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The Effect of Data Revisions on the Basic New Keynesian Model

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Abstract

This paper proposes an extended version of the basic New Keynesian monetary (NKM) model which contemplates revision processes of output and inflation data in order to assess the importance of data revisions on the estimated monetary policy rule parameters and the transmission of policy shocks. Our empirical evidence based on a structural econometric approach suggests that although the initial announcements of output and inflation are not rational forecasts of revised output and inflation data, ignoring the presence of non well-behaved revision processes may not be a serious drawback in the analysis of monetary policy in this framework. However, the transmission of inflation-push shocks is largely affected by considering data revisions. The latter being especially true when the nominal stickiness parameter is estimated taking into account data revision processes.

JEL Classification: NKM model, monetary policy rule, indirect inference, real-time data, (non-)rational forecast error
Keywords: C32, E30, E52

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1 Introduction

The importance of the timing and availability of data used in the empirical evaluation of policy rules has now become a crucial issue. While several studies have shown the considerable magnitude of the revision processes for key macroeconomic variables, there is large disagreement on whether considering or not these revisions has a significant effect on estimated monetary policy rules. However, no paper that we are aware of considers simultaneously the dynamics of revision processes with their impact on estimated monetary policy rules in a structural model. Therefore, in this paper we add to the literature by proposing an extended version of the New Keynesian monetary (NKM) model that includes revision processes of output and inflation data to assess the importance of data revisions on the estimated monetary policy rule parameters and the transmission of policy shocks.

One of the first studies to investigate the properties of revision process errors is Diebold and Rudebusch (1991). They show that the index of leading indicators does a much worse job in predicting future movements of output in real time than it does after data are revised. More recently, Aruoba (2008) investigates the empirical properties of revisions to major macroeconomic variables in the U.S. and finds out that they are not well-behaved. That is, they do not satisfy simple desirable properties such as zero mean, which indicates that the revisions of initial announcements made by statistical agencies are biased, and they might be predictable using the information set available at the time of the initial announcement. Moreover, Aruoba (2008) points out that if revisions of real-time data were rational forecast errors then the arrival of revised data would not be relevant for policy makers' decisions, and policy rule estimates would be rather similar regardless of whether revised or real-time data were used.

The impact of revision processes over the empirical evaluation of monetary policy has been largely documented in the literature.[1] An early study by Maravall and Pierce

[1] See Croushore (2011) for an excellent survey of the importance of data revisions not only on

(1986) investigates how preliminary and incomplete data affect monetary policy.[2] More precisely, they compare the results of a SURE system for the M1 and the Federal funds rate targets where the set of common explanatory variables includes the preliminary estimate of the rate of growth of seasonally adjusted M1 instead of its final revised data. Their empirical results show that revision errors have little impact on the setting targets if the Fed indeed ignores erratic, short term volatility in the rate of growth of seasonally adjusted M1 that is uncorrelated with the final revised ("true") variable. In short, they show that even if revisions to measures of money supply are large, monetary policy would not be much different if more accurate data were known.[3] More recently, Croushore and Evans (2006) present evidence suggesting that the use of revised data in VAR analyses of monetary policy may not be a serious limitation for recursively identified systems. However, their analysis also reveals that many simultaneous VAR systems identifiable when real-time data issues are ignored cannot be completely identified when these measures are considered.[4]

One of the best-known studies comparing results based on real-time data with those obtained with revised data in the context of monetary policy analysis is Orphanides (2001), which examines parameter as well as model specification uncertainty in the Taylor-rule by using data over a period of more than 20 years. The paper concludes that the Taylor principle does not hold when real-time data are used. This empirical evidence is in sharp contrast with that found in many papers that use only revised data (for instance, Clarida, Galí and Gertler (2000)). Moreover, Ghysels,

estimated policy rules, but more generally on structural modeling and forecasting.

[2] Another seminal paper is Mankiw, Runkle and Shapiro (1984). They develop a theoretical framework for analyzing initial announcements of economic data and apply that framework to the money stock.

[3] Maravall and Pierce (1986) base their reduced-form econometric analysis on the assumption that the initial release and the final revision of a variable are orthogonal (Assumption B in their paper). This assumption implies that the initial announcement of a variable is the (rational) optimal forecast of its final revised value and the final data revision is not predictable using the available information at the time the initial release was made. By contrast, our analysis allows for the possibility that data revisions are predictable.

[4] There are also a few articles reporting empirical evidence for the implications of using real-time macroeconomic data for research in empirical finance (Christoffersen, Ghysels and Swanson, 2002; Evans and Speight, 2006; Kizys and Pierdzioch, 2010).

Swanson and Callan (2002) find that a Taylor-type rule would have been significantly improved if policymakers had waited for data to be revised rather than reacting to newly released data. With regard to Taylor-rule parameters, Rudebusch (2002) indicates that data uncertainty potentially plays an important role in reducing the coefficients of the rule that characterize both policy inertia and shock persistence.[5] The main advantage of using real-time data in estimating policy rules is to reduce the effects of parameter uncertainty in actual policy settings since the researcher can estimate policy rules with data which were actually available at any given point in time. This is particularly important with seasonally adjusted data as such those subject to revisions based on two-sided filters.[6]

All the aforementioned literature on estimating policy rules with real-time data either uses reduced-form econometric approaches or VAR structural approaches. As an alternative, this paper adds to the literature by building on the basic New Keynesian monetary (NKM) model to include revision processes of output and inflation data. To the best of our knowledge, this is the first paper to analyze revised and real-time data together using a structural econometric approach. This approach allows for (i) a joint estimation procedure of both monetary policy rule and revision process parameters; (ii) an assessment of the interaction between these two sets of parameters; (iii) an alternative test of the null hypothesis establishing that real-time data are a rational forecast of revised data in the context of a dynamic structural general equilibrium (DSGE) model; and (iv) an analysis of how the reaction of the Fed funds rate to alternative shocks is affected by allowing for badly-behaved revision processes.

The use of real-time data in the estimation of a DSGE model may look tricky be-

[5] By using reduced-form estimation approaches some empirical studies, such as English, Nelson and Sack (2003) and Gerlach-Kristen (2004) have shown that both persistent shocks and policy inertia enter the U.S. estimated monetary policy rule. María-Dolores and Vázquez (2006, 2008) obtain similar results for the U.S. and the Eurozone using an econometric structural approach.

[6] Kavajecz and Collins (1995) conclude, using Monte Carlo simulations, that irrationality in seasonally adjusted data arises from the specific seasonal adjustment procedure used by the Federal Reserve.

cause decisions by private agents (households and firms) determine the true (revised) values of macroeconomic variables, such as output and inflation. However, these variables are not observable without error by policymakers in real time. The problem is easily solved by augmenting the NKM model with the revision processes of output and inflation. In particular, these revision processes are allowed to be determined by the information available at the time when the initial announcements of output and inflation are released.

The availability of real-time information motivates another distinctive feature of the augmented NKM model analyzed in this paper. In the model, the monetary policy rule has both backward- and forward-looking components. Apart from the standard policy inertia component, the backward-looking part of the policy rule captures the fact that the initial announcements of output and inflation are available to the Fed with a lag. The forward-looking components capture the idea that the Fed may take into account the possibility that real-time data are not rational forecasts of revised data and then the initial announcements may contain useful information for predicting future revisions of actual data introduced by statistical agencies.

We follow a classical approach based on the indirect inference principle suggested by Smith (1993, 2008) to estimate our extended version of the NKM model. In particular, we follow Smith (1993) by using an unrestricted VAR as the auxiliary model. More precisely, the distance function is built upon the coefficients estimated from a five-variable VAR that considers U.S. quarterly data on revised output growth, revised inflation, real-time output growth, real-time inflation and the Fed funds rate.

