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APPLICATION OF ADVANCED STATED PREFERENCE DESIGN METHODOLOGY

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APPLICATION OF ADVANCED STATED PREFERENCE DESIGN METHODOLOGY

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CONTENTS

ABSTRACT

This paper demonstrates the application of the design methodology developed in the Advanced Stated Preference Design project to stated preference experiments. The paper considers binary response experimental designs of two, three and four variables. In addition the special case of a two variable design with an alternative specific constant is also considered. Alternative optimality criteria are discussed. The paper concludes with recommendations on how to apply the design methodology successfully.

1 BACKGROUND

Many of the issues surrounding the current design process for stated preference (SP) techniques are discussed in Fowkes (1996) so only a brief overview is given here.

The form of the SP experiments considered here are binary response experiments. Here the respondent is presented with a small (typically between 9 and 16) number of scenarios. Each scenario consists of a pair of alternatives (typically, though not necessarily, between two modes), between which the respondent is invited to choose. Each choice is described by a number of attributes (typically including cost and time) which are presented as values. A typical two variable SP design, taken from Fowkes and Nash (1991), is given in table 1.

Scenario	Alternative A		Alternative B		Difference		
	COST (pence)	TIME (min)	COST (pence)	TIME (min)	COST (pence)	TIME (min)	BVoT (pence/min)
1	100	30	115	20	15	-10	1.50
2	100	30	125	20	25	-10	2.50
3	100	30	140	20	40	-10	4.00
4	150	45	165	30	15	-15	1.00
5	150	45	175	30	25	-15	1.67
6	150	45	190	30	40	-15	2.67
7	200	60	215	40	15	-20	0.75
8	200	60	225	40	25	-20	1.25
9	200	60	240	40	40	-20	2.00

Table 1 : A possible binary choice SP design

In this design, alternative B is always the faster but more expensive option. This need not always be the case, a mixture of either alternative being the faster, more expensive is acceptable. In fact it is possible to have one of the alternatives being both the faster and cheaper option.
Since individuals are choosing between alternatives, a more succinct representation would be to

express the attributes as the differences in their levels. Thus the question becomes a direct trade-off between savings in time and cost.

A feature of the design in table 1 is that the correlations between all the attribute differences is zero. Such a design is said to be orthogonal. This is the first of two widely used design criteria. The supposed reason for ensuring that this property exists is that this would produce the most efficient estimates from any model estimation procedure, primarily from an analogy with least squares regression (Fowkes, 1996).

Another item of information which can be extracted from the design in table 1 are the boundary values of time (BVoT), ie COST difference divided by TIME difference. These values show at what time valuation individuals are indifferent between alternatives. Thus for scenario 1, if an individual's value of time is 1.5 pence per minute then they are indifferent between alternatives A and B in this scenario. If their value of time is less than 1.5, then they would be expected to choose the slower but cheaper option (alternative A). If their value of time is greater than 1.5, then alternative B should be the preferred option. It is possible to plot a graphical representation of the spread of BVoT's. This plot for the design in table 1 is given in figure 1.

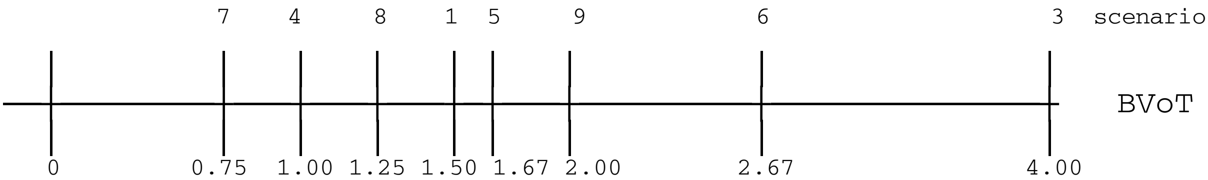


Figure 1 : Boundary value map of table 1 design

Figure 1 begins to show how effective a design should be at recovering a range of values of time. In the discussion which follows a perfect knowledge on the part of the respondent is assumed (ie deterministic choice). If the respondents are thought to follow compensatory choice processes, some form of randomness is incorporated into the decision process which represents incorrect (or inconsistent) choices.

With reference to the example design, if the value of time is greater than 4.00 then all the respondents will chose the faster, more expensive option. Thus the only clear result will be that the lower bound of value of time is 4.00. If the value is between 2.67 and 4.00 then all the respondents will chose the faster, more expensive alternative, except for scenario 3, in which case they would select the slower, cheaper option. In this case there is both a lower and upper bound on the value of time. The interval is however wide, at 1.33. If the value of time is between 1.50 and 1.67 then the choice will be the faster, more expensive mode for scenarios 7, 4, 8 and 1 and the slower, cheaper mode for scenarios 5, 9, 6 and 3. The interval is also narrow at 0.16. Intuitive inspection suggests that this design would perform well at recovering values of time in the range 1.00 to 2.00.

This methodology is the second design technique which is widely employed in SP design, namely trying to ensure that there is a reasonable coverage of boundary values near an expected value of time. There is an extension of this technique into a three variable case, where the boundary points become boundary rays, with an intercept and a slope (Fowkes 1991).

In summary, the suggested technique for designing an efficient SP is, up to now, to choose levels which give orthogonality and also give a reasonable coverage of boundary values.

2 MODELLING

Once an SP design has been designed it is used in an experiment to try and extract a valuation of the measure of interest (in the example given in section 1 it would be the value of time). A model of individuals behaviour is required from which parameter values can be estimated. An assumption used here is to derive a set of utilities, for each alternative, which is a linear combination of the attribute levels. The expression of this utility will be of the form:

$$U_a = \beta_1 COST_a + \beta_2 TIME_a + \varepsilon \tag{1a}$$

$$U_b = \beta_1 COST_b + \beta_2 TIME_b + \varepsilon \tag{1b}$$

An individual will be expected to choose the option which has the highest utility, U_a or U_b, depending on the values for the parameters β_1 and β_2. It is also worth noting that most estimation packages do not directly estimate the β_i's, but instead estimate a scaled β_i ie ΩB_i, where B_1 and B_2 are the 'true' β_i's. When estimating values of time (see below) these Ω's are irrelevant since they cancel out, but if the estimates are to be used for forecasting purposes then the true underlying β_i's will be required. In what follows β_1 and β_2 should be strictly interpreted as ΩB_1 and ΩB_2.

The expectations of (1) can be converted into probabilities such that an individual makes their decision in favour of alternative A if:

$$Pr(a) = Pr(U_a > U_b) \tag{2a}$$

$$Pr(a) = Pr(U_a - U_b > 0) \tag{2b}$$

An alternative expression for this choice utility is the utility difference expression:

$$\Delta U = \beta_1 \Delta COST + \beta_2 \Delta TIME + \Delta \varepsilon \tag{3}$$

Where $\Delta COST$ is $(COST_b - COST_a)$
$\Delta TIME$ is $(TIME_b - TIME_a)$

Both β_1 and β_2 are assumed negative since spending extra money or time on a given trip should cause dis-utility. If ΔU is greater than 0 then the individual prefers alternative A whilst if it is less than 0 then alternative B is the preferred choice. The values of β_1 and β_2 are usually estimated using maximum likelihood techniques.

The probability expression then becomes:

$$Pr(a) = Pr(\Delta U > 0) \quad (4)$$

Which, under certain assumptions about the distribution of the error terms, can be calculated from:

$$Pr(a) = \frac{1}{1 + e^{(\beta_1 \Delta COST + \beta_2 \Delta TIME)}} \quad (5)$$

Thus if this probability is greater than 0.5 then one would assume that the individual will chose alternative A and otherwise alternative B.

The expression (β_2 / β_1) gives a valuation for the overall value of time (VoT).

Expressions can be derived for the variances of the parameters β_1 and β_2 and the ratio β_2 / β_1 (Watson et al, 1996). These expressions involve: β_1, β_2, ΔCOST, ΔTIME and additionally in the later case, Var(β_1), Var(β_2) and Covariance(β_1 ,β_2.).

