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Topology from Art and Math to contemporary cities

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Abstract

Mathematics is not a mere tool of kitchen recipes, but has contributed to, if not determined, our way of conceiving space on Earth and even in the Universe. A lack of awareness of mathematics as an essential tool of our culture has led to a major delay in the use of instruments that mathematicians have used for decades. A case in point is topology, the science of transformations, the science of invariants. Its relatively recent discovery by architects has led to a remarkable transformation of the art of conceiving and building space. A notorious cultural leap has led to construction using techniques and material that allow realizing transformation, rendering it almost continuous, a sort of contradiction between the finished construction and its deformation. The so called fluid topological architecture is the final result of a set of new ideas of space. New topological ideas are used in art, and has been materialized in recent buildings in towns all around the world, such as the Guggenheim Museum in Bilbao, the National Library in Astana, the Moebius Bridge in Bristol, the MAXXI museum in Rome, the Max Reinhardt Haus of Eisemann, the Moebius House of Van Berkel. I will illustrate this story using various examples from the topological gold jewels in pre-Colombian culture in South America to the new development in modern cities. I conclude with a reflection on the ethical and artistic aspects of this new topological public architecture.

Keywords: mathematics, architecture, contemporary, topology.

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“Many of the great creative acts in art and science can be seen as fundamentally metamorphic, in the sense that they involve the conceptual re-shaping of ordering principles from one realm of human activity to another visual analogy. Seeing something as essentially similar to something else has served as a key tool in the fluid transformation of mental frameworks in every field of human endeavour. I used the expression *structural intuitions* to try to capture what I felt about the way in which such conceptual metamorphoses operate in the visual arts and the sciences. Is there anything that creators of artefacts and scientists share in their impulses, in their curiosity, in their desire to make communicative and functional images of what they see and strive to understand?

The expression *structural intuitions* attempts to capture what I tried to say in one phrase, namely that sculptors, architects, engineers, designers and scientists often share a deep involvement with the profound sense of involvement with the beguiling structures apparent in the configurations and processes of nature – both complex and simple. I think we gain a deep satisfaction from the perception of order within apparent chaos, a satisfaction that depends on the way that our brains have evolved mechanisms for the intuitive extraction of the underlying patterns, static and dynamic.”

These are the words of Martin Kemp, an art historian specialized in the relationship between art and science in the article *Intuizioni strutturali e pensiero metamorfico nell’arte, architettura e scienze*, in *Focus*, one of the volumes that make up the catalogue of the 2004 Venice International Architecture Exhibition[1].

In his article Kemp writes mainly about architecture. The image accompanying Kemp’s article is a project by Frank O. Gehry[2], an architect who obviously cannot be overlooked when discussing modern architecture, continuous transformation, unfinished architecture, and infinite architecture. Kurt W. Forster, curator of the exhibit, discusses the great complexity, the enormous number of variations developed through essential technological innovations, the continuous surfaces in transformation. He cites the mathematician Ian Stewart’s article entitled *Nature’s numbers: discovering order and pattern in the universe* (1995). Some key words: pattern, structure, motif, order, metamorphosis, variations, transformations, mathematics[3].

Forster writes:

> *“Recent buildings predicated upon continuous surfaces make clear that they depend in conception and realization on the use of computer technology. The single hinge upon which they turn is the computer. Any number of hybrid transformations and exchanges between traditional methods and rapidly developed software have multiplied and modified the process of elaboration and realization of projects. Hardly a method that cannot be integrated within the ‘loop’ of numeric calculations, but more consequential than the flexibility of elaboration and the constant back-and-forth between image and object, is the fact of architecture’s migration to the realm of the virtual and simulated.”*

Forster continues regarding Gehry:

> *"What really interests Gehry is the process, in the sense of dynamic process used to achieve a structural and aesthetic result."*

These words, projects, and ideas at the 2004 Exhibition were visually closely connected to the ties between mathematics, architecture, topology, and transformation. The layout of the pavilion of the Venice Exhibition was assigned to two famous architects: Hani Rashid and Lise Anne Couture. In an article for the catalogue entitled *Asymptote, the Architecture of Metamorph*, they summarized their project as follows:

> *"Asymptote's transformation of the Corderie in the Arsenale emerged from computer-generated morphing animation sequences derived from utilizing rules of perspective geometry with the actions and dynamics of torquing and stringing the space of the Corderie. The experience of Metamorph is spatial in that it is itself an architectural terrain of movement and flow. The exhibition architecture –from installation and exhibition design to graphic identity and catalogue design– provides for a seamless experience that fuses the Arsenale, Giardini and Venice, making explicit a contemporary reading of architecture where affinities and disparities co-mingle to produce the effects of flux and metamorphoses of form and thinking."* [4]

