v*}, \tag{2.20}$$

where dZ_t^{s*}, dZ_t^{v*} are the corresponding risk-neutral Brownian processes. The parameters θ^* and κ^* are, respectively, the long-term mean of the variance and the rate of mean reversion to θ^* under risk neutrality. It is verified that $\kappa^* = \kappa + \lambda$ and $\theta^* = (\kappa\theta)/\kappa^*$ with λ as the volatility risk premium.

Table 2.2 shows the same results as Table 2.1, but now they are about the fair value of our hypothetical ESO. In this case we assume that $\lambda = 0$, but later we relax this assumption to analyze the effects of the variance risk premium. We also include the expected life of the ESO in years, computed as the median of the exercise time of the different paths[4]. These values are implied in the subjective valuations of Table 2.1. First, we observe that the expected life of the ESO is larger under stochastic volatility. However, the differences between expected lifes are not very high. The American ESO objective value is lower under CV than under SV (except one case). We didn't expect this result because a lower expected life means a lower exercise threshold, thus the objective value also should be lower. This result differs from other works such as Brown and Szimayer (2008) or León

[4]we exclude 1% of the lowest observations

TABLE 2.2: Objective ESO valuation under stochastic volatility

BS=9.5964			Expected Life		American ESO	
γ	α	ω	CV	SV	CV	SV
2	0.333	0.333	3.5770	3.7329	8.6155	8.8394
			(0.0100)	(0.0094)	(0.0428)	(0.0397)
		0.05	3.8461	3.9251	9.2094	9.2473
			(0.0128)	(0.0107)	(0.0437)	(0.0408)
	0.25	0.25	3.7637	3.8473	9.0516	9.1100
			(0.0140)	(0.0118)	(0.0459)	(0.0419)
		0.05	4.0233	4.0379	9.4957	9.4434
			(0.0149)	(0.0144)	(0.0473)	(0.0439)
3	0.333	0.333	3.4235	3.6046	8.1506	8.4582
			(0.0064)	(0.0071)	(0.0412)	(0.0345)
		0.05	3.6174	3.7787	8.7144	8.9370
			(0.0077)	(0.0066)	(0.0375)	(0.0363)
	0.25	0.25	3.5436	3.6988	8.5267	8.7600
			(0.0085)	(0.0080)	(0.0405)	(0.0363)
		0.05	3.7633	3.8571	9.0460	9.1243
			(0.0104)	(0.0085)	(0.0416)	(0.0383)

This table shows the expected life in years and the objective valuation for a hypothetical ESO for different degrees of diversification, α, and risk-aversion, γ. CV and SV columns correspond to the constant and stochastic volatility cases, respectively. The relevant parameters are: $S_0 = 30$, $r = 0.03$, $\mu = 0.07$, $T = 5$, $\tau = 2$, $\theta = 0.09$, $\kappa = 1$, $\rho_{sv} = -0.5$ and $\sigma_v = 0.1$.

and Vaello-Sebastià (2009), who obtain that the ESO cost under time-varying volatility is lower than under CV with the same unconditional variance.

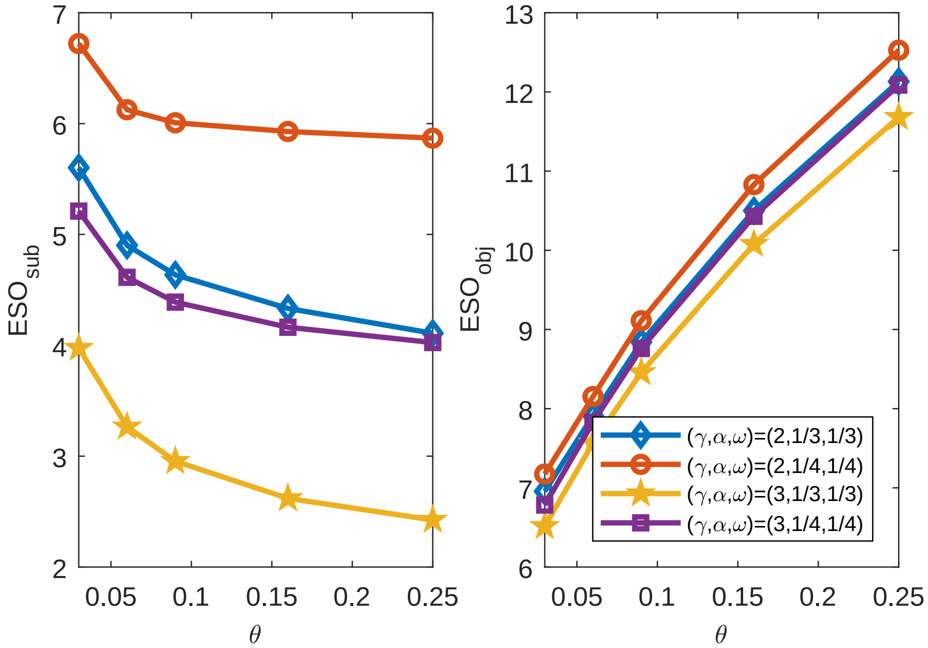
An interesting study is the influence of the volatility risk premium, λ, on ESO_{obj}. Notice that ESO_{sub} is independent of λ since it is obtained under the real measure. Meanwhile, the objective one is computed under the risk-neutral measure which clearly depends on λ, see equations (2.18) to (2.20). Figure 2.4 shows a monotone decreasing pattern of ESO_{obj} respecting λ (left-hand side graphic). Meanwhile, the relative bias measure, but now defined for the objective ESO price, increases with λ (right-hand side graphic). Note that depending on λ, the bias size can be higher than 20%. It also holds that the higher α and/or γ, the higher the size of the bias.

Figure 2.2 shows the analysis of the effects of the long-term variance, θ. At first sight, the results are surprising: an opposite behavior of ESO_{sub} respecting ESO_{obj} is observed when increasing θ. In Figure 2.2 we can see that, in general, ESO_{obj} (right-hand graphic) increases with θ as expected. This result agrees with the BS model, since it increases with the volatility. However, ESO_{sub} drops significantly (left-hand side graphic). The intuition is that a higher value of θ yields a higher volatility of the executive's wealth, since a large proportion of his wealth is invested in restricted stocks of the firm. Therefore, the higher variance for wealth leads to a lower expected utility, and hence a lower certainty equivalent value.

2.4.2 Misspecified volatility model

Previous analysis showed in Table 2.2 compares the ESO cost under SV and CV, holding both approaches the same unconditional variance. Nonetheless, in that case we are not analyzing the effects of the choice of an incorrect volatility model in the cost of the ESOs. Thus, now we conduct a Monte Carlo analysis assuming that the true data generating process (DGP) exhibits stochastic

FIGURE 2.2: *ESO* valuation and long-term variance, θ



This figure shows the objective valuation for our hypothetical ESO for different degrees of diversification, (α, ω) and risk aversion, γ, when changing the long-term variance, θ. The left-hand (right-hand) side graphics showa the subjective (objective) *ESO* valuation.

volatility, but the firm uses erroneously the *CV* model to obtain the ESO cost by mistake.

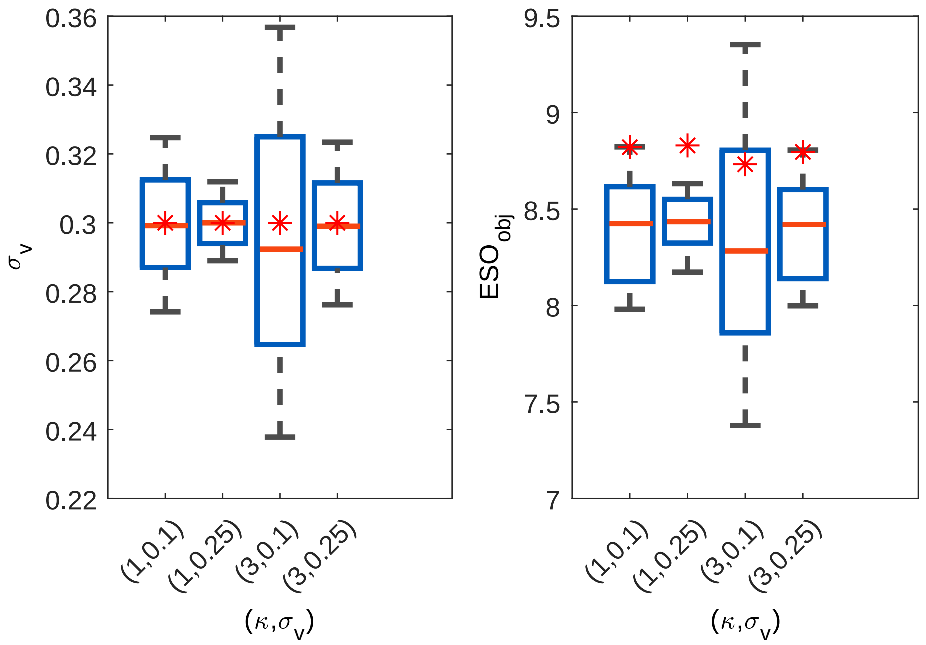
The methodology is described as follows. First, we assume a true DGP under *SV* and we obtain the correct ESO cost, ESO^{sv}_{obj}. Second, we simulate 10,000 price paths with 756 observations per path, corresponding to a total of 3 years, under the true process which is the *SV* model, and we estimate the standard deviations of the log-return series, $\hat{\sigma}_{sim}$, for all simulated paths. Third, for each value of $\hat{\sigma}_{sim}$, we obtain ESO^{cv}_{obj}. Finally, we compare ESO^{sv}_{obj}, or true cost, with ESO^{cv}_{obj}.

Our numerical example analyzes the mispricing effects under alternative values of κ and σ_v for the true DGP. Specifically, we consider the values of $\theta = 0.09$, $\kappa = \{1, 3\}$ and $\sigma_v = \{0.1, 0.25\}$ and the rest of parameters are identical to those in Section 2.3.

Figure 2.3 shows the boxplots of $\hat{\sigma}_{sim}$ and ESO^{cv}_{obj} for the different values of κ and σ_v. The left-hand side graphic exhibits the boxplots for the estimated standard deviations over the log-returns under SV, $\hat{\sigma}_{sim}$, for the previous parameters. The stars inside the boxes represent the square root of θ revealing that the boxes are quite centered around the unconditional value, that is, $\sqrt{\theta} = 0.3$. The dispersion of $\hat{\sigma}_{sim}$ becomes higher when $\kappa = 3$. We also appreciate that the lower σ_v, the wider the box. These results suggest that if the volatility of the variance is low and/or the speed of volatility reversion is big, the estimated values of $\hat{\sigma}_{sim}$ can be very different from the unconditional ones. Thus, the estimated ESO cost under CV can vary widely depending on the SV parameters.

The right-hand side graphic shows the boxplot for ESO^{cv}_{obj} by using the estimated $\hat{\sigma}_{sim}$. In this case, the stars inside the boxes of the boxplots represent ESO^{sv}_{obj}, the correct ESO cost. We can see that this value is over the median of the values obtained with the misspecified volatility model, ESO^{cv}_{obj}, suggesting an ESO overvaluation and hence, lower costs for the firm than the true ones. Furthermore, the boxes are more displaced with respect to the correct price (asterisk) for those cases with $\sigma_v = 0.1$, suggesting that the lower the volatility of the variance, the greater the undervaluation. Moreover, according with the findings of $\hat{\sigma}_{sim}$ (left-hand side graphic), the distribution of ESO^{cv}_{obj} becomes more disperse when σ_v (κ) is low (high).

FIGURE 2.3: Misspecified volatility model and *ESO* valuation



This figure shows the boxplots of $\hat{\sigma}_{sim}$ and ESO^{cv}_{obj} valuation for the parameters $\kappa = 1, 3$ and $\sigma_v = 0.1, 0.25$, to analyze the pricing errors when the firm estimates the ESO cost using CV (incorrect model) instead of SV (true model).

2.4.3 Accounting implications

This subsection is devoted to the analysis of the effects of stochastic volatility when using adjusted European-style approximations to account for the ESO cost as the main accounting standards suggest. Specifically, the FAS 123 suggests the use of the BS formula but introducing the expected life of the ESO rather than its maturity in order to account for the early exercise. The ESO cost according to this statement would be:

$$ESO_{FAS} = BS(S_0, K, r, d, \sigma, L) \tag{2.21}$$

where L is the expected life of the ESO. In our numerical approach, we estimate L as the average of the exercise times in the subjective valuation.

Notice that the pricing bias of ESO_{FAS} (approximation) with respect to ESO^{sv}_{obj} (true cost) may be different than the bias respect to ESO^{cv}_{obj}. These differences may arise from two sources. On one hand, if SV generates a different exercise behavior, the size of L under SV may be significantly different than under CV. Thus, it will yield a different value in the BS approximation. On the other hand, we have shown in the left-hand graphic of Figure 2.4 that ESO^{sv}_{obj} is quite sensitive to the variance risk premium, λ, while ESO^{cv}_{obj} remains unaltered. Then, differences between ESO^{sv}_{obj} and ESO_{FAS} can also be originated by λ. In consequence, due to the relevance of the topic, the analysis of the pricing errors when using European-style approximations under SV is worth.

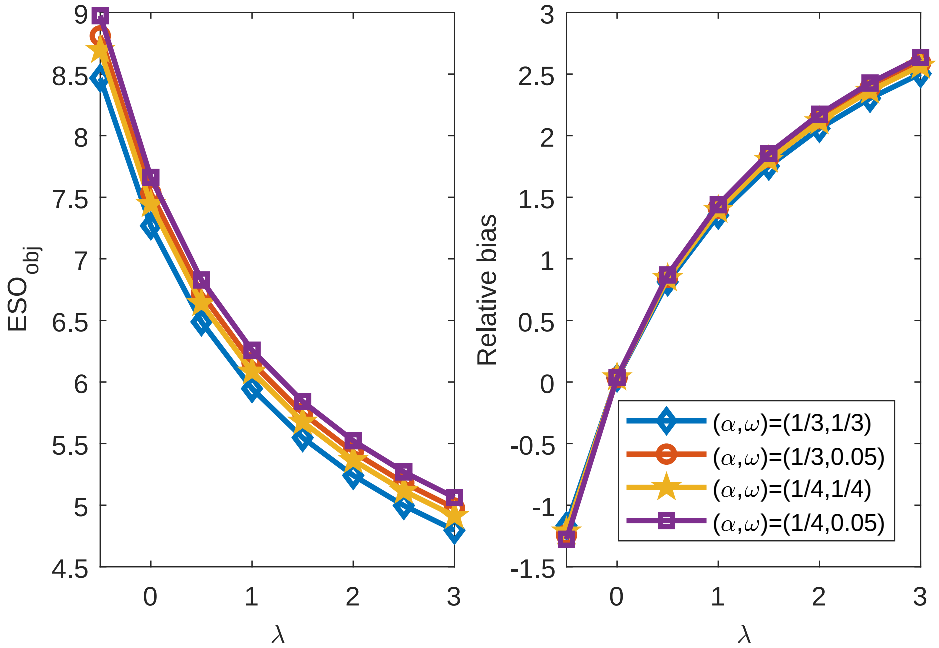
For a fair comparison, we also consider the European approximation under SV using the option pricing model of Heston (1993). Similar to equation (2.21), the ESO cost in this case would be

$$ESO^{sv}_{FAS} = H(\ldots, L, \lambda), \tag{2.22}$$

where H stands for Heston's formula. In this case, the possible biases should not be related with λ since it is now an input of the pricing model. In this case, we use the exact simulation scheme of Broadie and Kaya (2006) to compute ESO^{sv}_{FAS} by Monte Carlo, since for European-style contracts we only need simulated samples at maturity.

In Figure 2.5 we show the relative biases of both European approximations computed as $bias = (FAS - ESO^{sv}_{obj})/ESO^{sv}_{obj}$ where FAS becomes the price

FIGURE 2.4: Objective ESO valuation and volatility risk premium

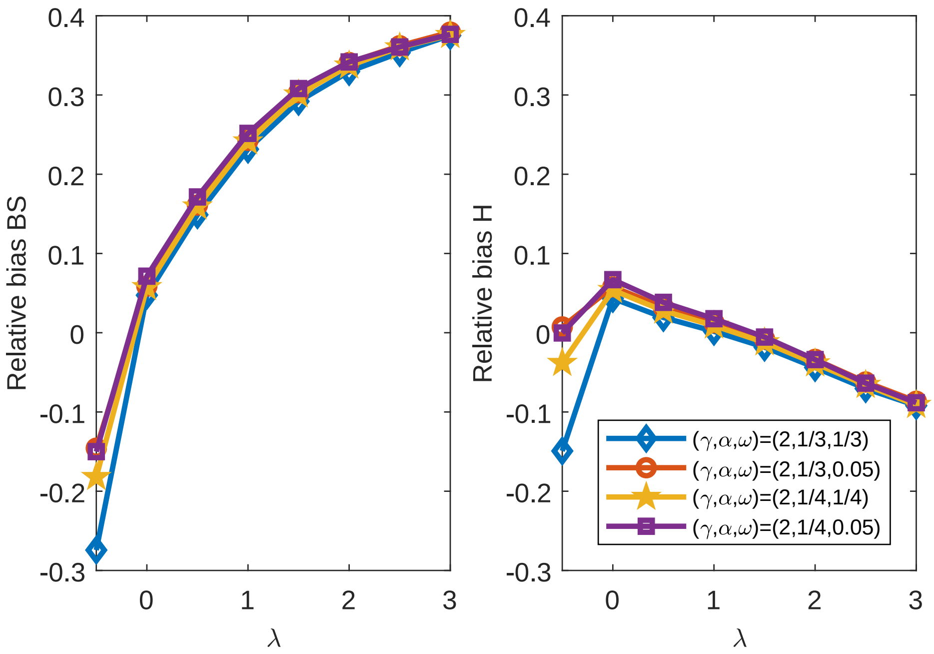


This figure shows the objective valuation for our hypothetical ESO for different degrees of diversification, (α, ω) and risk aversion, γ, when changing the volatility risk premium, λ. The left-hand side graphics exhibit the behavior of ESO_{obj} when λ changes, and the right- hand side graphics the relative biases $ESO^{cv}_{obj} - ESO^{sv}_{obj}$ for different volatility risk premium values. The remainder of the parameters are the same as in Table 1

according to either equation (2.21) or (2.22), see both left-hand and right-hand side graphics respectively. We consider our general parameter set with different values for λ. In the left-hand side graphic, we can appreciate that when using the BS approximation, the relative bias can be as higher as 30 %. Moreover, this bias is an increasing function of λ. Meanwhile, if we consider the Heston approximation for European options (right-hand side graphic), the biases are around 10%, and they remain quite stable for different levels of λ.