The estimates of the revision process parameters show that the initial announcements of output and inflation are not rational forecasts of revised data on output and inflation. For instance, a 1% increase in the initial announcement of inflation leads to a downward revision in output of -1.36%. These estimation results are in line with the empirical evidence provided by Aruoba (2008) mentioned above, who finds that data revisions are not well-behaved (i.e. they are not white noise processes).

However, the estimation results also provide evidence that the estimates of the monetary policy rule parameters are not too sensitive to allowing for the possibility of non-rational revision processes. Moreover, the impulse-response analysis shows that ignoring the presence of badly-behaved revision processes is not a serious drawback in the analysis of monetary policy. Only the responses to an inflation-push shock are sensitive to allowing for the presence of badly-behaved revision processes. The latter being especially true when the price stickiness parameter is not fixed in the estimation procedure.

The rest of the paper is organized as follows. Section 2 introduces the log-linearized approximation of an augmented version of the NKM model that includes the revision processes for output and inflation. Section 3 describes the structural estimation method used in this paper. Section 4 describes the data and discusses the estimation results. Section 5 concludes.

2 AN NKM MODEL AUGMENTED WITH DATA REVISION PROCESSES

The model analyzed in this paper is a basic NKM model augmented with data revision processes. We focus our attention on a simple version of the NKM model instead of a medium-scale NKM model such as in Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007) for two main reasons. First, our goal is to illustrate how monetary policy analysis is affected by allowing for deviations from well-behaved revision processes in a simple framework, i.e. without adding too much structure and too many restrictions in the characterization of the private sector of the economy. Second, by considering a basic NKM model we can deal with a small set of observable (revised and real-time) variables and treat all parameters characterizing private agent decisions as fixed in order to focus on the characterization of monetary policy and revision process parameters.

The augmented NKM model considered in this paper is given by the following set

of equations:

$$x_t = E_t x_{t+1} - \tau(i_t - E_t \pi_{t+1}) - \phi(1 - \rho_\chi)\chi_t, \tag{1}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + z_t, \tag{2}$$

$$i_t = \rho i_{t-1} + (1-\rho)[\psi_1(\pi^r_{t-1} + E_t r^\pi_{t-1}) + \psi_2(x^r_{t-1} + E_t r^x_{t-1})] + v_t, \tag{3}$$

$$x_t \equiv x^r_t + r^x_t, \tag{4}$$

$$\pi_t \equiv \pi^r_t + r^\pi_t, \tag{5}$$

$$r^x_t = b_{xx} x^r_t + b_{x\pi} \pi^r_t + \epsilon^r_{xt}, \tag{6}$$

$$r^\pi_t = b_{\pi x} x^r_t + b_{\pi\pi} \pi^r_t + \epsilon^r_{\pi t}, \tag{7}$$

where x denotes revised output gap (that is, the log-deviation of output with respect to the level of output under flexible prices), π and i denote the deviations from the steady states of revised inflation and nominal interest rate, respectively. E_t denotes the conditional expectation based on the agent information set at time t. π^r_t and x^r_t are real-time data for inflation and output gap, respectively. r^π_t and r^x_t are the (final) revisions associated with inflation and output gap, respectively. Notice that the subindexes of π^r_t (x^r_t) and r^π_t (r^x_t) are associated with the period in which the corresponding revised value π_t (x_t) is determined by private agent decisions. Thus, as discussed below, the initial announcements π^r_t (x^r_t) arrive with a lag and the revisions r^π_t (r^x_t) can take several periods to be released by statistical agencies. This fact explains why the conditional expectation operator also applies to r^π_t and r^x_t in (3). χ, z and v denote aggregate productivity, cost-push inflation and monetary policy shocks, respectively. Each of these shocks is further assumed to follow a first-order autoregressive process as follows:

$$\chi_t = \rho_\chi \chi_{t-1} + \epsilon_{\chi t}, \tag{8}$$

$$z_t = \rho_z z_{t-1} + \epsilon_{zt}, \tag{9}$$

$$v_t = \rho_v v_{t-1} + \epsilon_{vt}, \tag{10}$$

where $\epsilon_{\chi t}$, ϵ_{zt} and ϵ_{vt} denote i.i.d. random innovations associated with these shocks and their standard deviations are denoted by σ_χ, σ_z and σ_v, respectively.

Equation (1) is the log-linearized consumption first-order condition obtained from the representative agent optimization plan. The parameter $\tau > 0$ represents the intertemporal elasticity of substitution obtained when assuming a standard constant relative risk aversion utility function. $\phi = (1+\eta)/(\tau^{-1}+\eta)$, where η denotes the Frisch elasticity.

Equation (2) is the standard New Phillips curve that is obtained in a sticky price model *à la* Calvo (1983) where monopolistically competitive firms produce (a continuum of) differentiated goods and each firm faces a downward sloping demand curve for its produced good. The parameter $\beta \in (0,1)$ is the agent discount factor, and κ measures the slope of the New Phillips curve, which is related to other structural parameters as follows:

$$\kappa = \frac{[(1/\tau)+\eta](1-\omega)(1-\omega\beta)}{\omega},$$

where ω denotes Calvo's probability. In particular, κ is a decreasing function of ω. The parameter ω is a measure of the degree of nominal rigidity. A larger ω implies that fewer firms adjust prices in each period and that the expected time between price changes is longer.[7] At this point, it is worthwhile emphasizing that the IS and Phillips curve equations are described in terms of the revised output and inflation data since they are indeed determined by the optimal choices of private agents (households and firms).

Equation (3) describes a Taylor-type monetary policy rule (Taylor, 1993). In contrast to Equations (1) and (2), Equation (3) only considers real-time data on output and inflation actually available at the time of implementation of monetary policy. As pointed out by Aruoba (2008), the initial announcement of quarterly

[7] See, for instance, Walsh (2003, chapter 5.4) for a detailed analytical derivation of the New Phillips curve and the flexible-price level of output considered below.

(monthly) macroeconomic variables corresponding to a particular quarter (month) appears in the vintage of the next quarter (month), roughly 45 (at least 15) days after the end of the quarter (month). Since the initial announcements might not be rational forecasts of revised data, the Fed may take into account this feature to predict the actual revisions of these announcements. Notice that according to equation (3) and taking into account equations (4) and (5) described below, the Fed is assumed to react to expected revised values of inflation ($E_t\pi_{t-1} \equiv \pi^r_{t-1} + E_t r^{\pi}_{t-1}$) and output ($E_t x_{t-1} \equiv x^r_{t-1} + E_t r^x_{t-1}$). Moreover, the nominal interest rate exhibits smoothing behavior captured by the size of ρ.

The NKM model is extended to incorporate the revision processes of output and inflation data, respectively. Equations (4) and (5) are identities showing how revised data of output (x_t) and inflation (π_t) are related to the initial announcements of output (x^r_t) and inflation (π^r_t), respectively. Then, r^x_t (r^{π}_t) denotes the revision of output (inflation). By adding the log of potential output (i.e. the level of output under flexible prices) on both sides of (4), we have that r^x_t also denotes the revision of the log of output. Equations (6) and (7) describe the revision processes associated with output and inflation, respectively. These processes allow for the existence of non-zero correlations between output and inflation revisions and the initial announcements of these variables.[8] ϵ^r_{xt} and $\epsilon^r_{\pi t}$ denote i.i.d. random innovations associated with the revision processes where the corresponding standard deviations are denoted by σ^r_x and σ^r_{π}, respectively. Furthermore, notice that equations (6) and (7) imply that

$$r^x_t = E_{t+1} r^x_t + \epsilon^r_{xt} = b_{xx} x^r_t + b_{x\pi} \pi^r_t + \epsilon^r_{xt},$$

$$r^{\pi}_t = E_{t+1} r^{\pi}_t + \epsilon^r_{\pi t} = b_{\pi x} x^r_t + b_{\pi\pi} \pi^r_t + \epsilon^r_{\pi t}.$$

These two equations further motivate the timing assumption used in the forward-looking components of the monetary policy rule (3). That is, the initial announcements of the variables determined at time t will be released at time $t+1$.