3 NEW DESIGN METHODOLOGY

Given an expression for the variance of the parameters, a sensible approach is to derive a design which, for a given β_1 and β_2, chooses ΔCOST and ΔTIME to minimise these variances. This is essentially the new methodology. In reality the values of β_1 and β_2 will be unknown until the survey is conducted which is something of a drawback, however, information from pilot or previous full studies may inform the choice of β_1 and β_2, thereby overcoming this drawback.

It has been shown that the adoption of this methodology will produce a design with certain properties (Wardman and Toner, 1996):

- The Pr(a) = p* which will equal 0.9168 or 0.0832; and
- The t-ratio of the parameters will be given by the expression:

$$t^* = \frac{\sqrt{n}}{2} \sqrt{(u^2 - 4)} \quad (6)$$

where u is the utility difference which produces p* in (5), ie ±2.399;
n is the number of scenarios.

In the case under consideration here there are two parameters whose variance can be minimised. When one variable is at its minimum variance, the other may not be. Thus a number of approaches suggest themselves:

(1) Successive minimisation of Var(β_1) and Var(β_2);
(2) Successive minimisation of t(β_1) and t(β_2) (minimisation since β_1 and β_2 are negative) ;
(3) Weighted minimisation of:

$$\sum_{i=1}^{2}(w_i Var(\beta_i))$$

(4) Weighted minimisation of:

$$\sum_{i=1}^{2}(w_i (t^* - | t(betasubi) |$$

Cases (1) and (2) would require iterative minimisation loops, whilst cases (3) and (4) require only one minimisation. Other minimisation criteria are also possible, eg involving the ratio of $t(\beta_i)$ to t* or a weighted sum of the $Var(\beta_i)$'s and Var(VoT). Since Var(VoT) is unbounded, however, constraints may be required here. The results presented in this paper were obtained from FORTRAN programs which used the NAG (Ford and Pool, 1984) minimisation routine E04JAF. Similar minimisation routines to perform these tasks can be found in popular spreadsheets.

4 TWO VARIABLES

4.1 PRODUCING THE DESIGN

The initial design used to illustrate the application of the new methodology is that given in table 1. As a first step towards the application of this new methodology an exercise was conducted to ensure that the expressions for the variance parameters were correct. The responses of 20 individuals, with values of β_1=-0.1 and β_2=-0.2, to the design in table 1 were simulated. The ALOGIT package (1992) was then used to estimate the $\hat{\beta}_1$, $\hat{\beta}_2$, se($\hat{\beta}_1$) and se($\hat{\beta}_2$) values from this simulation. These se values are then compared with the same information from the analytical variance expressions. This comparison is given in table 2:

Method	$\hat{\beta}_1$	$\hat{\beta}_2$	se($\hat{\beta}_1$)	se($\hat{\beta}_2$)
ALOGIT	-0.1236	-0.2390	0.0194	0.0367
Analytical	-0.1236	-0.2390	0.01936	0.03666

Table 2 : Comparison of ALOGIT and analytical expression results

For this section it has been decided to optimise around given values of β_1=-0.1 and β_2=-0.2. The initial design and the final optimal design for cases (1) and (2) as outlined in section 3, are given in figure 2. The t* value for a nine scenario design with one individual is 1.9882. The starting point for both cases is the initial design. Each case has produced a different solution, demonstrating that there is no unique optimal design. In practice, only integer values of TIME and COST differences are of use so the final optimal designs are integerised in figure 2. Both cases have produced near p* and t* values and if non-integer variables are allowed p* values are guaranteed. With the t-ratios only the last optimised parameter, β_2 in this experiment will be at t*. Each t-ratio is based on one replication of the survey and if many individuals were interviewed then these t-ratios would increase.
The t-ratio of VoT has increased from 3.70 to 12.86 or 12.55 and the correlation between the cost and time difference has also departed from 0.

Initial design

COST	TIME	Pn	(1-Pn)	BVoT
15.	-10.	0.6225	0.3775	1.5000
25.	-10.	0.3775	0.6225	2.5000
40.	-10.	0.1192	0.8808	4.0000
15.	-15.	0.8176	0.1824	1.0000
25.	-15.	0.6225	0.3775	1.6667
40.	-15.	0.2689	0.7311	2.6667
15.	-20.	0.9241	0.0759	0.7500
25.	-20.	0.8176	0.1824	1.2500
40.	-20.	0.5000	0.5000	2.0000

CORR (COST,TIME) = 0.0000

	estimate	variance	t-ratio
COST	-0.1000	0.0058	-1.3152
TIME	-0.2000	0.0211	-1.3769
VoT	2.0000	0.2926	3.6976

Case (1)
Design after 10 iterations of [min Var(β_1) and then min Var(β_2)]:

COST	TIME	Pn	(1-Pn)	BVoT
104.	-64.	0.9168	0.0832	1.6250
118.	-47.	0.0832	0.9168	2.5106
177.	-77.	0.0911	0.9089	2.2987
112.	-68.	0.9168	0.0832	1.6471
164.	-94.	0.9168	0.0832	1.7447
183.	-79.	0.0759	0.9241	2.3165
123.	-73.	0.9089	0.0911	1.6849
169.	-96.	0.9089	0.0911	1.7604
193.	-85.	0.0911	0.9089	2.2706

CORR (COST,TIME) = -0.7122

	estimate	variance	t-ratio
COST	-0.1000	0.0026	-1.9623
TIME	-0.2000	0.0101	-1.9868
VoT	2.0000	0.0242	12.8606

Case (2)
Design after 10 iterations of [min t(β_1) and then min t(β_2)]:

COST	TIME	Pn	(1-Pn)	BVoT
89.	-57.	0.9241	0.0759	1.5614
109.	-42.	0.0759	0.9241	2.5952
171.	-74.	0.0911	0.9089	2.3108
126.	-75.	0.9168	0.0832	1.6800
164.	-94.	0.9168	0.0832	1.7447
190.	-83.	0.0832	0.9168	2.2892
116.	-70.	0.9168	0.0832	1.6571
162.	-93.	0.9168	0.0832	1.7419
187.	-82.	0.0911	0.9089	2.2805

CORR (COST,TIME) = -0.7433

	estimate	variance	t-ratio
COST	-0.1000	0.0026	-1.9656
TIME	-0.2000	0.0101	-1.9870
VoT	2.0000	0.0254	12.5484

Figure 2 : Initial and final designs for cases (1) and (2)

To illustrate the performance of these two essentially similar approaches, information at each iteration for case (2) is displayed in figures 3, 4 and 5. In these figures the x-axis shows the iteration stage, 0 is the starting point, 1 is after minimisation of t(β_1), 2 is after minimisation of t(β_2) and 3 is the final result.

Figure 3 shows that the parameter being optimised reaches the t* value, whilst the other loses the t* value. As the iterations progress, however, the extent of this loss deteriorates. The t-ratio for VoT after an initial dip, rises with each iteration. Figure 5 shows the nature of the design at each iteration. The positive line is the maximum cost difference across all nine scenarios whilst the negative line is the minimum time difference across all nine scenarios.

A clear saw-tooth pattern is apparent in this figure. A minimisation of t(β_1) increases the maximum COST difference whilst decreasing the *absolute* value of the TIME difference. A minimisation of t(β_2) produces the opposite effect. The more iterations, the larger these maximum and minimum differences become and the larger the resultant t(VoT). These large differences may be impractical. If the maximum permissable COST difference was set at +100 and the minimum permissable TIME difference at -50, then the result after the second minimisation of t(β_2) would be selected, with a t(VoT)=5.6758, which is still an improvement on the starting value of t(VoT)=3.6976. The actual design is provided in figure 6.

COST	TIME	Pn	(1-Pn)	BVoT
46.	-35.	0.9168	0.0832	1.3143
53.	-14.	0.0759	0.9241	3.7857
80.	-28.	0.0832	0.9168	2.8571
51.	-37.	0.9089	0.0911	1.3784
73.	-49.	0.9241	0.0759	1.4898
81.	-29.	0.0911	0.9089	2.7931
55.	-40.	0.9241	0.0759	1.3750
76.	-50.	0.9168	0.0832	1.5200
88.	-32.	0.0832	0.9168	2.7500

CORR (COST,TIME) = -0.1317

	estimate	variance	t-ratio
COST	-0.1000	0.0028	-1.8804
TIME	-0.2000	0.0101	-1.9868
VoT	2.0000	0.1242	5.6758

Figure 6 : Final Design with 'reasonable' differences

The final designs in cases (3) and (4), with equal weight given to COST and TIME are given in figure 7.