In short, when using BS approximation we may incur in large (positive) biases, that is, an overvaluation of the ESO cost, but, when using Heston approximation

FIGURE 2.5: European approximation biases



This figure shows the relative biases for both European approximations computed as $bias = (FAS - ESO^{sv}_{obj})/ESO^{sv}_{obj}$ where FAS can be the price according to equation (2.21) or (2.22). That is, BS and Heston models respectively. We consider our general parameter set with different values for λ

we may incur in undervaluation, in a less pronounced bias. This result also agrees with Brown and Szimayer (2008) and León and Vaello-Sebastià (2009), who obtain significant discounts for the ESO cost when considering SV and GARCH effects respectively.

2.5 Concluding Remarks

In this article we tackle the valuation of ESOs considering that the firm's stock price is driven by the model of stochastic volatility of Heston (1993). This paper analyzes both executive's subjective valuations and the firm cost (or objective valuation), that is relevant for accounting purposes. In both valuations we obtain significant differences with respect to the constant volatility case, although both models hold the same unconditional variance. Moreover, we show that the executive's subjective valuation depends on the dynamics (parameters) of the variance process in a sensitivity analysis of the model. In general, the differences between constant and stochastic volatility models are larger for less diversified and/or more risk averse executives.

In the objective valuation case, we must emphasize the effect that volatility risk premium has on the cost of the ESOs, obtaining biases larger than 20% with respect to the constant volatility case. Another important result is the effects of long-term variance in the ESO valuation. In general, the objective valuation increases when increasing the long-term variance whereas the subjective one behaves on the opposite way. A higher unconditional variance of the stock price also implies a higher variance for the executive's wealth, then it yields a lower expected utility and hence a lower certainty equivalent.

We carry out a Monte Carlo experiment to assess the mispricing when a firm uses, by mistake, the constant volatility model to compute the ESO cost, but the true variance process is stochastic. Our findings reveal that the standard deviations estimated over return series simulated with stochastic volatility may be very different, yielding a very disperse distribution of possible cost for the ESO assuming constant volatility. This finding is especially relevant for high values of the volatility of the variance.

Finally, we analyze European approximations using the ESO expected life, as the main accounting standards suggests. In this case we also obtain significant relative biases, higher than 20%.

Appendices

2.A The pricing algorithm

Let Φ be a vector that collects the optimal utilities of the executive's terminal wealth for each path of the simulated stock prices according to equations (2.14) to (2.17). The optimal value for each path is initially estimated at maturity, T, and it is updated backwards in time where the option is exercised from time $T-1$ to time 0.

At maturity, the optimal exercise rule becomes obvious: the ESOs are exercised for all those paths in-the-money. Thus, the vector Φ is computed by evaluating equation (2.5) in $W_{T|T}$. For the next periods, $T-1$, $T-2$, ..., 0, the executive must decide for each simulated stock price between exercising or holding the options in his portfolio. If the expected utility of exercise outweighs the continuing one, the executive will exercise the ESOs. Otherwise, he will hold them in his portfolio at least until the next period back. Given simulated samples of S_{T-1} and V_{T-1}, the utility of the terminal wealth if the executive decides to exercise the ESO is computed using equations (2.4) and (2.5), and the conditional expectation is obtained by least-squares:

$$\mathbb{E}_{T-1}\left[U(W_{T|T-1})|S_{T-1},V_{T-1}\right] \approx \sum_{i=1}^{M} b_{T-1,i}\Psi_i(S_{T-1},V_{T-1}), \tag{2.23}$$

where it is assumed that $\Psi_i\left(S_{T-1},V_{T-1}\right)$ can be expressed as a linear combination of orthonormal basis functions of S_{T-1} and V_{T-1} with $b_{T-1,i}$ as constant coefficients to be determined.

The expected utility of continuing with the ESOs, conditional on the current values of S_{T-1} and V_{T-1}, is $\mathbb{E}_{T-1}\left[\Phi|S_{T-1},V_{T-1}\right] \equiv \widetilde{\Phi}$. Similarly to the previous case, this expectation is also estimated by least-squares regressing Φ on some basis functions of the state variables.

Once both expectations are calculated, the ESO is exercised at $T-1$ for those paths that verify

$$\mathbb{E}_{T-1}\left[U(W_{T|T-1})|S_{T-1},V_{T-1}\right] > \mathbb{E}_{T-1}\left[\Phi|S_{T-1},V_{T-1}\right]. \tag{2.24}$$

that is, those paths where exercising the ESOs provides a higher expected utility than holding them. Finally, for those paths in which the inequality (2.24) is verified, the corresponding elements in the maximizing utility vector, Φ, are updated with $\widetilde{\Phi}(W_{T|T-1})$. We can work backwards $T-2$, $T-3$, ...until the initial time. Then, the average of the vector Φ is the expected utility of the executive's wealth at grant date, and we can compute the amount CE that minimizes the equation (2.9) to finally obtain the ESO value.

3

Stock option incentives for loss-averse executives

3.1 Introduction

Executive Stock Options (ESOs hereafter) are typically an important part of executive compensation packages. Hall and Murphy (2002) report that in 1999, 94% of S&P 500 companies granted ESOs to their top executives, and these grants accounted for 47% of total pay of S&P 500 CEOs. Murphy (2013) reveals that in 2011 option-based compesation accounts for the 21% of the CEO compensation. The literature covering different features of ESOs has grown dramatically in recent years. But, from the ESO valuation problem to the optimal design of the executive's compensation package, in all cases the standard model has largely assumed an executive who is a Von Neumann-Morgenstern expected utility maximizer.

Allais (1953) and Ellsberg (1961) were among the first to question the predictions of the expected utility theory. Since then, many influential experimental studies have questioned its ability to explain many observed phenomena endorsing the need for new theoretical developments. The most influential among all alternative models of choice under uncertainty is the prospect theory of Kahneman and Tversky (1979). Here, agent's preferences are defined over gains and losses relative to some reference point, in such a way that the agent is more sensitive to losses than to gains. Then, the utility function representing these preferences has a kink at the reference point, with the slope in the loss region larger than in the gain region. This is what is known as loss aversion which is an essential element of the prospect theory. Its relative merits in explaining what seem to be anomalies for the expected utility theory are documented in Camerer (2000) and Subrahmanyam (2007) provides a survey of the literature of behavioral finance. de Meza and Webb (2007) analyze option-based compensation under loss aversion finding that there will be situations where pay is non sensitive to firm performance. In this line, Dittmann et al. (2010) calibrates a principal-agent model where agent (executive) is loss averse and they show that their model can explain some stylized facts, such as, observed option holdings and large base salaries.

As it is well known in the literature on ESOs, see for instance Ingersoll (2006), we can distinguish three different ESO's valuation: the risk-neutral valuation corresponding to an unconstrained agent; the subjective valuation made by a constrained executive, who has not a fully diversified portfolio since he cannot trade his holdings of ESOs and firm's stocks; and the objective valuation, the cost to the firm of granting the ESOs, as measured by the value attached by an agent with a fully diversified portfolio who is restricted to follow the executive's exercise policy. In this paper, we will concentrate on the subjective valuation for loss-averse executives and the incentives.

To obtain the executive's subjective value we follow the literature that begins with Lambert et al. (1991), and later continued by Hall and Murphy (2002), Tian (2004) and Cai and Vijh (2005) among others, that uses the certainty equivalent approach to obtain it. Our results suggest that the executive's asset allocation and the subjective valuation exhibits a different pattern with respect to the risk averse case. Therefore, the subjective sensitivities (*delta, vega*), commonly used to measure the incentives, are also different.

This subjective valuation, on the other hand, is affected by the menu of assets that the executive has available to hold his wealth. In this regard we will consider a risk free asset and a market index as his alternatives for holding wealth. This leads naturally to an examination of the executive's wealth allocation decision as part of our examination of the subjective ESO valuation, an issue which we also examine in the paper. Once we obtain this subjective valuation, we can discuss the incentives provided by ESOs to loss-averse executives.

The rest of the paper is organized as follows. Section 3.2 makes a detailed presentation of our model. In the first subsection, we describe the executive's environment given by the value of his initial wealth, the compensation package paid by the firm and the available menu of assets for allocating that part of his wealth which is not held in the form of ESOs. Next, we introduce the utility function that describes the preferences of the representative loss-averse executive, paying particular attention to his reference level. In the final subsection we introduce the approach to measure the executive valuation of ESOs which will be extensively used to obtain our results. These are described in section 3.3 where we perform a sensitivity analysis of how several variables of interest are affected by his degree of loss aversion and the number of ESOs in his compensation packages. These variables are the allocation of the executive's unrestricted wealth, discussed in the first subsection, the subjective valuation of the average ESO, in the next one and the executive's perceived incentives, in the last subsection. Section 3.4 concludes.

3.2 The model

3.2.1 Executive's environment

Let W_0 denote the executive's total initial wealth. We assume that the fraction ω of W_0 is made up of N European call options on the firm's stock having all the same maturity, T, for

$$N = \frac{\omega W_0}{V_0} \ , \tag{3.1}$$

and V_0 denoting the cost of the option to the firm at the grant date, $t = 0$. Since the firm is not restricted and it can hedge its positions, this value will be calculated using the Black-Scholes pricing formula. Moreover, another fraction α of wealth are restricted stocks of the company also granted to the executive. The remaining fraction, $(1-\alpha)W_0$, is made up of his cash compensation from the firm and his non-firm related wealth accumulated from previous time periods. This unrestricted wealth is distributed between the risk-free asset and a market index so that the value of the executive's end of period wealth is given by

$$W(\eta) = W_0 \left[\alpha \frac{S_T}{S_0} e^{d_S T} + (1-\alpha-\omega)\left(\eta \frac{M_T}{M_0} + (1-\eta)e^{rT}\right)\right] + N\Big[S_T - K\Big]^+ \quad (3.2)$$

where $\Big[S_T - K\Big]^+ \equiv \max\Big[S_T - K, 0\Big]$, r is the risk-free rate, M_T and S_T are, respectively, the value of the market index and the price of one stock of the firm at maturity, K is the exercise price of the options and η is the weight of the non restricted wealth invested in the market index.

We assume that the stock price and the market index follow a joint geometric Brownian motion. Thus, given the starting values S_0 and M_0, the distributions of S_τ and M_τ, for any instant τ, are jointly log-Normal obeying

$$S_\tau = S_0 \exp\left\{\left(\mu_S - \frac{1}{2}\sigma_S^2\right)\tau + \sigma_S\sqrt{\tau}Z_S\right\} \quad (3.3)$$

$$M_\tau = M_0 \exp\left\{\left(\mu_M - \frac{1}{2}\sigma_M^2\right)\tau + \sigma_M\sqrt{\tau}Z_M\right\} \quad (3.4)$$

where μ_S and μ_M are the expected returns of the firm's stock and the market index, σ_S and σ_M are the corresponding volatilities, and Z_S and Z_M are standard normal innovations with correlation ρ.

3.2.2 Executive's preferences

Our representative executive is of the loss averse type and his preferences are described by the concave kinked utility function postulated by Berkelaar et al. (2004).

$$U\big(W_T\big) = \begin{cases} A\dfrac{1}{1-\gamma}W_T^{1-\gamma} + \big(B-A\big)\dfrac{1}{1-\gamma}\theta^{1-\gamma}\,, & \text{for } W_T \leq \theta \\ B\dfrac{1}{1-\gamma}W_T^{1-\gamma}\,, & \text{for } W_T > \theta \end{cases} \tag{3.5}$$

Essentially it is a power utility function with different curvature depending of whether the wealth is above or below the reference point, that is, on whether the executive is experiencing gains or losses. The first part provides the utility of losses, wealth's values below the reference level, given by the value of the parameter θ, whereas the second part provides the utility of gains. Then, for loss aversion to hold, $A > B$ so that the curvature of the utility function in the region of losses is greater than in the region of gains. Finally, we assume that $B = 1$ so that equation (3.5) nests the case of constant relative risk aversion, $CRRA$, when $A = 1$.

This function differs from the one used in prospect theory in that the latter is convex in the region of losses. Therefore, although our representative executive is loss averse he does not exhibit a risk-seeking behavior. Nevertheless, these preferences still have the property of first-order risk aversion (see Segal and Spivak (1990)) due to the kink in the utility function at the reference point.

Therefore, the executive's problem consist on selecting that η which maximizes his expected utility conditioned to the grant date, i.e.[1]

$$\max_{\eta} \mathbb{E}_0 \Big(U\Big(W_T(\eta)\Big)\Big) \ , \tag{3.6}$$

subject to the wealth constraint given by equation (3.2).

3.2.3 Executive's Subjective Valuation. The Certainty Equivalent Approach

As it is well known, the ESO's subjective value is the value attached by the executive to each one of the N ESOs granted. Because executives are by law unable of shorting the firm stock and using other hedging strategies, the subjective and the objective value will be, in general, different (see, for instance Ingersoll, 2006). In this subsection we describe the way in which the subjective valuation of ESOs is computed. This is done by using the certainty equivalent, CE, approach. Specifically, CE is the amount of cash payment that, if received at the ESOs' grant date, allows the executive to achieve the same expected utility that the given amount N of ESOs with exercise price K and maturity T. In such a case, the value of executive's wealth at option maturity, that we shall denote as W_T^{CE}, is given by

$$W_T^{CE} = W_0 \left[\alpha \frac{S_T}{S_0} e^{d_S T} + \left((1-\alpha-\omega) + \frac{\omega}{V_0} \cdot CE \right) \left[\eta \frac{M_T}{M_0} + (1-\eta) e^{rT} \right] \right] \tag{3.7}$$

[1]For simplicity, we abstract from any consideration of the related consumption decision.

Then, the CE is found by solving the following optimization problem

$$\max_{\eta_1} E_0\left(U\Big(W_T\Big)\right) = \max_{\eta_2,} E_0\left(U\Big(W_T^{CE}\Big)\right) \tag{3.8}$$

Observe that, the optimal share of the market index in the executive's unrestricted wealth will be, in general, different according to the definition of W_T. Because of this, we have distinguished with a different sub-index for the optimal composition of the executive's unrestricted wealth.

In practice, the subjective value has been obtained by choosing that CE for which the quadratic relative distance

$$\left(1 - \frac{\max_{\eta_1} \mathbb{E}_0\left(U\Big(W_T\Big)\right)}{\max_{\eta_2} \mathbb{E}_0\left(U\Big(W_T^{CE}\Big)\right)}\right)^2$$

is below some preset tolerance.[2]

3.3 Discussion of results

In this section we present the results concerning the asset allocation of his unrestricted wealth, the executive's subjective discount and, finally, the incentives to raise shareholders' wealth and to invest in risky projects.

Our benchmark case assumes the following values for the parameters: $S_0 = K = 30$, $r = 0.03$, $\mu_S = \mu_M = 0.07$, $\sigma_S = 0.3$, $\sigma_M = 0.15$, $W_0 = \$5,000,000$, $\rho = 0.5$ and $T = 5$. In our simulations we have used $500,000$ paths for the stock

[2]Further details are available from the authors upon request.

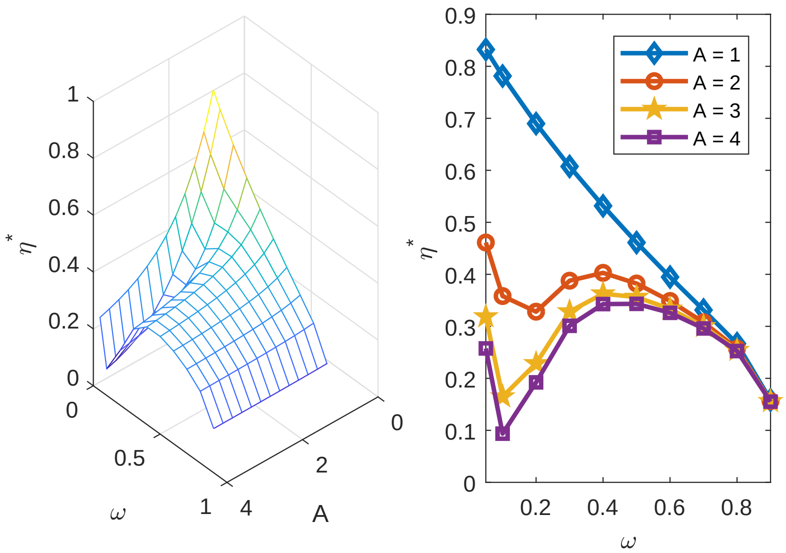
price and the market index (including antithetic). As our reference point we have postulated the initial level of wealth, $\theta = W_0$. In this regard, we have also analyzed the case in which the reference point is the value of the initial wealth at ESO's maturity when it is fully allocated in the sure asset, $\theta = W_0 e^{rT}$. However we have chosen the former reference point because the percentage of simulated paths for which W_T is above θ, the percentage of paths in gains for short, turns out to be an important element to understand the executive valuation. We shall return to this issue in the next subsection.

We have considered different configurations of the executive's compensation package. Specifically, we propose a grid of values for the share of ESOs, ω, in the interval $\left[0.10\ ,\ 0.90\right]$. The loss aversion parameter, A, ranges from 1 to 4. The case $A = 1$ represents a risk-averse executive which will serve for comparison purposes. In all cases we have used $B = 1$ and $\gamma = 2$ for the curvature of the utility function. We have also explored the case $\gamma = 3$ but it has been omitted here since it provides similar qualitative results. Similarly we have studied other cases, like the amount of restricted stock, $\alpha = 1/3, 1/4$ or the maturity $T = 10$ with similar results.