[8] The two revision processes assumed are not intended to provide a structural characterization of the revision processes followed by statistical agencies, but to provide a simple framework to assess whether the nature of the revision processes might affect the estimated monetary policy rule.

Finally, the model is completed by the following identities involving forecast errors:

$$x_t = E_{t-1}x_t + (x_t - E_{t-1}x_t),$$

$$\pi_t = E_{t-1}\pi_t + (\pi_t - E_{t-1}\pi_t).$$

The system of equations (1)-(10) together with the latter four identities can be written in matrix form as follows:

$$\Gamma_0 Y_t = \Gamma_1 Y_{t-1} + \Psi\epsilon_t + \Pi\eta_t, \tag{11}$$

$$Y_t = (x_t, \pi_t, i_t, E_t x_{t+1}, E_t\pi_{t+1}, \chi_t, z_t, v_t, x_t^r, \pi_t^r, r_t^x, r_t^\pi, E_{t+1}r_t^x, E_{t+1}r_t^\pi)',$$

$$\epsilon_t = (\epsilon_{\chi t}, \epsilon_{zt}, \epsilon_{vt}, \epsilon_{xt}^r, \epsilon_{\pi t}^r)',$$

$$\eta_t = (x_t - E_{t-1}x_t, \pi_t - E_{t-1}\pi_t)'.$$

Equation (11) represents a linear rational expectations system that can be solved using standard routines.[9,10] The model's solution yields the output gap, x_t. This measure is not observable. In order to estimate the model by simulation, the output gap must be transformed into a measure that has an observable counterpart such as output growth. This is a quite straightforward exercise since the log-deviation of output from its steady state can be defined as the output gap plus the (log of the) flexible-price equilibrium level of output, y_t^f, and the latter can be expressed as a linear function of the productivity shock:

$$y_t^f = \phi\chi_t.$$

[9] Alternatively, the matrix system (11) can be expressed by eliminating variables $E_{t+1}r_t^x$ and $E_{t+1}r_t^\pi$ from Y_t and the associated identities $r_t^x = E_{t+1}r_t^x + \epsilon_{txt}^r$ and $r_t^\pi = E_{t+1}r_t^\pi + \epsilon_{\pi t}^r$ by substituting the identities $E_t r_{t-1}^x = b_{xx}x_{t-1}^r + b_{x\pi}\pi_{t-1}^r$ and $E_t r_{t-1}^\pi = b_{\pi x}x_{t-1}^r + b_{\pi\pi}\pi_{t-1}^r$ into the policy rule (3).

[10] We use the solution algorithm suggested by Lubik and Schorfheide (2003). The matrices in equation (11) are described in detail in the Appendix.

The log-deviation of output from its steady state is also unobservable. However, the growth rate of output is observable and its model counterpart is obtained from the first-difference of the log-deviation of output from its steady state.

Similarly, the solution of the model yields the deviations in inflation and interest rate from their respective steady states. In order to obtain the levels of inflation and nominal interest rate, we first calibrate the steady-state value of inflation as the sample mean of the inflation rate. Second, using the calibrated value of steady-state inflation and the definition of the steady-state value of real interest rate, the steady-state value of the nominal interest rate can be easily computed. Third, the level of the nominal interest rate is obtained by adding the deviation (from its steady-state value) of the nominal rate to its steady-state value computed in the previous step. Finally, since a period is identified with a quarter and the nominal interest rate is then measured in quarterly values, the quarterly interest rate is transformed into an annualized value as in the actual data.

3 ESTIMATION PROCEDURE

In order to carry out a joint estimation of the NKM model augmented with the revision processes using both revised and real-time data, we follow a classical approach based on the indirect inference principle suggested by Smith (1993, 2008). In particular, we follow Smith (1993) by first using an unrestricted VAR as the auxiliary model. More precisely, the distance function is built upon the coefficients estimated from a five-variable VAR with four lags that considers U.S. quarterly data on revised output growth, revised inflation, real-time output growth, real-time inflation and the Fed funds rate. The lag length considered is fairly reasonable when using quarterly data. Second, we apply the simulated moments estimator (SME) suggested by Lee and Ingram (1991) and Duffie and Singleton (1993) to estimate the parameters of the

model. In this context, we believe that it is useful to consider an unrestricted VAR (which imposes mild restrictions) as the auxiliary model, which lets the data speak more freely than other estimation approaches such as maximum-likelihood.[11]

This estimation procedure starts by constructing a $p\times 1$ vector with the coefficients of the VAR representation obtained from actual data, denoted by $H_T(\theta_0)$ where p in this application is 120. We have 105 coefficients from a four-lag, five-variable system and 15 extra coefficients from the non-redundant elements of the variance-covariance matrix of the VAR residuals. T denotes the length of the time series data, and θ is a $k \times 1$ vector whose components are the model parameters. The true parameter values are denoted by θ_0. Since our main goal is to estimate the policy rule and revision process parameters, we split the model parameters into two groups prior to estimation. The first group is formed by the pre-assigned structural parameters β, τ, η and ω. We set $\beta = 0.995$, $\tau = 0.5$, $\eta = 2.0$ and $\omega = 0.75$, corresponding to standard values assumed in the relevant literature for the discount factor, intertemporal elasticity of consumption, the Frisch elasticity and Calvo's probability, respectively.[12] The second group, formed by policy, shock and revision process parameters, is the one being estimated. In the augmented NKM model, the estimated parameters are $\theta = (\rho, \psi_1, \psi_2, \rho_\chi, \rho_z, \rho_v, b_{xx}, b_{x\pi}, b_{\pi x}, b_{\pi\pi}, \sigma_\chi, \sigma_z, \sigma_v, \sigma_x^r, \sigma_\pi^r)$ and then $k = 15$.

As pointed out by Lee and Ingram (1991), the randomness in the estimator is derived from two sources: the randomness in the actual data and the simulation. The importance of the randomness in the simulation to the covariance matrix of the estimator is decreased by simulating the model a large number of times. For each simulation a $p \times 1$ vector of VAR coefficients, denoted by $H_{N,i}(\theta)$, is obtained from the simulated time series of output growth, inflation and the Fed funds interest rate generated from the NKM model, where $N = nT$ is the length of the simulated data. By averaging the m realizations of the simulated coefficients, i.e. $H_N(\theta) =$

[11] For a detailed description of this estimation procedure see María-Dolores and Vázquez (2006, 2008).

[12] We also run our estimation procedure by considering Calvo's probability, ω, as a free parameter.

