Valuation of wind energy projects: A real options approach

Luis M. Abadie and José M. Chamorro

October 2012

LOW CARBON PROGRAMME

BC3 WORKING PAPER SERIES

2012-11

The Basque Centre for Climate Change (BC3) is a Research Centre based in the Basque Country, which aims at contributing to long-term research on the causes and consequences of Climate Change in order to foster the creation of knowledge in this multidisciplinary science.

The BC3 promotes a highly-qualified team of researchers with the primary objective of achieving excellence in research, training and dissemination. The Scientific Plan of BC3 is led by the Scientific Director, Prof. Anil Markandya.

The core research avenues are:

- Adaptation to and the impacts of climate change
- Measures to mitigate the amount of climate change experienced
- International Dimensions of Climate Policy
- Developing and supporting research that informs climate policy in the Basque Country

This Working paper has been produced under the Low Carbon Programme initiative:
http://www.lowcarbonprogramme.org/

The BC3 Working Paper Series is available on the internet at
http://www.bc3research.org/lits_publications.html

Enquiries (Regarding the BC3 Working Paper Series):

Roger Fouquet

Email: roger.fouquet@bc3research.org

www.bc3research.org

Valuation of wind energy projects: A real options approach

Luis M. Abadie
Basque Centre for Climate Change (BC3)
Alameda Urquijo 4, 4^o
48008, Bilbao, Spain
Tel. +34 94-401 46 90
E-mail: lm.abadie@bc3.research.org

José M. Chamorro
University of the Basque Country
Av. Lehendakari Aguirre, 83
48015 Bilbao, Spain
Tel. +34-946013769
E-mail: jm.chamorro@ehu.es

October 14, 2012

Abstract

This paper addresses the valuation of an operating wind farm and the finite-lived option to invest in such a farm under different reward and/or support schemes. They range from a feed-in tariff to a premium on top of electricity market price, to a transitory subsidy to capital expenditure. The availability of futures contracts on electricity with ever longer maturities allows to undertake valuations based on market data. The model considers two sources of uncertainty, namely the future electricity price (which shows seasonality) and the level of wind generation (which is intermittent in addition to seasonal). Lacking analytical solutions we resort to a trinomial lattice (which supports mean reversion in prices) combined with Monte Carlo simulation at each of the nodes in the lattice. Our data set refers to the UK. The numerical results show the impact of a number of factors involved in the decision to invest: the subsidy per unit of electricity generated, the initial lump-sum subsidy, the investment option's maturity, and price volatility.

Keywords: wind farms, electricity, stochastic load factor, futures markets, real options.

1 Introduction

Public support to renewable energies is usually justified on three grounds: climate change, security of supply, and industrial policy. Some of the positive effects from renewables' development are global, e.g. the abatement of greenhouse gas emissions, and the reduction of investment unit costs (because of the learning effect). Impacts from enhanced energy security and industrial policy, instead, are derived at the national level.

Renewable sources are getting ever more relevant in the generation of electric energy. Major drivers are the decreasing costs of renewable technologies and strong support from government agencies. This trend is expected to continue in the years ahead (European Commission [5]). Pérez-Arriaga and Batlle [15] analyze the impact of a strong penetration of renewable, intermittent generation on the planning, operation, and control of power systems. See also EWEA [6] and NREL [12].

Within this set of technologies wind stands apart, with solar photovoltaic (PV) and concentrated solar power (CSP) somewhat behind. The increasing role of these intermittent generation technologies gives rise to important challenges in the operation of the electric system. Regarding solar energy, it is more predictable than wind over short periods of time. It also displays a diurnal seasonality which overlaps with the hours of strongest load thus coinciding with the times of highest prices. This suggests that the prices at the times of strongest operation of solar plants will approach peak prices.

One of the problems afflicting wind energy is certainly intermittence. However, this problem is less acute when dealing with a large balancing area since the behavior of wind correlates less than perfectly across all the sites in the area (provided there is enough transmission capacity). Further deployment of renewable energies (wind in particular) would also benefit significantly from greater storage capacities. A minor concern is that wind energy is not quite carbon free.[1] Large-scale deployment of turbines can also disrupt local wildlife and fauna, affect local temperature and even global weather. These negative impacts are hard to quantify but this does not render them less real.

Despite these shortcomings the fact remains that in principle the potential of wind goes well beyond global needs. Marvel et al. [11] use a climate model to estimate the amount of power that can be extracted from both surface and high-altitude winds, considering only geophysical limits. According to their results, surface wind turbines alone could extract kinetic energy at a rate of at least 400 terawatts (TW, one trillion watts) while the level of present global primary power demand approaches 18 TW. On the other hand, Jacobson and Archer [8] define the saturation wind power potential as the maximum wind power that can be extracted upon increasing the number of wind turbines over a large geographic region, independent of societal, environmental, climatic, or economic

[1] For example, the very construction of a wind turbine consumes energy (fossil to a large extent). Ortegon et al. [13] report a CO2 emission factor for wind power in the range 20-38 gCO2/kWh and 9-13 gCO2/kWh for on-shore and offshore applications, respectively. O focurse, this consideration also applies to coal stations or nuclear plants.

considerations. This saturation potential is over 250 TW at 100 meters up globally (100 m above ground is the hub height of most modern wind turbines), assuming conventional wind turbines distributed everywhere on Earth.

Empirical evidence shows, though, that actual deployment of this technology is well below that potential. Several barriers (whether economic, social, or other type) are probably playing a role in hampering adoption across the globe. Regarding economic barriers, casual observation allows to identify a number of support schemes which are presumably aimed at providing greater certainty to potential investors in this technology; see Klessmann et al. [9]. In other words, uncertain returns on these investments are considered a major cause for concern.

A number of financial incentives have been put in place; an overview can be found in Daim et al. [4] and Snyder and Kaiser [17]. Feed-in tariffs are a guaranteed payment to generators of renewable electricity (say 90 €/MWh) over a certain period of time (e.g. 20 years). This instrument is typical in several EU countries, among them Germany and Spain. The UK instead incentivizes renewable electricity through the use of renewable energy credits (the so-called Renewables Obligation Certificates, or ROCs) which are further traded in their specific market. EU nations also grant some tax exemptions (for instance, from carbon taxes) and subsidies (to capital expenditure). In the US there is a production tax credit at the federal level. The fact that it has expired three times over the last ten years is not reassuring, however. A number of States have set renewable portfolio standards whereby a certain fraction of the State's electricity must come from renewable sources. Some States also take part in a regional greenhouse gas initiative, a cap-and-trade market for carbon. Regarding subsidies, they are both lower and less certain than those in Europe.

A suitable valuation approach for wind projects must not only account for intermittence and uncertainty. It must also take account of their irreversible character and the flexibility enjoyed by project managers (e.g. the option to delay investment). Under these circumstances, traditional valuation techniques based on discounted cash flows have been found inferior to contingent claims or real options analysis.

Following the latter approach, Boomsma et al. [1] assess both the time and the size of the investment in renewable energy projects under different support schemes. They consider up to three sources of uncertainty: steel price, electricity price, and subsidy payment, all of which are assumed to follow uncorrelated geometric Brownian motions (with the last one modulated by Markov switching). For illustration purposes, they focus on a Norwegian case study. According to their results, a fixed feed-in tariff encourages earlier investment than renewable energy certificates. The latter, though, create incentives for larger projects.