3.3.1 Executive's portfolio allocation

We begin by discussing how the executive's degree of loss aversion, measured by the parameter A, and his exposure to the firm's risk, measured by the parameter ω, affect to his decision of wealth allocation, measured by the value of η. This is shown in Figure 3.1. The case of a risk-averse executive $A = 1$ will be taken as a basis for comparisons. For this case, the share of the market index in the

FIGURE 3.1: Optimal executive's portfolio allocation: η^*



This figure plots η^* as a function of the ESOs' share in the executive's initial wealth, for different values of the parameter A. The case $A = 1$ describes a risk-averse executive and the others belongs to executives with increasing order of loss-aversion. Loss-averse executives have the initial level wealth as their reference point. The graph has been obtained for the following values of the parameters:$\gamma = 2$, $\alpha = 0.05$, $W_0 = 5,000,000$, $S_0 = K = 30$, $r = 0.03$, $\mu_S = \mu_M = 0.07$, $\sigma_S = 0.3$, $\sigma_M = 0.15$, $\rho = 0.5$ and $T = 5$.

executive's portfolio decreases monotonically with his exposure to the firm's risk. By contrast, the share of the market portfolio for the loss-averse executive is not monotone, exhibiting an inverted U-shape. For low size ESO packages (lower ω) the executive increases the exposure to the market portfolio as the size of the ESO package increases, but for a given threshold value, $\omega = 0.4$ in our set-up, then begins to decrease until convergence with the value associated to a risk averse executive. This change of behaviour is noticeable when $A = 4$ for which we can observe a clear kink around $\omega = 0.4$.

Why is there such differential behaviour of loss-averse executives respecting risk-averse ones? The answer turns out to be related with the reference point used to define the executive's preferences. When the loss-averse executive expects to suffer a relatively large number of losses from his exposure to the firm's risk he decides to increase his exposure to the market index as a way of reducing the probability of experimenting losses[3]. This argument can be confirmed with the help of Figure 3.2. As the executive's exposure to the firm's risk (ω) increases, the percentage of paths in gains, those for which the value of wealth at maturity are above its initial value, decreases. This result holds for different values of parameter A (degree of loss aversion) and is linked with the progressive reduction in the share of the market index in the executive's wealth. However, given the chosen reference point, a loss-averse executive experiments a higher percentage of paths in gains than the risk-averse one. This situation is, nonetheless, reversed once the threshold level of ω is reached. Given that both parts of the utility function are concave, the executive's does not show a risk-seeking behaviour. Therefore, the only plausible explanation is that he is trying to compensate the experienced increase in losses by taking a larger exposure to the risk of the market index. We have also explored the hedging argument, experimenting with different values for the correlation between the firm's stock and the market index [4] but we did not find any change in the qualitative behaviour of the loss-averse executive. He always increased his exposure to the market index once the threshold ω, in all cases the same, was reached.[5]

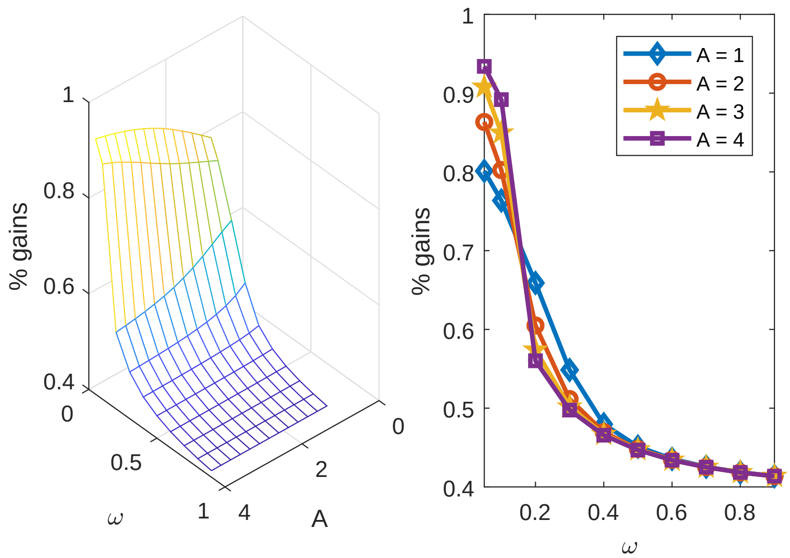
In this moment, we should return to our choice of W_0 as the executive's reference point. As we have mentioned before, we also considered the case $\theta = W_0 e^{rT}$, which could be considered as a more reasonable choice. However, for this value of θ, the percentage of paths in gains for loss-averse executives was always below the value

[3]Note that the proportion of unrestricted wealth of the agent decreases as the proportion of stock options increases

[4]In particular, we considered the cases $\rho = 0$ and $\rho = 1$

[5]Of course there was some differences in the percentage of paths in gains for each case but the threshold ω was the same in each case.

FIGURE 3.2: Percentage of paths for which $W_T > \theta$



This figure plots the percentage of simulated paths for which W_T is above W_0, the reference level for the loss-averse type of executives. Notice that, for each value of A, W_T has been computed by taking the corresponding optimal composition of the executive portfolio, η^*. The parameter's values used for the computations are: $\gamma = 2$, $\alpha = 0.05$, $W_0 = 5,000,000$, $S_0 = K = 30$, $r = 0.03$, $\mu_S = \mu_M = 0.07$, $\sigma_S = 0.3$, $\sigma_M = 0.15$, $\rho = 0.5$ and $T = 5$.

for risk-averse ones. This would have provided a plot like that of Figure 3.2 but in which we had only observed the paths to the right of the threshold ω, missing relevant information to understand the executive's decision. Since all other results concerning the executive's discount and his incentives were qualitatively similar, we have preferred to chose the initial wealth as the reference point.[6]

To end this subsection we observe that, although η^* is chosen at the beginning of period, the final wealth distribution is random. Then the average share of ESOs, the market index and the riskless asset might be different from what the executive initially intended. To check this point and provide additional evidence of the executive's portfolio decision, we show the average composition of the executive's

[6]The results for the reference point $\theta = W_0 e^{rT}$ are available from the authors upon request.

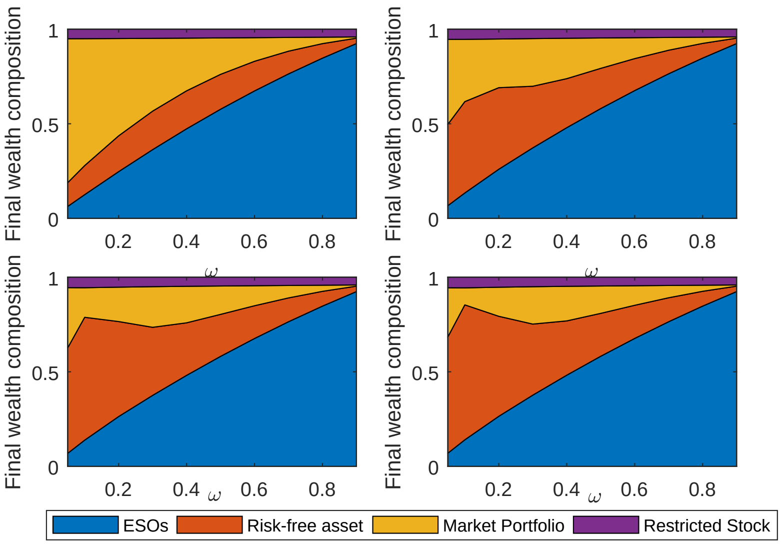
final wealth for each one of the four values of A in the corresponding panels of Figure 3.3. Leaving aside the upper band that represents the restricted stock, and remember that it's sale is not allowed until maturity, the remaining final wealth is distributed as follows. The area plotted in lower part of each panel represents the average share of ESOs in the final wealth. Similarly, the area plotted in the upper part represents the average share of the market index and the remaining area between the other two represents the share of the riskless asset in total wealth. We see that the results are similar to those portrayed in Figure 3.1. For a risk-averse executive, the share of ESOs increases along with a reduction in the share of the market index. However, a loss-averse executive always put more weight to the risk-free asset and the average share of the risky asset decrease until some threshold value of ω and then its increases.. The case of $A = 4$ provides the most clear evidence in this regard.

3.3.2 Executive's Subjective Discount

In this subsection we report results concerning the executive's subjective valuation of ESOs, as measured by CE. Nevertheless, a more informative measure of the executive's valuation is provided by the executive's discount, which measures the proportion of the ESO cost that its perceived by the executive. It has been calculated as the ratio of the executive's subjective valuation (CE) over the market fair value (Black-Scholes price) and the specific results are reported in Table 3.1 for different shares of the ESOs in the compensation package (ω) and different degrees of loss aversion (A).

Again considering as our reference the risk-averse executive, we see that the

FIGURE 3.3: Average composition of final wealth.



In this figure we represent the average composition of W_T for each one of the four values considered for the parameter of loss-aversion, A. In each one of the four panels, the area plotted in lower part represents the average share of ESOs in the final wealth. The area plotted in the upper part represents the average share of the market index and the remaining one, the share of the riskless asset in total wealth. All graphs have been obtained for the following values of the parameters:$\gamma = 2$, $\alpha = 0.05$, $W_0 = 5,000,000$, $S_0 = K = 30$, $r = 0.03$, $\mu_S = \mu_M = 0.07$, $\sigma_S = 0.3$, $\sigma_M = 0.15$, $\rho = 0.5$ and $T = 5$.

greater exposure to the firm's risk makes the loss-averse executive to feel worse and his ESOs' valuation decreases comparatively more than the corresponding one for a risk-averse executive. However, these differences in valuation disappears as the compensation package consists almost completely of ESOs. This result holds for different values of α and γ.

3.3.3 Executive's Incentives

In this final subsection we present the results concerning the executive's incentives. The literature has paid great attention to the optimal design of the

Table 3.1: Executive's ESO discount

	A						
ω	1	1.5	2	2.5	3	3.5	4
0.10	0.7382	0.7058	0.6873	0.6798	0.6799	0.6834	0.6877
0.20	0.6199	0.5596	0.5021	0.4458	0.3928	0.3461	0.3083
0.30	0.5239	0.4439	0.3801	0.3333	0.2999	0.2756	0.2573
0.40	0.4400	0.3608	0.3119	0.2809	0.2600	0.2451	0.2340
0.50	0.3629	0.2979	0.2637	0.2433	0.2297	0.2202	0.2130
0.60	0.2890	0.2433	0.2211	0.2082	0.1997	0.1937	0.1892
0.70	0.2158	0.1890	0.1763	0.1689	0.1641	0.1607	0.1582
0.80	0.1402	0.1285	0.1229	0.1196	0.1174	0.1159	0.1148
0.90	0.0581	0.0557	0.0546	0.0539	0.0534	0.0531	0.0528

This table reports the executive's ESO discount calculated as the ratio of the subjective valuation (CE) over the Black-Scholes price. In the table, ω measures the weight of the ESOs in the initial wealth, and A is the parameter for loss aversion. The case $A = 1$ describes the risk-averse executive. Loss-averse executives have the initial level wealth as their reference point. Other parameter's values are: $\gamma = 2$, $\alpha = 0.05$, $W_0 = 5,000,000$, $S_0 = K = 30$, $r = 0.03$, $\mu_S = \mu_M = 0.07$, $\sigma_S = 0.3$, $\sigma_M = 0.15$, $\rho = 0.5$ and $T = 5$.

executive compensation package as a way of affecting their incentives. These compensation packages typically consist of a fixed cash component, restricted stocks and ESOs. Given that the available empirical evidence shows a greater share of ESOs at the expense of restricted stocks, we have decided to omit them practically for simplicity, fixing $\alpha = 0.05$.

Typically in the literature, the role of ESOs on executives' incentives has been approached by examining the sign and size of the subjective ESO greeks, since it is the executive's perception of incentives what matters. In this regard, there are two ways in which ESOs may affect executives' incentives. First, as part of an agency problem, they align executives and shareholders interests in raising the firm's stock price. The perceived reward from acting this way is measured by the subjective *delta*. And second, when a new investment project is taken, not only the firm's expected return tends to increase but also its volatility. Given the executive limitation to hedge this risk, this may affect executive's incentives to adopt such investment projects. Therefore, the perceived reward form adopting risky investment projects is the subjective *vega*.

Following Tian (2004) we distinguish between the average and the aggregate subjective *delta*, Δ_{sub} and Δ^+_{sub} respectively. Δ_{sub} is defined as the partial derivative of the subjective valuation of the average ESO with respect to the initial firm's stock price S_0, whereas the aggregate subjective *delta*, Δ^+_{sub}, is the partial derivative of the subjective valuation of the total ESOs granted with respect to the initial firm's stock price S_0.

Both sensitivities have been calculated numerically by finite-differences. Specifically,

$$\Delta_{sub} = \frac{\partial CE}{\partial S_0} \approx \frac{CE(S_0+\varepsilon_s) - CE(S_0)}{\varepsilon_s} \tag{3.9}$$

$$\Delta^+_{sub} = \frac{\partial (N\cdot CE)}{\partial S_0} \approx \frac{N(S_0+\varepsilon_s)\cdot CE(S_0+\varepsilon_s) - N(S_0)\cdot CE(S_0)}{\varepsilon_s} \tag{3.10}$$

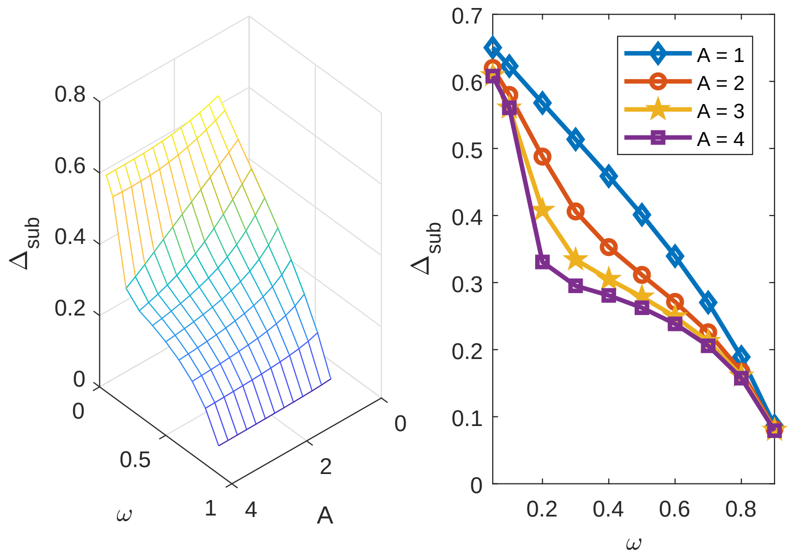
where $\varepsilon_s = 1$ is the increment in the stock price. Respect to Δ^+_{sub}, notice that changes in S_0 will also affect the number of ESOs, N. According to equation (3.1), the number of ESOs depends on their market value (Black-Scholes price), V_0. Thus increments in S_0 will also suppose increments in V_0, yielding a lower number of ESOs.

Figure 3.4 displays the subjective *delta* for different sizes of the ESO package

(ω) and different degrees of loss aversion (A). As in Tian (2004), the average incentive provided by an option is positive but declines quickly as ω increases, independently of the executive preference's type. But the interesting point is that the executive's perceived incentives are systematically lower when he is of the loss-averse type. These differences according to the degree of loss aversion increase up to the level of the treshold, $\omega = 0,4$, from this point these differences decreases until disappearing when the compensation package is almost completely made of ESOs. These behaviour is also present in Figure 3.5, where the aggregate subjective *delta* of the entire option grant, Δ^+_{sub} is showed. Again the Δ^+_{sub} for a loss-averse executive is always below the corresponding one to a risk-averse executive. The relationship between Δ^+ and ω has an inverted U-shaped form when the executive is of the risk-averse type, and the same thing happens in the case of loss-averse executive. The difference between an risk-averse executive and another loss-averse is greater when ω is above the treshold $\omega = 0,4$.

Turning now to the executive's incentives to adopt investment projects that increase firm's volatility, some preliminary comments are in order. In the literature it is conventional to distinguish between the systematic and the idiosyncratic component of the firm's total volatility, σ_S. Although this distinction can be performed independently of the specific set-up considered, it is typically associated to the Capital Asset Pricing Model (CAPM). This is so because, any investor can eliminate idiosyncratic risk through diversification and any change in the systematic risk is positively related with the firm's expected return. Therefore, this decomposition can affect the executive's incentives to take on higher risks, differently. However, in the context of the loss-averse theory, the CAPM does not hold and the rationale for this distinction disappears. As a result we have focused on the total firm's volatility when

FIGURE 3.4: Average subjective Delta : Δ_{sub}



Both, left and right panel of the figure, plot the executive's incentives to raise the firm's stock price as a function of both A and ω. These incentives are evaluated using the average *delta*, Δ_{sub}, which is measured in the vertical axis of both panels. Equation (3.9) provides its numerical definition. The values of the parameters used in the computations have been:$\gamma = 2$, $\alpha = 0.05$, $W_0 = 5,000,000$, $S_0 = K = 30$, $r = 0.03$, $\mu_S = \mu_M = 0.07$, $\sigma_S = 0.3$, $\sigma_M = 0.15$, $\rho = 0.5$ and $T = 5$.

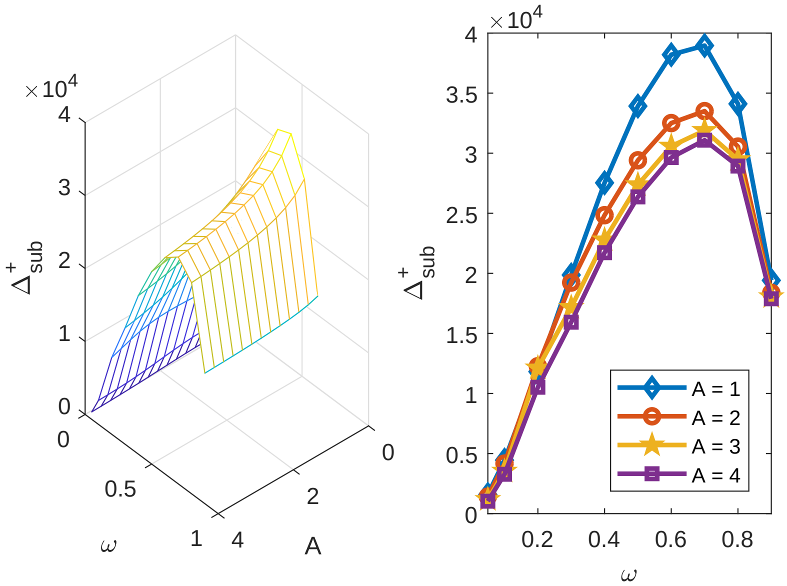
computing the corresponding ESO *vega*.