$\frac{1}{m}\sum_{i=1}^{m} H_{Ni}(\theta)$, we obtain a measure of the expected value of these coefficients, $E(H_{Ni}(\theta))$. The choice of values for n and m deserves some attention. Gouriéroux, Renault and Touzi (2000) suggest that it is important for the sample size of synthetic data to be identical to T (that is, $n = 1$) to get a finite sample bias of identical size in estimators of the auxiliary parameters computed from actual and synthetic data. We make $n = 1$ and $m = 500$ in this application. To generate simulated values of (revised and real-time) output growth, (revised and real-time) inflation and interest rate time series we need the starting values of these five variables. For the SME to be consistent, the initial values must have been drawn from a stationary distribution. In practice, to avoid the influence of starting values, we generate a realization from the stochastic processes of the five variables of length $200 + T$, discard the first 200 simulated observations, and use only the remaining T observations to carry out the estimation. After 200 observations have been simulated, the influence of the initial conditions must have disappeared.

The SME of θ_0 is obtained from the minimization of a distance function of VAR coefficients from actual and synthetic data. Formally,

$$\min_{\theta} \; J_T(\theta) = [H_T(\theta_0) - H_N(\theta)]' W [H_T(\theta_0) - H_N(\theta)],$$

where W is the optimal weighting matrix containing the inverse of the covariance matrix associated with the VAR coefficients and the non-redundant elements of the covariance matrix of the VAR residuals.

Denoting the solution of the minimization problem by $\hat{\theta}$, Lee and Ingram (1991) and Duffie and Singleton (1993) prove the following asymptotic results:

$$\sqrt{T}(\hat{\theta} - \theta_0) \rightarrow N\left[0, \left(1 + \frac{1}{m}\right)(B'WB)^{-1}\right],$$

$$\left(1 + \frac{1}{m}\right) T J_T(\hat{\theta}) \rightarrow \chi^2(p - k), \tag{12}$$

where B is a full rank matrix given by $B = E(\frac{\partial H_{Ni}(\theta)}{\partial \theta})$.

The objective function J_T is minimized using the optimization package OPTMUM programmed in GAUSS language. We apply the Broyden-Fletcher-Goldfard-Shanno algorithm. To compute the covariance matrix we need to obtain B. Computation of B requires two steps: first, obtaining the numerical first derivatives of the coefficients of the VAR representation with respect to the estimates of the structural parameters θ for each of the m simulations; second, averaging the m-numerical first derivatives to get B.

4 DATA AND ESTIMATION RESULTS

We consider quarterly U.S. data for the growth rate of output, the inflation rate obtained from the first-difference of the log of the implicit GDP deflator and the Fed funds rate during the post-Volcker period (1983:1-2008:1). In addition, we consider real-time data on output growth and inflation as reported by the Federal Reserve Bank of Philadelphia.[13,14] Figure 1 shows the five-time series considered in the paper.

We focus on the post-Volcker period for two main reasons. First, the Taylor rule seems to fit better in this period than in the pre-Volcker era. Second, considering the pre-Volcker era opens the door to many issues studied in the relevant literature, such as the presence of macroeconomic switching regimes and the existence of switches in monetary policy (see, for instance, Sims and Zha, 2006), which are beyond the scope of this paper.

In this section we first motivate the inclusion of real-time data in the estimated policy rule. In particular, we first use a reduced form approach to analyze whether

[13] See Croushore and Stark (2001) for the details of the real-time data set.

[14] We follow Aruoba (2008) by considering the growth rates of the initial announcement of real GDP and GDP deflator in order to isolate our analysis from the presence of benchmark revisions, which are defined as the changes introduced by statistical agencies when they change their methodologies or make statistical changes such as change of base years or seasonal weights. As pointed out by Arouba (2008), benchmark revisions are problematic for the users of the data because they contaminate real time information available at each point of time. More precisely, the benchmark revisions for GDP and GDP deflator take place about every five years. Given our 25-year sample, the GDP growth and the inflation rates are contaminated each one with only five jumps due to benchmark revisions. We eliminate each of these jumps by substituting the jumping value of the corresponding variable with the average value obtained from the two observations released just before and after the jump.

real-time data are rational forecasts of revised data. In the second subsection, we use our structural estimation approach to study the augmented NKM model using both revised and real-time data. Related to the evidence of non-rational forecast found in the first subsection, we analyze the effects of ignoring the presence of badly-behaved revisions on estimated policy parameters and the transmission of policy shocks. Finally, in the third subsection, we explore the robustness of the results by leaving Calvo's probability parameter free in the estimation procedure.

(Insert Figure 1)

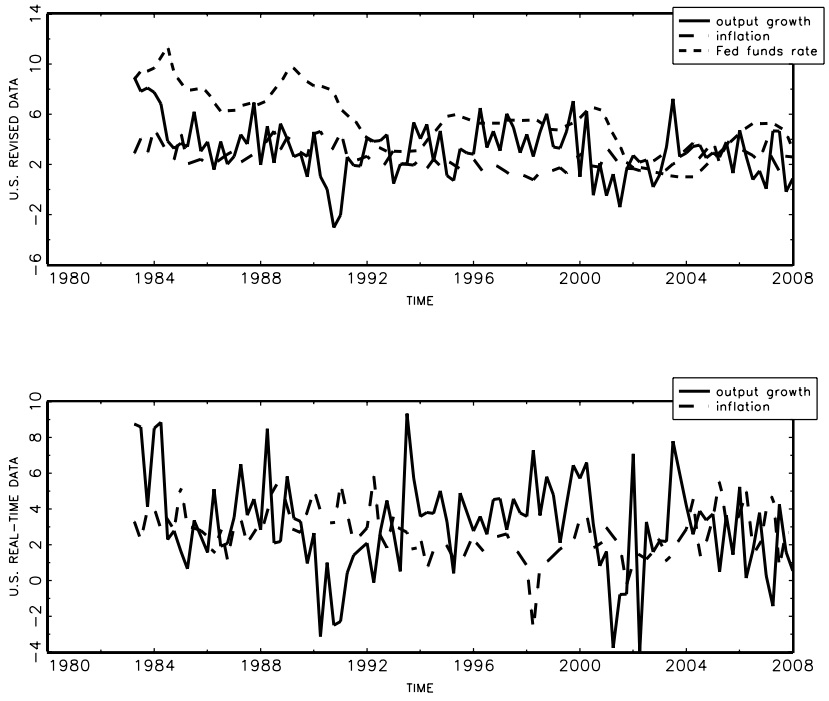

Figure 1: U.S. Time Series

4.1 Preliminary Evidence for the Revision Processes

As a preliminary step, we investigate whether real-time data are a rational forecast of revised data. Following Aruoba (2008), Panel A of Table 1 shows a set of summary statistics as well as some tests that allow us to assess whether revision processes for output growth and inflation are well-behaved. For both revision processes, we cannot reject the null hypothesis that the unconditional mean is zero. However, the standard deviation for the two revision processes is quite large, although in both cases not larger than that for the revised data (i.e. the noise/signal parameter). The evidence that revisions are not rational forecast errors is further supported by the statistics displayed in Panel B. Either output growth or inflation revision processes are not orthogonal to the initial announcements and their conditional means are not null. In particular, the estimated coefficient of the variable being forecasted in each case is negative and significant suggesting that the two variables tend to be revised back towards their respective means in line with the intuition in Dynan and Elmendorf (2001).[15] In sum, these preliminary estimation results are in line with the empirical evidence provided by Aruoba (2008) who also finds that data revisions for these variables are not white noise.

The non-rational features of revision processes suggest that analyzing policymakers' decisions based only on revised data could be misleading. Next, we explore the implications of this issue by estimating the extended version of the NKM model using both revised and real-time data together.

4.2 Data Revisions and Monetary Policy

We now analyze the importance of data revisions on the estimated monetary policy rule parameters and the transmission of policy shocks. Table 2 shows the estimation results obtained using both revised and real-time data. The estimate of the inflation parameter, ψ_1, is one, which suggests that the Taylor principle may not hold during

[15] A more comprehensive analysis of the relation between revision processes and the business cycle can be found in Croushore (2011) and papers cited therein.

