Multi-attribute Aggregation Operators

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Abstract

This paper deals with the idea of aggregation. A new, enlarged notion of aggregation operator is given, along with a classification of classical properties into some main groups. The concept of multi-attribute aggregation operator, which incorporates many classical aggregation methods, is provided. An extension of different properties to multi-attribute aggregation operators is proposed.

Key words: Aggregation operator; OWA operator; multi-attribute aggregation; weighted aggregation.

1 Introduction

Though the fusion of observed numerical input values into a single output value can be found in the majority of areas dealing with quantitative information, the theory of aggregation can be dated to the last decade only. The axiomatic framework for aggregation proposed in [9], not limited to a fixed number of inputs, was related to earlier ideas on n-ary aggregation functions discussed in [8], and we call this approach "classical". For an overview of results linked to classical aggregation functions we recommend a recent monograph ([6]). The classical approach covers huge classes of aggregation techniques with different roots, such as different kinds of means, conjunctive and disjunctive operators,

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and usually parametric classes of such operators are exploited to model an appropriate aggregation in each discussed domain, often based on fitting the optimal parameters to a sample space. A recursive approach proposed in [3] is based on a consecutive application of binary aggregation operators, which may differ in different steps, and thus reflect a development of an observed process. However, the basic theory of aggregation seems to be incomplete, because it does not adequately contemplate the crucial role played in many cases by some collateral parameters, called in the sequel *attributes*. Note that some particular methods based on attributes are well-known, especially various kinds of weighted aggregation ([2]) or induced aggregation ([14]), but a unified theory of multi-attribute aggregation operators is not yet known in literature. The aim of our paper is to fill this gap.

The paper is organized as follows: in Section 2, we recall the classical notion of aggregation operator and we propose a new, enlarged one, along with two types of classification of the properties which may characterize any operator into some main groups, possibly overlapping. In Section 3, we formally introduce the notion of multi-attribute aggregation operator, able to incorporate some prominent aggregation methods, as shown with explicit examples. Finally, in Section 4, we extend many typical properties of classical aggregation operators to multi-attribute ones: at the same time, we explain why such generalization is not able to cover any kind of property.

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