Reuter et al. [16] instead pick Germany as a case study. In their model the electric utility decides whether to add new generation capacity or not once a year over the planning horizon. The new capacity can be either a fossil fuel power plant (with a constant load factor) or a wind power plant (with a normally distributed load factor), both equally sized. The yearly electricity price is subject to (normally distributed) exogenous shocks (assumed independent from wind load factor). The third source of uncertainty concerns climate policy; it is represented by the feed-in tariff which is a Markov chain with two possible values and a given transmission matrix. This risk factor is also assumed independent from the other two. Their results stress the importance of explicitly modeling the variability of renewable loads owing to their impact on profit distributions and the value of the firm. Besides, greater uncertainty about the future behavior of the feed-in tariff requires much higher trigger tariffs for which renewable investments become attractive (i.e. equally profitable as a coal-fired station of equal capacity).

Here we address the present value of an investment in a wind park and the optimal time to invest under different payment settings: (a) A fixed feed-in tariff for renewable electricity over 20 years of useful life. (b) Electricity price as determined by the market. (c) A combination of the market price and a constant premium. (d) A transitory subsidy available only at the initial time. We also develop sensitivity analyses with respect to changes in the investment option's maturity and electricity price volatility.

Our paper differs from others in several respects. We consider two sources of uncertainty. We assume more general stochastic processes for the state variables; in particular, we account for mean reversion in commodity prices (this fits better the sample data as shown in Appendix A). We develop a trinomial lattice that supports this behavior. We also make room for seasonal behavior in the price of electricity and in wind load factor. Indeed, they turn out to be correlated to some degree, which has been typically overlooked despite its impact on project value. The underlying dynamics in the price of electricity is estimated from observed futures contracts with the longest maturities available (namely, up to five years into the future); this includes the market price of electricity price risk. The dynamics of wind load factor is also estimated from actual (monthly) time series alongside seasonality. The riskless interest rate is also taken from (financial) markets. Both the project's life and the option's maturity are finite; in our simulations below the size of the time step is not $\Delta t = 1$ (or one step per year), but a much shorter $\Delta t = 1/60$ (five steps per month). In addition to a fixed feed-in tariff and a premium over electricity price, another support scheme that we consider is an investment subsidy that is only available at the initial time but is foregone otherwise. We further provide numerical estimates of the trigger investment cost below which it is optimal to invest immediately.

The paper is organized as follows. First we introduce the stochastic processes for electricity price and wind load factor. Next we estimate these processes with sample data from the UK. Valuation is then undertaken under two scenarios. The first one adopts a now-or-never perspective. This is the setting where the traditional Net Present Value rule applies. Numerical solutions are derived from exact formulas when possible but also from Monte Carlo simulation. The second scenario allows for optimally choosing the time to invest. In our case this is accomplished by means of a trinomial lattice which supports mean reversion. Several cases and sensitivity analyses are then addressed. A number of them involve running whole simulations of electricity price and load factor at each node in the lattice. We thus combine two numerical methods that are frequently used in isolation. A section with the main results concludes.

2 Stochastic models

2.1 Electricity price

We specify the long-term price of electricity in a risk-neutral world as a mean reverting stochastic process governed by the following differential equation:

$$dE_t = df(t) + [k_E(E_m - (E_t - f(t))) - \lambda_E(E_t - f(t))]dt + \sigma_E(E_t - f(t))dW_t^E,$$

or, rearranging,

$$dE_t = df(t) + [k_E E_m - (k_E + \lambda_E)(E_t - f(t))]dt + \sigma_E(E_t - f(t))dW_t^E. \quad (1)$$

E_t is the time-t price of electricity while E_m is the level to which the deseasonalized price tends in the long run. $f(t)$ is a deterministic function that captures the effect of seasonality in electricity prices. This function is defined as $f(t) = \gamma \cos(2\pi(t + \varphi))$, where the time t is measured in years and the angle in radians; when $t = -\varphi$ we have $f(t)=\gamma$ and the seasonal maximum value is reached. k_E is the speed of reversion towards the "normal" level E_m. It can be computed as $k_E = \ln 2/t^E_{1/2}$, where $t^E_{1/2}$ is the expected half-life, i.e. the time required for the gap between $E_0 - f(0)$ and E_m to halve. σ_E is the instantaneous volatility of electricity price changes; it determines the variance of E_t at t. And dW_t^E is the increment to a standard Wiener process; it is normally distributed with mean zero and variance dt. Last, $\lambda_E E_t$ is the market price of electricity price risk.

The mathematical expectation (under the risk-neutral probability measure Q) at time t_0, or equivalently the futures price with maturity t, is:

$$F(E_{t_0}, t) = E^Q(E_t) = f(t) + \frac{k_E E_m}{k_E + \lambda_E}[1 - e^{-(k_E+\lambda_E)(t-t_0)}] + \\ +(E_{t_0} - f(t_0))e^{-(k_E+\lambda_E)(t-t_0)}. \quad (2)$$

For a time arbitrarily far into the future ($t \to \infty$) we have $F(E_{t_0}, \infty) - f(\infty) = \frac{k_E E_m}{k_E+\lambda_E}$. Thus, (deseasonalised) electricity price in the long run is anticipated to reach the long-term equilibrium level.

2.2 Wind electricity

Reuter et al. [16] consider wind stations and address the impact of uncertainty in the load factor on their profits. As expected, the distribution of yearly profits is more variable than under a constant load factor (equal to the long-term average). In addition, the expected profit is smaller under a changing load factor. They find similar evidence when analyzing the value of the firm. Thus assuming a constant load factor leads to overestimating this technology's profitability.

Table 1. Summary statistics for UK electricity futures (ICE). Daily data from 12/01/2009 to 03/30/2012			
	Observations	Avg. Price (£/MWh)	Std. Dev.
All contracts	26,057	54.88	7.69
1 Month	604	44.95	6.03
6 Months	604	47.53	7.31
12 Months	594	49.68	5.60
24 Months	422	54.80	3.82
36 Months	422	58.34	4.21
48 Months	422	61.83	4.30
60 Months	25	68.59	0.59

Intermittence *per se* drives a sizeable wedge between installed capacity and metered electricity; it can be measured through the load factor, W. We explicitly recognize the uncertain character of wind energy. All the interruptions (whatever their reasons) are modeled through the stochastic behavior of the load factor. The theoretical model assumed is:

$$W_t = g(t) + W_m + \sigma_W W_m dW_t^W. \tag{3}$$

Generation from wind stations shows a seasonal pattern. Our simulations below assume this behavior in wind electricity, $g(t)$, so the seasonality in the load factor must be previously identified (from historical time series). W_t evolves around a long-run average value W_m. And dW_t^W is the increment to a standard Wiener process; it is normally distributed with mean zero and variance dt.

3 Estimation

3.1 Electricity price process

We have 26,057 prices of monthly UK Base Electricity Futures from the Intercontinental Exchange (ICE, London). The sample period goes from 12/01/2009 to 03/30/2012 thus comprising 604 trading days; see Table 1. The number of traded contracts on the last day of the sample is 59, i.e. we use futures contracts with maturities up to five years from now (thus they are long-term futures prices instead of short-term forward prices or day-ahead prices). These prices for successive months are assumed to reflect all the information available to the market about generation costs and profit margins of power plants. In particular, they take account of fuel prices, allowance prices, decommissioning of old plants, new starts, etc.