Table 1: Reduced-form analysis for revision processes.

Panel A: Summary Statistics		
	r_t^y	r_t^π
Mean	0.077	−0.055
Median	0.033	0.034
Min	−7.286	−3.694
Max	6.164	3.357
St. dev.	2.202	1.046
Noise/Signal	0.996	0.388
corr. with initial	0.612	0.653
AC(1)	−0.147	−0.017
t-stat. $(E(r_t) = 0)$	0.379	−0.537
Panel B: Conditional Mean		
	r_t^y	r_t^π
	Coef.	Coef.
constant	0.906^{**}	1.196^{***}
$(y_t^r - y_{t-1}^r) * 400$	-0.470^{***}	0.033
$(\pi_t^r) * 400$	0.243^{**}	-0.521^{***}
$F_{3,90}$	31.305^{***}	118.596^{***}

Note: Revisions are calculated over (annualized) quarterly GDP growth and inflation respectively. Since revisions are likely to have a first-order autocorrelation pattern, t-statistics for testing whether the conditional or unconditional mean is null are calculated based on Newey-West corrected standard deviations (1 lag). $^{*},^{**},^{***}$ represent significance at the standard 1, 5 and 10% confidence levels. The noise/signal statistic is calculated as the standard deviation of the revision over the standard deviation of the revised data. The null hypothesis for computing the F-test in Panel B or conditional mean hypothesis is that all coefficients associated with real-time information are null.

the post-Volcker period, somewhat in line with the evidence provided by Orphanides (2001) using a reduced-form estimation approach and only real time data.[16] The output gap parameter is small ($\psi_2 = 0.31$) in line with the estimated values reported in previous papers. Moreover, our estimation results show that the policy inertia parameter estimate ($\rho = 0.90$) is larger than in previous studies mentioned above whereas the policy shock persistence parameter is large ($\rho_v = 0.77$). The estimates of the remaining shock parameters all display high levels of persistence. The high persistence estimates of supply and demand shocks are in line with the estimation results found in Smets and Wouters (2007).

Our estimation results also show that many revision process parameters are significant, suggesting that real-time data are not rational forecasts, in line with the evidence from reduced-form approaches in Dynan and Elmendorf (2001), Aruoba (2008), and that shown in Table 1 (Panel B). In particular, the initial announcements of inflation are the most important piece of information for predicting the actual revisions of the two variables ($b_{x\pi} = -1.36$, $b_{\pi\pi} = -0.07$). The inflation coefficients are negative and significant suggesting that a higher-than-average initial announcement anticipates a downward revision in both output and inflation.[17]

In order to investigate whether the characteristics of revision processes have an effect on estimated policy rule parameters, we next estimate the system under the null hypothesis that r_t^x and r_t^π are rational forecast errors. r_t^x and r_t^π are viewed as rational forecast errors under the null hypothesis, $H_0 : b_{xx} = b_{x\pi} = b_{\pi x} = b_{\pi\pi} = 0$. This hypothesis implies that the two revision processes r_t^x and r_t^π are characterized by two white noise processes: ϵ_{xt}^r and $\epsilon_{\pi t}^r$, respectively.

[16] By using the methodology suggested by Lubik and Schorfheide (2003), we also tried to estimate the model by assuming that the Taylor principle did not hold (i.e. $\psi_1 < 1.0$) and allowing for the existence of sunspots. The estimation algorithm did not reach convergence in this case and the associated distance function value, $J_T(\theta)$, was always larger than the value obtained when $\psi_1 \geq 1.0$.

[17] It is important to emphasize that the results from our structural estimation are not directly comparable with those from reduced-form approaches. On the one hand, the output gap is the explanatory variable in Equations (6)-(7), whereas the output growth is the explanatory variable for the regressions in panel B of Table 1. On the other hand, the revision coefficients become harder to interpret as they interact with the parameters in the structural model. Moreover, as mentioned above, analyzing the actual method used by statistical agencies goes beyond the scope of this paper.

Table 2: Joint estimation of the NKM model and the revision processes using both revised and real-time data.

$J_T(\hat{\theta})$	7.5998				
Policy parameter	Estimate	Shock parameter	Estimate	Revision parameter	Estimate
ρ	0.9042	ρ_χ	0.9845	b_{xx}	0.1083
	(0.0131)		(0.0325)		(0.0339)
ψ_1	1.0000	ρ_z	0.8834	$b_{x\pi}$	-1.3563
	(0.0511)		(0.0137)		(0.1681)
ψ_2	0.3070	ρ_v	0.7707	$b_{\pi x}$	0.0159
	(0.0816)		(0.0405)		(0.0070)
		σ_χ	$2.1e-04$	$b_{\pi\pi}$	-0.0712
			$(1.2e-04)$		(0.0382)
		σ_z	$3.3e-04$	σ_x^r	$2.7e-04$
			$(5.6e-05)$		$(7.8e-05)$
		σ_v	$7.2e-05$	σ_π^r	$1.7e-03$
			$(1.4e-05)$		$(1.5e-05)$

Table 3 shows the estimation results when H_0 is imposed. It is well known that the null hypothesis H_0 can be tested using the following Wald statistic:

$$F_1 = \left(1 + \frac{1}{m}\right) T \left[J_T(\hat{\theta}') - J_T(\hat{\theta})\right] \rightarrow \chi^2(4),$$

where $J_T(\hat{\theta}')$ denotes the value of the distance function under H_0. The F_1-statistic takes the value 162.59. Therefore, we can reject the joint hypothesis that the revision processes of output and inflation are both white noise processes at any standard level of significance. Nevertheless, by comparing the estimation results of Tables 2 and 3, it is interesting to observe that monetary policy rule parameter estimates (ρ, ψ_1, ψ_2, σ_v) are similar regardless of whether or not the restriction that the two revision processes are well-behaved, i.e. when H_0, is imposed. The exception is the estimate of ρ_v, which is less than one third as large when H_0 is imposed.

The relative unimportance of imposing H_0 is further revealed by carrying out an impulse-response analysis. Figures 2-4 show the impulse responses of the endogenous

Table 3: Joint estimation of the NKM model assuming that the revision processes are well-behaved.

$J_T(\hat{\theta}')$	9.2901				
Policy parameter	Estimate	Shock parameter	Estimate	Revision parameter	Estimate
ρ	0.9036	ρ_χ	0.9346	σ_x^r	$3.9e-03$
	(0.0107)		(0.0124)		$(2.8e-04)$
ψ_1	1.0851	ρ_z	0.9900	σ_π^r	$1.4e-03$
	(0.0755)		(0.0308)		$(2.3e-04)$
ψ_2	0.0000	ρ_v	0.1997		
	(0.0207)		(0.0464)		
		σ_χ	0.0030		
			$(5.2e-04)$		
		σ_z	$2.2e-04$		
			$(1.8e-04)$		
		σ_v	$2.9e-04$		
			$(4.2e-05)$		

variables of the extended NKM model (11) to a productivity shock, an inflation shock and a monetary policy shock, respectively, using the estimates displayed in Table 2. In these figures, the solid line represents the impulse response implied by the NKM model augmented with revision processes, whereas the dashed lines are the corresponding 95% confidence bands. The diamond-dashed lines represent the impulse responses implied by the model under H_0 (i.e. using the estimates displayed in Table 3). The size of the shock in each case is determined by its estimated standard deviation obtained when H_0 is not imposed. We observe that the impulse-responses of the three endogenous variables to all three shocks are not so different whether H_0 is imposed or not. There are two exceptions. First, the response of nominal interest rate to a monetary policy shock is smoother when H_0 is imposed (Figure 4). Second, the responses of the inflation and interest rates to a positive inflation shock (Figure 3) are significantly more persistent when H_0 is imposed, which is due to the high estimated persistence of inflation shocks (i.e. the estimated value of ρ_z is very close to one when H_0 is imposed). In short, we can conclude that the empirical evidence

suggests that ignoring the presence of deviations from well-behaved revision processes may not be an important limitation in the analysis of monetary policy in simple NKM frameworks.