Consequently, we have calculated both the average subjective *vega*, Λ_{sub}, defined as the partial derivative of the subjective valuation of the average ESO with respect to the firm's stock volatility and the aggregate subjective *vega*, Λ^+_{sub}, defined as the partial derivative of the subjective valuation of the total ESOs granted with respect to the firm's stock volatility. Both sensitivites have been calculated by finite-differences similar to equations (3.9) and (3.10),

$$\Lambda_{sub} = \frac{\partial CE}{\partial \sigma_S} \approx \frac{CE(\sigma_S + \varepsilon_{\sigma_S}) - CE(\sigma_S)}{\varepsilon_{\sigma_S}} \tag{3.11}$$

$$\Lambda^+_{sub} = \frac{\partial (N \cdot CE)}{\partial \sigma_S} \approx \frac{N(\sigma_S + \varepsilon_{\sigma_S}) \cdot CE(\sigma_S + \varepsilon_{\sigma_S}) - N(\sigma_S) \cdot CE(\sigma_S)}{\varepsilon_{\sigma_S}} \tag{3.12}$$

FIGURE 3.5: Aggregate Subjective Delta: Δ^+_{sub}



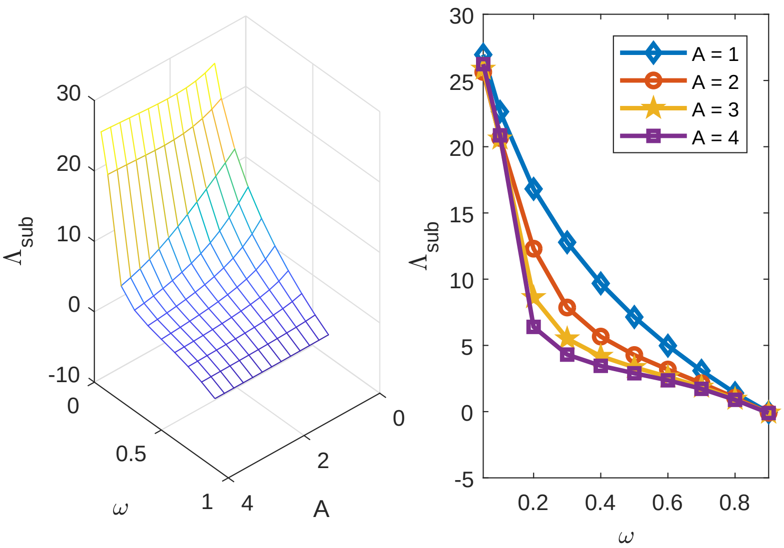
As in Figure 3.4, the executive's incentives to raise the firm's stock price as a function of both A and ω are represented using now the aggregate *delta*, Δ^+_{sub}, as our measure. Its numerical definition is provided by equation (3.10) in the main text. The values of the parameters used in the calculations have been:$\gamma = 2$, $\alpha = 0.05$, $W_0 = 5,000,000$, $S_0 = K = 30$, $r = 0.03$, $\mu_S = \mu_M = 0.07$, $\sigma_S = 0.3$, $\sigma_M = 0.15$, $\rho = 0.5$ and $T = 5$.

where $\varepsilon_{\sigma_S} = 0.05$ is the increment in the stock volatility. Similar to equation (3.10), changes in σ_S will also affect the number of ESOs, N, through the ESOs market value, V_0.

The results are depicted in Figures 3.6 and 3.7. When the perceived incentives are measured by the average *vega*, Λ_{sub}, executives are willing to increase firm's volatility but these incentives decrease with ω, independently of the type of preferences considered ($\Lambda_{sub} > 0$). However, when ω reaches its threshold value, the loss-averse executive perceives lower incentives to raise firm's volatility respecting those perceived by a risk-averse executive. However, when we use the aggregate *vega* as a our measure of the executive's perceived incentives to increase firm's volatility there are some interesting differences. Firstly, when the number of ESOs granted implies a large proportion of the executive's wealth, his incentives are to reduce the firm's volatility, not to increase it ($\Lambda^{+}_{sub} > 0$).

This effect is much more significant for a loss-averse executive. We interpret this result in connection with the order of risk aversion for loss and risk averse executives. Loss-averse executives exhibit first-order risk aversion, so that the risk premium of small gambles is proportional to the standard deviation. Risk-averse executives exhibit second-order risk aversion and the risk premium turns out to be proportional to the variance. As a result, the increase in the firm's volatility has greater impact for the loss-averse executive. However, this negative impact only appears to affect differentially the incentives of each type of executive, when the expected losses for loss-averse executives become higher than those experienced for a risk-averse one and this occurs when ω reaches its threshold value. Notice that this threshold value for ω depends upon the reference point considered. When $\theta = W_0 e^{rT}$, Λ^{+}_{sub} is always below for loss-averse executives.

FIGURE 3.6: Average Subjective Vega: Λ_{sub}

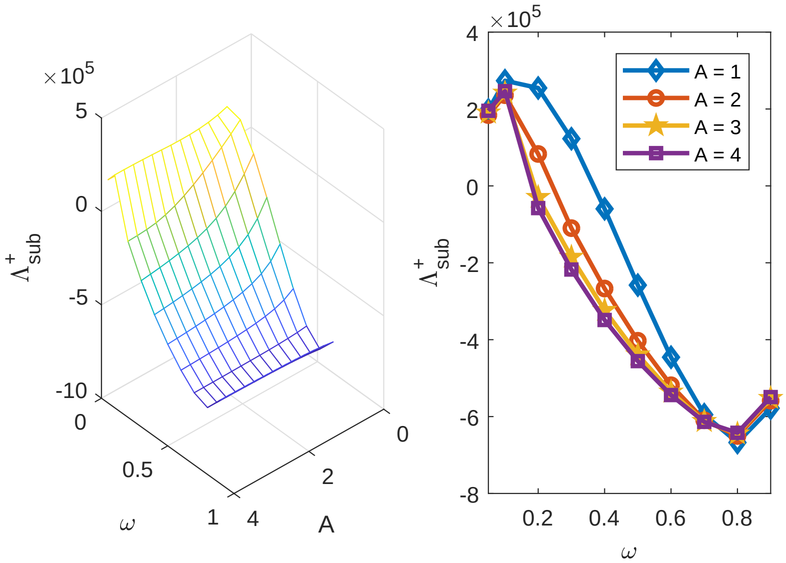


The figure plots the executive's incentives to raise firm's volatility by taking on risky investment projects. This incentives are measured using the average *vega*, Λ_{sub}, which is measured in the vertical axis of both panels. Λ_{sub} has been computed by finite-differences, taking $\epsilon_{\sigma_S} = 0.05$. See equation (3.12) in the main text for its precise definition. The values of the parameters used in the computations have been:$\gamma = 2$, $\alpha = 0.05$, $W_0 = 5,000,000$, $S_0 = K = 30$, $r = 0.03$, $\mu_S = \mu_M = 0.07$, $\sigma_S = 0.3$, $\sigma_M = 0.15$, $\rho = 0.5$ and $T = 5$.

3.4 Conclusions

We have shown that when executives are of the loss-averse type their ESO's valuation is substantially affected. Results depend upon the reference point considered. To highlight some interesting features for the executive's portfolio allocation, we have set this reference point to the initial level of wealth. As a result we have shown that the optimal share of the market index is not monotone: initially, the relationship is negative, decreasing as the size of the ESOs package (ω) increases, but there is a threshold value for which it begins to increase as a way to reduce the expected losses from a reduced weight of his unrestricted wealth, that is not allocated in ESOs.

FIGURE 3.7: Aggregate Subjective Vega: Λ^+_{sub}



The figure depicts the executive's incentives to raise the firm's volatility as a function of both A and ω. These incentives are evaluated now using the aggregate *vega*, Λ^+_{sub}, which has been computed by finite-differences, taking $\epsilon_{\sigma_S} = 0.05$. Its precise numerical definition appears in equation (3.12). The values of the parameters used in the calculations have been:$\gamma = 2$, $\alpha = 0.05$, $W_0 = 5,000,000$, $S_0 = K = 30$, $r = 0.03$, $\mu_S = \mu_M = 0.07$, $\sigma_S = 0.3$, $\sigma_M = 0.15$, $\rho = 0.5$ and $T = 5$.

Regarding the incentives, we have found that the executive's subjective valuation and subjective *delta* are lower when he is of the loss averse type. These differences may be substantial when the expected losses are substantial. Similar results have been obtained for the executive perceived risk incentives as measured by the ESO *vega*. Moreover, contrary to the risk-neutral paradigm, we show that the aggregate *Vega*, Λ^+_{sub}, can be negative, rejecting the idea that ESO compensation will lead an increase of the firm risk.

As suggestions for future research, it would be very interesting to extend the analysis to the case of risk-seeking loss averse executives along the lines of Gomes (2005), who postulates an utility function partially convex in the region of losses. Also, considering transformations of the probability distribution as the cumulative prospect theory of Tversky and Kahneman (1992). A final step in this line of research would be to examine the more realistic case of American ESOs as in León and Vaello-Sebastià (2010). This should be a fruitful line of research to improve our understanding of the role of ESOs in modifying executive's incentives.

Appendices

3.A Anothers utility functions

In this context, we study how the investor, holder of ESO, evaluates the potential gains or losses. In 1979, Tversky and Kahneman (1979) presented a model of choice with the name of prospect theory to explain some violations of expected utility theory. In recent decades has shown repeatedly that the theory of expected utility does not adequately explain individual choice. Prospect theory allows us to describe the behavior of people when they have to choose between risky alternatives, such as in certain financial decisions. Across the board, the two key points of this theory are: 1) a value function concave for gains, convex for losses and steeper for losses than for gains (risk-seeking behavior) and 2) a nonlinear transformation of the probability scale.

According to this theory, we study how the choice of the utility function affects the European ESO subjective value using 3 different utility functions, each of which represents one of the features set theory prosper.

First, we calculate the value of the European ESO from the perspective of an investor averse to losses, like in Chapter3. Subsequently, we propose a nonlinear transformation of probabilities of the usual CRRA utility function. Thirdly, we use a utility function concave for gains and convex-concave in losses.

3.A.1 Loss averse utility function (Kinked power-utility function)

Most people worry about the negative impacts on her portfolio for any loss that obtaining higher gains. This behavior is known as loss aversion and it is because a loss has a negative feeling whose impact is greater than the pleasured that would get with a gain of equal magnitude.

According to this theory, investors make their decisions based on gains or losses obtained, rather than as a function of final wealth. While the power utility function (or constant relative risk aversion (CRRA)) only depends on a parameter, which defines the curvature of the same, the loss aversion utility function is a kinked power function, which involved 3 parameters. Two of them determine the levels of curvature of the losses and gains and the third is the coefficient of risk aversion. In our case, we will use the utility function given by Berkelaar et al. (2004), a concave loss aversion utility function with a kink that determines the reference point.

We have considered this utility function rather than a utility function convex in losses, to study separately the effect of first-order aversion in decision making (sensitivity to small deviations around the reference point).

3.A.2 Transformed probabilities-utility function

In classical theory, the utility of a prospect uncertain is calculated as the sum of the utilities of outcomes, each weighted by its probability. In the case of equal weighted probabilities, the utility is the arithmetic mean or average. In this case, we consider the standard power utility function, and we apply the idea of the cumulative prosper theory, Tversky and Kahneman (1992), in which, instead of transforming each probability separately it transforms the cumulative distribution function. This transformation captures the experimental evidence that people tend to overestimate small probabilities and are more sensitive to small changes in higher probabilities.

The Tversky and Kahneman probability-weighting function is defined as follows:

$$\sum U(W_{T_i})\pi_i(p) \tag{3.13}$$

where $U(W)$ is any utility function (in our case the CRRA utility function), and π_i is a set of decision weighs. In this case, the decision weights depend on the probability distribution and are calculated as the difference of a probability-weighting function, $\pi_i(p) = \Omega(p_i) - \Omega(p_{i-1})$, with

$$\Omega(p) = \frac{p^\delta}{[p^\delta + (1-p)^\delta]^{1/\delta}} \quad 0.28^{7} < \delta \leq 1 \tag{3.14}$$

These weights must be positive because they behave like probabilities of the

[7] technical condition that ensures that the first derivative with respect to parameter p is positive

expected utility function. Noted that when $\delta = 1$, we have CRRA utility function. For values $\delta < 1$, the weighting function is concave, which implies that the investor is more averse to sudden changes in their wealth, so he gives more weight to more stable situations. The closer to 0.28 is the parameter δ, the tails of the probability distribution are thicker.

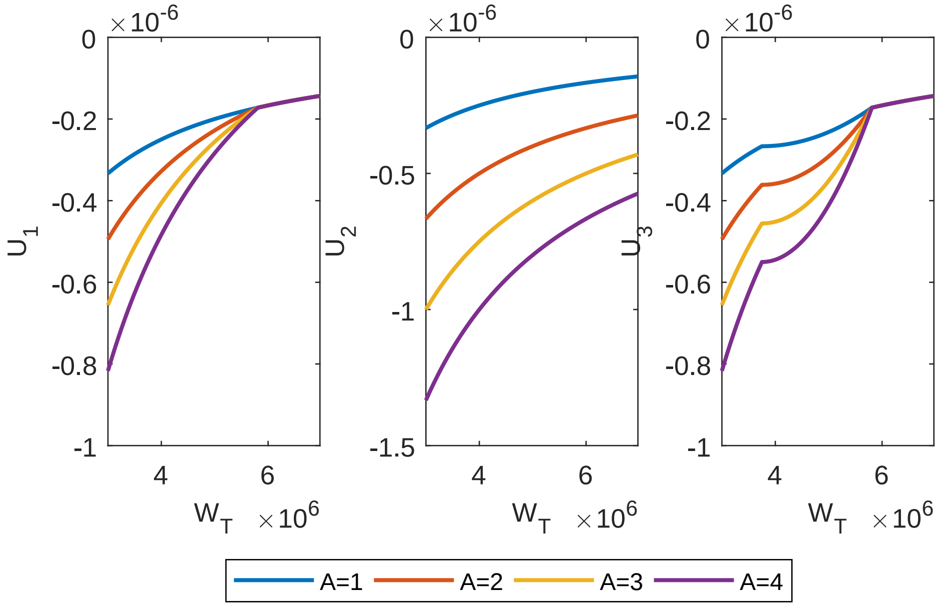
3.A.3 Convex utility function

Following Gomes (2005), we define a new piecewise utility function in which the stretch of losses is convex, i.e. the investor is risk loving in the domain of moderate losses, except for extreme losses where function is again concave (to avoid optimization problems). In this case, we considered $\underline{W} = W_0$ and $\theta = W_0 e^{rT}$ (the investor's initial wealth invested at the risk-free interest over the life of the option) as threshold value between losses and gains. The parameter of the utility function indicates the degree of convexity of the stretch of losses bearable, the higher A higher "disutility", in other words, the higher A, more loss-averse is the investor.

$$U(W) = \begin{cases} A\frac{1}{1-\gamma}W^{1-\gamma} + (B-A)\frac{1}{1-\gamma}\theta^{1-\gamma} & W < \underline{W} \\ A\frac{1}{1-\gamma}\frac{\theta^{1-\gamma}-\underline{W}^{1-\gamma}}{(\theta-\underline{W})^2}\left(W-\underline{W}\right)^2 A\frac{1}{1-\gamma}\underline{W}^{1-\gamma} + (B-A)\frac{1}{1-\gamma}\theta^{1-\gamma} & \underline{W} \leq W \leq \theta \\ B\frac{1}{1-\gamma}W^{1-\gamma} & W > \theta \end{cases} \tag{3.15}$$

where $A > B$ to hold for aversion and $A, B > 0$ to ensure that $U(W)$ is a increasing

FIGURE 3.A.1: Utilities functions



These figures show the utilities aversion with parameter values $A = \{1, 2, 3, 4\}$.

function, θ is the wealth threshold (reference point), $\underline{W}$ is the point where function is again concave and $\gamma > 0$ to impose risk aversion.

Figure 3.A.1 shows the loss aversion utility functions used in this section, where the reference point is equal to the investor's initial wealth invested at the risk free rate during the option's life, $W_0 e^{rT}$, and there are 4 values of A which determines the curvature of the region of losses. For $A = B = 1$ we have the CRRA function. The higher A the larger is the curvature of the function in the region of losses, i.e., the investor is more loss-averse.