We estimate the parameters underlying the stochastic model using all the futures prices on each day by non-linear least-squares. Table 2 shows the results. All the estimates are statistically significant. We get a coefficient of determination $R^2 = 0.8579$; the log-likelihood of this model is -64,707.64. See Appendix A for a formal test of this model against the null hypothesis of a geometric

Table 2. Non-linear least-squares estimates of the price process.				
Parameter	Estimate	Std. error	t-ratio	p-value
$k_E + \lambda_E$	0.1134	0.001939	58.47	0.000
$\frac{k_E E_m}{k_E+\lambda_E}$	85.9128	0.542854	158.3	0.000
γ	3.02281	0.020658	146.3	0.000
φ (years)	0.03139	0.0010417	30.13	0.000

Brownian motion; the test results show that the mean-reverting process is a much better choice than the standard GBM. For the last day in the sample we compute $E_0 - f(t_0) = 48.9135$ £/MWh; this price is the starting point for estimations of the electricity price in the future.

Figure 1 displays the futures prices actually observed on the last day of the sample (03/30/2012) along with those implied by our numerical estimates using all the contracts traded every day. We can estimate the spot electricity price for day t_0 from the futures contract with the nearest maturity using equation (2):

$$E_{t_0} = \left[F(E_{t_0}, t) - f(t) - \frac{k_E E_m}{k_E + \lambda_E}\right] e^{(k_E+\lambda_E)(t-t_0)} + \frac{k_E E_m}{k_E + \lambda_E} + f(t_0).$$

The seasonally adjusted spot price is: $E_{t_0} - f(t_0)$. Thus we compute a spot price for every day. They behave more smoothly (or are less bumpy) than actual futures prices.

Using the differential equation describing price behavior in the physical (as opposed to risk neutral) world we get:

$$\frac{d(E_{t_0} - f(t_0))}{E_{t_0} - f(t_0)} = \left[\frac{k_E E_m}{E_{t_0} - f(t_0)} - k_E\right] dt + \sigma_E dW_t^E.$$

Discretizing this formula we derive a regression model whose residuals allow us to compute their volatility:

$$\sigma_E = 0.255045.$$

On the other hand, the risk-free interest rate considered is $r = 2.05$ %, which corresponds to the 10-year UK government debt in January 2012.

3.2 Wind electricity: load factor, seasonality, and drift rate

In discrete time we have:

$$W_{t+\Delta t} = g(t) + W_m + \sigma_W \sqrt{\Delta t} W_m \epsilon_t^2.$$

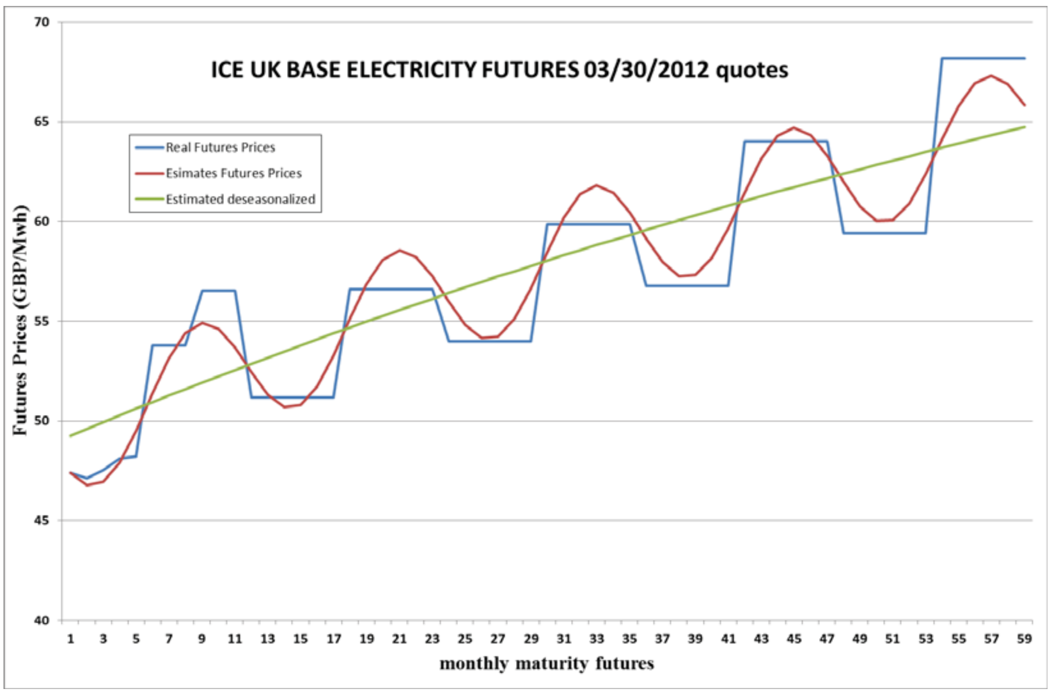

Figure 1: UK base electricity futures prices on London ICE, 03/30/2012.

Table 3. Seasonal (OLS) estimates in wind load factor.

Dummy	Coeff.	t-ratio	Dummy	Coeff.	t-ratio
d_1	8.7442	9.1273	d_7	-8.8292	-10.3039
d_2	-2.0608	-2.1511	d_8	-3.8895	-4.5392
d_3	6.2505	6.5244	d_9	1.4574	1.7009
d_4	-4.1947	-4.8954	d_{10}	1.7411	2.0320
d_5	-4.6595	-5.4378	d_{11}	12.4732	14.5565
d_6	-11.3065	-13.1949	d_{12}	4.4757	5.2232

We are implicitly assuming that beyond (deterministic) seasonality the electricity price and the wind load factor are uncorrelated; in other words, they can be correlated but only through their seasonal patterns. Based on past (say, monthly) data one can get a numerical estimate of the above parameters $\{g(t), W_m, \sigma_W\}$. Later on they can be used to simulate random paths over a number of periods.

The sample comprises the monthly ratios between output electricity and installed capacity for the whole UK from April 2006 to December 2010, i.e. 57 observations.[2] As a first step the seasonal component is taken out of the original series. Estimation then proceeds on the deseasonalised series. The estimate of the average value is $\widehat{W}_m = 24.0899$ %. The results for the (dummy) monthly variables appear in Table 3.[3] They are depicted in Figure 2.

3.3 The joint effect of seasonalities in electricity price and wind generation

As Table 3 suggests, the periods with (statistically) highest wind generation fall in January and November. The highest prices of electricity are reached between October and March; thus there is some overlapping. This time coincidence allows UK farms' owners to get a greater profitability from wind generation. Other papers overlook this feature [4] yet our model takes it into account.

[2] The maximum possible output for each month is calculated from the installed capacity of the wind farm: Maximum output (MWh) = Installed capacity (MW) * number of days * 24. The actual output is then expressed as a percentage of the maximum possible output over the same time interval. Source: CLOWD [2].

[3] This value of $\widehat{W}_m$ is slightly higher than the average of 23 % cited for Germany by Reuter et al. [16]. The dummy variables here do not display a symmetrical behavior; this is in contradiction with their assumption of a normal distribution.

[4] This is the case, for example, when a constant annual capacity factor is chosen, say 35%.