(Insert Figure 2)

(Insert Figure 3)

(Insert Figure 4)

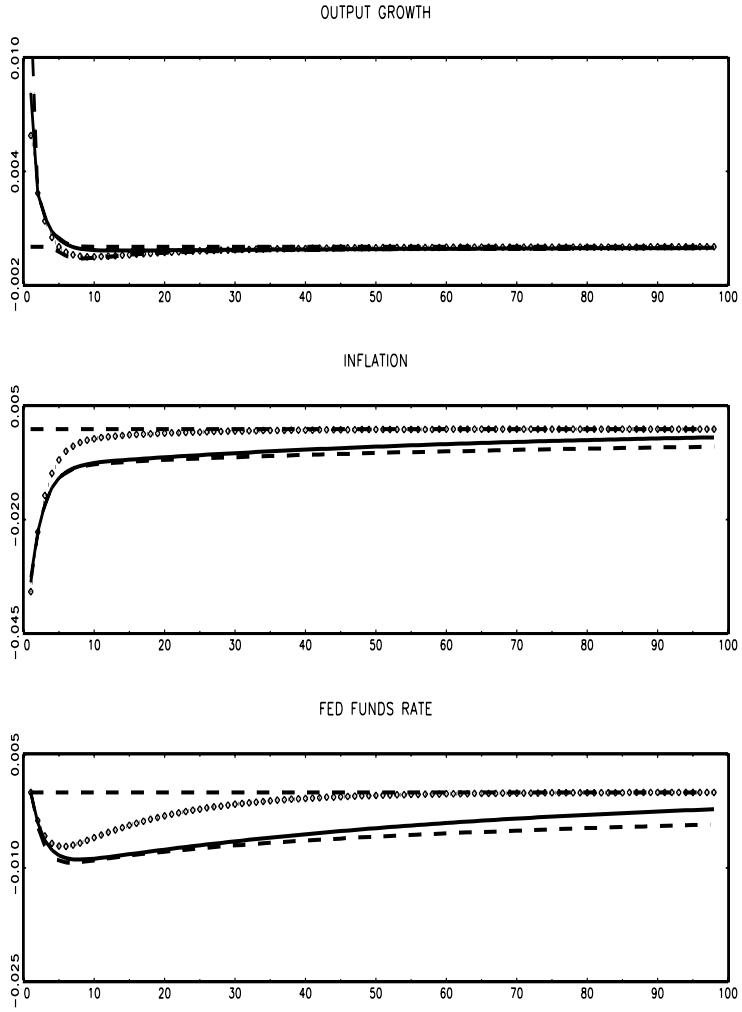

Figure 2: Impulse responses to a productivity shock

Notes to Figures 2-4: The solid line represents the impulse response implied by the NKM model augmented with revision processes, whereas the dashed lines are the corresponding 95% confidence bands obtained using Monte Carlo methods (Hamilton (1994, p.337)). The diamond-dashed line represents the impulse response implied by the model under H_0 (i.e. using the estimates displayed in Table 3).