Case (3)
Design after one minimisation of Σ t(β_i):

COST	TIME	Pn	(1-Pn)	BVoT
696.	-360.	0.9168	0.0832	1.9333
628.	-302.	0.0832	0.9168	2.0795
1227.	-601.	0.0759	0.9241	2.0416
763.	-394.	0.9241	0.0759	1.9365
1075.	-550.	0.9241	0.0759	1.9545
1310.	-643.	0.0832	0.9168	2.0373
842.	-433.	0.9168	0.0832	1.9446
1150.	-587.	0.9168	0.0832	1.9591
1466.	-721.	0.0832	0.9168	2.0333

CORR (COST,TIME) = -0.9968

	estimate	variance	t-ratio
COST	-0.1000	0.0025	-1.9873
TIME	-0.2000	0.0101	-1.9872
VoT	2.0000	0.0005	86.2491

Case (4)

Design after one minimisation of $\Sigma\ (t^*-t(\beta_i))^2$:

COST	TIME	Pn	(1-Pn)	BVoT
351.	-188.	0.9241	0.0759	1.8670
317.	-147.	0.0911	0.9089	2.1565
555.	-266.	0.0911	0.9089	2.0865
416.	-220.	0.9168	0.0832	1.8909
481.	-253.	0.9241	0.0759	1.9012
685.	-331.	0.0911	0.9089	2.0695
426.	-225.	0.9168	0.0832	1.8933
620.	-322.	0.9168	0.0832	1.9255
703.	-340.	0.0911	0.9089	2.0676

CORR (COST,TIME) = -0.9852

	estimate	variance	t-ratio
COST	-0.1000	0.0025	-1.9859
TIME	-0.2000	0.0101	-1.9862
VoT	2.0000	0.0021	44.1309

Figure 7 : Initial and final designs for cases (3) and (4)

Both these cases have quickly produced higher t(VoT) values than those seen for cases (1) and (2). Case (4) has near p* across all scenarios and t* values for both parameters. The drawback, especially in case (3), is much higher COST and TIME differences.

4.2 TESTING THE DESIGN

The results in figures 2 and 6 show how well the design performs at recovering values of β_1 and β_2 around which the design is optimised. The next question is how an optimised design will perform when recovering other combinations of β_1 and β_2? Three situations may arise:

(1) It is known with a fair degree of confidence the vicinity of the β_1 and β_2 values;
(2) It is known with a great deal of confidence a range of β_1 and β_2 values
(3) Nothing is known about the location of the β_1 and β_2 values.

To explore these situations three experiments are conducted. The first is to sample alternative β_1 and β_2 values in the neighbourhood of the design values, and test them with the design (situation 1). The second is to use the methodology to try and recovering different combinations of β_1 and β_2 values (situation 2). The final experiment is to construct a grid of β_1 and β_2 values and test the performance of the design on this grid (situation 3).

4.2.1 What's happening in the neighbourhood?

An optimal design is constructed, based on the second iteration of β_2 in case (2).

A large sample of five hundred alternative values of β_1 and β_2 are randomly sampled from the triangular distributions in the upper portion of figure 8. These values produce the distribution of VoT given in the lower portion of figure 8. Extremes of as large as 5.0 have been allowed. The t-ratios for these 500 alternative values are then calculated on the separate assumptions of the use of the initial, (orthogonal) design and the optimal design.

The distribution of the $t(\beta_1)$, $t(\beta_2)$ and t(VoT) under these two assumptions are given in figures 9 to 11.

The optimal design has produced a more uniform distribution of t-ratios for β_1 and β_2 in comparison with the more peaked distribution provided by the orthogonal design. The optimal design has produced fewer small t-ratios and more high t-ratios for t(VoT) than the orthogonal design.

4.2.2 Divide and conquer

Instead of using all nine scenarios to try and recover a fixed combination of β_1 and β_2 values, it may be more efficient to partition the scenarios. Thus the first three scenarios could be used to recover β_1^a and β_2^a, the next three β_1^b and β_2^b and the last three β_1^c and β_2^c values. Careful consideration needs to be given to the approach adopted. Issues worth considering are:

(a) Should the exercise treat each design as an series of independent mini-SP's? This would involve an approach similar to that used above but only using the appropriate scenarios during each optimisation. The scenarios would then be assembled for the full SP.

(b) Would the allocation of scenarios to β_1 and β_2 combinations be significant?

(c) An integrated SP may be required, were the full design is used to calculate the variance expressions during optimisation (unlike (a) above) but only the relevant scenarios are changed during optimisation.

(d) In this case, is the order in which each combination is optimised significant?

To explore issue (a) the nine scenario design of table 1 is used to recover β_1 and β_2 values of (-0.1,-0.2), (-0.1, -0.1), (-0.1, -0.3). The first three scenarios in the design are used to optimise around (-0.1,-0.2). When this is complete, these scenarios are put to one side and the next three scenarios are used to optimise around (-0.1,-0.1). The third set of parameters, (-0.1,-0.3) are similarly used for the final set of three scenarios. When this last stage has been completed all three sets of three scenarios are brought together in one design. The detailed output of this exercise is given in the appendix. The results are compared with the performance of the orthogonal design and summarised as Optimal (1) in table 3.

In all but one case (given in italics) this new design has produced an improvement in the t-ratios, and always an improvement for t(VoT).

The rows labelled Optimal (2) shows the effect of allocating the parameter combinations to different scenarios (issue b above). Here (-0.1,-0.2) has been allocated to scenarios 4,5 and 6; (-0.1,-0.1) to scenarios 7,8 and 9 and (-0.1,-0.3) to 1, 2 and 3. Clearly this has an effect since Optimal (1) is different to Optimal (2) but the improvement over the optimal design is still present.

(β_1, β_2)		t(β_1)	t(β_1)	t(VoT)
(-0.1,-0.2)	Orthogonal	-1.3152	-1.3769	3.6976
	Optimal (1)	-1.3634	*-1.3702*	5.6892
	Optimal (2)	-1.3919	-1.3914	5.7753
(-0.1,-0.1)	Orthogonal	-1.1956	-0.7963	1.6792
	Optimal (1)	-1.7896	-1.5661	3.5299
	Optimal (2)	-1.6186	-1.4278	3.9043
(-0.1,-0.3)	Orthogonal	-1.1264	-1.4273	3.0628
	Optimal (1)	-1.4679	-1.4602	4.9610
	Optimal (2)	-1.2521	*-1.3885*	5.4198

Table 3 : Comparison of Orthogonal and Optimal designs

The alternative approach suggested in (c) above is where the full design is used to calculate the variance values, but only a subset of the scenarios are allowed to change during optimisation.

The first three scenarios are once again optimised around (-0.1,-0.2) as above, but all nine scenarios are used to calculate the variances during optimisation. When this stage has been completed the next three scenarios are used to optimise for (-0.1,-0.1), again changing only these scenarios but using the full design to calculate the variances. After stage three, where the design is around (-0.1, -0.3) the final design is complete.

A fuller account of this complex process, with only two iterations, is show in appendix A. Adopting this approach gives the summary results presented as Optimal (1) in table 4. This approach has produced an improvement in the t-ratios over the orthogonal design. No consistent pattern emerges when the optimal results in table 3 are compared with those in table 4.

The Optimal (2) rows in table 4 show the change when a different ordering is used in the optimisation process. The parameter pairs are still associated with the same scenarios, but the pair (-0.1,-0.3) is optimised first, then (-0.1,-0.2) and finally (-0.1,-0.1). With a non-integrated design this subtle change in the ordering would have no effect, however, as can be seen in table 4, the integrated case this has produced different results.