One of the studies of the layout was described quite significantly as follows:

> *"Study of the topological surface that develops in the space of the Corderie and determines the movements and the curvatures used in designing levels."*

Let's backtrack a bit, to the early 1990s. In 1992 the architect Eisenmann (who won the *Leone d'Oro* for his architectures at the 2004 exhibition) and his collaborators projected a skyscraper in Berlin, the *Max Reinhardt Haus*. The structure of the enormous building is based on a topological surface, the Moebius strip. In 1993 Ben van Berkel planned and built the *Moebius House*. So these two projects held the place of honor in the large hall of the *Corderie*, as if a reminder of an important step in contemporary architecture, in the idea of transformation, of metamorphosis. An explicit reference to topology.

Also interesting is what Hana Rashid writes in the catalog of the Biennale[5]:

> *"With the help of computers in all its forms developments of a new architecture, an architecture influenced and modulated by the infinite and provocative possibilities offered by these technological tools, beyond the simple promise of greater efficiency and production capacity, are emerging. These new processes and methodologies associated with history, theory, conceptual thinking, experimentation and production are radically changing not only the way we see and think about space, but also the means by which we can occupy and inhabit the territory. In one form or another, it is now within the reach of artists and architects to discover and evoke digitally induced spatial deliria in which the merging simulation and effect with*

> *physical reality creates the possibility of a sublime digital metamorphosis from thought to its realization."*

Until a few decades ago these were utopian projects, and many still are, as architects also have fun designing projects that will not be realized.

The fascination of topology

Jules Henri Poincaré held that:

> *"the geometrical axioms are neither synthetic a priori intuitions nor experimental facts. They are conventions. Our choice among all possible conventions is guided by experimental facts; but it remains free, and is only limited by the necessity of avoiding every contradiction, and thus it is that postulates may remain rigorously true even when the experimental laws which have determined their adoption are only approximate.*
>
> *In other words the axioms of geometry are only definitions in disguise. What then are we to think of the question: Is Euclidean geometry true? It has no meaning. We might as well ask if the metric system is true and if the old weights and measures are false; if Cartesian coordinates are true and polar coordinates are false. One geometry cannot be more true than another; it can only be more convenient. Euclidean geometry is and will remain the most convenient."*

Poincaré, in Analysis Situs6 (Latin translation of the Greek τὸττς, luogo, e λὸγος, studio), published in 1895, is responsible for the official birth of the sector of mathematics which today is called *Topology*:

> *"As far as I am concerned, all of the various research that I have performed has brought me to Analysis Situs."*

Poincaré defined topology as the science that introduces us to the qualitative properties of geometric figures not only in ordinary space but also in more than 3-D space. Adding the geometry of complex systems, fractal geometry, chaos theory, and all of the *mathematical* images discovered (or invented) by mathematicians in the last 30 years using computer graphics, it is easy to see how mathematics has contributed to changing our concept of space – the space in which we live and the idea of space itself[7]. Because mathematics is not merely a means of measurement in recipes but has contributed, if not determined, the way in which we understand space on earth and in the universe, specifically in regard to topology, the science of transformations, and the science of invariants. It is important to mention that the discovery (or invention) of non-Euclidean geometry, the higher dimensions (from the fourth on) and topology, the new idea of space to summarize, is one of the most interesting examples of the profound repercussions that mathematical ideas will have on humanistic culture, art and architecture[8].

The key word is geometrical intuition. Some of the topological ideas were sensed by artists and architects in the past decades, first by artists, then much later by architects[9].

Mark Burry, who is in charge to complete the *Sagrada Familia* by Gaudì in Barcelona[10] dedicated a chapter on topology in his recent book[11] *The New Mathematics in Architecture*. He wrote[12]:

> *"The freedon that topology affords in architecture as a more generalized framework that geometry has received greater appreciation in the post-digital age…The essence of architectural and urban planning is also captive in such non-geometrical diagrams, as are the relationships between component spaces or activities of building. This is regardless of how building itself may solidify through the process of design and construction into a static, unchanging form that is also subject to detailed geometrical description. It is possible that the organization of the early development world of our childhood is a similar network of connections between significant places and things, and it is only later and gradually that the absolute reference of metrical Cartesian space is superimposed on our established perception of proximities and relationships…*
>
> *What is it about topology and its freedom of description that has seized modern architectural production, long after the underlying ideas were in common domain? One possible answer is the confluence of unimaged new levels of computer graphical representations with the transition of non-rational basis splines, or NURBS, from the automotive industry into other computer-asided design software…The dynamism of systems could not only be represented in truly dynamic models, but their manifestations could now be understood visually. Truly visual feedback changed everything. It became possible to model surfaces that could change, stretch, adopt free from curvature, or conform to a geometrical rationale without loosing their integrity – wonderful surfaces that, plastically and geometrically at least, exceeded the behaviour of any known material and could be given visual material qualities at a whim…Topological description is being adopted as the means of mapping architectural intention, and with it arrives the progressive discovery of how to map this onto the frozen Euclidean moment in the physical world."*

Topological and Fluid Surfaces

In the chapter Topological Surfaces Alicia Imperiale writes[13]:

> *"The architects Ben van Berkel and Caroline Bos of UN Studio discuss the impact of new scientific discoveries on architecture. The scientific discoveries have radically changed the definition of the word Space, attributing a topological shape to it. Rather than a static model of constitutive elements, space is perceived as something malleable, mutating, and its organization, its division, its appropriation become elastic."*

And the role of topology, from the architect's perspective:

> *"Topology is the study of the behaviour of a structure of surfaces which undergo deformations. The surface registers the changes in the differential space time leaps in a continuous deformation. This entails further potential for architectural deformation. Continuous deformation of a surface can lead to the intersection of external and internal planes in a continuous morphological mutation, exactly like in the Moebius Strip. Architects use this topological form to design houses, inserting differential fields of space and time into an otherwise static structure."*

Naturally some words and ideas are changed in switching from a strictly scientific field to an artistic and architectonic one. But this is not a problem, nor a criticism. Ideas move freely and each person has the right to interpret and attempt, as with topology, to capture the essence. The role of computer graphics in all of this is essential, it allows the insertion of that deformation-time variable that would otherwise be unthinkable, not to mention unattainable.

Imperiale continues regarding the Moebius Strip:

> *"Van Berkel's house, inspired by the Moebius Strip (Moebius House), was designed as a programmatically continuous structure, that combines the continuous mutation of the dialectic sliding couples that flow into each other, from the interior to the exterior, from the activity of work to that of free time, from the fundamental to the non-fundamental structure.*
>
> *The building, composed of arches, made up of intersecting and overlapping forms, presents a unified structure that separates, that compresses, transforms and finally comes back together on the horizontal plane at the height of the attic. The origin of the form is represented in the Moebius Strip, a three-dimensional geometric form characterized by a unique, unending surface that undergoes three iterative operations. In the first, the planes are generated from the extension of the vectors and triangulations of the surfaces… The second iteration overturns the strip, causing an operation similar to that in the first phase, and then appends these surfaces on top of the original form, thus creating a ghost form. The third phase applies an element of Berlinese history to the form itself, wrapping up vast public spaces between the grilled and the base of the ground floor of an already folded surface.*
>
> *Just as the Moebius Strip folds two sides into one surface by folding on itself, the Max Reinhardt Haus denies the dialectic tradition between internal and external and confuses the distinction between public and private."*

Van Berkel writes that another famous topological object, the Klein bottle, can be translated into a canal system that incorporates all of the elements that it encounters and causes them to precipitate into a new type of internally connected integral organization. Note that the words integral and internally connected have precise meanings in mathematics.

But this is not a problem because *"the diagrams of these topological surfaces are not used in architecture in a rigorously mathematically way, but constitute abstract diagrams, three-di-*

mensional models that consent the architects to incorporate differential ideas of space and time into architecture."

As I mentioned before, architects became aware (albeit rather late) of the new scientific discoveries in the field of topology. And not only did they begin to design and build but also to reflect.

In the section on Installations of the Biennale of Architecture in Venice in 2008 was presented a Zaha Hadid and Patrick Schumacher's project, Lotus[14] was presented in a hall of the Arsenale and the Malcontenta Villa, one of the most famous buildings by Palladio on the Brenta River, far away from Venice.