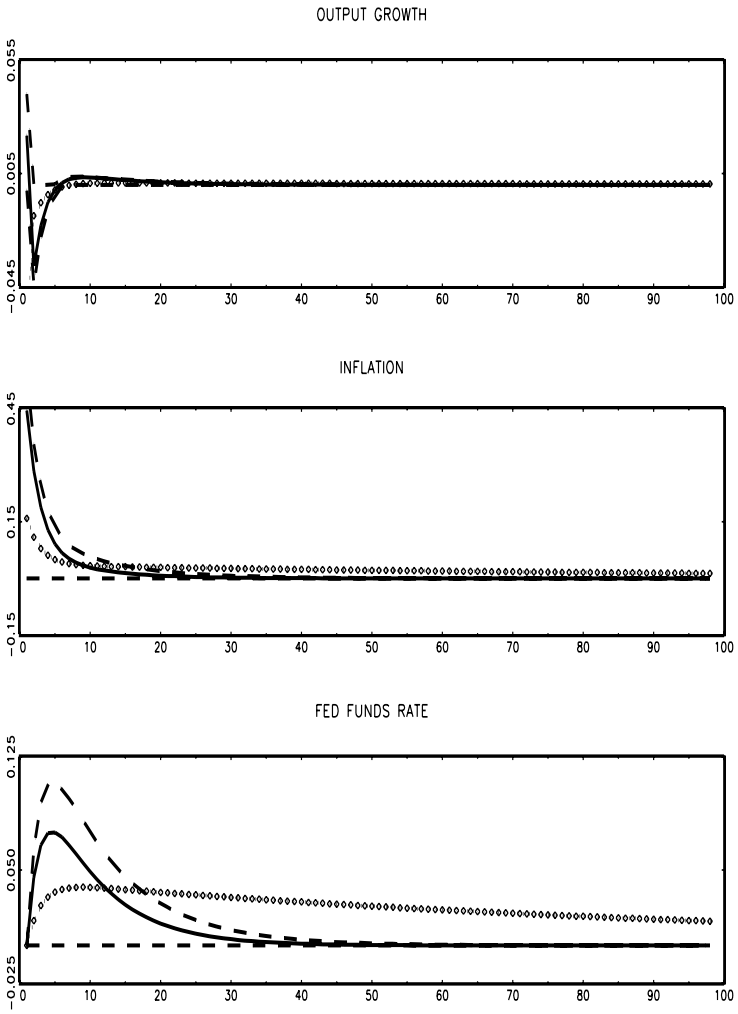

Figure 3: Impulse responses to an inflation-push shock

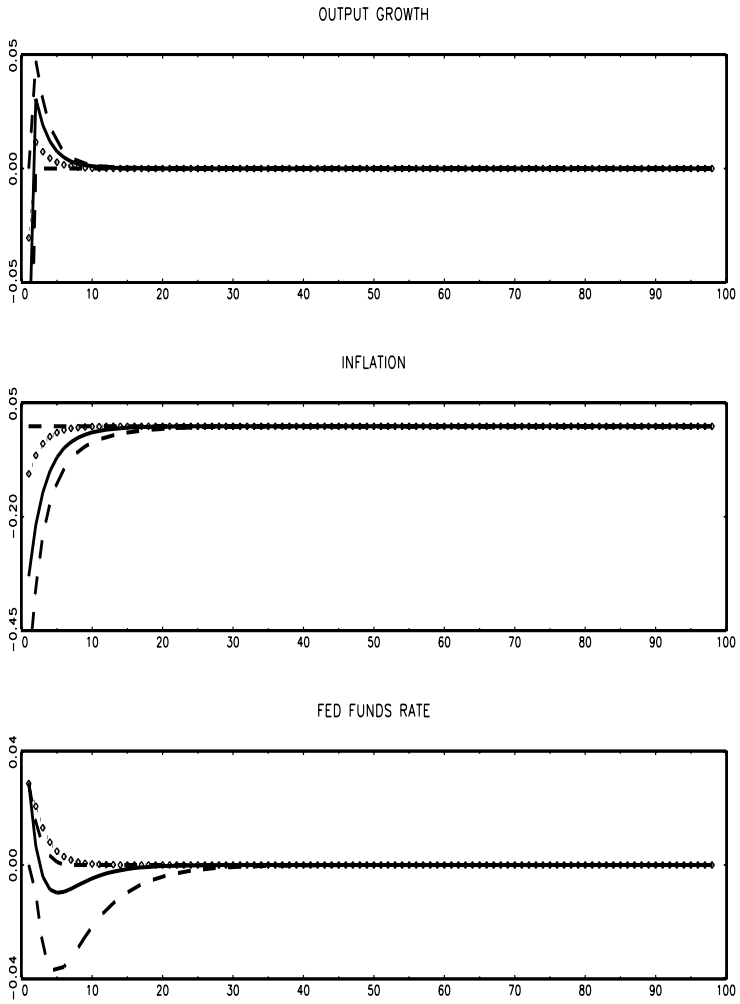

Figure 4: Impulse responses to a monetary policy shock

4.3 Estimating the Nominal Stickiness Parameter

Arguably, Calvo's probability parameter, ω, may not be considered a deeper parameter because it is likely to depend on macroeconomic outcomes such as inflation stability. In this subsection, we also estimate Calvo's parameter to analyze (i) how sensitive policy rule parameter estimates are when this parameter is left free in the estimation procedure; (ii) how sensitive the ω estimate is to ignoring the fact that the initial announcements of output growth and inflation are not rational forecast of re-

Table 4: Joint estimation of the NKM model and the revision processes when estimating Calvo's probability parameter.

$J_T(\hat{\theta})$	7.5458				
Policy parameter	Estimate	Shock parameter	Estimate	Revision parameter	Estimate
ω	0.6501	ρ_χ	0.9845	b_{xx}	0.1285
	(0.0458)		(0.0180)		(0.0446)
ρ	0.8938	ρ_z	0.8904	$b_{x\pi}$	-1.4166
	(0.0109)		(0.0116)		(0.1667)
ψ_1	1.0000	ρ_v	0.7544	$b_{\pi x}$	0.0212
	(0.0278)		(0.0350)		(0.0049)
ψ_2	0.2116	σ_χ	$2.1e-04$	$b_{\pi\pi}$	-0.1510
	(0.0465)		$(8.5e-05)$		(0.0571)
		σ_z	$5.7e-04$	σ_x^r	$3.0e-04$
			$(1.3e-04)$		$(8.8e-05)$
		σ_v	$5.4e-05$	σ_π^r	$1.6e-03$
			$(1.3e-05)$		$(1.4e-04)$

vised data; and (iii) whether the sensitiveness of ω has an impact on the transmission of shocks.

Tables 4-5 show the estimation results obtained by estimating Calvo's probability parameter, ω, in addition to policy, revision process and shock parameters when allowing for deviations from well-behaved revision errors and imposing H_0, respectively. When comparing the estimation results displayed in these two tables with those obtained by fixing $\omega = 0.75$ shown in Tables 2-3, two main conclusions emerge. First, Calvo's probability estimate is much larger when H_0 is imposed. Therefore, allowing for the presence of badly-behaved revision errors decreases substantially the estimated expected time between firm price changes from $(1-0.899)^{-1} = 9.92$ quarters to $(1-0.650)^{-1} = 2.86$ quarters. Second, policy rule parameter estimates (ρ, ψ_1, ψ_2, ρ_v, σ_v) are quantitatively similar regardless of whether or not the Calvo parameter is estimated.

By looking at the impulse-response functions displayed in Figures 5-7, it is ob-

Table 5: Estimation results assuming that the revision processes are well-behaved and estimating Calvo's probability parameter.

$J_T(\hat{\theta}')$	9.1407				
Policy parameter	Estimate	Shock parameter	Estimate	Revision parameter	Estimate
ω	0.8992	ρ_χ	0.9338	σ_x^r	$3.9e-03$
	(0.0269)		(0.0185)		$(2.9e-04)$
ρ	0.9149	ρ_z	0.9900	σ_π^r	$1.6e-03$
	(0.0103)		(0.0470)		$(2.1e-04)$
ψ_1	1.0509	ρ_v	0.2086		
	(0.0796)		(0.0516)		
ψ_2	0.0000	σ_χ	0.0100		
	(0.0167)		$(3.9e-03)$		
		σ_z	$6.2e-05$		
			$(8.5e-04)$		
		σ_v	$3.7e-04$		
			$(4.8e-05)$		

served that the sensitivity of Calvo's parameter estimates depending on whether H_0 is imposed does not lead to much different responses of the endogenous variables to productivity and monetary policy innovations. However, the analysis of the transmission of inflation-push shocks is largely affected by ignoring the possibility of deviations from well-behaved revision processes when the Calvo parameter is estimated.

(Insert Figure 5)

(Insert Figure 6)

(Insert Figure 7)

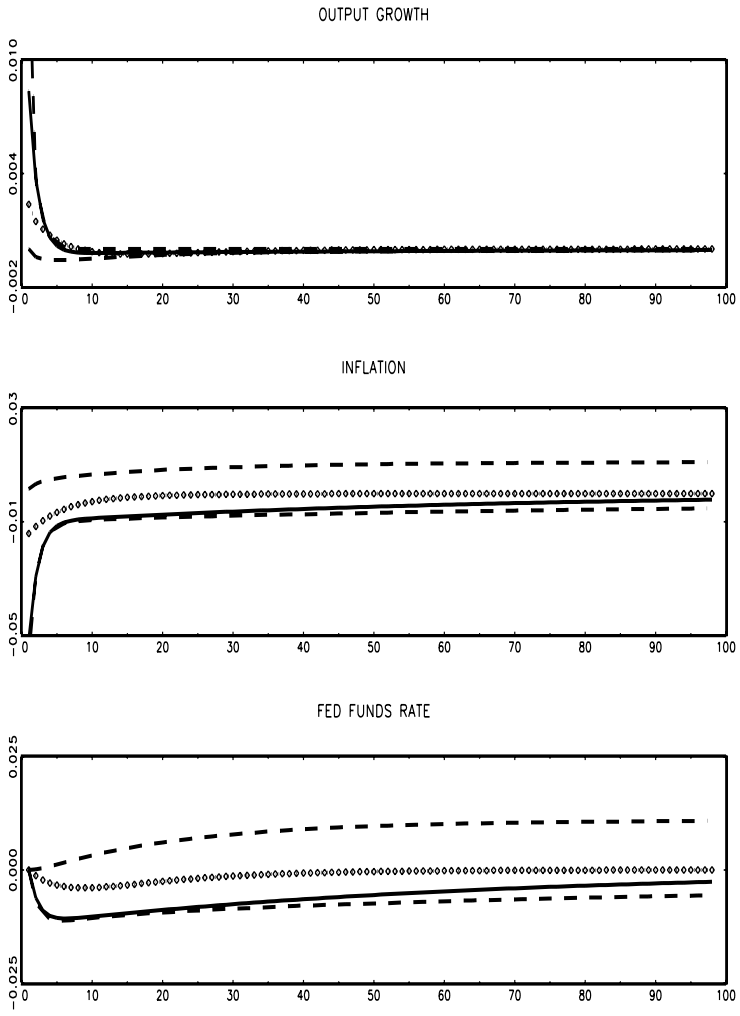

Figure 5: Impulse responses to a productivity shock (estimating ω)

Notes to Figures 5-7: The solid line represents the impulse response implied by the NKM model augmented with revision processes, whereas the dashed lines are the corresponding 95% confidence bands. The diamond-dashed line represents the impulse response implied by the model under H_0 (i.e. using the estimates displayed in Table 5).