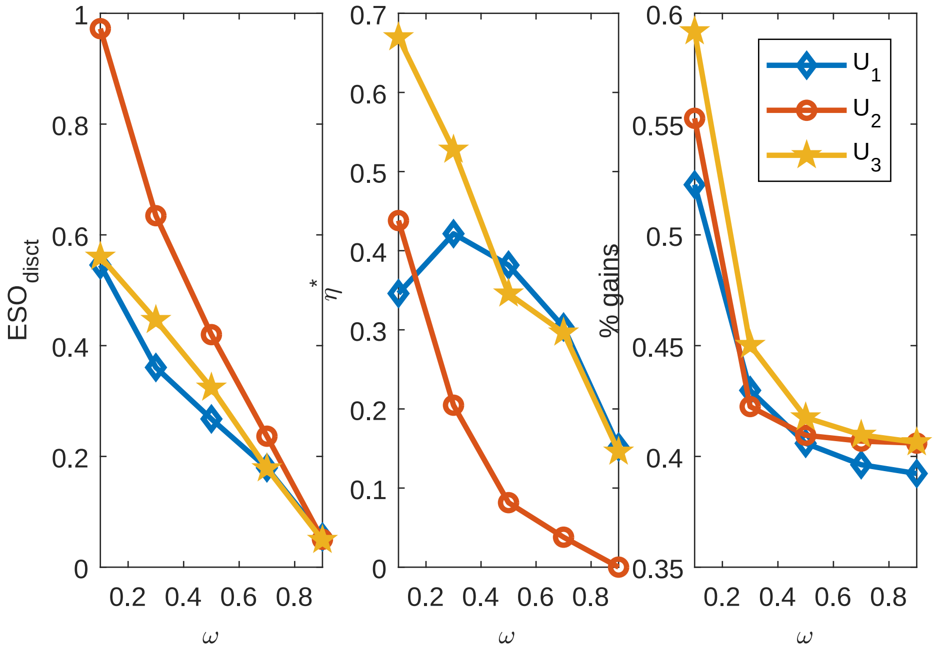
3.A.4 Numerical results

In this section we will comment briefly on the differences obtained in subjective valuation when we consider the three types of executives mentioned above (risk averse, risk-loss averse (concave - convex - concave utility) and transformed risk averse). Figure 3.A.2 presents the three most important results, the ESO subjective value relative to fair value, the percentage of wealth allocated to the market portfolio and the percentage of gains. We can conclude that a transformed risk averse agent values ESOs more than either of the two loss averse agents (left panel), but, nevertheless, this same agent invests less in the market portfolio than the other two. As for the percentage of profit trails, the pattern followed by the three agents is similar, although slightly higher in the case of the risk-loss averse agent, which indicates that the fact that the agent varies its aversion to risk according to the degree of losses incurred affects its final behavior.

If we look at the wealth final composition of each agent (see Figure 3.A.3), we see that, as we commented before, the third type of agent invests more of its wealth in the market portfolio, while the other two are more conservative, which could be interpreted as considering that they assume greater potential losses with ESOs since the percentage of each in profits is similar in all cases.

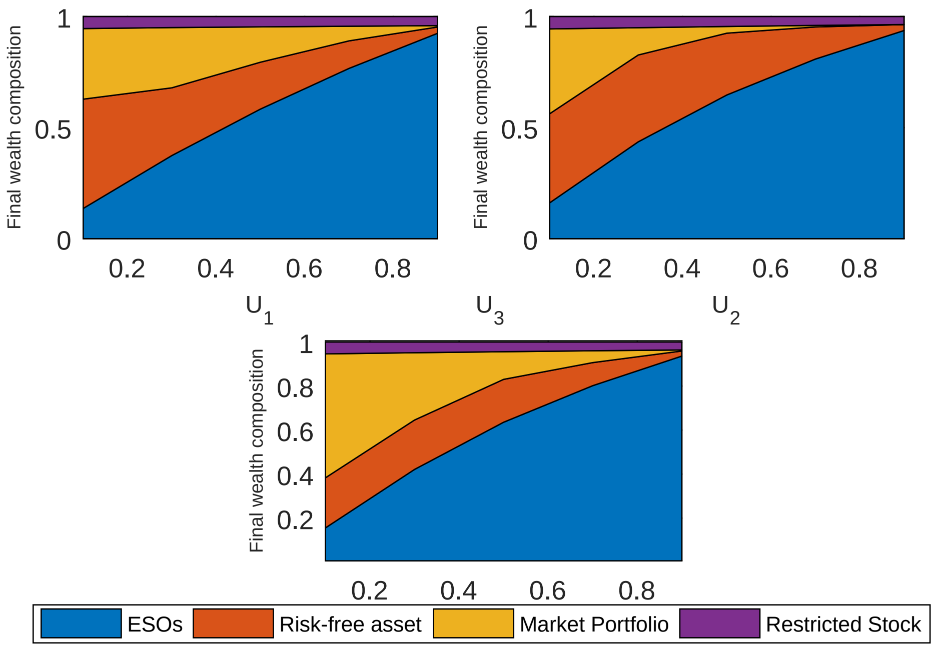
Finally, we calculate the sensitivities of firm's stock price and its volatility (see Figure 3.A.4). The executive's perceived incentives are lower in the case of the loss averse agent, however, if we consider the total amount of ESO (aggregate value) this pattern is reversed, being greater incentives for this agent, whatever the proportion invested in ESO.

FIGURE 3.A.2: Results for various utilities functions



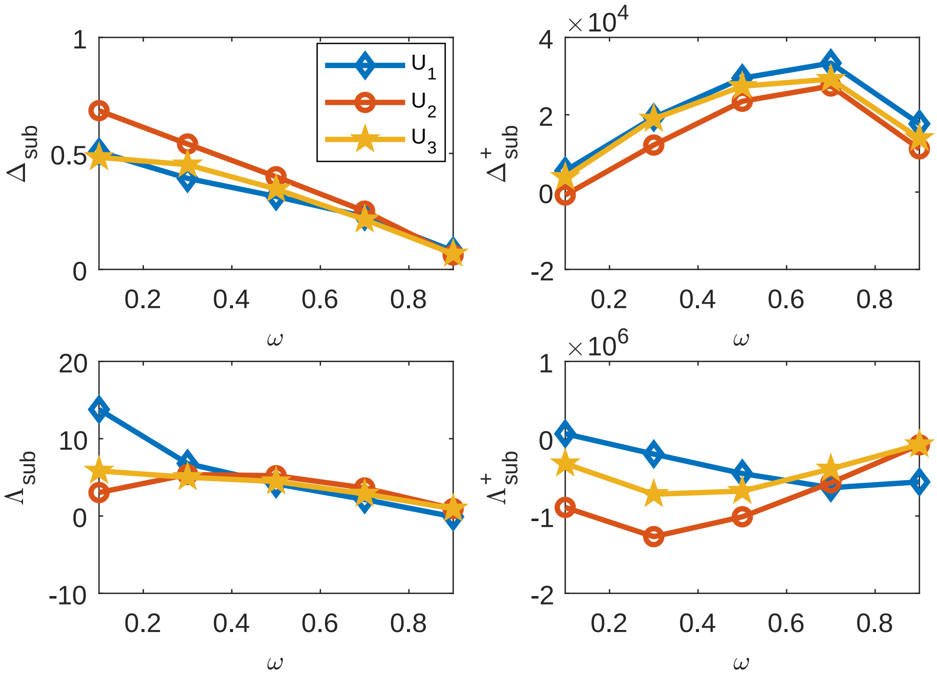
This figure plots η^* as a function of the ESOs' share in the executive's initial wealth,the percentage of simulated paths for which W_T is above W_0 and the average composition of W_T for different values of the parameter ω and utilities functios. The graphs has been obtained for the following values of the parameters:$\gamma = 2$, $\alpha = 0.05$, $A = 3$, $\delta_1 = \delta_2 = 0.7$, $\Theta = W_0 * \exp(rT)$, $\bar{W} = W_0$, $W_0 = 5,000,000$, $S_0 = K = 30$, $r = 0.03$, $\mu_S = \mu_M = 0.07$, $\sigma_S = 0.3$, $\sigma_M = 0.15$, $\rho = 0.5$ and $T = 5$.

FIGURE 3.A.3: Average composition of final wealth.



In this figure we represent the average composition of W_T for each one of the four utilities. In each one of the four panels, the area plotted in lower part represents the average share of ESOs in the final wealth. The area plotted in the upper part represents the average share of the market index and the remaining one, the share of the riskless asset in total wealth. All graphs have been obtained for the following values of the parameters:$\gamma = 2$, $\alpha = 0.05$, $A = 3$, $\delta_1 = \delta_2 = 0.7$, $\Theta = W_0 * \exp(rT)$, $\bar{W} = W_0$, $W_0 = 5,000,000$, $S_0 = K = 30$, $r = 0.03$, $\mu_S = \mu_M = 0.07$, $\sigma_S = 0.3$, $\sigma_M = 0.15$, $\rho = 0.5$ and $T = 5$.

FIGURE 3.A.4: Results for various utilities functions: Average subjective Delta and Vega



This figure plots the executive's incentives to raise the firm's stock price and the executive's incentives to raise firm's volatility by taking on risky investment projects as a function of both ω and utilities functios. The graphs has been obtained for the following values of the parameters:$\gamma = 2$, $\alpha = 0.05$, $A = 3$, $\delta_1 = \delta_2 = 0.7$, $\Theta = W_0 * \exp(rT)$, $\bar{W} = W_0$, $W_0 = 5,000,000$, $S_0 = K = 30$, $r = 0.03$, $\mu_S = \mu_M = 0.07$, $\sigma_S = 0.3$, $\sigma_M = 0.15$, $\rho = 0.5$ and $T = 5$.

4

Executive effort, compensation & Loss-Aversion

4.1 Introduction

It is well known that the subjective valuation of ESOs is performed by an agent that does not have its wealth completely well diversified, since the part invested in ESO and restricted stocks are not negotiable. In particular, the manager is endowed with an initial wealth that includes the investment in his own company through ESO and restricted stocks. The remaining part of his wealth is freely invested between the market portfolio and the riskless asset. The subjective value of the ESO is calculated using certainty equivalent approach, as Lambert et al. (1991), Hall and Murphy (2002), León and Vaello-Sebastià (2009) and others already did. To simplify our model and the results obtained, we will only focus on the case of European ESOs.

The aim of the executive is to maximize the expected utility of its final wealth considering that, in addition to being a loss averse agent, the effort it makes to increase its wealth causes on one hand, some negative utility and on the other, an increase in value's company.

Since the 60's have been many authors who have published various articles related to expected utility theory. Many of them, such as Kahneman and Tversky (1979) have proposed different ways allowing to model the behavior of the agent when there are losses in its portfolio. Our model is characterized by a concave kinked power function, where the curvature of losses is greater than in gains.

Following Desmettre S. (2010) the proposed model corresponds to an executive who has the possibility of influencing the price of the company's shares thanks to the effort made with his work, that is, we consider that if the agent works well (makes a greater effort), the company will improve its results and with it the price of the company's shares will rise. But, on the other hand, greater effort in turn leads to disutility: the greater effort at work, the larger the disutility.

4.2 The model

4.2.1 Executive's environment

Let W_0 denote the executive's total initial wealth. Following Tian (2004), we assume that the fraction ω of W_0 is made up of N European call options on the firm's stock having all the same maturity, T, for

$$N = \frac{\omega \cdot W_0}{V_0} \ , \tag{4.1}$$

and V_0 denoting the cost of the option to the firm at the grant date, $t = 0$. Another fraction, α, of W_0 is invested in restricted stock of the firm, which can not be sold until the ESO maturity, T. The degree of diversification of the executive depends on α and ω. The remaining fraction, $(1 - \alpha - \omega)W_0$, is made up of his cash compensation from the firm and his non-firm related wealth accumulated from previous time periods. This unrestricted wealth is distributed between the risk-free asset and a market index so that the value of the executive's end of period wealth is given by

$$W_T = W_0(1-\alpha-\omega)\left(\eta\frac{M_T}{M_0} + \left(1-\eta\right)e^{rT}\right) + \frac{\omega \cdot W_0}{V_0}\left[S_T - K\right]^+ + \alpha \cdot W_0\frac{S_T}{S_0} \tag{4.2}$$

where $\left[S_T - K\right]^+ \equiv \max\left[S_T - K, 0\right]$, r is the risk-free rate, M_T and S_T are, respectively, the value of the market index and the price of one stock of the firm at maturity, K is the exercise price of the options and η is the weight of the non restricted wealth invested in the market index.

We assume that the stock price and the market index follow a joint geometric

Brownian motion. Thus, given the starting values S_0 and M_0, the distributions of S_τ and M_τ, for any instant τ, are jointly log-Normal obeying

$$S_\tau = S_0 \exp\left\{\left(\mu_S - \frac{1}{2}\sigma_S^2\right)\tau + \sigma_S\sqrt{\tau}Z_S\right\} \quad (4.3)$$

$$M_\tau = M_0 \exp\left\{\left(\mu_M - \frac{1}{2}\sigma_M^2\right)\tau + \sigma_M\sqrt{\tau}Z_M\right\} \quad (4.4)$$

where μ_S and μ_M are the expected returns of the firm's stock and the market index, σ_S and σ_M are the corresponding volatilities, and Z_S and Z_M are standard normal innovations with correlation ρ. Since the firm is not restricted and it can hedge its positions, the fair value of the ESOs, V_0 in equation 4.1, can be calculated using the Black-Scholes-Merton pricing formula.

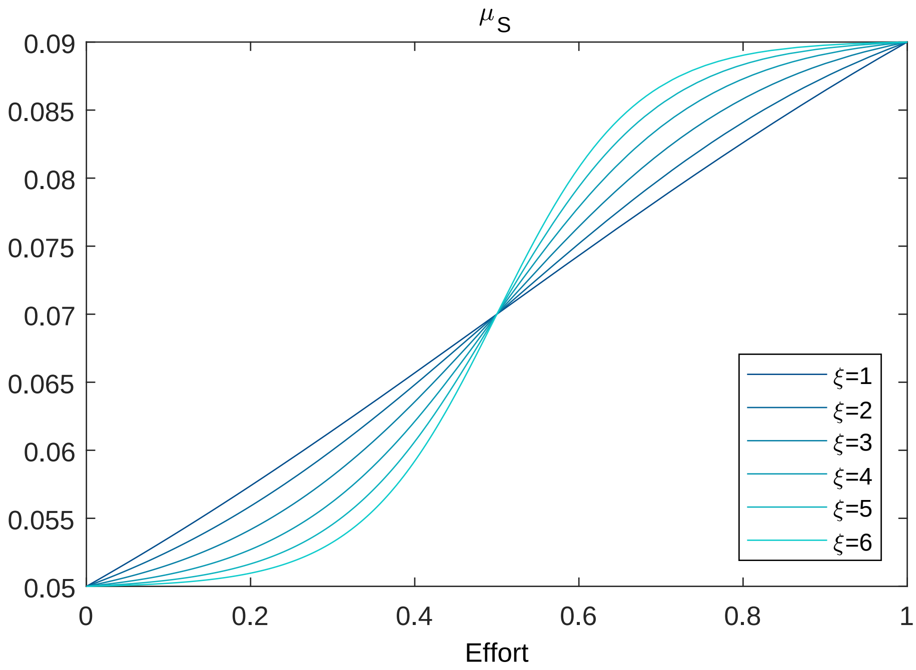
The executive can exert different amounts of effort, y, and that effort improves the performance of the firm through the drift of the shares of the firm, which reverberates in the final value that the executive gives to the stocks and options he gets from the company. Specifically, we propose the following function for the stocks drift,

$$\mu_S = \mu_L + (\mu_U - \mu_L)\frac{e^\xi}{(1-e^\xi)}\frac{(e^{-2y\xi}-1)}{(1+e^{\xi-2y\xi})} \quad (4.5)$$

where μ_L is a lower bound and μ_U is the upper bound of the drift of S_T. The previous equation (4.5) only represents a generalized logistic function (sigmoide) between the values μ_L (minimum) and μ_U (maximum) of the company, depending on the parameter ξ, which drives the shape (curvature) of the drift function.

Figure 4.1 displays different shapes of the drift function for different values of the shape parameter ξ, and assuming that $\mu_L = 0.05$, $\mu_U = 0.09$ and the effort is normalized to lie in the range $[0, 1]$. For $\xi = 1$ we see that the effort has an linearly increasing impact in the company's price. The greater ξ more curvature has the function, i.e., the initial effort of the actor has less effect to, subsequently, fast forward until it reach a point of exhaustion, in that the impulse of the effort decreases.

FIGURE 4.1: Drift of S logistic



Value of the shares's drift over the effort for different values of logistic curvature with bounds $\mu_L = 0.05$, $\mu_U = 0.09$.

4.2.2 Executive's preferences

In this paper we adapt the preferences proposed by Berkelaar et al. (2004), described by a concave kinked power function. Specifically, we incorporate the diss-utility of effort as in Desmettre S. (2010). The equation of the kinked power utility function of Berkelaar et al. (2004) is as follows,

$$U\Big(W_T\Big) = \begin{cases} A\dfrac{1}{1-\gamma}W_T^{1-\gamma} + \Big(B - A\Big)\dfrac{1}{1-\gamma}\theta^{1-\gamma}\,, & \text{for } W_T \leq \theta \\ B\dfrac{1}{1-\gamma}W_T^{1-\gamma}\,, & \text{for } W_T > \theta \end{cases} \tag{4.6}$$

This utility function considers a threshold θ, which separates the region of gains and the region of losses. The first part provides the utility of losses, wealth's values below the reference level, whereas the second part provides the utility of gains. Then, for loss aversion to hold, $A > B$ so that the curvature of the utility function in the region of losses is greater than in the region of gains. Finally, we assume that $B = 1$ so that the equation (4.6) nests the case of CRRA when $A = 1$.

The instantaneous cost of effort (or disutility) is assumed to satisfy:

$$c(t, u, y) = \kappa \cdot u^{1-\gamma}\frac{y^{\zeta}}{\zeta} \tag{4.7}$$

The parameter u serves to scale the cost of effort y to the level of wealth utility, i.e., in our case the executive's wealth, W_T. The coefficient $\zeta > 2$ serves to scale the diss-utility stress and $\kappa > 0$ is the inverse work productivity. ζ governs the curvature of the disutility function of executive's effort. Note that the greater ζ the lower the disutility of effort. The greater ζ, the lower the effort required of the executive to increase their wealth is less useful. That is, we seek to maximize profit potential with minimal effort.[1]

Then, executive's preferences incorporating the diss-utility of effort will be:

$$J\Big(W_T\left(\eta,\, y, \ldots\right),\, y\Big) = U\Big(W_T\Big) - \int_0^T c(t, u, y)du \tag{4.8}$$

[1]To notice that ξ is different that ζ, at first the ideal thing would be that ζ was minor that ξ.