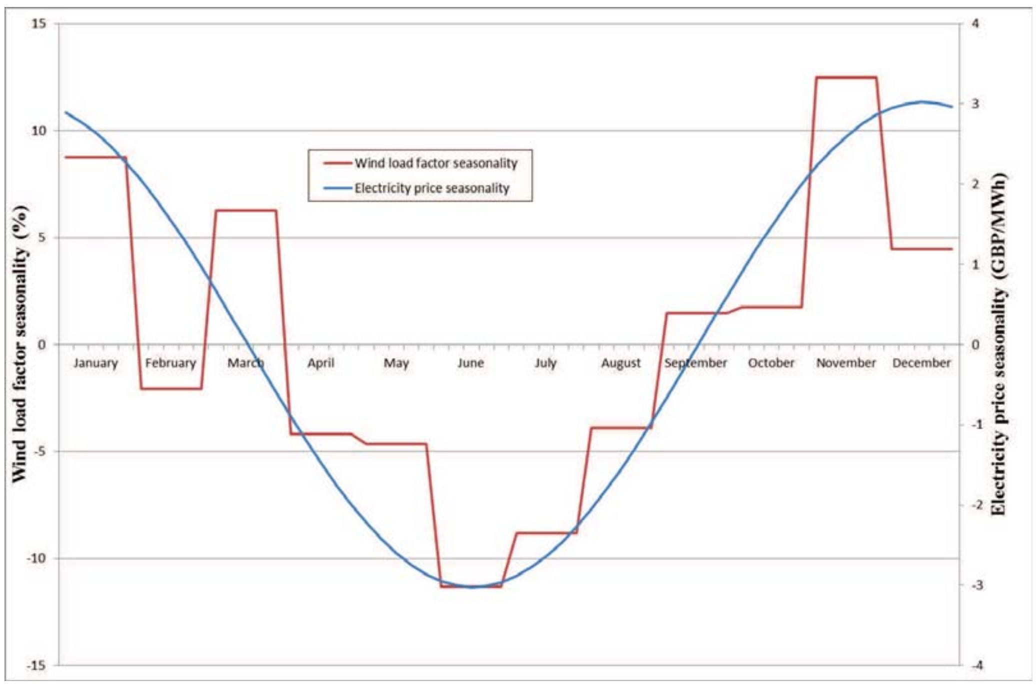

Figure 2: Monthly load factor of UK wind farms 2006-10.

4 Valuation in a now-or-never setting: Monte Carlo simulation

Uncorrelated random variables are generated according to the following discrete-time schemes (see equations (2) and (3)):

$$E_{t+\Delta t} = f(t+\Delta t) + \frac{k_E E_m}{k_E + \lambda_E}(1 - e^{-(k_E+\lambda_E)\Delta t}) + (E_t - f(t))e^{-(k_E+\lambda_E)\Delta t} + \sigma_E \sqrt{\Delta t}(E_t - f(t))\epsilon_t^E, \tag{4}$$

$$W_{t+\Delta t} = g(t) + W_m + \sigma_W \sqrt{\Delta t} W_m \epsilon_t^W. \tag{5}$$

Note that in both cases we start from known values, e.g. $E_t - f(t)$ or $g(t)$, and then add a random component ϵ_t.

Let us consider a wind farm with installed capacity $C = 50$ MW (think of a set comprising 25 turbines each 2 MW). The average load factor is $\widehat{W}_m = 24.0899$ % (see Table 3). Seasonality comes on top of this. Thus the expected availability in January would be $\widehat{W}_m + \widehat{d}_1 = 24.0899 + 8.7442 = 32.8341$ %; or, in absolute terms $50 \times 24 \times 31 \times 0.328341 = 12{,}214.29$ MWh.[5] In general, wind generation over a time period Δt amounts to:

$$C \times 24 \times 365.25 \times \Delta t \times W_t.$$

Now, if there is a fixed feed-in tariff p in place then the present value of production in that period is computed as:

$$V_t = p \times C \times 24 \times 365.25 \times \Delta t \times W_t \times e^{-rt}. \tag{6}$$

If, instead, the farm owner receives as a payment the market price E_t then the present value of the revenues is given by:

$$V_t = E_t \times C \times 24 \times 365.25 \times \Delta t \times W_t \times e^{-rt}. \tag{7}$$

Note that our simulations below are based on a risk-neutral drift. Consequently future cash flows can be discounted at the risk-free rate r.

Each simulation run s (with $s = 1, ..., m$) comprises a number of time steps denoted by j (with $j = 1, ..., n$). We denote the value of the wind park at any step by V_{sj}. These values are aggregated over the n steps to derive the value under simulation s, denoted V_s. Then we compute the average value over all the m simulations:

$$V_s = \sum_{j=1}^{j=n} V_{sj} \Rightarrow V = \frac{1}{m}\sum_{s=1}^{s=m} V_s. \tag{8}$$

[5] This is equivalent to saying that January generation amounts to $24 \times 31 \times 0.328341 = 244.28$ MWh per MW of capacity installed. This figure changes from one month to another. Instead, Boomsma et al. [1] consider a constant annual capacity factor of 35 %, which translates into a flat generation of 255.5 MWh per MW of capacity installed every month.

Table 4. Present value of a 50 MW wind park under a constant feed-in tariff.		
Tariff p (£/MWh)	Exact V (£)	Monte Carlo V (£)
50	86,654,277	86,638,266
60	103,985,132	103,965,920
70	121,315,988	121,293,573
80	138,646,843	138,621,226
90	155,977,698	155,948,880

We undertake $m = 1{,}000$ simulation runs, and consider a useful time of 20 years. The step size is $\Delta t = 1/60$, thus each simulation comprises $n = 1{,}200$ steps.

4.0.1 A constant feed-in tariff

The feed-in tariff is generally claimed to be the most effective method for promoting renewable energy. Let p denote the tariff applied to the electricity generated. A given month x (with $x = 1, ..., 12$) comprises a number of days x_i. Since the useful life of the facility stretches over 20 years (i.e. $y = 1, ..., 19$) the present value V of the investment under this scheme is:[6]

$$V = p \sum_{y=0}^{y=19} \sum_{x=1}^{x=12} C \times 24 \times x_i \times (d_m + d_i) e^{-r(12y+x)/12}. \tag{9}$$

Table 4 shows the present value for a range of potential tariffs when the riskless interest rate is $r = 0.0205$. The second column is directly computed from the above exact formula. These sums of money are to be set against the investment cost and the present value of fixed costs.[7]

For consistency with next sections, the numerical estimates of the parameters in wind load factor $\{g(t), W_m, \sigma_W\}$ are also used here to simulate random paths, month after month, over a number of years. The third column in Table 4 comes from this Monte Carlo approach. It results from running 1,000 simulations each comprising 1,200 time steps (i.e. five steps per month) and then taking the average value. The amounts resemble pretty much those in the second column.

4.0.2 The market price

Assume that the unit payment to the owner of the wind park strictly amounts to the market price of electricity; this can be thought of as the case of a generator who is ineligible for renewable energy support (or the feed-in tariff suddenly ceases to apply). In this case we resort to simulation in order to take account

[6] For simplicity each cash flow is assumed to be received at the end of the month.