(β_1, β_2)		$t(\beta_1)$	$t(\beta_1)$	t(VoT)
(-0.1,-0.2)	Orthogonal Optimal (1) Optimal (2)	-1.3152 -1.6737 -1.5126	-1.3769 -1.6380 -1.6157	3.6976 4.4300 5.4991
(-0.1,-0.1)	Orthogonal Optimal (1) Optimal (2)	-1.1956 -1.6402 -1.3823	-0.7963 -1.7142 -1.4469	1.6792 5.0985 4.6902
(-0.1,-0.3)	Orthogonal Optimal (1) Optimal (2)	-1.1264 -1.3555 -1.4544	-1.4273 -1.4075 -1.4500	3.0628 4.9854 6.4594

Table 4 : Comparison of Orthogonal and Optimal integrated designs

4.2.3 The wider picture

An optimal design is constructed, based on the second iteration of β_2 in case (2). This design was then used to calculate a grid of t(VoT) values based on values of β_1 and β_2 in the range [-0.05,-1.00] in steps of -0.05. Figure 12 shows the 3D plot for the orthogonal design whilst figure 13 shows the corresponding plot for the optimal design.

Figure 12 is characterised by a shallow but wide plateau, whilst figure 13 has two sharper, more concentrated, ridges. Inspection of these two graphs suggests that if the actual β_1 and β_2 values fall within either of these two ridges then the optimal design is best, otherwise the orthogonal design may be better.

5 THREE VARIABLES

The three variable design is a natural extension to that of two variables. Here the utility equations are given by the expressions:

$$U_a = \beta_1 COST_a + \beta_2 TIME_a + \beta_3 DEPARTURE_a + \varepsilon$$

$$U_b = \beta_1 COST_b + \beta_2 TIME_b + \beta_3 DEPARTURE_b + \varepsilon$$

A complicating factor is that the construction of point based boundary values are no longer possible. By way of example consider the SP design given in table 5, taken from a study by Preston and Wardman (1991).

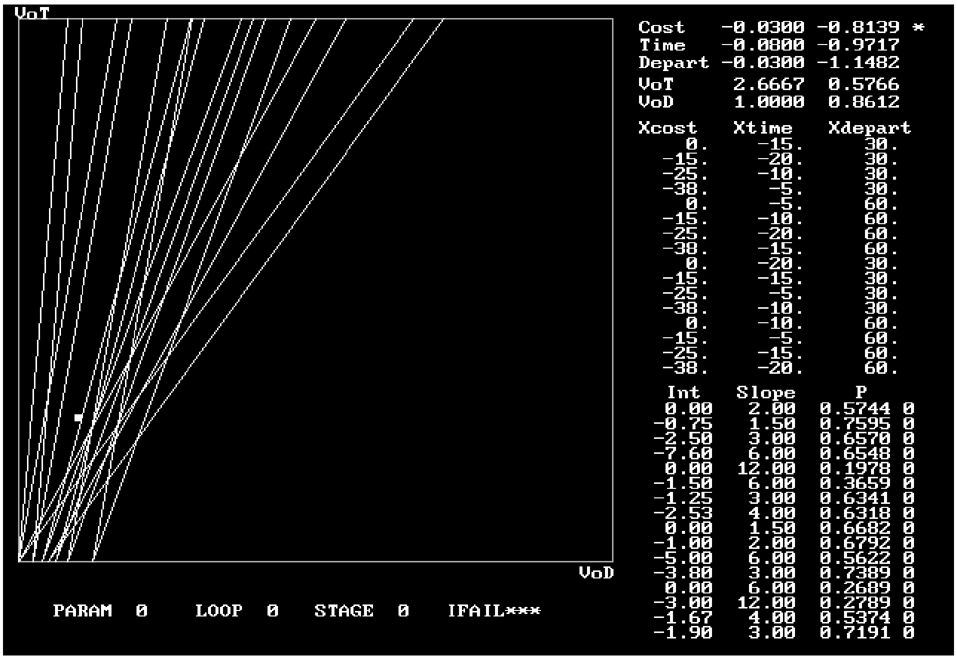
	Alternative A			Alternative B			Difference (B-A)				
	COST (pence)	TIME (min)	DEPART (min)	COST (pence)	TIME (min)	DEPART (min)	COST (pence)	TIME (min)	DEPART (min)	Intercept	Slope
1	50	40	0	50	25	30	0	-15	30	0.00	2.0
2	30	45	0	0	25	30	-30	-20	30	-1.50	1.5
3	100	45	0	50	35	30	-50	-10	30	-5.00	3.0
4	75	40	0	0	35	30	-75	-5	30	-15.0	6.0
5	0	40	0	0	35	60	0	-5	60	0.00	12.0
6	80	45	0	50	35	60	-30	-10	60	-3.00	6.0
7	50	45	0	0	25	60	-50	-20	60	-2.50	3.0
8	125	40	0	50	25	60	-75	-15	60	-5.00	4.0
9	50	45	0	50	25	30	0	-20	30	0.00	1.5
10	30	40	0	0	25	30	-30	-15	30	-2.00	2.0
11	100	40	0	50	35	30	-50	-5	30	-10.0	6.0
12	75	45	0	0	35	30	-75	-10	30	-7.50	3.0
13	0	45	0	0	35	60	0	-10	60	0.00	6.0
14	80	40	0	50	35	60	-30	-5	60	-6.00	12.0
15	50	40	0	0	25	60	-50	-15	60	-3.33	4.0
16	125	45	0	50	25	60	-75	-20	60	-3.75	3.0

Table 5 : A possible binary choice three variable SP design

Fowkes (1991) proposes that a boundary ray map may be constructed from this design, where the intercept and slope of the ray are given by the following expression.

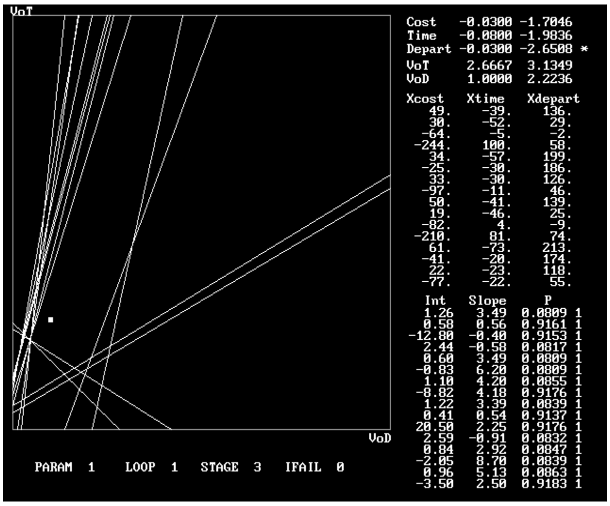
$$BVoT = \frac{\Delta COST}{-\,DELTATIME} + VoD\,\frac{\Delta DEPARTURE}{-\,\Delta TIME} \tag{8}$$

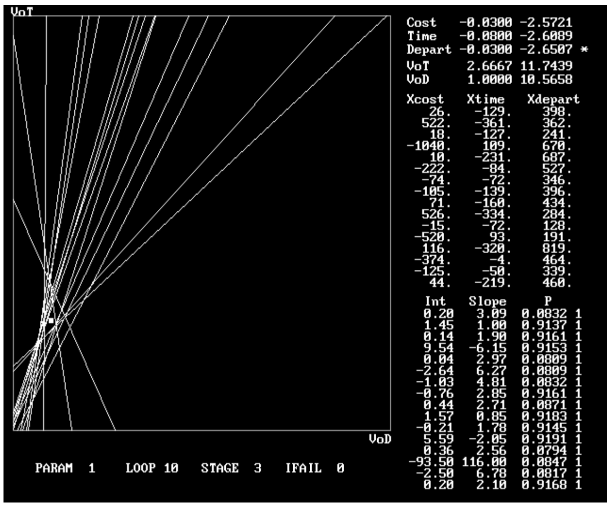
The boundary value map for the design in table 5 is given in figure 14.



The units of journey time and departure time change are in single legs of a journey whilst the cost is for a round trip of two legs. To keep the units consistent the costs have been halved. This map is characterised by non-positive intercepts with the VoT axis and positive slopes. Note that for the t-ratios, the information presented is based on only one replication of the survey. The information in figure 14 does not suggest that the t-ratios of VoT or VoD from the surveys will be as low as 0.5731 or 0.8555

If the methodology of case (2) is adopted to optimise this design then after the first iteration the map in figure 15 results and the final map after 10 iterations is given in figure 16.