In the left side of the previous equation, notice that the executive's effort has an impact in the terminal wealth due to the effect in the drift (μ_S, see equation 4.5), but also in the utility function J because of the effort disutility. In our case, we use the trapezoidal rule for approximating the definite integral

$$\int_0^T c(t,u,y)du = \kappa \frac{y^\zeta}{\zeta} \frac{T}{2} \left(W_T^{1-\gamma} + W_0^{1-\gamma} \right) \tag{4.9}$$

Therefore, the executive problem consist on selecting the amount of effort she wants to exert, y, and her asset allocation through the parameter η (market portfolio weight), which maximize the expected utility conditioned to the grant date, i.e,

$$\max_{\eta,\, y} \mathbb{E}_0 J \Big(W_T \left(\eta,\, y, \ldots \right),\, y \Big)\ , \tag{4.10}$$

subject to the wealth constraint given by equation (4.2).

4.2.3 Executive's Subjective Valuation. The Certainty Equivalent Approach

In this section we explain how to calculate the subjective value of ESO. There are three ways of valuing stock options: subjective valuation, objective valuation and neutral risk valuation (Ingersoll, 2006, see). The subjective valuation is the one carried out by a non-diversified executive who depends on its characteristics as an employee: loss aversion, misdiversification, effort made.... To calculate it we use the certainty equivalence principle, that is, we calculate the amount of money, CE, that the agent is willing to receive in compensation for the ESO at the end date. In this case, the value of the executive's wealth at maturity is given by the following expression:

$$W_T^{CE} = W_0 \left(\eta \frac{M_T}{M_0} + \Big(1-\eta\Big) e^{rT}\right) \left(1 - \omega - \alpha + CE \frac{w}{V_0}\right) + W_0 \alpha \frac{S_T}{S_0}. \tag{4.11}$$

Therefore, the CE value is calculated by solving the optimization problem:

$$\max_{\eta_1, y} \mathbb{E}_0 J\Big(W_T\left(\eta,\, y\right),\, y\Big) = \max_{\eta_2} \mathbb{E}_0 J\Big(W_T^{CE}\left(\eta_2,\, y\right),\, y\Big) \tag{4.12}$$

Note that the effort made by the agent, y, is the same in both cases. We do so to avoid distortions in the final solution, that is, we want to know what the agent's valuation is assuming the same effort.

The subjective valuation of the total variable compensation package, CE_T, is obtained by aggregation of the subjective valuation of the ESOs and the subjective valuation of the restricted stocks. The subjective valuation of the restricted stocks, S_r, is obtained through the certainty equivalent: the amount of cash at the grant date, that in substitution of the restricted stocks provides the same expected utility. In this case, the final wealth of the executive is

$$W_T^{S_r} = W_0 \left(\eta \frac{M_T}{M_0} + \Big(1-\eta\Big) e^{rT}\right) \left(1 - \omega - \alpha + Sr \frac{\alpha}{S_0}\right) + \frac{\omega W_0}{V_0} \Big[S_T - K\Big]^+. \tag{4.13}$$

The amount S_r is obtained by solving the following optimization problem:

$$\max_{\eta_3, y} \mathbb{E}_0 J\Big(W_T\left(\eta,\, y\right),\, y\Big) = \max_{\eta_4} \mathbb{E}_0 J\Big(W_T^{S_r}\left(\eta_4,\, y\right),\, y\Big) \tag{4.14}$$

Then, the subjective value of the total variable compensation package is

$$CE_T = W_0 \left(\omega \frac{CE}{bs} + \alpha \frac{S_r}{So}\right) \tag{4.15}$$

4.3 Numerical Results and Discussion

In this section we present the results obtained by analyzing the behavior of the agent, its effort, asset allocation of unrestricted wealth and the subjective value with regard to both parameters that govern the effort, ξ (which models the impact of the effort on the value of the company) and (ζ, κ) (which models the impact of the agent's effort on the negative utility). We have analyzed two cases, in the first one the agent has practically no restricted stock (only 5%)[2]. In the second case, the restricted stock acounts for a third of the executive's wealth (the same proportion that she has in ESOs).[3]

Our benchmark case assumes the following values for the usual parameters $S_0 = K = 30$, $r = 0.03$, $\mu_M = 0.07$, $\mu_U = 0.09$, $\mu_L = 0.05$,$\sigma_M = 0.15$, $\sigma_S = 0.3$, $T = 5$, $\rho = 0.5$, $\gamma = 2$, $\omega = 1/3$, $A = 3$, $B = 1$ and $W_0 = 500,000$. We used for our simulation a total of 500000 simulations for the stock price and the market index (including antithetic). In the case of the utility function our kinked point is $\theta = W_0 e^{rT}$, i.e. below the value of the final wealth that the agent invested all it's money in the risk-free asset, it is considered to have losses.[4]

[2]We avoid the case without restricted stocks, $\alpha = 0$, to avoid problems in the final optimization.

[3]Since the conclusions that can be drawn are similar in both cases, we omit the latter case in our analysis. The results are available on request.

[4]Benchmark case without effort for various values of μ_S

μ_S	0.05	0.06	0.07	0.08	0.09
eso_{disc}	0.2607	0.2904	0.3227	0.3574	0.3948
η^*	0.4603	0.4325	0.4029	0.3725	0.3408

TABLE 4.1: Executive's ESO discount without effort

Firstly, we will comment on the effects that effort parameters have on the study of the problem. To do this, we will analyze the different pairs of values of the three possible parameters and the effect that their variation has on the estimation of the subjective evaluation and on the optimal effort of the agent.

In Tables 4.2 to 4.4 we display the results obtained with respect to the subjective valuation over the usual one (Black-Scholes), while in Table 4.5 the results for the optimal efforts are presented. In both cases, below each result, between parentheses, there is the trend of the share price of the company obtained (the company value assigned by the agent).

Table 4.2 represents the results of the subjective valuation when we vary the disutility stress parameter, ζ, and the impact in the firm value, ξ. When the stress parameter increases (lower disutility), keeping the impact on the company's value fixed, the subjective value that the agent gives to the stock options also increases: her effort yields a higher firm value, but the utility cost of that effort is lower. However, the impact that has the increase in the company's value depends on the magnitude of the disutility. For small values of this parameter ($\zeta = 3, 4$), if we increase the company value the subjective value of the ESO decreases slightly, while for larger values the opposite occurs.

In Table 4.3 we keep constant the stress parameter, ζ, and vary the work inverse parameter κ[5], which also affects disutility but in opposite effect. We can observe that if the value of the company remains fixed, the effect of κ remains stable, and only varies when ξ is small. In other words, the effect of the company value prevails over the work inverse. We can also appreciate that, in this case, the effort

[5]Theoretically, higher work inverse, greater disutility.

TABLE 4.2: Executive's ESO discount: ξ and ζ effects

$\zeta \backslash \xi$	3	4	5	6
3	0.6515	0.6428	0.6394	0.6384
	(0.052)	(0.051)	(0.050)	(0.050)
4	0.6981	0.6907	0.6864	0.6799
	(0.060)	(0.059)	(0.058)	(0.058)
5	0.7486	0.7594	0.7783	0.7967
	(0.068)	(0.069)	(0.072)	(0.074)
6	0.7838	0.8002	0.8180	0.8348
	(0.073)	(0.075)	(0.077)	(0.079)

This table reports the executive's ESO discount calculated as the ratio of the subjective valuation (CE) over the Black-Scholes price. Below each price, in parenthesis, we show the corresponding company's value μ_S. The relevant parameters are:$\gamma = 2$, $\omega = 1/3$, $W_0 = 5,000,000$, $S_0 = K = 30$, $r = 0.03$, $\mu_M = 0.07$, $\sigma_S = 0.3$, $\sigma_M = 0.15$, $\mu_U = 0.09$, $\mu_L = 0.05$, $A = 3$, $\rho = 0.5$, $\kappa = 3$ and $T = 5$.

chosen is quite similar across the different alternatives, since the drifts achieved in each case are rather the same.

Finally, comment on the effect that κ has on the agent's assessment of ESOs. In Table 4.4 we observe that when this parameter increases (disutility of effort also increases), there is a small reduction in subjective value, just the opposite of what happens when disutility stress parameter increases.

Table 4.3: Executive's ESO discount: ξ and κ effects

$\xi \backslash \kappa$	3	4	5	6
3	0.6528	0.6496	0.6478	0.6467
	(0.052)	(0.052)	(0.051)	(0.051)
4	0.6442	0.6430	0.6424	0.6420
	(0.051)	(0.051)	(0.050)	(0.050)
5	0.6408	0.6405	0.6403	0.6402
	(0.050)	(0.050)	(0.050)	(0.050)
6	0.6398	0.6397	0.6397	0.6397
	(0.050)	(0.050)	(0.050)	(0.050)

This table reports the executive's ESO discount calculated as the ratio of the subjective valuation (CE) over the Black-Scholes price. Below each price, in parenthesis, we show the corresponding company's value μ_S. The relevant parameters are:$\gamma = 2$, $\omega = 1/3$, $W_0 = 5,000,000$, $S_0 = K = 30$, $r = 0.03$, $\mu_M = 0.07$, $\sigma_S = 0.3$, $\sigma_M = 0.15$, $\mu_U = 0.09$, $\mu_L = 0.05$, $A = 3$, $\rho = 0.5$, $\zeta = 3$ and $T = 5$.

Table 4.4: Executive's ESO discount: ζ and κ effects

$\zeta \backslash \kappa$	3	4	5	6
3	0.6442	0.6430	0.6424	0.6420
	(0.051)	(0.051)	(0.051)	(0.050)
4	0.6932	0.6736	0.6639	0.6586
	(0.059)	(0.056)	(0.054)	(0.053)
5	0.7617	0.7397	0.7251	0.7131
	(0.069)	(0.066)	(0.064)	(0.062)
6	0.8022	0.7865	0.7739	0.7632
	(0.075)	(0.073)	(0.071)	(0.069)

This table reports the executive's ESO discount calculated as the ratio of the subjective valuation (CE) over the Black-Scholes price. Below each price, in parenthesis, we show the corresponding company's value μ_S. The relevant parameters are:$\gamma = 2$, $\omega = 1/3$, $W_0 = 5,000,000$, $S_0 = K = 30$, $r = 0.03$, $\mu_M = 0.07$, $\sigma_S = 0.3$, $\sigma_M = 0.15$, $\mu_U = 0.09$, $\mu_L = 0.05$, $A = 3$, $\rho = 0.5$, $\xi = 4$ and $T = 5$.

The effects on the agent's effort that have the variation of these three parameters are presented in Table 4.5. In general, the conclusions are the same as those we can draw with respect to subjective value. An increase in disutility, either by an increase in inverse work productivity parameter or by a decrease in disutility stress, causes the agent's effort to fall. The effect of increasing the degree of sigmoid curvature, the so-called company value makes the effort also decrease when disutility increases.

To conclude, two general comments on the results appearing in these tables. On the one hand, the final average value assigned to the company's shares (μ_S) increases when there is less disutility and, in any scenario, a subjective valuation obtained is higher than that obtained effortlessly, maintaining in a range between 0.05 and 0.07, the first tranches of the logistics function. On the other hand, if we consider any of the possible scenarios with respect to the three parameters, the ESO's subjective value is higher than that obtained when we consider that the agent does not make any effort (see Table 4.1). Henceforth, we focus on studying the impact of the decrease in disutility (through the disutility stress parameter) and the variation of the effect on the company's value in the results, leaving the inverse work productivity parameter fixed as $\kappa = 3$.

TABLE 4.5: Executive's effort

(A) $\kappa = 3$

$\zeta \backslash \xi$	3	4	5	6
3	0.1364	0.0975	0.0621	0.0383
	(0.052)	(0.051)	(0.050)	(0.050)
4	0.3402	0.3531	0.3704	0.3777
	(0.060)	(0.059)	(0.058)	(0.057)
5	0.4650	0.4902	0.5167	0.5334
	(0.068)	(0.069)	(0.072)	(0.074)
6	0.5394	0.5580	0.5714	0.5799
	(0.073)	(0.075)	(0.077)	(0.079)

(B) $\zeta = 3$

$\xi \backslash \kappa$	3	4	5	6
3	0.1363	0.1111	0.0957	0.0851
	(0.052)	(0.052)	(0.052)	(0.051)
4	0.0974	0.0787	0.0676	0.0600
	(0.051)	(0.051)	(0.050)	(0.050)
5	0.0620	0.0509	0.0440	0.0393
	(0.050)	(0.050)	(0.050)	(0.050)
6	0.0382	0.0319	0.0278	0.0250
	(0.050)	(0.0500)	(0.0500)	(0.050)

(C) $\xi = 4$

$\zeta \backslash \kappa$	3	4	5	6
3	0.0974	0.0787	0.0676	0.0600
	(0.051)	(0.051)	(0.051)	(0.050)
4	0.3564	0.2949	0.2537	0.2251
	(0.059)	(0.056)	(0.054)	(0.053)
5	0.4917	0.4538	0.4266	0.4023
	(0.069)	(0.066)	(0.064)	(0.062)
6	0.5587	0.5328	0.5121	0.4945
	(0.075)	(0.073)	(0.071)	(0.069)

This table reports the executive's optimal effort. Below each price, in parenthesis, we show the corresponding company's value μ_S. The relevant parameters are:$\gamma = 2$, $\omega = 1/3$, $W_0 = 5,000,000$, $S_0 = K = 30$, $r = 0.03$, $\mu_M = 0.07$, $\sigma_S = 0.3$, $\sigma_M = 0.15$, $\mu_U = 0.09$, $\mu_L = 0.05$, $A = 3$, $\rho = 0.5$, $\kappa = 3$ and $T = 5$.

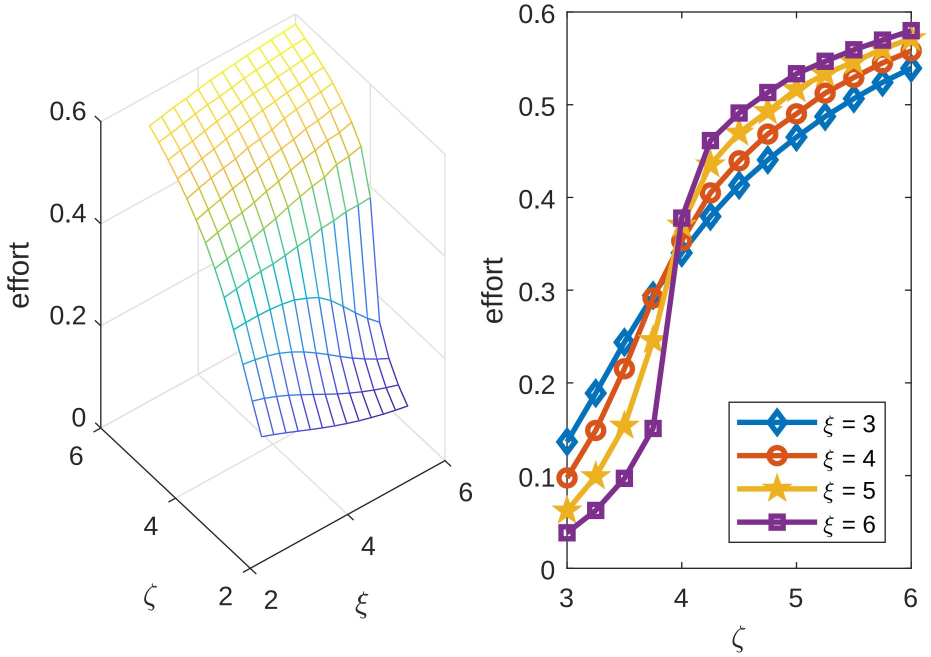
In Figure 4.2 we can observe the behaviour of the effort with respect to the parameters of company value and stress disutility. As we have commented previously, effort increases when stress is reduced (higher ζ) or the value of the company is increased (higher ξ), except when disutility is greater, in which case it decreases. This result may be due to the fact that for these values the resulting disutility is so high that the agent's effort does not compensate and compensate more if his investment in the market portfolio increases (see Figure 4.4). On the other hand, when the ζ parameter increases, the disutility is lower so the agent will be more willing to make an effort to obtain greater benefits, that is, higher expected utility. In the right panel of the same figure (Figure 4.2) we can see how the value of the logistic curve affects the effort (the greater ξ parameter, the greater logistic curvature).

In the same way, if we analyze the CE (over BS), the money that the agent is willing to receive in exchange for the ESO, that is, the subjective value that the agent assigns to this product, we can observe that this amount increases in the same way as the optimum effort (when increasing both ζ and ξ) (see Figure 4.3, central panel). This result, together with the previous one of the optimal effort, indicates that the agent is willing to make more effort, that is, he values more the ESO assigned, when this effort increases the value of the company more and causes less negative utility. In the same figure we represent the subjective values that the agent assigns to the sum of both (ESO and restricted stock), in percentage and the value assigned to the restricted stock with respect to the market price, from which we can obtain similar conclusions.[6]

Another parameter that is interesting to analyze is the optimal executive's

[6] All the subjective values presented are calculated with the same effort but the optimal proportion invested in the market portfolio is recalculated.