[7] Boomsma et al. [1] set the initial level of p at 50 €/MWh with an annual percentage increase of 2%. Unlike us, they also consider vaiable costs (14.50 €/MWh on average). The level of p in Reuter et al. [16] goes from 70 €/MWh to 110 €/MWh.

of the situations in which high electricity prices (due to strong demand) coincide with high wind generation (owing to seasonal weather). Discretization of equation (1) and equation (3) yields:

$$E_{t+\Delta t} = E_t + (f_E(t+\Delta t) - f_E(t)) + [k_E E_m - (k_E + \lambda_E)(E_t - f_E(t))]\Delta t + +\sigma_E(E_t - f_E(t))\sqrt{\Delta t} u_{E,t},$$

$$W_{t+\Delta t} = g(t) + W_m + \sigma_W \sqrt{\Delta t} W_m u_{W,t}.$$

We use the parameter values in Table 2 for generating electricity price paths. The (average) present value turns out to be:

$$V = 122,196,833 \text{ £}.$$

With a total investment cost I = 66,000,000 £ (see LGA [10]) the net present value amounts to $V - I$ = 56,196,833 £.

For a now-or-never investment this present value V is equivalent to a fixed feed-in tariff of 70.52 £/MWh. Note in Table 4 that, for $p = 70$, the corresponding values are slightly lower than present value stated here V = 122,196,833 £. So a small increase in the level of p suffices to reach that figure.

4.0.3 The market price plus a fixed premium

Here we assume that the farm owner gets a payment that is composed of the electricity price plus an extra premium for each megawatt-hour generated.[8] Again we run 1,000 simulations with 1,200 steps. Table 5 displays the results.

Each amount in the second column consists of two parts. The first one comes from MC simulation, namely V = 122,196,833. The second is derived as in Table 4; thus, with $p = 50$ we get some 86.6 M£, so with a fraction 0.1 of that p we would get 10 % of that amount, or 8.66 M£. In sum, for a price premium of 5 £/MWh we derive V = 130,860,523 £. Similarly for other premium levels.

5 Valuation and investment timing: Trinomial lattice with mean reversion

The investment time horizon T is subdivided in n steps, each of size $\Delta t = T/n$. Starting from an initial electricity price E_0, in a trinomial lattice one of three possibilities will take place: either the price jumps up (by a factor u to E^+), remains the same ($E^=$), or jumps down (by a factor d to E^-). At time i, after j positive increments, the price is given by $E_0 u^j d^{i-j}$, where $d = 1/u$.

[8] Boomsma et al. [1] consider an initial level of the price premium of 10 €/MWh with an annual percentage increase of 2%.

Table 5. Present value of a wind farm under market price plus a premium.	
Premium (£/MWh)	Present Value V (£)
5	130,860,523
10	139,524,214
15	148,187,904
20	156,851,594
25	165,515,285
30	174,178,975
40	191,506,356
50	208,833,737

Table 6. Formulae for the probabilities in the trinomial lattice.			
Case	p_u	p_m	p_d
Normal	$\frac{1}{6}+\frac{M^2+M}{2}$	$\frac{2}{3}-M^2$	$\frac{1}{6}+\frac{M^2-M}{2}$
High X ($p_u<0$)	$\frac{7}{6}+\frac{M^2+3M}{2}$	$-\frac{1}{3}-M^2-2M$	$\frac{1}{6}+\frac{M^2+M}{2}$
Low X ($p_d<0$)	$\frac{1}{6}+\frac{M^2-M}{2}$	$-\frac{1}{3}-M^2+2M$	$\frac{7}{6}+\frac{M^2-3M}{2}$

Consider an asset whose risk-neutral, seasonally-adjusted behavior follows the differential equation:

$$dE_t = (k_E(E_m - E_t) - \lambda_E E_t)dt + \sigma_E E_t dW_t^E. \tag{10}$$

This can also be written as:

$$dE_t = \left(\frac{k_E(E_m - E_t)}{E_t} - \lambda_E\right) E_t dt + \sigma_E E_t dW_t^E. \tag{11}$$

Since it is usually easier to work with the processes for the natural logarithms of asset prices, we carry out the following transformation: $X = \ln E$. Thus $X_E = 1/E$, $X_{EE} = -1/E^2$, and $X_t = 0$. By Ito's Lemma:

$$dX = (\frac{k_E(E_m - E_t)}{E_t} - \lambda_E - \frac{1}{2}\sigma_E^2)dt + \sigma dZ = \mu_E dt + \sigma_E dZ, \tag{12}$$

where $\mu_E \equiv \frac{k_E(E_m-E_t)}{E_t} - \lambda_E - \frac{1}{2}\sigma_E^2$ depends at each moment on the asset price E_t (so strict notation would read $\mu_E(t)$). See Appendix B for further details on this lattice.

In a trinomial lattice, there are three probabilities p_u, p_m, and p_d associated with a rise, maintenance, and a fall in the (seasonally adjusted) price of electricity. In comparison to a binomial lattice, we can choose the size of the time step Δt so as to avoid negative probabilities. If, despite this choice, they do appear then we adopt the formulae in Table 6.

Now, at the end of the investment horizon (time T) the value of the investment option in each of the final nodes is given by the maximum of two

Table 7. Option value (M£) as a function of the investment cost I (M£).				
	$I = 75$	$I = 100$	$I = 125$	$I = 150$
Investment value	47.2	22.2	-2.8	-27.8
Continuation value	58.9	37.5	18.2	7.7
Option value	58.9	37.5	18.2	7.7

quantities, namely the value of an immediate investment (which presumes that we have not invested yet) and zero. As before, the present value of investing immediately is determined through MC simulation. This means that we run 1,000 simulations of 1,200 steps at each final node. Since the option to invest is akin to a "call" option we denote its value by C:

$$C_T = \max\left[V(i,j), 0\right]$$

At earlier times, however, the option to invest is worth the maximum of two other values: that of investing immediately and that of waiting to invest for one more period (thus keeping the option alive):

$$C = \max\left[V(i,j), (p_u C^{+} + p_m C^{=} + p_d C^{-})e^{-r\Delta t}\right].$$

$V(i,j)$ is derived by simulation at each node. Here the symbols $+$, $=$, and $-$ stand for a rise, no change, and a fall in the price of the asset.

6 Valuation of the option to invest: Case studies

All the cases that follow rest on the same starting values of the underlying variables; see Table 2. We assume that the investment option expires 10 years from now. When building the lattice we take a time step $\Delta t = 1/4$. From Section 3.1 the price change volatility is $\sigma_E = 0.255045$.

6.1 A constant tariff

The argument here is straightforward: all the relevant information is available at the very outset. The (gross) values in Table 4 outweigh the continuation value.

6.2 The market price

Let I denote the present value of all the costs (fixed and variable) incurred by the investment owner over the whole useful life of the wind farm. Table 7 shows the value of investing immediately (NPV) alongside that of investing at the optimal time. The former can take on negative values (it decreases linearly as I increases), while the latter is bounded from below at zero. As usual, the value of the option to invest is the maximum of both amounts (bottom row).

Table 8. Trigger cost I^* (M£) as a function of the subsidy S (M£).

	$S = 5$	$S = 10$	$S = 15$	$S = 20$
I^*	19.4	61.9	98.1	122.4

For low investment costs ($I = 75$ and $I = 100$) the net present value of the immediate investment is positive ($NPV > 0$). Indeed it remains positive as long as $I \leq 122.2$ M£. Therefore, if there is no option to wait the right decision is to rush for the investment provided I does not surpass that threshold. Yet, if the investment can be delayed, investing immediately is far from optimal. For all the investment costs considered in the table, waiting for the optimal time to invest increases the value of the project. In fact, as suggested by the last two columns ($I = 125$ and $I = 150$), the value of waiting can be so high as to turn an otherwise uninteresting project ($NPV < 0$) into an attractive one. Of course, I might rise so high that it renders the option to invest worthless. And conversely, it could be so low that the NPV is higher than the continuation value in which case delay makes no sense. We can resort to continuity arguments and claim that there is some threshold or "trigger" investment cost I^* below which immediate investment becomes optimal. Clearly this I^* is lower than any of the values considered in Table 7.