After only one iteration, the t-ratio of the last optimised parameter (ΔDEPARTURE) is near its t* value of 2.6510, and the t-ratios of VoT and VoD have shown considerable improvement. The p values are also close to p*.

During the iterative process the same features as were seen for the two variable case are apparent, namely: optimised parameter near t*; other parameters sub-t* but improving; p's at or near p* and increases in the magnitude of the differences. After ten iterations, the t-ratios for VoT and VoD are high at 11.7439 and 10.5658.

Much of the discussion of section 4.2 with regard to the testing of the design is relevant to a three variable design. The performance will be good in the neighbourhood of the design point and divide and conquer approaches are equally applicable to a three variable design. An idea of the wider picture is difficult to gain since a 4D plot would be required to show the performance of each design at distant points.

6 FOUR VARIABLES

The application of this methodology to a four variable design begins to show its utility over traditional approaches for designing SP experiments. Clearly a graphical representation of the design is difficult to envisage, requiring a 3D plot of graphical planes.

The test design for a binary choice case is shown in table 6. This design is taken from Toner (1991). For space considerations, only the difference values for the variables are shown.

	Fare (pence)	Walk (min)	Wait (min)	IVTime (min)
1	150	-12	-10	-10
2	150	-7	-6	-7
3	150	-4	-3	-4
4	50	-12	-6	-4
5	50	-7	-3	-10
6	50	-4	-10	-7
7	80	-12	-3	-7

8	80	-7	-10	-4
9	80	-4	-6	-10

Table 6 : A possible binary choice four variable SP design (in differences)

The initial design and the optimised designs which result after the first iteration and after 10 iterations are given in figure 17.

Initial design

COST	WALK	WAIT	IVTIME	Pn	(1-Pn)
150.	-12.	-10.	-10.	0.1516	0.8484
150.	-7.	-6.	-7.	0.1000	0.9000
150.	-4.	-3.	-4.	0.0718	0.9282
50.	-12.	-6.	-4.	0.4611	0.5389
50.	-7.	-3.	-10.	0.4502	0.5498
50.	-4.	-10.	-7.	0.4693	0.5307
80.	-12.	-3.	-7.	0.3189	0.6811
80.	-7.	-10.	-4.	0.3208	0.6792
80.	-4.	-6.	-10.	0.3165	0.6835

	COST	WALK	WAIT	IVTIME
COST	1.0000	0.0000	0.0000	0.0000
WALK	0.0000	1.0000	0.0000	0.0000
WAIT	0.0000	0.0000	1.0000	0.0000
IVTIME	0.0000	0.0000	0.0000	1.0000

	estimate	t-ratio	
Cost	-0.0200	-0.8976	*
Walk T	-0.0340	-0.1778	
Wait T	-0.0440	-0.1965	
IVTime	-0.0430	-0.1815	

	estimate	t-ratio
Walk/C	1.7000	0.1878
Wait/C	2.2000	0.2077
Time/C	2.1500	0.1940

Design after min(t(β_1)),min(t(β_2)),min(t(β_3)),min(t(β_4))

COST	WALK	WAIT	IVTIME	Pn	(1-Pn)
144.	-81.	16.	36.	0.0849	0.9151
141.	-23.	-9.	17.	0.0853	0.9147
138.	-9.	-18.	18.	0.0805	0.9195
91.	-303.	69.	182.	0.0847	0.9153
211.	16.	74.	-242.	0.9158	0.0842
155.	117.	-170.	66.	0.0805	0.9195
377.	-211.	-25.	-39.	0.9177	0.0823
129.	7.	-98.	-20.	0.9133	0.0867
48.	112.	62.	-118.	0.0815	0.9185

	COST	WALK	WAIT	IVTIME
COST	1.0000	-0.3459	-0.1527	-0.2405
WALK	-0.3459	1.0000	-0.3670	-0.4856
WAIT	-0.1527	-0.3670	1.0000	-0.2846
IVTIME	-0.2405	-0.4856	-0.2846	1.0000

	estimate	t-ratio	
Cost	-0.0200	-1.5593	
Walk T	-0.0340	-1.6787	
Wait T	-0.0440	-1.5530	
IVTime	-0.0430	-1.9880	*

	estimate	t-ratio
Walk/C	1.7000	2.8066
Wait/C	2.2000	2.2372
Time/C	2.1500	2.5254

Design after min(t(β_1)),min(t(β_2)),min(t(β_3)),min(t(β_4))
10 times

COST	WALK	WAIT	IVTIME	Pn	(1-Pn)
550.	-238.	-321.	317.	0.0820	0.9180
585.	67.	-473.	215.	0.0823	0.9177
350.	-61.	-387.	337.	0.0840	0.9160
-91.	-702.	113.	538.	0.0818	0.9182
1039.	265.	209.	-850.	0.0805	0.9195
335.	799.	-914.	92.	0.9164	0.0836
1645.	-1007.	224.	-254.	0.9171	0.0829
516.	-12.	-147.	-136.	0.9171	0.0829
121.	469.	8.	-379.	0.0816	0.9184

	COST	WALK	WAIT	IVTIME
COST	1.0000	-0.3821	0.3581	-0.5315
WALK	-0.3821	1.0000	-0.5898	-0.2778
WAIT	0.3581	-0.5898	1.0000	-0.4592
IVTIME	-0.5315	-0.2778	-0.4592	1.0000

	estimate	t-ratio	
Cost	-0.0200	-1.9576	
Walk T	-0.0340	-1.9609	
Wait T	-0.0440	-1.9610	
IVTime	-0.0430	-1.9881	*

	estimate	t-ratio
Walk/C	1.7000	10.6499
Wait/C	2.2000	10.0233
Time/C	2.1500	10.7921

Figure 17 : Initial, first optimised and final designs for table 6

Once again all the features seen for the two variable situation occur here. The p values are close to p*, as seen previously.

7 TWO VARIABLES PLUS ASC

The equation for the utility of each mode given in (1) can be modified to include an alternative specific constant (ASC). The role of this constant is to account for factors which are not specifically included in the design when determining the attractiveness of one mode over another. The revised form of equation (1) becomes:

$$U_a = ASC + \beta_1 COST_a + \beta_2 TIME_a + \varepsilon$$

$$U_b = \beta_1 COST_b + \beta_2 TIME_b + \varepsilon$$

If the ASC is estimated to be negative then, all other things being equal, $U_a < U_b$ and alternative B would be preferred over A. If the ASC is positive then A would be the preferred mode. This revised form can be cast into the general form of an SP by setting one of the variables to a constant value. Table 7 gives an illustrative example of a two variable design with a range of ASC's, taken from Fowkes (1991).

	Alternative A	Alternative B	Difference (A-B)	ASC for option A

	COST (£)	TIME (min)	COST (£)	TIME (min)	ΔCOST	ΔTIME	BVoT (pence/min) ASC=0	ASC=2	ASC=5
1	40	460	20	260	20.00	240.00	10.00	9.00	7.50
2	60	460	50	260	10.00	250.00	5.00	4.00	2.50
3	60	610	50	210	10.00	200.00	2.50	2.00	1.25
4	40	310	20	210	20.00	190.00	20.00	18.00	15.00
5	60	460	20	360	40.00	320.00	40.00	38.00	35.00
6	40	310	50	360	-10.00	370.00	20.00	24.00	30.00

Table 7 : Two variable with an ASC design

If there is an expectation that the ASC is zero then the methodology used in section 4 can be applied. Otherwise the optimisation process must take account of the presence of the ASC but must not alter its value since it is, like β_1 and β_2, a given parameter.