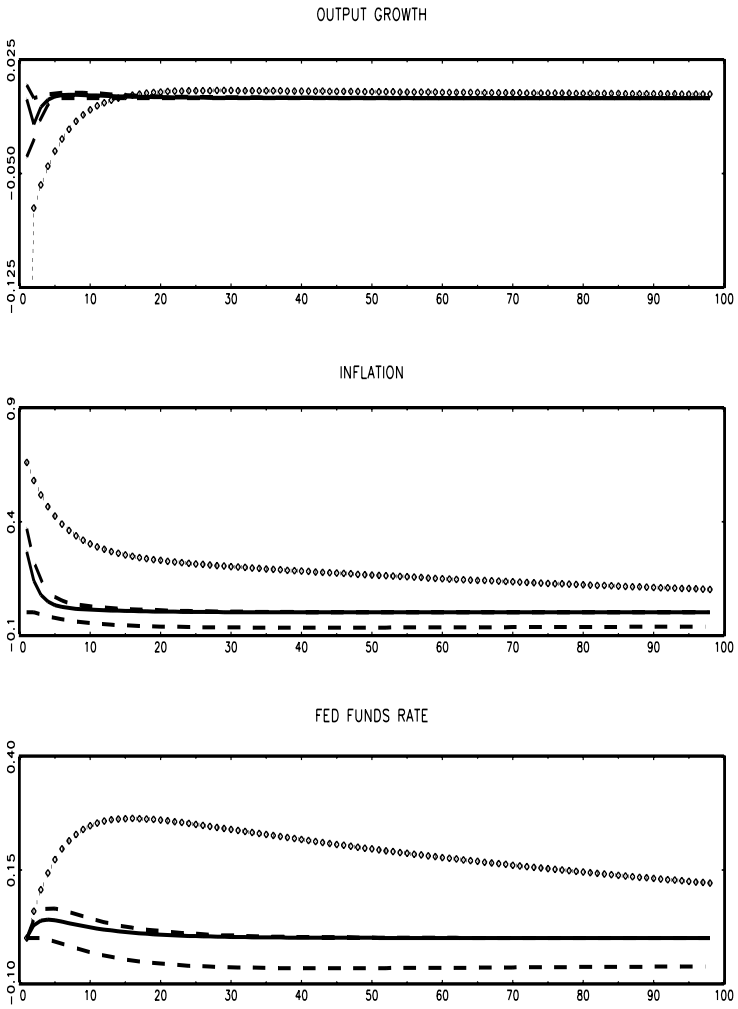

Figure 6: Impulse responses to an inflation-push shock (estimating ω)

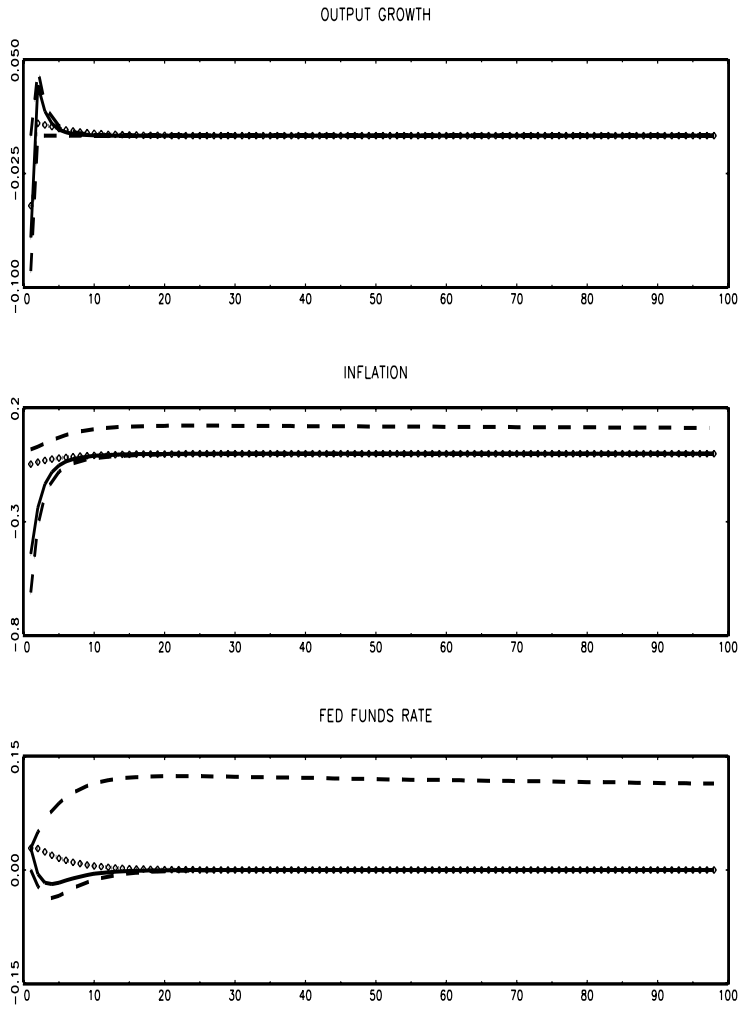

Figure 7: Impulse responses to a monetary policy shock (estimating ω)

5 CONCLUSIONS

This paper suggests an augmented version of the basic New Keynesian monetary (NKM) model which contemplates revision processes of output and inflation data in order to assess the influence of deviations in real time data from being a rational forecast of revised data on the estimated monetary policy rule parameters and the transmission of shocks.

Empirical evidence based on a structural econometric approach suggests that the estimated policy rule parameters are not too sensitive to allowing for the possibility of non-rational revision processes. Moreover, our empirical analysis shows that ignoring the presence of badly-behaved revision processes may not be a serious drawback in the analysis of monetary policy in this framework. However, the responses of the three endogenous variables (output, inflation and nominal interest rate) to inflation-push innovations are rather sensitive to allow for the presence of non-rational revision processes. The latter being especially true when the nominal stickiness parameter is estimated.

In this paper, we have assumed that decisions by private agents (consumers and firms) are not affected by real-time data issues. This would not be the case in more general versions of the NKM model. For instance, when there are price and wage indexation rules that force firms, which are not able to choose their prices (wages) optimally, to take into account real-time lagged inflation to adjust their prices (wages) instead of revised lagged inflation. The results of this paper can be viewed as initial step to understand the implications of extending a medium-scale NKM model of the type analyzed by Smets and Wouters (2007) with revision processes. This more challenging exercise is left for future research.

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APPENDIX

This appendix shows the matrices in equation (11).

$$\Gamma_0 = \begin{pmatrix}
1 & 0 & \tau & -1 & -\tau & \phi(1-\rho_\chi) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\kappa & 1 & 0 & 0 & -\beta & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & $-b_{xx}$ & $-b_{x\pi}$ & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & $-b_{\pi x}$ & $-b_{\pi\pi}$ & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix},$$

$$\Gamma_1 = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & ρ & 0 & 0 & 0 & 0 & 0 & $\Gamma_1^{3,9}$ & $\Gamma_1^{3,10}$ & 0 & 0 & $\Gamma_1^{3,9}$ & $\Gamma_1^{3,10}$ \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & ρ_χ & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & ρ_z & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & ρ_v & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix},$$

where

$$\Gamma_0^{3,9} = (1-\rho)\psi_2,\ \Gamma_0^{3,10} = (1-\rho)\psi_1.$$

$$\Psi = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \Pi = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}.$$