The Aura installation for the 2008 Venice Biennale represents a dialogue between the fluid contemporary language of the Zaha Hadid studio and the mathematical principles of harmonious architectural composition of Andrea Palladio, on the 500th anniversary of his birth. The work focuses on the piano nobile of Palladio's Villa Foscari *La Malcontenta*, which encapsulates his theory of perfect form. Accordingly, the proportions of the sequence of spaces provided the starting point for Zaha Hadid and Patrik Schumacher's study. [14]

Changing, transforming the rules, *"instead of representing a system already domesticated through internal rules, the Lotus room (author's note: that of the Arsenale) seduces through the folds of undulating rhythm, its exclusions, its reconfigurability and its ability to remain outside of categories."* In November 2009, a new space for contemporary art and architecture in Rome, MAXXI was inaugurated.

This how the project is presented at the site of the study of Zara Hadid[15]: "MAXXI supercedes the notion of museum as *object* or fixed entity, presenting instead a field of buildings accessible to all, with no firm boundary between what is within and what without. Central to this new reality – its primary force – is a confluence of lines – walls that constantly intersect and separate to create indoor and outdoor spaces.

MAXXI integrates itself with its surrounds, re-interpretation urban grids to generate its own geometric complexity. Through the flow of its walls it defines major streams – the galleries – and minor streams – interconnections and bridges, delighting in a peculiar L-shape footprint which in this context becomes 'liberation' – a freedom to bundle, twist and turn through existing buildings. In this very meandering MAXXI both draws on and feeds the cultural vitality of its mother city… MAXXI expresses itself through glass, steel and cement – delighting in neutrality, achieving great curatorial flexibility and variety. To wander through, to experience this place – these spaces – is to encounter constantly changing vistas and surprises.

At the inauguration of MAXXI Zaha Hadid said that first of all she had to decide whether or not to keep all existing buildings. Once the decision, she began to study the geometries that would replaced, them orthogonal, parallel or diagonal. *"What appeared was a confluence of lines of different geometrie present on the site. This way it started and a fluid interpretation of the space emerged."*

The fluidity is now one of the keywords of contemporary architecture. Among other things, Zaha Hadid has a degree in mathematics.

The Topological Tendency in Architecture

In her 1999 doctoral thesis Architettura e Topologia: per una teoria spaziale della architettura, Giuseppa Di Cristina writes[16]:

> *"Architecture's final conquest is space: this is generated through a sort of positional logic of the elements, that is through the arrangement that spatial relationships generate; the formal value is thus substituted by the spatial value of the configuration: the external aspect of the form is not as important as the spatial quality. And thus topological geometry, without 'measure' and characteristic of non-rigid figures, is not something purely abstract that comes before architecture, but a trace left by that modality of action in the spatial concretization of architecture."*

In 2001 Di Cristina edited a book on Architecture and Science[17]. In her introduction *The Topological Tendency in Architecture* Di Cristina clarifies that:

> *"The articles that are included here bear witness to the interweaving of this architectural neo-avant-garde with scientific mathematical thought, in particular topological thought: although no proper theory of topological architecture has yet been formulated, one could nevertheless speak of a topological tendency in architects at both the theoretical and operative levels. In particular, developments in modern geometry or mathematics, perceptual psychology and computer graphics have an influence on the present formal renewal of architecture and on the evolution of architectural thought. What mainly interests architects theorizing the logic of curvability and pliability is the significance of the 'event', of 'evolution', of 'process', or the innate dynamism in the fluid and flexible configurations of what is now called topological architecture.*
>
> *Topological architecture means that dynamic variation of form, facilitated by information technology, by computerassisted design, by animation software. The topologification of architectonic forms according to dynamic and complex configurations leads architectural design to a new and often spectacular plasticity, in the footsteps of the Baroque or organic Expressionism."*

These are observations in which ideas on geometry, topology, computer graphics, space-time, and so on converge. The reasons why the links, the Ariadne's threads of culture over the years have worked: new words, new meanings, new links, new influences, new forms, new spaces. In a never-ending process, in which the mathematical ideas on space have played an important part.

A last example: Armani Fifth Avenue, NYC, 2009

This is how it is described the incredible staircase inside the building, realizied by the Studio of Massimiliano and Dorina Fuksas in 2009 [18]:

> *"Situated in the centre of New York, the showroom develops on four different levels and it is conceived as a single space, without clear distinctions, a space in harmony connected with the power generated by the vortex that is the staircase. The heart of the building is, in fact, epitomized by the staircase. Structure in rolled calendar steel and cladded in a plastic layer that highlights its exceptional scultural presence. It is an entity that it is almost impossible to convey in terms of any normal geometric shape that originate from a vortex with great dynamism, surrounded by the different levels …The movement of the ribbons that constitues the staircase… The fluidity of the internal space is rendered by the wall of the continuous threads."*

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