Figure 18 shows the results after 15 iterations of min(t(β_1)) and min(t(β_2)) with ASC's of 2.00 and 5.00.

```
Initial design with ASC=2.00

   ASC   COST   TIME       Pn       (1-Pn)      BVoT
     2.    20.  -200.    0.0573    0.9427    0.0900
     2.    10.  -200.    0.5498    0.4502    0.0400
     2.    10.  -400.    0.8022    0.1978    0.0200
     2.    20.  -100.    0.0323    0.9677    0.1800
     2.    40.  -100.    0.0001    0.9999    0.3800
     2.   -10.    50.    0.9910    0.0090    0.2400

CORR (COST,TIME) =  -0.5247

ASC     2.00000     6.63744    0.77630
COST   -0.30000     0.08949   -1.00285
TIME   -0.00600     0.00017   -0.45634
T/C     0.02000 15450.01465    0.00016

Design after 10 iterations

   ASC   COST   TIME       Pn       (1-Pn)      BVoT
     2.   178.  -214.    0.0000    1.0000    0.8224
     2.     5.  -283.    0.9001    0.0999    0.0106
     2.    19.  -977.    0.8968    0.1032    0.0174
     2.    19.  -160.    0.0607    0.9393    0.1062
     2.    18.  -128.    0.0671    0.9329    0.1250
     2.    -1.   814.    0.0702    0.9298    0.0037

CORR (COST,TIME) =  -0.2230

ASC     2.00000    2.70369    1.21633
COST   -0.30000    0.08331   -1.03938
TIME   -0.00600    0.00002   -1.46319
T/C     0.02000 1111.05481    0.00060

Initial design with ASC=5.00

   ASC   COST   TIME       Pn       (1-Pn)      BVoT
     5.    20.  -200.    0.5498    0.4502    0.0750
```

```
      5.    10.  -200.     0.9608    0.0392    0.0250
      5.    10.  -400.     0.9879    0.0121    0.0125
      5.    20.  -100.     0.4013    0.5987    0.1500
      5.    40.  -100.     0.0017    0.9983    0.3500
      5.   -10.    50.     0.9995    0.0005    0.3000

CORR (COST,TIME) =  -0.5247

ASC      5.00000      4.88852    2.26142
COST    -0.30000      0.21312   -0.64985
TIME    -0.00600      0.00059   -0.24766
T/C      0.02000 59243.35547    0.00008

Design after 10 iterations

   ASC   COST   TIME       Pn       (1-Pn)      BVoT
     5.   193.  -178.     0.0000    1.0000    1.0562
     5.    12.  -163.     0.9151    0.0849    0.0429
     5.    28.  -958.     0.9128    0.0872    0.0240
     5.   -82.  -121.     1.0000    0.0000   -0.7190
     5.    24.    20.     0.0895    0.9105   -0.9500
     5.    15.   461.     0.0940    0.9060   -0.0217

CORR (COST,TIME) =  -0.1552

ASC      5.00000     1.93686    3.59270
COST    -0.30000     0.13150   -0.82730
TIME    -0.00600     0.00002   -1.32280
T/C      0.02000 2065.62158    0.00044
```

Figure 18 : Initial and final designs for ASC=2.00 and ASC=5.00.

In both cases the final design has produced improvements in the t-ratios for all parameters. For the case where ASC=2.00, the final optimised design does not possess p* values, the first time this feature has been noted. When ASC=5.00 the design does contains some near p* but also some 1.0 or 0.0 p's. The final $t(\beta_2)$ value in this design, 1.32280 corresponds to a t*=1.32548 with n=4, ie the number of scenarios with p*'s.

8 CONSTRAINTS

One undesirable feature of this methodology is the tendency to produce large magnitude differences in the variables. This may be practically impossible or infeasible. One approach is to set limits on these differences. The optimisation process can either be stopped when any of these limits are exceeded or constrained to operate within these limits. The first approach was adopted in section 4 where the design after only two iterations was chosen as the best. This design still gave a reasonable increase in all the t-ratio's over the initial design. The second approach is to specify constraints in the optimisation process. By way of example, the ΔCOST variable can be constrained to lie within [1,100] and the ΔTIME to be within [-50,-1]. When this modification is applied, the results are as given in figure 19.

```
Initial design (as given in figure 2)

  COST   TIME       Pn       (1-Pn)      BVoT
    15.   -10.    0.6225    0.3775    1.5000
    25.   -10.    0.3775    0.6225    2.5000
    40.   -10.    0.1192    0.8808    4.0000
```

15.	-15.	0.8176	0.1824	1.0000
25.	-15.	0.6225	0.3775	1.6667
40.	-15.	0.2689	0.7311	2.6667
15.	-20.	0.9241	0.0759	0.7500
25.	-20.	0.8176	0.1824	1.2500
40.	-20.	0.5000	0.5000	2.0000

CORR (COST,TIME) = 0.0000

	estimate	variance	t-ratio
COST	-0.1000	0.0058	-1.3152
TIME	-0.2000	0.0211	-1.3769
VoT	2.0000	0.2926	3.6976

Final design after 10 iterations of min(t(β_1)) and min(t(β_2))

COST	TIME	Pn	(1-Pn)	BVoT
76.	-50.	0.9168	0.0832	1.5200
95.	-35.	0.0759	0.9241	2.7143
95.	-36.	0.0911	0.9089	2.6389
76.	-50.	0.9168	0.0832	1.5200
76.	-50.	0.9168	0.0832	1.5200
95.	-35.	0.0759	0.9241	2.7143
76.	-50.	0.9168	0.0832	1.5200
76.	-50.	0.9168	0.0832	1.5200
95.	-36.	0.0911	0.9089	2.6389

CORR (COST,TIME) = 0.9989

	estimate	variance	t-ratio
COST	-0.1000	0.0027	-1.9123
TIME	-0.2000	0.0101	-1.9871
VoT	2.0000	0.0806	7.0445

Final design after 1 iteration of Σ (t*-t(β_i))^2

COST	TIME	Pn	(1-Pn)	BVoT
40.	-32.	0.9168	0.0832	1.2500
100.	-38.	0.0832	0.9168	2.6316
100.	-38.	0.0832	0.9168	2.6316
55.	-40.	0.9241	0.0759	1.3750
76.	-50.	0.9168	0.0832	1.5200
100.	-38.	0.0832	0.9168	2.6316
61.	-42.	0.9089	0.0911	1.4524
76.	-50.	0.9168	0.0832	1.5200
100.	-38.	0.0832	0.9168	2.6316

CORR (COST,TIME) = -0.0461

	estimate	variance	t-ratio
COST	-0.1000	0.0026	-1.9595
TIME	-0.2000	0.0103	-1.9721
VoT	2.0000	0.0889	6.7096

Figure 19 : Constrained two variable design

The final design does yield higher t-ratios for the parameters and the VoT than those in the initial design. For comparison purposes the t(VoT) value after the second iteration of an unconstrained optimisation was 5.6758. Notice some redundancy in the scenarios with some BVoT's making multiple appearances.

9 CONCLUSIONS

This paper has demonstrated that the methodology devised can be applied to practical binary choice Stated Preference designs. To summarise, the methodology is:

- simple in its application;
- able to deliver real, quantifiable benefits over traditional SP design methodologies;
- is applicable to an n-variable design, n>2;
- can accommodate designs with alternative specific constants;
- flexible enough to code an incorporate a variety of user requirements;
- works within constraints;
- simple to implement in spreadsheets or FORTRAN code.