FIGURE 4.2: Optimal executive's portfolio allocation: effort

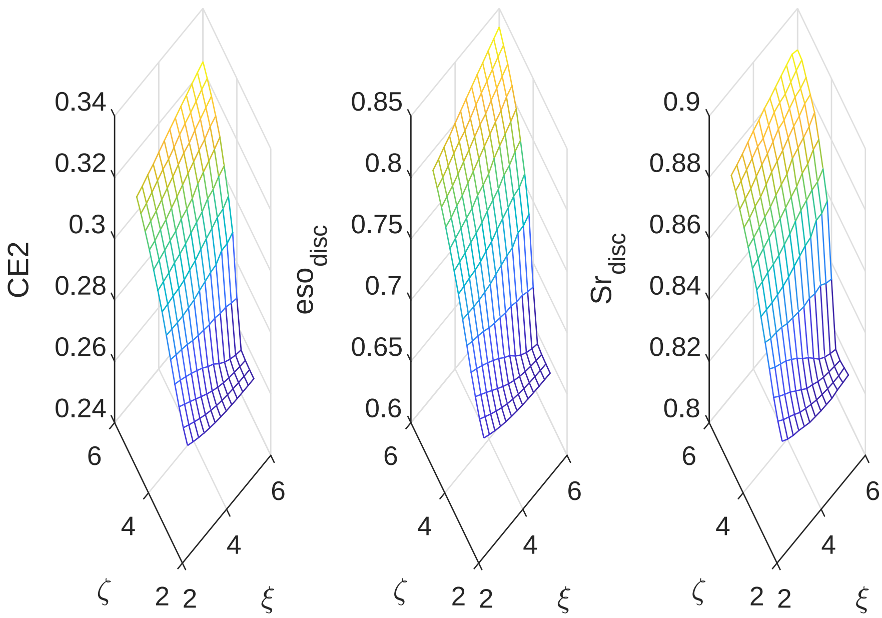


This figure plots effort as a function of the ESOs' share in the executive's initial wealth, for different values of the parameters ξ and ζ. The graph has been obtained for the following values of the parameters:$\gamma = 2$, $\omega = 1/3$, $W_0 = 5,000,000$, $S_0 = K = 30$, $r = 0.03$, $\mu_M = 0.07$, $\sigma_S = 0.15$, $\sigma_M = 0.2$, $\mu_U = 0.09$, $\mu_L = 0.05$, $A = 3$, $\rho = 0.5$, $\kappa = 3$ and $T = 5$.

portfolio allocation, the amount of money that the restricted agent freely allocates to the market portfolio (see Figure 4.4). If we calculate this value optimally together with the agent's effort we can observe that it is not very stable although the changes are minimal. However, when we calculate the subjective value of the ESO we can conclude that both parameters are inversely influential. The lower they are, the greater the proportion of free wealth that the agent invests in the market portfolio, assuming greater risk, although the variation may be insignificant in the context of total wealth.

If we analyze the composition of the agent's final wealth (see Figure 4.5), we

FIGURE 4.3: Certainty Equivalent Approach.

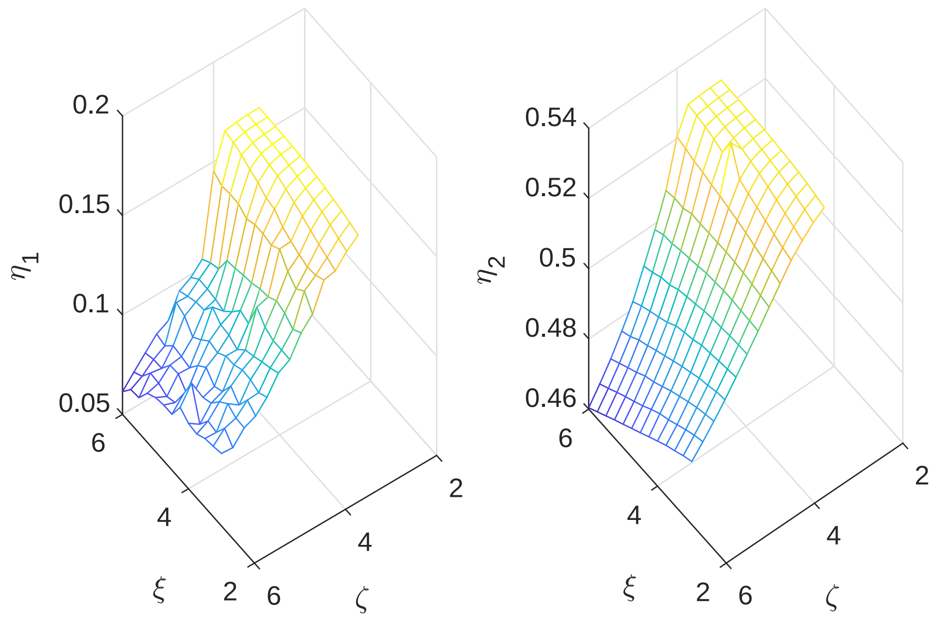


In this figure we represent the Certainty Equivalence CE for each one of the four values considered for the parameter of company's value ξ or κ,. In each one of the four panels we represente differents ways of calculate the CE. All graphs have been obtained for the following values of the parameters:$\gamma = 2$, $\omega = 1/3$, $W_0 = 5,000,000$, $S_0 = K = 30$, $r = 0.03$, $\mu_M = 0.07$, $\sigma_S = 0.3$, $\sigma_M = 0.15$, $\mu_U = 0.09$, $\mu_L = 0.05$, $A = 3$, $\rho = 0.5$, $\kappa = 3$ and $T = 5$.

observe that for any value of the company, the proportion invested in ESO increases as the disutility caused by optimal effort decreases, while the percentage invested in the market portfolio hardly varies as we have mentioned when analyzing Figure 4.4.

Likewise, when we study the percentage of paths for which wealth is above the kinked point (see Figure 4.6), which are considered gains, we see that these increase when the effect of the effort decreases in the negative utility and when that same effect increases in the value of the company, in this last case, in a constant way. Finally, comment on the last of the figures (see Figure 4.7), related to the final

FIGURE 4.4: Optimal executive's portfolio allocation: η^*



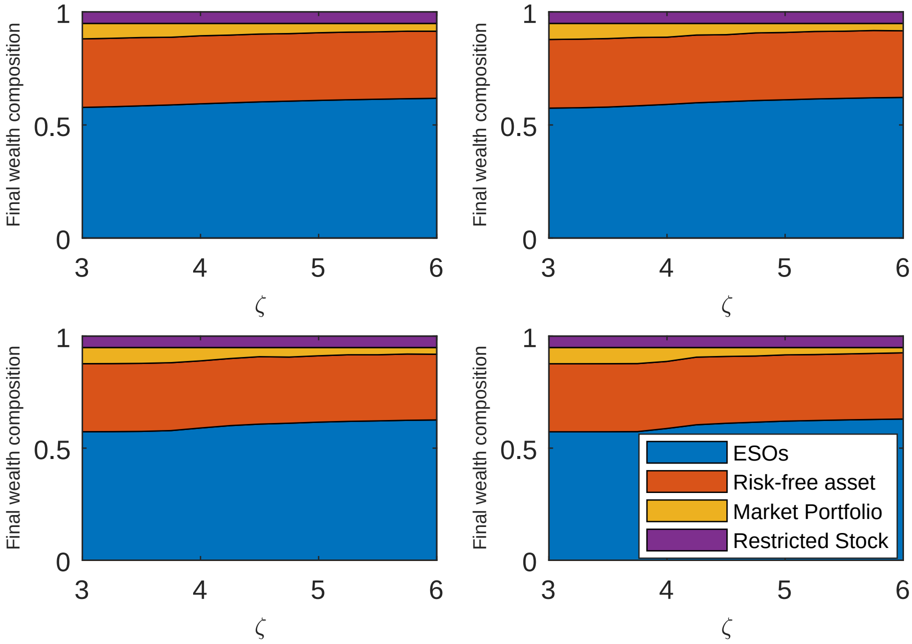
This figure plots η^* as a function of the ESOs' share in the executive's initial wealth, for different values of the parameters ξ and ζ. The graph has been obtained for the following values of the parameters:$\gamma = 2$, $\omega = 1/3$, $W_0 = 5,000,000$, $S_0 = K = 30$, $r = 0.03$, $\mu_M = 0.07$, $\sigma_S = 0.3$, $\sigma_M = 0.15$, $\mu_U = 0.09$, $\mu_L = 0.05$, $A = 3$, $\rho = 0.5$, $\kappa = 3$ and $T = 5$.

value that the agent gives to the company, thanks to his effort. It can be observed, as in the previous cases, that the lower the disutility, the higher the value assigned to the company's shares.

4.3.1 Executive's Incentives

Finally, we will focus on analyzing the results obtained in relation to the agent's incentives. In the existing literature these incentives are usually calculated through the sign and size of the subjective ESO greeks. With respect to this point, there are two different ways in which ESOs affect executive incentives. First, it considers the

FIGURE 4.5: Average composition of final wealth.

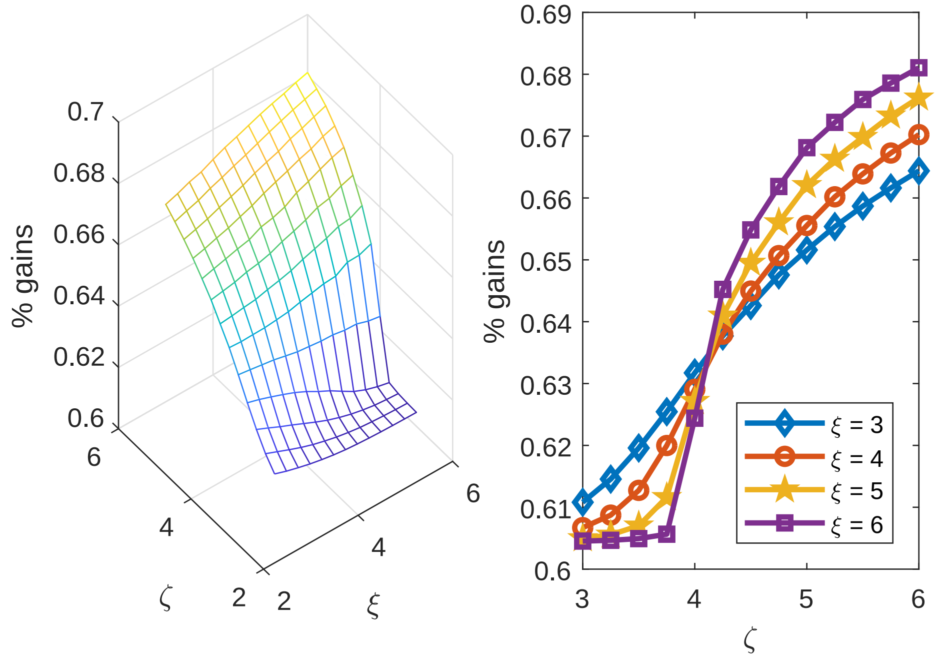


In this figure we represent the average composition of W_T for each one of the four values considered for the parameter of company's value ξ,. In each one of the four panels, the area plotted in lower part represents the average share of ESOs in the final wealth. The area plotted in the upper part represents the average share of the market index and the remaining one, the share of the riskless asset in total wealth. All graphs have been obtained for the following values of the parameters:$\gamma = 2$, $\omega = 1/3$, $W_0 = 5,000,000$, $S_0 = K = 30$, $r = 0.03$, $\mu_M = 0.07$, $\sigma_S = 0.3$, $\sigma_M = 0.15$, $\mu_U = 0.09$, $\mu_L = 0.05$, $A = 3$, $\rho = 0.5$, $\kappa = 3$ and $T = 5$.

reward that both the executive and the company (through its shareholders) obtain by increasing the price of the company's shares. Secondly, the risk associated with the volatility of the shares also influences the incentives of the agent, so we must consider both effects, price increase and volatility increase of shares, calculating the subjective ESO greeks.

In the same way as Tian (2004), we will calculate the average and the aggregate subjective *delta*, Δ_{sub} and Δ^+_{sub}. The first is calculated using finite differences to approximate the partial derivation of the subjective value of the ESO average with

FIGURE 4.6: Percentage of paths for which $W_T > \theta$



This figure plots the percentage of simulated paths for which W_T is above W_0, the reference level for the loss-averse type of executives. Notice that, for each value of ξ, W_T has been computed by taking the corresponding optimal composition of the executive portfolio, η^*. The parameter's values used for the computations are:$\gamma = 2$, $\omega = 1/3$, $W_0 = 5,000,000$, $S_0 = K = 30$, $r = 0.03$, $\mu_M = 0.07$, $\sigma_S = 0.3$, $\sigma_M = 0.15$, $\mu_U = 0.09$, $\mu_L = 0.05$, $A = 3$, $\rho = 0.5$, $\kappa = 3$ and $T = 5$.

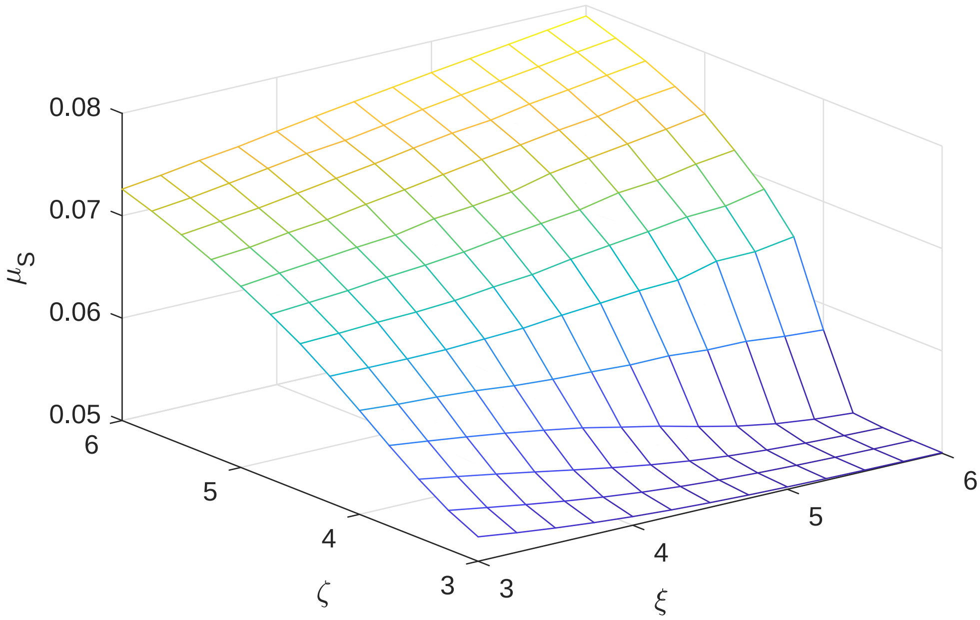
respect to the firm's stock price S_0. The second is calculated as the finite difference approximation of the partial derivation of the total ESO value with respect to the stock price. Specifically,

$$\Delta_{sub} = \frac{\partial CE}{\partial S_0} \approx \frac{CE(S_0 + \varepsilon_s) - CE(S_0)}{\varepsilon_s} \tag{4.16}$$

$$\Delta_{sub}^{+} = \frac{\partial (N \cdot CE)}{\partial S_0} \approx \frac{N(S_0 + \varepsilon_s) \cdot CE(S_0 + \varepsilon_s) - N(S_0) \cdot CE(S_0)}{\varepsilon_s} \tag{4.17}$$

where $\varepsilon_s = 1$ is the increase in the stock price. Since the number of ESOs granted depends on than their market value V_0, an increase in the stock price S_0 leads to an increase in the market value V_0, so the number of ESOs decreases (see equation

FIGURE 4.7: The value of company μ_S.



In this figure we represent the value of the company for the differents values of ξ, ζ and κ. All graphs have been obtained for the following values of the parameters:$\gamma = 2$, $\omega = 1/3$, $W_0 = 5,000,000$, $S_0 = K = 30$, $r = 0.03$, $\mu_M = 0.07$, $\sigma_S = 0.3$, $\sigma_M = 0.15$, $\mu_U = 0.09$, $\mu_L = 0.05$, $A = 3$ $\rho = 0.5$, $\kappa = 3$ and $T = 5$.

(4.1)).

As for the increase in volatility ,σ_S, we must remember that, in a context of loss aversion, the Capital Asset Pricing Model (CAPM) is not satisfied, so a distinction between systematic risk and idiosyncratic risk is not valid. Because of this, we will only consider the total volatility of the firm when analysing the agent's incentives with respect to the variation of this parameter.

In the same way as before, we have calculated the average and the aggregate subjective with respect to volatility, *vega*. To calculate both values, Λ_{sub} and Λ^+_{sub}, we use a finite difference approximation of the partial derivative of the subjective valuation and the total subjective valuation of the ESO, respectively, with respect to the total volatility of the firm's stock price.

$$\Lambda_{sub} = \frac{\partial CE}{\partial \sigma_S} \approx \frac{CE(\sigma_S + \varepsilon_{\sigma_S}) - CE(\sigma_S)}{\varepsilon_{\sigma_S}} \tag{4.18}$$

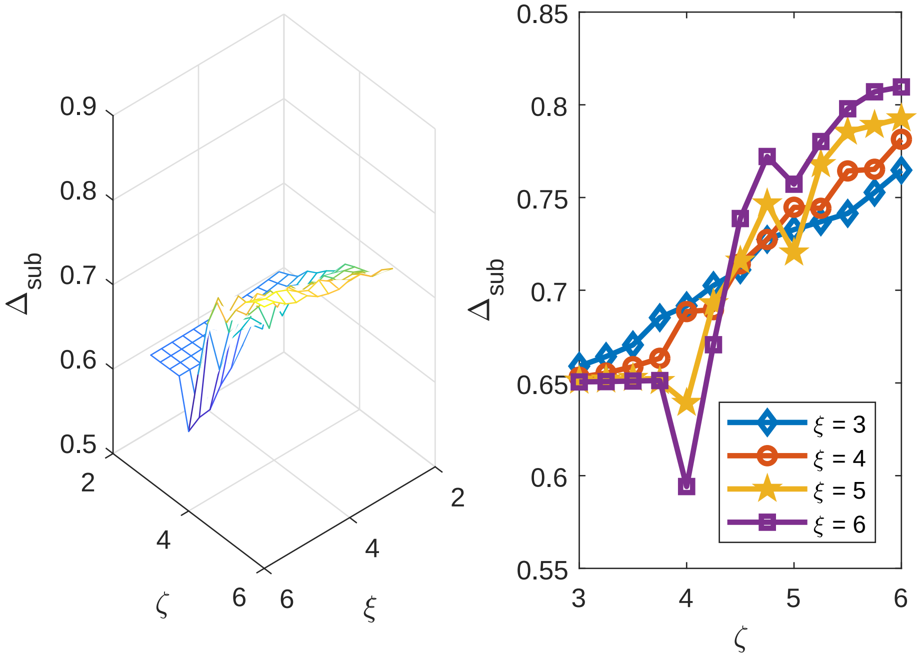
$$\Lambda^{+}_{sub} = \frac{\partial (N \cdot CE)}{\partial \sigma_S} \approx \frac{N(\sigma_S + \varepsilon_{\sigma_S}) \cdot CE(\sigma_S + \varepsilon_{\sigma_S}) - N(\sigma_S) \cdot CE(\sigma_S)}{\varepsilon_{\sigma_S}} \tag{4.19}$$

where $\varepsilon_{\sigma_S} = 0.05$ is the increase in stock volatility. As before, when calculating the aggregate subjetive *vega*, Λ^{+}_{sub}, an increase in volatility, σ_S, has an impact on the total number of ESOs through their market value V_0.