6.3 The effect of an initial subsidy

Sometimes policy makers grant different subsidies to developers of renewable energy (e.g. to help pay for the capital costs of offshore wind farms). They are meant to enhance the appeal of investments which in their absence would not seem to pay off. The impact of these measures depends on their specific terms and the institutional environment in place. Now we check how the decision to invest reacts to a public subsidy S ranging from 5 M£ to 20 M£ which is only available at the initial time; in other words, if the decision maker opts for postponing the investment the subsidy is foregone. Specifically, we look at the threshold I^* that triggers immediate investment under different values of S. Table 8 displays the numerical results. A subsidy $S = 10$ M£ prompts the option holder to invest immediately whenever the investment cost falls below 61.9 M£. Note, though, that this would not be the case if the subsidy were available at any time over the whole 10-year investment horizon; though not shown in Table 7, even for values of I as low as 25 M£ it is better to wait.

6.4 Sensitivity to changes in the maturity of the option to invest

Intuitively, if the investment option is available over a shorter time frame there is less to be gained from waiting to invest. As a consequence the continuation value will fall and investment will take place earlier. Table 9 shows the impact

Table 9. Option value (M£) as a function of the option's maturity T (years).				
$T = 5$	$I = 75$	$I = 100$	$I = 125$	$I = 150$
Investment Value	47.2	22.2	-2.8	-27.8
Continuation Value	54.7	31.8	11.8	3.3
Option Value	54.7	31.8	11.8	3.3
$T = 2.5$	$I = 75$	$I = 100$	$I = 125$	$I = 150$
Investment Value	47.2	22.2	-2.8	-27.8
Continuation Value	51.5	27.7	7.3	1.0
Option Value	51.5	27.7	7.3	1.0
$T = 1$	$I = 75$	$I = 100$	$I = 125$	$I = 150$
Investment Value	47.2	22.2	-2.8	-27.8
Continuation Value	49.1	24.6	3.7	0.1
Option Value	49.1	24.6	3.7	0.1

Table 10. Trigger investment cost I^* for different subsidies S (M£) with $T = 1$.								
	$S = 1$	$S = 2$	$S = 3$	$S = 4$	$S = 5$	$S = 10$	$S = 15$	$S = 20$
I^* (M£)	26.0	77.8	114.9	119.1	122.5	130.7	136.4	142.0

of a shorter maturity T on the option value for different levels of investment cost I.

As the time that the option is available shortens, the difference between the continuation value (always positive) and the investment value falls. Consider, for example, $I = 100$. With $T = 5$ the difference amounts to 31.8 - 22.2 = 9.6. Instead, with $T = 1$ it drops to 24.6 - 22.2 = 2.4.

A combination of short option maturities and transitory public subsidies only available at $t = 0$ can bring forward investments in wind energy. See Table 10, where expiration of the option is assumed to take place at $T = 1$. Initial subsidies of certain amount are very effective in that they raise the investment threshold below which it is optimal to invest. Note, however, that the marginal effect of each additional monetary unit decreases significantly.

On the other hand, potential improvements in wind technology with their ensuing drops in facilities' costs would lead to delaying investments. Yet this effect could be offset by other factors such as rising financial or personnel costs, or prior occupation of the best sites for wind farms.

6.5 Sensitivity to changes in electricity volatility

Again we consider $T = 10$ and $\Delta t = 1/4$; price volatility in the base case is $\sigma_E = 0.255045$. To the extent that investments in wind energy are highly irreversible the volatility of electricity prices can be anticipated to play a major role. Thus,

Table 11. Option value (M£) as a function of price volatility σ_E.				
$\sigma_E = 0.20$	$I = 75$	$I = 100$	$I = 125$	$I = 150$
Investment Value	47.2	22.2	-2.8	-27.8
Continuation Value	57.6	36.3	16.2	5.1
Option Value	57.6	36.3	16.2	5.1
$\sigma_E = 0.10$	$I = 75$	$I = 100$	$I = 125$	$I = 150$
Investment Value	47.4	22.4	-2.6	-27.6
Continuation Value	55.6	34.4	13.6	1.2
Option Value	55.6	34.4	13.6	1.2

a low volatility pushes in favor of deploying wind turbines while the opposite is true for high volatilities. Unless there are good reasons for assuming that future volatility will deviate significantly from past volatility (e.g. owing to regulatory or structural changes), an initial assessment based on historical volatility seems reasonable.[9]

Volatility can be caused by a number of reasons many of them falling beyond the realm of policy makers. One such example is the price of natural gas in the international markets, as long as it serves sometimes as a reference for establishing the price of electricity (in conjunction with other factors like the emission allowance price in those countries where electric utilities are subject to carbon restrictions).

Regarding the numerical results in Table 11, whatever the value of I assumed, the NPV rises when volatility falls. Thus, for $I = 100$ the NPV goes from 22.2 ($\sigma_E = 0.20$) to 22.4 ($\sigma_E = 0.10$). This effect, however, is not strong enough to offset the incentive to wait: the value of investing immediately falls short of the continuation value in all the cases considered. The reason is that the value of waiting is quite significant. This being clear for $\sigma_E = 0.10$ and $\sigma_E = 0.20$, it is easy to anticipate the results with $\sigma_E = 0.255045$ or even higher volatilities.

Unlike the NPV, the continuation value rises with volatility. Take, for example, $I = 100$; it goes from 34.4 ($\sigma_E = 0.10$) to 36.3 ($\sigma_E = 0.20$). This effect becomes stronger as the investment cost increases. With $I = 150$, it goes from 1.2 ($\sigma_E = 0.10$) to 5.1 ($\sigma_E = 0.20$).

6.6 The market price plus a fixed premium

Consider the case in which the owner of the wind farm receives the market price of electricity augmented by a fixed premium. Table 12 shows the value of the option to invest for two different levels, namely 10 £/MWh and 20 £/MWh.

[9] Nonetheless, Pérez-Arriaga [14] anticipates the volatility of marginal prices to increase in deregulated electricity markets with substantial penetration of renewables.

Table 12. Option value (M£) as a function of the premium (£/MWh).				
premium = 20	$I = 75$	$I = 100$	$I = 125$	$I = 150$
Investment Value	81.8	56.8	31.8	6.8
Continuation Value	89.1	67.3	45.7	24.8
Option Value	89.1	67.3	45.7	24.8
premium = 10	$I = 75$	$I = 100$	$I = 125$	$I = 150$
Investment Value	64.5	39.5	14.5	-10.5
Continuation Value	74.0	52.3	31.0	14.0
Option Value	74.0	52.3	31.0	14.0

For $I = 75$, the presence of a premium raises the value of investing immediately in 64.5 - 47.2 = 17.3 M£ ($p = 10$) and 81.8 - 47.2 = 34.6 M£ ($p = 20$), respectively. However, this does not lead to bringing forward the decision to invest in the wind farm since the continuation values is higher in both cases. In other words, a subsidy granted over the whole useful life with a present value of 17.3 M£ (i.e. more than 25 % of the total disbursement) falls short of triggering the immediate investment when $I = 75$. Note, though, that with a subsidy $S = 15$ M£ which is available only initially the trigger investment cost I^* goes as high as 98.4 M£ (see Table 8). Thus we infer that a subsidy per generated MWh which is spread over the farm's life (20 years) and is available up to the investment option's maturity (10 years) is less effective than a subsidy at $t = 0$ available only at the initial time.