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APPENDIX

Non-integrated

The three parameter pairs (-0.1,-0.2), (-0.1,-0.1) and (-0.1,-0.3) with the initial design

COST	TIME	Pn	(1-Pn)	BVoT
15.	-10.	0.6225	0.3775	1.5000
25.	-10.	0.3775	0.6225	2.5000
40.	-10.	0.1192	0.8808	4.0000
15.	-15.	0.8176	0.1824	1.0000
25.	-15.	0.6225	0.3775	1.6667
40.	-15.	0.2689	0.7311	2.6667
15.	-20.	0.9241	0.0759	0.7500
25.	-20.	0.8176	0.1824	1.2500
40.	-20.	0.5000	0.5000	2.0000

CORR (COST,TIME) = 0.0000

COST	-0.1000	0.0058	-1.3152
TIME	-0.2000	0.0211	-1.3769
VoT	2.0000	0.2926	3.6976

COST	TIME	Pn	(1-Pn)	BVoT
15.	-10.	0.3775	0.6225	1.5000
25.	-10.	0.1824	0.8176	2.5000
40.	-10.	0.0474	0.9526	4.0000
15.	-15.	0.5000	0.5000	1.0000
25.	-15.	0.2689	0.7311	1.6667
40.	-15.	0.0759	0.9241	2.6667
15.	-20.	0.6225	0.3775	0.7500
25.	-20.	0.3775	0.6225	1.2500
40.	-20.	0.1192	0.8808	2.0000

CORR (COST,TIME) = 0.0000

	estimate	variance	t-ratio
COST	-0.1000	0.0070	-1.1956
TIME	-0.1000	0.0158	-0.7963
VoT	1.0000	0.3546	1.6792

COST	TIME	Pn	(1-Pn)	BVoT
15.	-10.	0.8176	0.1824	1.5000
25.	-10.	0.6225	0.3775	2.5000
40.	-10.	0.2689	0.7311	4.0000
15.	-15.	0.9526	0.0474	1.0000
25.	-15.	0.8808	0.1192	1.6667
40.	-15.	0.6225	0.3775	2.6667
15.	-20.	0.9890	0.0110	0.7500
25.	-20.	0.9707	0.0293	1.2500
40.	-20.	0.8808	0.1192	2.0000

CORR (COST,TIME) = 0.0000

	estimate	variance	t-ratio
COST	-0.1000	0.0079	-1.1264
TIME	-0.3000	0.0442	-1.4273
VoT	3.0000	0.9594	3.0628

Optimise (-0.1,-0.2)

COST	TIME	Pn	(1-Pn)	BVoT
61.	-42.	0.9089	0.0911	1.4524
22.	1.	0.0832	0.9168	-22.0000
38.	-7.	0.0832	0.9168	5.4286

	estimate	variance	t-ratio
COST	-0.1000	0.0100	-0.9994
TIME	-0.2000	0.0304	-1.1470
VoT	2.0000	0.8794	2.1328

COST	TIME	Pn	(1-Pn)	BVoT
90.	-57.	0.9168	0.0832	1.5789
33.	-4.	0.0759	0.9241	8.2500
57.	-17.	0.0911	0.9089	3.3529

	estimate	variance	t-ratio
COST	-0.1000	0.0086	-1.0775
TIME	-0.2000	0.0304	-1.1469
VoT	2.0000	0.4153	3.1033

Optimise (-0.1,-0.1)

COST	TIME	Pn	(1-Pn)	BVoT
59.	-83.	0.9168	0.0832	0.7108
24.	1.	0.0759	0.9241	-24.0000
35.	-11.	0.0832	0.9168	3.1818

	estimate	variance	t-ratio
COST	-0.1000	0.0102	-0.9915
TIME	-0.1000	0.0076	-1.1474
VoT	1.0000	0.2505	1.9980

COST	TIME	Pn	(1-Pn)	BVoT
89.	-113.	0.9168	0.0832	0.7876
35.	-11.	0.0832	0.9168	3.1818
54.	-30.	0.0832	0.9168	1.8000

	estimate	variance	t-ratio
COST	-0.1000	0.0087	-1.0736
TIME	-0.1000	0.0076	-1.1479
VoT	1.0000	0.1087	3.0328

Optimise (-0.1,-0.3)

COST	TIME	Pn	(1-Pn)	BVoT
32.	-19.	0.9241	0.0759	1.6842
47.	-24.	0.9241	0.0759	1.9583
79.	-18.	0.0759	0.9241	4.3889

	estimate	variance	t-ratio
COST	-0.1000	0.0091	-1.0475
TIME	-0.3000	0.0685	-1.1465
VoT	3.0000	1.3551	2.5771

COST	TIME	Pn	(1-Pn)	BVoT
42.	-22.	0.9168	0.0832	1.9091
62.	-29.	0.9241	0.0759	2.1379
104.	-27.	0.0911	0.9089	3.8519

	estimate	variance	t-ratio
COST	-0.1000	0.0082	-1.1022
TIME	-0.3000	0.0685	-1.1464
VoT	3.0000	0.6931	3.6036

All three segments are assembled to give the final design and the t-ratios are calculated.

COST	TIME	Pn	(1-Pn)	BVoT
90.	-57.	0.9168	0.0832	1.5789
33.	-4.	0.0759	0.9241	8.2500
57.	-17.	0.0911	0.9089	3.3529
89.	-113.	1.0000	0.0000	0.7876
35.	-11.	0.2142	0.7858	3.1818
54.	-30.	0.6457	0.3543	1.8000
42.	-22.	0.5498	0.4502	1.9091
62.	-29.	0.4013	0.5987	2.1379
104.	-27.	0.0067	0.9933	3.8519

CORR (COST,TIME) = -0.6348

	estimate	variance	t-ratio
COST	-0.1000	0.0054	-1.3634
TIME	-0.2000	0.0213	-1.3706
VoT	2.0000	0.1236	5.6893

COST	TIME	Pn	(1-Pn)	BVoT
90.	-57.	0.0356	0.9644	1.5789
33.	-4.	0.0522	0.9478	8.2500
57.	-17.	0.0180	0.9820	3.3529
89.	-113.	0.9168	0.0832	0.7876
35.	-11.	0.0832	0.9168	3.1818
54.	-30.	0.0832	0.9168	1.8000
42.	-22.	0.1192	0.8808	1.9091
62.	-29.	0.0356	0.9644	2.1379
104.	-27.	0.0005	0.9995	3.8519

CORR (COST,TIME) = -0.6348

	estimate	variance	t-ratio
COST	-0.1000	0.0031	-1.7896
TIME	-0.1000	0.0041	-1.5661
VoT	1.0000	0.0803	3.5299

COST	TIME	Pn	(1-Pn)	BVoT
90.	-57.	0.9997	0.0003	1.5789
33.	-4.	0.1091	0.8909	8.2500
57.	-17.	0.3543	0.6457	3.3529
89.	-113.	1.0000	0.0000	0.7876
35.	-11.	0.4502	0.5498	3.1818
54.	-30.	0.9734	0.0266	1.8000
42.	-22.	0.9168	0.0832	1.9091
62.	-29.	0.9241	0.0759	2.1379
104.	-27.	0.0911	0.9089	3.8519

CORR (COST,TIME) = -0.6348

	estimate	variance	t-ratio
COST	-0.1000	0.0046	-1.4679
TIME	-0.3000	0.0422	-1.4602
VoT	3.0000	0.3657	4.9610

Integrated

The starting designs are the same as those for the non-integrated process

Optimise (-0.1,-0.2)

COST	TIME	Pn	(1-Pn)	BVoT	
104.	-64.	0.9168	0.0832	1.6250	Only change
47.	-11.	0.0759	0.9241	4.2727	these three
94.	-35.	0.0832	0.9168	2.6857	scenarios
15.	-15.	0.8176	0.1824	1.0000	
25.	-15.	0.6225	0.3775	1.6667	
40.	-15.	0.2689	0.7311	2.6667	
15.	-20.	0.9241	0.0759	0.7500	
25.	-20.	0.8176	0.1824	1.2500	
40.	-20.	0.5000	0.5000	2.0000	

CORR (COST,TIME) = -0.8316

	estimate	variance	t-ratio
COST	-0.1000	0.0041	-1.5593
TIME	-0.2000	0.0149	-1.6375
VoT	2.0000	0.1505	5.1550

Optimise (-0.1,-0.1)

COST	TIME	Pn	(1-Pn)	BVoT	
104.	-64.	0.0180	0.9820	1.6250	
47.	-11.	0.0266	0.9734	4.2727	
94.	-35.	0.0027	0.9973	2.6857	
163.	-187.	0.9168	0.0832	0.8717	Only change
5.	19.	0.0832	0.9168	-0.2632	these three
34.	-10.	0.0832	0.9168	3.4000	scenarios
15.	-20.	0.6225	0.3775	0.7500	
25.	-20.	0.3775	0.6225	1.2500	
40.	-20.	0.1192	0.8808	2.0000	

CORR (COST,TIME) = -0.9099

	estimate	variance	t-ratio
COST	-0.1000	0.0043	-1.5333
TIME	-0.1000	0.0039	-1.6025
VoT	1.0000	0.0363	5.2499

Optimise (-0.1,-0.3)