In general, the results obtained when studying the incentives of the agent with respect to the three parameters that influence the effort are not as stable as they should be, producing some kind of peak that is difficult to explain. In spite of this, it can be concluded that in the case of the subjective *delta*, Δ_{sub} (see Figure 4.8), the symoidal pattern is broadly repeated and for almost all cases, the greater the stress, the greater the incentives with respect to an increase in the price of stocks.

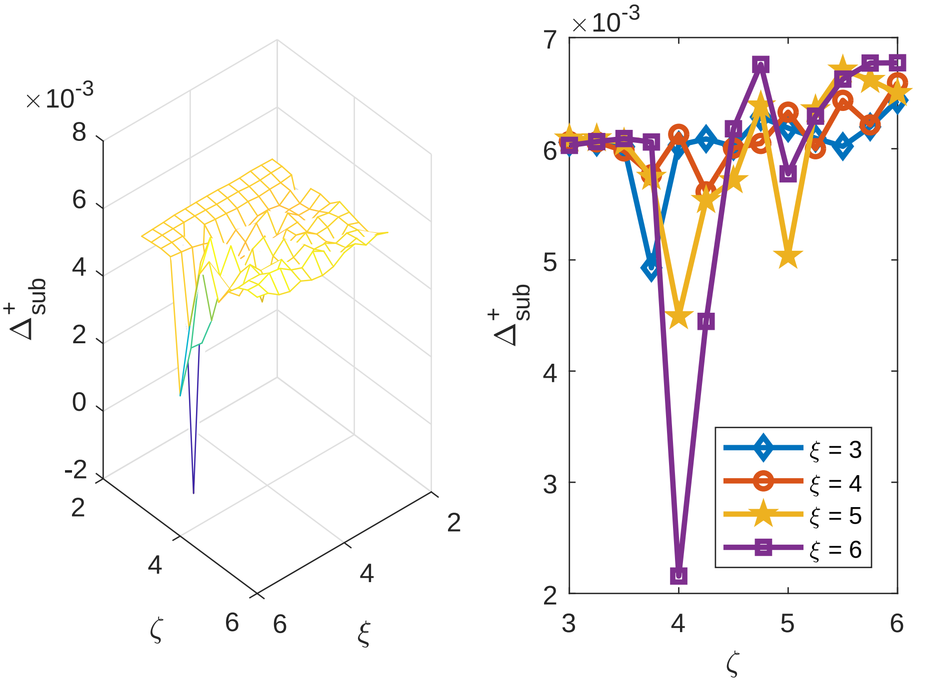
With respect to the incentives that the agent receives through the variation of the volatility of the stock's price, the average subjective with respect to volatility, *vega*, Λ_{sub}, except for a few isolated cases, it seems that the company value parameter is not so relevant since the incentives are practically the same for any value of the parameter ζ (see Figure 4.10).

FIGURE 4.8: Average subjective Delta : Δ_{sub}



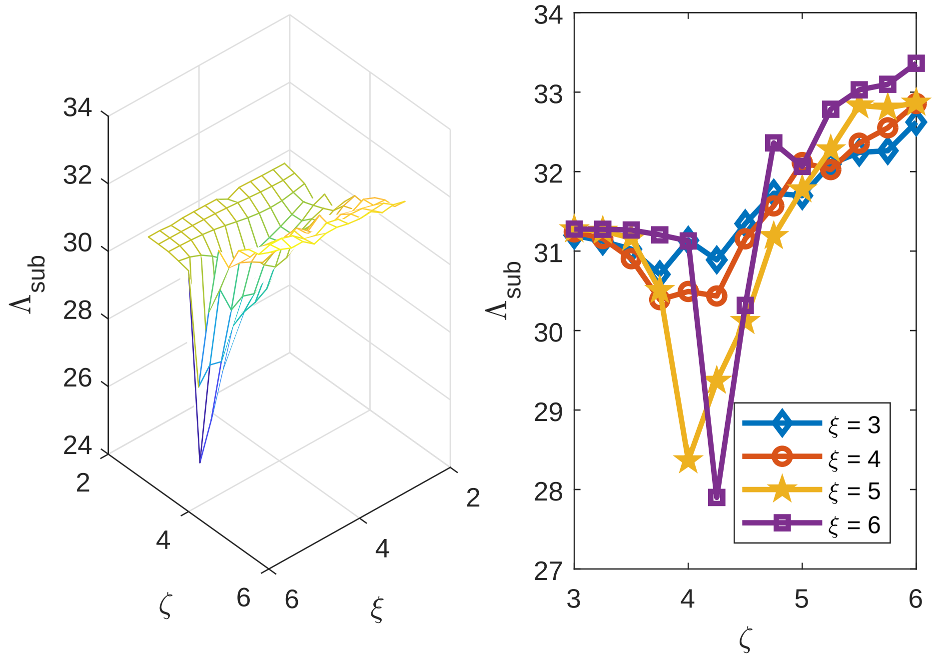
Both, left and right panel of the figure, plot the executive's incentives to raise the firm's stock price as a function of both A and ω. These incentives are evaluated using the average *delta*, Δ_{sub}, which is measured in the vertical axis of both panels. Equation (4.16) provides its numerical definition. All graphs have been obtained for the following values of the parameters:$\gamma = 2$, $\omega = 1/3$, $W_0 = 5,000,000$, $S_0 = K = 30$, $r = 0.03$, $\mu_M = 0.07$, $\sigma_S = 0.3$, $\sigma_M = 0.15$, $\mu_U = 0.09$, $\mu_L = 0.05$, $A = 3$, $\rho = 0.5$, $\kappa = 3$ and $T = 5$.

FIGURE 4.9: Aggregate Subjective Delta: Δ^+_{sub}



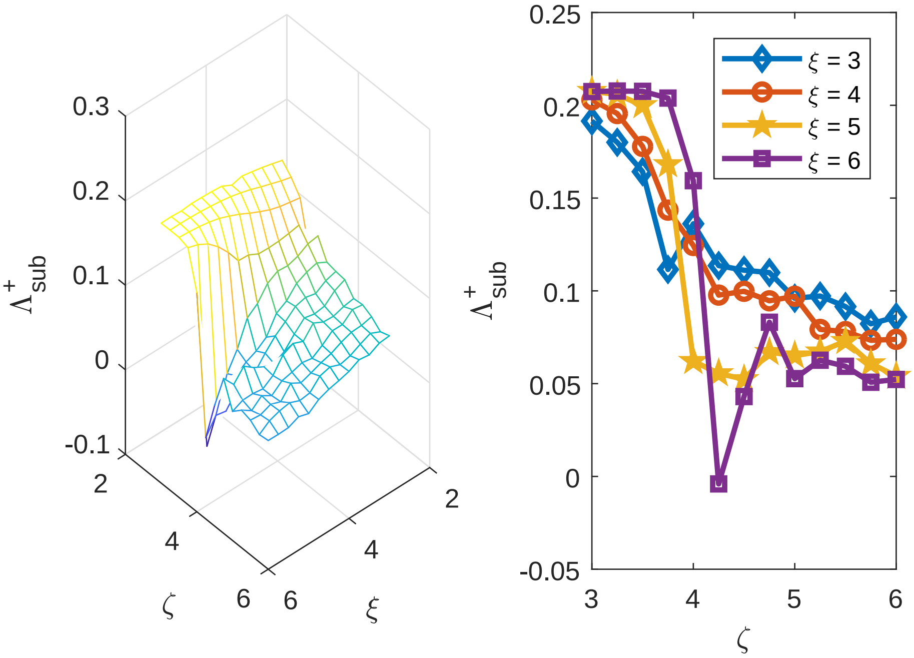
As in Figure 4.8, the executive's incentives to raise the firm's stock price as a function of both A and ω are represented using now the aggregate *delta*, Δ^+_{sub}, as our measure. Its numerical definition is provided by equation (4.17) in the main text. All graphs have been obtained for the following values of the parameters:$\gamma = 2$, $\omega = 1/3$, $W_0 = 5,000,000$, $S_0 = K = 30$, $r = 0.03$, $\mu_M = 0.07$, $\sigma_S = 0.3$, $\sigma_M = 0.15$, $\mu_U = 0.09$, $\mu_L = 0.05$, $A = 3$, $\rho = 0.5$, $\kappa = 3$ and $T = 5$.

FIGURE 4.10: Average Subjective Vega: Λ_{sub}



The figure plots the executive's incentives to raise firm's volatility by taking on risky investment projects. This incentives are measured using the average *vega*, Λ_{sub}, which is measured in the vertical axis of both panels. Λ_{sub} has been computed by finite-differences, taking $\epsilon_{\sigma_S} = 0.05$. See equation (4.19) in the main text for its precise definition.All graphs have been obtained for the following values of the parameters:$\gamma = 2$, $\omega = 1/3$, $W_0 = 5,000,000$, $S_0 = K = 30$, $r = 0.03$, $\mu_M = 0.07$, $\sigma_S = 0.3$, $\sigma_M = 0.15$, $\mu_U = 0.09$, $\mu_L = 0.05$, $A = 3$, $\rho = 0.5$, $\kappa = 3$ and $T = 5$.

FIGURE 4.11: Aggregate Subjective Vega: Λ^{+}_{sub}



The figure depicts the executive's incentives to raise the firm's volatility as a function of both A and ω. These incentives are evaluated now using the aggregate *vega*, Λ^{+}_{sub}, which has been computed by finite-differences, taking $\epsilon_{\sigma_S} = 0.05$. Its precise numerical definition appears in equation (4.19). The values of the parameters used in the calculations have been:$\gamma = 2$, $\alpha = 0.05$, $W_0 = 5,000,000$, $S_0 = K = 30$, $r = 0.03$, $\mu_S = \mu_M = 0.07$, $\sigma_S = 0.3$, $\sigma_M = 0.15$, $\rho = 0.5$ and $T = 5$.

4.4 Conclusions

We have studied the effects that the effort of the loss averse agent has on the ESO's subjective valuation. For this, we have analyzed different values of the most relevant parameters in the calculation of optimal effort and we can conclude that both parameters of disutility and company value's parameter influence the results, each in its own way. The greater the disutility, the lower the value of the ESO and inversely with the company value. In other words, the optimal effort made by the agent positively influences the value of the company but causes the expected utility of this agent to decrease, which penalizes the subjective valuation made. In the same way, the choice of these parameters influences the allocation of the agent's portfolio through the optimal effort.

For future research, it is necessary to carry out a more detailed study of the incentives perceived by the investor and compare the results obtained with those of a risk averse agent.

Bibliography

Maurice Allais. Le comportement de l'homme rationnel devant le risque: critique des postulats et axiomes de l'école américaine. *Econometrica*, 21(4):503–546, 1953.

Arjan B. Berkelaar, Roy Kouwenberg, and Thierry Post. Optimal portfolio choice under loss aversion. *The Review of Economics and Statistics*, 86:973–987, 2004.

F. Black and M. Scholes. The pricing of options and corporate liabilities. *Journal of Political Economy*, 81(3):637–659, 1973.

M. Broadie and Ö. Kaya. Exact simulation of stochastic volatility and other affine jump diffusion processes. *Operations Research*, 54:217–231, 2006.

P. Brown and A. Szimayer. Valuing executive stock options: performance hurdles, early exercise and stochastic volatility. *Accounting and Finance*, 48: 363–389, 2008.

Jie Cai and Anand M. Vijh. Executive stock and option valuation in a two state-variable framework. *Journal of Derivatives*, 12(3):9–27, 2005.

A. Louis Calvet and Abdul H. Rahman. The subjective valuation of indexed executive stock options and their incentive effects. *The Financial Review*, 41: 205–227, 2006.

Colin F. Camerer. Choice, values and frames. In Daniel Kahneman and Amos Tervsky, editors, *Choice, Values and Frames*, chapter Prospect Theory in the Wild: Evidence from the Field, pages 288–300. Cambridge University Press, 2000.

J. N. Carpenter. The exercise and valuation of executive stock options. *Journal of Financial Economics*, 48(2):127–158, 1998.

David de Meza and David C. Webb. Incentive design under loss aversion. *Journal of the European Economic Association*, 5(1):66–92, 2007.

Szimayer A. Desmettre S. Work effort, consumption, and portfolio selection: When the occupational choice matters. *Berichte des Fraunhofer ITWM, Nr. 188*, 2010.

I. Dittmann, E. Maug, and O. Spalt. Sticks or carrots? optimal ceo compesation when managers are loss-averse. *Journal of Finance*, 65(6), 2010.

J. C. Duan. The GARCH option pricing model. *Mathematical Finance*, 5(1): 13–32, 1995.

Daniel Ellsberg. Risk, ambiguity, and the savage axioms. *Quarterly Journal of Economics*, 75(4):643–669, 1961.

Financial Accounting Standard Board. Statement of financial accounting standards nº 123 (revised 2004): Share-based payments. *FASB*, 2004.

Francisco J. Gomes. Portfolio choice and trading volume with loss-averse investors. *The Journal of Business*, 78:675–706, 2005.

Brian J. Hall and Kevin J. Murphy. Stock options for undiversified executives. *Journal of Accounting and Economics*, 33(1):3–42, 2002.

S. Heston. A closed-form solutions for options with stochastic volatility. *Review of Financial Studies*, 6:327–343, 1993.

S. Hwang and S. E. Satchell. How loss averse are investors in financial markets? *Journal of Banking & Finance*, 2010.

J. Ingersoll. Non-monotonicity of the tversky-kahneman probability-weighting function: A cautionary note. *European Financial Management*, 00:1–6, 2008.

J. Edward Ingersoll. The subjective and objective evaluation of incentive stock options. *Journal of Business*, 79(2):453–487, 2006.

International Accounting Standard Board. International financial reporting standard nº 2: Share-based payment: 2004. *IASB*, 2004.

J. Martin Bizjak J. Carr Bettis and M. Louis Lemmon. Exercise behavior, valuation, and the incentive effects of employee stock options. *Journal of Financial Economics*, 76(2):445–470, 2005.

George J. Jiang and Yisong S. Tian. Forecasting volatility using long memory and comovements: An application to option valuation under sfas 123r. *Journal of Financial and Quantitatives Analysis*, 45 (2):503–533, 2010.

S. Johnson and Y. Tian. Indexed executive stock options. *Journal of Financial Economics*, 57(1):35–64, 2000.

Daniel Kahneman and Amos Tversky. Prospect theory: An analysis of decision under risk. *Econometrica*, 47(4):263–291, 1979.

N. Kulatilaka and A. J. Marcus. Valuing employee stock options. *Financial Analysts Journal*, 50(6):46–56, 1994.

Richard A. Lambert, David F. Larcker, and Robert E. Verrecchia. Portfolio considerations in valuing executive compensation. *Journal of Accounting Research*, 29(1):129–149, 1991.

L.Andersen. Simple and efficient simulation of the Heston stochastic volatility model. *Journal of Computational Finance*, 11, 2008.

A. León and A. Vaello-Sebastià. American GARCH employee stock option valuation. *Journal of Banking and Finance*, 33(6):1129–1143, 2009.

Angel León and Antoni Vaello-Sebastià. A simulation-based algorithm for American executive stock option valuation. *Finance Research Letters*, 7:14–23, 2010.

F. Longstaff and E. Schwartz. Valuing American options by simulation: A simple least-squares approach. *Review of Financial Studies*, 14(1):113–147, 2001.

F. A. Longstaff. Optimal portfolio choice and the valuation of illiquid securities. *Review of Financial Studies*, 14(2):407–431, 2001.

J. Liu M. Kahl and F. A. Longstaff. Paper millionaires: how valuable is stock to a stockholder who is restricted from selling it? *Journal of Financial Economics*, 67(3):385–410, 2003.

P. Santa-Clara M. W. Brandt, A. Goyal and J. R. Stroud. A simulation approach to dynamic portfolio choice with an application to learning about return predictability. *Review of Financial Studies*, 18(3):831–873, 2005.

Kevin J. Murphy. *Handbook of the Economics of Finance*, chapter Executive Compensation: Where we are and how we got there., pages 211–356. Elsevier Science North Holland, 2013.

D. F. Larcker R. A. Lambert and R. E. Verrecchia. Portfolio considerations in valuing executive compensation. *Journal of Accounting Research*, 29(1): 129–149, 1991.

R. Koekkoek R. Lord and D. van Dijk. A comparison of biased simulation schemes for stochastic volatility models. *Quantitative Finance*, 10:177–194, 2010.

M. Rubinstein. On the accounting valuation of employee stock options. *Journal of Derivatives*, 3(1):8–24, 1995.

Uzi Segal and Avia Spivak. First order versus second order risk aversion. *Journal of Economic Theory*, 51:111–125, 1990.

L. Stentoft. Assessing the least squares Monte-Carlo approach to American option valuation. Review of Derivatives Research, 7:129–168, 2004.

Avanidhar Subrahmanyam. Behavioural finance: A review and synthesis. *European Financial Management*, 14(1):12–29, 2007.

S. R. Thorley. The time-diversification controversy. *Financial Analysts Journal*, 51(3):68–76, 1995.

Y. Tian and G. Jiang. Using historical volatility for stock option expensing under sfas 123r: Improving forecasting performance with long memory and comovements. European Financial Management Association, 2008 Annual Meetings, 2008.

Y. S. Tian. Too much of a good incentive? The case of executive stock options. *Journal of Banking and Finance*, 28(6):1225–1245, 2004.

A. Tversky and D. Kahneman. Prospect theory: An analysis of decision under risk. *Econometrica*, 47(2), 1979.

A. Tversky and D. Kahneman. Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty*, 5:297–323, 1992.