7 Conclusions

We have developed a valuation model for investments in wind energy in deregulated electricity markets when there are futures markets with long maturities. The results are thus focused on developed electricity markets where short- and long-term transactions take place regularly and it is possible to reward wind generation through a 'pure' scheme (i.e. at market rates) or a 'mixed' scheme (with some subsidies).

Looking at the UK futures market we find that contracts on electricity display mean reversion; this in turn has some implications for the valuation model. The parameters underlying the stochastic behavior of prices have been estimated from actual data, including the seasonal effect. We have also estimated another stochastic model (with seasonality) for electricity wind generation at any time as a function of the availability of wind.

The option to invest in a wind farm can be exercised up to some point into the future; thus it is an American-type option. Maximizing its value calls for exercising it at the optimal time. To assess this option we have built a trinomial lattice which supports mean reversion in prices. A new feature (to our knowledge) here is that the values involved in the decision to invest at each node are derived from Monte Carlo simulations where stochastic realizations of electricity price are combined with those of wind availability (and thus generation level) at any time.

Our numerical results show the impact of a number of factors involved in the decision to invest in a wind farm. Among them we have: the investment option's maturity, the initial lump-sum subsidy, the subsidy per unit of electricity generated (feed-in tariff or premium), and price volatility. Different combinations of variables can have an influence in bringing forward the investments in wind generation. One such example is a short decision time frame and an initial subsidy available only for limited time. The results also show the stronger impact of one-time policies, e.g. the $t = 0$ subsidy with respect to a premium per unit produced.

8 Acknowledgements

We gratefully acknowledge the Spanish Ministry of Science and Innovation for financial support through the research project ECO2011-25064, and Fundación Repsol through the Low Carbon Programme joint initiative, http://www.lowcarbonprogramme.org

References

[1] Boomsma, T.K., Meade, N., Fleten S.-T., (2012) Renewable energy investments under different support schemes: A real option approach, European Journal of Operational Research, 220, 225–237.

[2] Campaign to Limit Onshore Windfarm Developments. UK Metered Wind Energy Data. 2011.

[3] Clewlow, L.,& Strickland, C. (1998). Implementing Derivatives Models. Sussex, UK: John Wiley & Sons.

[4] Daim T.U., Amer M., Brenden R. (2012), Technology Roadmapping for wind energy: case of the Pacific Northwest, Journal of Cleaner Production 20(1), 27-37.

[5] European Commission, (2011) Communication from the Commission to the European Parliament, the Council, the European Economic and Social Committee and the Committee of Regions. Energy Roadmap 2050. Available at http://ec.europa.eu/energy/energy2020/roadmap/index_en.htm.

[6] European Wind Energy Association (2010), Wind Barriers: Administrative and grid access barriers to wind power. Brussels, Belgium. Available at: http://www.windbarriers.eu/fileadmin/WB_docs/documents/WindBarriers_report.pdf. Last accessed: October 8th 2012.

[7] Hull, J., White, A. (1994): Numerical procedures for implementing term structure models I: Single-factor models. Journal of Derivatives 2(1), 7-16.

[8] Jacobson M.Z., Archer C.L. (2012): "Saturation wind power potential and its implications for wind energy". PNAS Vol. 109,No. 39, 15679–15684.

[9] Klessmann C., Nabe C., Burges K. (2008), Pros and cons of exposing renewables to electricity market risks - A comparison of the market integration approaches in Germany, Spain, and the UK. Energy Policy 36, 3646-3661.

[10] Local Government Association (2012): How much do wind turbines cost and where can I get funding? Available at: http://www.local.gov.uk/web/guest/home/-/journal_content/

[11] Marvel K., Kravitz B., Caldeira C. (2012): "Geophysical limits to global wind power". Nature Climate Change, published on line 9 September.

[12] National Renewable Energy Laboratory (2010), Western Wind and Solar Integration Study. Prepared by GE Energy. Available from: http://www.nrel.gov/docs/fy10osti/47781.pdf. Last Accessed: October 8th 2012.

[13] Ortegon K., Nies L.F., Sutherland J.W. (2012) Preparing for end of service life of wind turbines, Journal of Cleaner Production, article in press. Retrieved: October 2012.

[14] Pérez-Arriaga, I.J. (2011) Managing large scale penetration of intermittent renewables, 2011 MITEI Symposium, Framework Paper, Cambridge MA.

[15] Pérez-Arriaga, I.J., Batlle, C. (2012) Impacts of Intermittent Renewables on Eelectricity Generation System Operation, Economics of Energy & Environmental Policy, 1(2), 3-17.

[16] Reuter W.H., Szolgayová J., Fuss S., Obersteiner M. (2012), Renewable energy investment: Policy and market impacts, Applied Energy 97, 249-254.

[17] Snyder B., Kaiser M.J. (2009), A comparison of offshore wind power development in Europe and the US: Patterns and drivers of development. Applied Energy 86(10), 1845-1856.

A Formal test of the GBM hypothesis for the electricity price

As already mentioned, we have estimated the parameters underlying the stochastic model using all the futures prices on each day by non-linear least-squares. Table 2 shows the results. All the estimates are statistically significant. We get a coefficient of determination $R^2 = 0.8579$; the log-likelihood of this (alternative) model is -64,707.64. We have also estimated a geometric Brownian model (GBM), which is a special case of the above mean-reverting model (i.e., it is the null model where $\frac{k_E E_m}{k_E+\lambda_E} = 0$). In this case we get an $R^2 = 0.7803$ and a log-likelihood of -70,386.18.

Availability of the log likelihoods from both models allows us to perform a likelihood ratio test. We compute a likelihood-ratio

$$LR = -2(LL \text{ GBM Model} - LL \text{ Mean Reverting Model}).$$

In our case we have $LR = +2(70{,}386.18 - 64{,}707.64) = 11{,}357.08$. The probability of this difference is approximately that of a chi-squared distribution with one degree of freedom. Upon computation of the test statistic and the associated p-value we reject the null hypothesis that $\frac{k_E E_m}{k_E+\lambda_E} = 0$. In sum, the mean reverting model represents a significant improvement with respect to the GBM process.

B Setting up the trinomial lattice

In a trinomial lattice, there are three probabilities p_u, p_m, and p_d associated with a rise, maintenance, and a fall in the (seasonally adjusted) price of electricity. Following Euler-Maruyama's discretization, these probabilities must satisfy three conditions:

a) $p_u + p_m + p_d = 1$.

b) $E(\Delta X) = p_u \Delta X + p_m \times 0 - p_d \Delta X = \mu_E \Delta t$. The aim is to equate the first moment of the binomial lattice ($p_u \Delta X - p_d \Delta X$) to the first moment of the risk-neutral underlying variable ($\mu_E \Delta t$).

c) $E(\Delta X^2) = p_u \Delta X^2 + p_m \times 0 + p_d \Delta X^2 = \sigma_E^2 \Delta t + \mu_E^2 (\Delta t)^2$. In this case the equality refers to the second moments. For small values of Δt, we have $E(\Delta X^2) \approx \sigma_E^2 \Delta t$.