COST	TIME	Pn	(1-Pn)	BVoT	
104.	-64.	0.9998	0.0002	1.6250	
47.	-11.	0.1978	0.8022	4.2727	
94.	-35.	0.7503	0.2497	2.6857	
163.	-187.	1.0000	0.0000	0.8717	
5.	19.	0.0020	0.9980	-0.2632	
34.	-10.	0.4013	0.5987	3.4000	
-42.	6.	0.9168	0.0832	7.0000	Only change
-16.	-3.	0.9241	0.0759	-5.3333	these three
26.	-17.	0.9241	0.0759	1.5294	scenarios

CORR (COST,TIME) = -0.8693

	estimate	variance	t-ratio
COST	-0.1000	0.0054	-1.3555
TIME	-0.3000	0.0454	-1.4075
VoT	3.0000	0.3621	4.9854

The three parameter pairs with optimal design

COST	TIME	Pn	(1-Pn)	BVoT
104.	-64.	0.9168	0.0832	1.6250
47.	-11.	0.0759	0.9241	4.2727
94.	-35.	0.0832	0.9168	2.6857
163.	-187.	1.0000	0.0000	0.8717
5.	19.	0.0134	0.9866	-0.2632
34.	-10.	0.1978	0.8022	3.4000
-42.	6.	0.9526	0.0474	7.0000
-16.	-3.	0.9002	0.0998	-5.3333
26.	-17.	0.6900	0.3100	1.5294

CORR (COST,TIME) = -0.8693

	estimate	variance	t-ratio
COST	-0.1000	0.0036	-1.6737
TIME	-0.2000	0.0149	-1.6380
VoT	2.0000	0.2038	4.4300

COST	TIME	Pn	(1-Pn)	BVoT
104.	-64.	0.0180	0.9820	1.6250
47.	-11.	0.0266	0.9734	4.2727
94.	-35.	0.0027	0.9973	2.6857
163.	-187.	0.9168	0.0832	0.8717
5.	19.	0.0832	0.9168	-0.2632
34.	-10.	0.0832	0.9168	3.4000
-42.	6.	0.9734	0.0266	7.0000
-16.	-3.	0.8699	0.1301	-5.3333
26.	-17.	0.2891	0.7109	1.5294

CORR (COST,TIME) = -0.8693

	estimate	variance	t-ratio
COST	-0.1000	0.0037	-1.6402
TIME	-0.1000	0.0034	-1.7142
VoT	1.0000	0.0385	5.0985

COST	TIME	Pn	(1-Pn)	BVoT
104.	-64.	0.9998	0.0002	1.6250
47.	-11.	0.1978	0.8022	4.2727
94.	-35.	0.7503	0.2497	2.6857
163.	-187.	1.0000	0.0000	0.8717
5.	19.	0.0020	0.9980	-0.2632
34.	-10.	0.4013	0.5987	3.4000
-42.	6.	0.9168	0.0832	7.0000
-16.	-3.	0.9241	0.0759	-5.3333
26.	-17.	0.9241	0.0759	1.5294

CORR (COST,TIME) = -0.8693

	estimate	variance	t-ratio
COST	-0.1000	0.0054	-1.3555
TIME	-0.3000	0.0454	-1.4075
VoT	3.0000	0.3621	4.9854

```
Change the sequence of parameter pairs

Optimise (-0.1,-0.3)

  COST   TIME      Pn      (1-Pn)     BVoT
    15.   -10.    0.8176    0.1824    1.5000
    25.   -10.    0.6225    0.3775    2.5000
    40.   -10.    0.2689    0.7311    4.0000
    15.   -15.    0.9526    0.0474    1.0000
    25.   -15.    0.8808    0.1192    1.6667
    40.   -15.    0.6225    0.3775    2.6667
  -145.    40.    0.9241    0.0759    3.6250    Only change
   -71.    16.    0.9089    0.0911    4.4375    these three
  -159.    61.    0.0832    0.9168    2.6066    scenarios

CORR (COST,TIME) =  -0.9782

      estimate  variance   t-ratio
COST   -0.1000    0.0038   -1.6221
TIME   -0.3000    0.0322   -1.6708
VoT     3.0000    0.1888    6.9036

Optimise (-0.1,-0.2)

  COST   TIME      Pn      (1-Pn)     BVoT
   140.   -82.    0.9168    0.0832    1.7073    Only change
    -9.    16.    0.0911    0.9089    0.5625    these three
    28.    -2.    0.0832    0.9168   14.0000    scenarios
    15.   -15.    0.8176    0.1824    1.0000
    25.   -15.    0.6225    0.3775    1.6667
    40.   -15.    0.2689    0.7311    2.6667
  -145.    40.    0.9985    0.0015    3.6250
   -71.    16.    0.9802    0.0198    4.4375
  -159.    61.    0.9759    0.0241    2.6066

CORR (COST,TIME) =  -0.9528

      estimate  variance   t-ratio
COST   -0.1000    0.0042   -1.5390
TIME   -0.2000    0.0155   -1.6090
VoT     2.0000    0.1438    5.2734

Optimise (-0.1,-0.1)

  COST   TIME      Pn      (1-Pn)     BVoT
   140.   -82.    0.0030    0.9970    1.7073
    -9.    16.    0.3318    0.6682    0.5625
    28.    -2.    0.0691    0.9309   14.0000
   145.  -169.    0.9168    0.0832    0.8580    Only change
    37.   -13.    0.0832    0.9168    2.8462    these three
    67.   -43.    0.0832    0.9168    1.5581    scenarios
  -145.    40.    1.0000    0.0000    3.6250
   -71.    16.    0.9959    0.0041    4.4375
  -159.    61.    0.9999    0.0001    2.6066

CORR (COST,TIME) =  -0.8878

      estimate  variance   t-ratio
COST   -0.1000    0.0052   -1.3823
TIME   -0.1000    0.0048   -1.4469
VoT     1.0000    0.0455    4.6902
```

The three parameter pairs with optimal design

COST	TIME	Pn	(1-Pn)	BVoT
140.	-82.	1.0000	0.0000	1.7073
-9.	16.	0.0198	0.9802	0.5625
28.	-2.	0.0998	0.9002	14.0000
145.	-169.	1.0000	0.0000	0.8580
37.	-13.	0.5498	0.4502	2.8462
67.	-43.	0.9980	0.0020	1.5581
-145.	40.	0.9241	0.0759	3.6250
-71.	16.	0.9089	0.0911	4.4375
-159.	61.	0.0832	0.9168	2.6066

CORR (COST,TIME) = -0.8878

	estimate	variance	t-ratio
COST	-0.1000	0.0047	-1.4544
TIME	-0.3000	0.0428	-1.4500
VoT	3.0000	0.2157	6.4594

COST	TIME	Pn	(1-Pn)	BVoT
140.	-82.	0.9168	0.0832	1.7073
-9.	16.	0.0911	0.9089	0.5625
28.	-2.	0.0832	0.9168	14.0000
145.	-169.	1.0000	0.0000	0.8580
37.	-13.	0.2497	0.7503	2.8462
67.	-43.	0.8699	0.1301	1.5581
-145.	40.	0.9985	0.0015	3.6250
-71.	16.	0.9802	0.0198	4.4375
-159.	61.	0.9759	0.0241	2.6066

CORR (COST,TIME) = -0.8878

	estimate	variance	t-ratio
COST	-0.1000	0.0044	-1.5126
TIME	-0.2000	0.0153	-1.6157
VoT	2.0000	0.1323	5.4991

COST	TIME	Pn	(1-Pn)	BVoT
140.	-82.	0.0030	0.9970	1.7073
-9.	16.	0.3318	0.6682	0.5625
28.	-2.	0.0691	0.9309	14.0000
145.	-169.	0.9168	0.0832	0.8580
37.	-13.	0.0832	0.9168	2.8462
67.	-43.	0.0832	0.9168	1.5581
-145.	40.	1.0000	0.0000	3.6250
-71.	16.	0.9959	0.0041	4.4375
-159.	61.	0.9999	0.0001	2.6066

CORR (COST,TIME) = -0.8878

	estimate	variance	t-ratio
COST	-0.1000	0.0052	-1.3823
TIME	-0.1000	0.0048	-1.4469
VoT	1.0000	0.0455	4.6902