Solving the system for the three probabilities (Hull and White [7]) we get:

$$\begin{aligned}
p_u &= \frac{1}{2}\left[\frac{\sigma_E^2 \Delta t + \mu_E^2 (\Delta t)^2}{(\Delta X)^2} + \frac{\mu_E \Delta t}{\Delta X}\right], \\
p_m &= 1 - \frac{\sigma_E^2 \Delta t + \mu_E^2 (\Delta t)^2}{(\Delta X)^2}, \\
p_d &= \frac{1}{2}\left[\frac{\sigma_E^2 \Delta t + \mu_E^2 (\Delta t)^2}{(\Delta X)^2} - \frac{\mu_E \Delta t}{\Delta X}\right].
\end{aligned}$$

The particular values depend on μ_E, which changes from one node to the next. Specifically:

$$\mu_E(i,j) \equiv \frac{k_E(E_m - E(i,j))}{E(i,j)} - \lambda_E - \frac{1}{2}\sigma_E^2. \tag{13}$$

So the three probabilities also change from one node to the next.

In a trinomial lattice after n periods we have $2n+1$ final nodes. This holds true irrespective of the initial (deseasonalised) electricity price. In case of investing in the wind farm, we would compute its value over the 20-year period of life. The present value $V(i,j)$ of immediate investment (i.e. we are in the now-or-never setting) will be derived by Monte Carlo simulation following the above approach starting from the electricity price in place $E(i,j)$ and calendar time $t = i\Delta t$. Thus we run 1,000 simulations with 1,200 steps at each node (i,j) throughout the lattice.

In a trinomial lattice, as compared to a binomial one, there is an additional degree of freedom (there is a third possibility -the price to stay the same- while the three conditions remain unchanged). Thus we can choose the size of the time step Δt; it is particularly convenient to choose its value in such a way that negative probabilities are avoided. Given that a trinomial lattice is basically an explicit difference scheme (Clewlow and Strickland [3]), convergence and stability reasons suggest to adopt $\Delta X = \sigma_E \sqrt{3\Delta t}$ (Hull and White [7]). In this case:

$$
\begin{aligned}
p_u &= \frac{1}{6} + \frac{M^2 + M}{2}, \quad \text{where } M \equiv \frac{\mu_E \Delta t}{\sigma_E \sqrt{3\Delta t}}, \\
p_m &= \frac{2}{3} - M^2, \\
p_d &= \frac{1}{6} + \frac{M^2 - M}{2}.
\end{aligned}
$$

When, in principle, $p_u < 0$, the three possibilities that we choose for the asset price are: stay unchanged, fall by $-\Delta X$, and fall by $-2\Delta X$ in which case:

a) $p_u + p_m + p_d = 1$.

b) $E(\Delta X) = p_u \times 0 - p_m \times \Delta X - 2p_d \Delta X = \mu_E \Delta t$.

c) $E(\Delta X^2) = p_u \times 0 + p_m \times \Delta X^2 + 4p_d \Delta X^2 = \sigma_E^2 \Delta t + \mu_E^2 (\Delta t)^2$.

The solution is then:

$$
\begin{aligned}
p_u &= \frac{7}{6} + \frac{M^2 + 3M}{2}, \\
p_m &= -\frac{1}{3} - M^2 - 2M, \\
p_d &= \frac{1}{6} + \frac{M^2 + M}{2}.
\end{aligned}
$$

If, instead, we have $p_d < 0$, then the price can either remain the same, rise by ΔX, and rise by $2\Delta X$. In this case:

a) $p_u + p_m + p_d = 1$.

b) $E(\Delta X) = p_u \times 2\Delta X + p_m \times \Delta X + p_d \times 0 = \mu_E \Delta t$.

c) $E(\Delta X^2) = 4p_u \Delta X^2 + p_m \times \Delta X^2 + p_d \times 0 = \sigma_E^2 \Delta t + \mu_E^2 (\Delta t)^2$.

The probabilities that solve this system are:

Table 6. Formulae for the probabilities in the trinomial lattice.			
Case	p_u	p_m	p_d
Normal	$\frac{1}{6}+\frac{M^2+M}{2}$	$\frac{2}{3}-M^2$	$\frac{1}{6}+\frac{M^2-M}{2}$
High X ($p_u<0$)	$\frac{7}{6}+\frac{M^2+3M}{2}$	$-\frac{1}{3}-M^2-2M$	$\frac{1}{6}+\frac{M^2+M}{2}$
Low X ($p_d<0$)	$\frac{1}{6}+\frac{M^2-M}{2}$	$-\frac{1}{3}-M^2+2M$	$\frac{7}{6}+\frac{M^2-3M}{2}$

$$\begin{aligned}
p_u &= \frac{1}{6}+\frac{M^2-M}{2}, \\
p_m &= -\frac{1}{3}-M^2+2M, \\
p_d &= \frac{7}{6}+\frac{M^2-3M}{2}.
\end{aligned}$$

Table 6 summarizes the above formulae.

BC3 WORKING PAPER SERIES

Basque Centre for Climate Change (BC3), Bilbao, Spain

The BC3 Working Paper Series is available on the internet at the following addresses:

http://www.bc3research.org/lits_publications.html

http://ideas.repec.org/s/bcc/wpaper.html

BC3 Working Papers available:

2011-11 Roger Fouquet: *The Demand for Environmental Quality in Driving Transitions to Low Polluting Energy Sources*

2011-12 Joseph V. Spadaro: *The Uniform World Model: A Methodology for Predicting the Health Impacts of Air Pollution*

2011-13 Thomas Heyd: *Climate Change, Responsibilities, and Defeatism and Complacency*

2012-01 Roger Fouquet: *Trends in Income and Price Elasticities of Transport Demand (1850-2010)*

2012-02 Sébastien Foudi: *Exploitation of soil biota ecosystem services in agriculture: a bioeconomic approach*

2012-03 Mikel González-Eguino, Anil Markandya and Marta Escapa: *From shadow to green: Linking environmental fiscal reform and the informal economy*

2012-04 Anil Markandya, Mikel González-Eguino and Marta Escapa: *Environmental fiscal reform and unemployment in Spain*

2012-05 Dirk Rübbelke and Stefan Vögele: *Effects of Carbon Dioxide Capture and Storage in Germany on European Electricity Exchange and Welfare*

2012-06 Eneko Garmendia and Gonzalo Gamboa: *Weighting social preferences in participatory multi-criteria evaluations: a case study on sustainable natural resource management.*

2012-07 Ferdinando Villa, Ken Bagstad, Gary Johnson, Brian Voigt: *Towards a Comprehensive Approach to Quantifying and Mapping Ecosystem Services*

2012-08 Roger Fouquet: *Economics of Energy and Climate Change: Origins, Developments and Growth*

2012-09 Maria-Angeles Diez, Iker Etxano, Eneko Garmendia: *Evaluating Governance and Participatory Processes in Natura 2000: Lessons Learned and Guidance for Future Prospects*

2012-10 Iker Etxano, Eneko Garmendia, Unai Pascual. David Hoyos, Maria-Angeles Diez, José A. Cadiñanos, Pedro J. Lozano: *Towards a Participatory Integrated Assessment Approach for Planning and Managing Natura 2000 Network Sites*

2012-11 Luis M. Abadie, José M. Chamorro: *Valuation of wind energy projects: A real options approach*