Lumped Parameter Model for the Time-Domain Soil-Structure Interaction Analysis of Structures on Pile Foundations

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ABSTRACT
A lumped parameter model for the time domain inertial soil-structure interaction analysis is proposed with reference to square pile group foundations. Simplified formulas are presented for estimating its parameters. The model is able to reproduce the coupled rotational-translational behaviour of the soil-foundation system. Formulas are calibrated from results of an extensive non-dimensional parametric analysis considering head-bearing pile groups. The closed-form expressions may be readily adopted to define the compliant base restraints of a generic structure for the non linear dynamic analysis carried out with commercial software.

1 INTRODUCTION

Dynamic Soil-Structure Interaction (SSI) is usually neglected in the design practice since it is traditionally considered to have beneficial effects on the seismic performance of structures. This arises from the convincement that compliant-base structures are subjected to lower inertia forces and have enhanced dissipation capacity due to the radiation damping of the soil-foundation system. However, in some circumstances SSI may significantly affects the seismic response of structures (Mylonakis, 2000).

SSI analysis may be performed by adopting the direct approach, which consists in studying structure, foundation and soil in a unique model (usually a finite element model), or the sub-structure approach, which consists in studying separately the soil-foundation system and the superstructure on compliant-base subjected to the foundation input motion (Wolf 1988). The second method is certainly more practical than the direct method since it may be applied using different modelling techniques for each subsystem. Making use of the superposition principle, this approach can of course be applied in the case of linear behaviour of soil, foundation and superstructure but, under the assumption of linear behaviour of the soil-foundation, can also be used in the case of nonlinear behaviour of the structure (Ciampoli and Pinto 2005, Carbonari et al, 2012). This simplifying assumption is generally acceptable if foundations are designed according to hierarchy principles. In the framework of the substructure approach, the analysis of the soil-foundation system has a twofold aim: (*i*) to evaluate the kinematic soil-foundation interaction (e.g. foundation input motion and stress resultants due to the propagation of seismic waves) and (*ii*) to define the soil-foundation dynamic impedance functions. The latter are complex-valued relationships between forces and displacements which have to be used in the inertial interaction analysis of the superstructure to define the foundation frequency-dependent compliance.

With reference to pile foundations, different models have been developed over the years to evaluate their dynamic stiffness. Numerical methods, generally requiring high computational efforts, are preferred to solve such a fully coupled problem; in particular, approaches based on the

Boundary Element Method (BEM) are the most rigorous ones (Kaynia, 1983); Finite Element Method (FEM) is generally adopted by introducing different level of simplifications (Waas et al., 1984; Takemiya, 1986) to avoid modelling the far-field, or even the whole soil. However, from an engineering perspective, methods based on simplified formulas have received more attention. Formulas generally derive from physical considerations and weight parameters are introduced to account for different soil conditions. Among the others, Gazetas and Dobry (1984) presented simple methods for estimating damping properties of horizontally loaded piles in layered soils; Dobry and Gazetas (1988) proposed a simplified formulation for the dynamic stiffness of floating pile groups starting from the dynamic stiffness of a single pile and adopting interaction factors to account for pile-to-pile interactions due to waves radiation. Interaction factors, derived from the solution of simplified wave propagation problems, have been also adopted by Banerjee and Sen (1987) and Makris and Gazetas (1992).

More recently, Dezi et al. (2009) have proposed a numerical model for the 3D kinematic interaction analysis of pile groups in horizontally layered soils, based on the BEM-FEM coupling in which piles are modelled with beam elements while the soil is schematized with horizontal independent infinite layers. The dynamics of the soil layer is described by means of Green's functions which allow accounting for both pile-soil-pile interactions and radiation damping. These derive from a simplification of the plane strain model of Novak et al. (1978) also developed by Gazetas and Dobry (1984). The model allows evaluating all the components of the frequency dependent foundation impedance matrix (e.g. translational, rotational and roto-traslational impedances).

Since foundation impedances are frequency-dependent functions, all previous procedures furnish results that may be adopted directly in frequency-domain interaction analysis of the superstructure (inertial interaction). However, such a kind of analysis may be adopted only if the superstructure behaves linearly and is less familiar to professional engineers than time-domain analysis. If inertial interaction analyses are performed in time-domain, as in the case of nonlinear behaviour of the structure, suitable Lumped Parameter Models (LPMs), obtained by assembling springs, masses and dashpots, have to be introduced to reproduce the foundation dynamic behaviour.

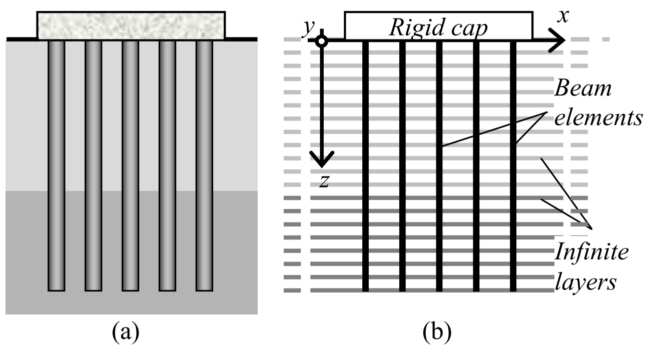
Dynamic SSI analyses are therefore computationally demanding procedures since they require performing dynamic soil-foundation interaction analyses and the calibration of suitable LPMs. From a practical point of view, simple engineering procedures may be of particular interest in view of the increasing attention that the SSI problem is receiving in modern codes. According to this need, Taherzadeh et al. (2009) presented simple formulas for the dynamic stiffness of large pile groups (greater than 50 piles).

In this paper simplified formulas for estimating the parameters of an LPM, able to reproduce the coupled rotational-translational behaviour of pile groups with small and medium dimensions, are presented. Formulas are calibrated from results of a non-dimensional parametric investigation considering head-bearing pile groups. Analyses are performed by means of the numerical model developed by the authors (Dezi et al. 2009). Formulas may be readily adopted by engineers to define restraint systems that may be implemented in common software for structural analysis.

2 FOUNDATION IMPEDANCES

The numerical model of Dezi et al. (2009), in which piles are modelled with beam elements and the soil is schematized with independent horizontal infinite layers, is used for the evaluation of the foundation impedances (Figure 1). With reference to a generic group of piles embedded in a horizontally layered soil, the dynamic stiffness matrix of the system, is written as

$$\mathbf{Z}(\omega) = \mathbf{K}_P - \omega^2 \mathbf{M}_P + \Re_P(\omega) \tag{1}$$



(a) (b)

Figure 1. (a) Pile group in a horizontally layered soil; (b) soil-foundation model.

where $\mathbf{K}_P$, $\mathbf{M}_P$ are the overall stiffness and mass matrices of the piles, respectively, and $\Re_P$ is the overall impedance matrix of the soil, accounting for pile-soil-pile interactions as well as hysteretic and radiation damping. The piles are connected at the head by a rigid cap having six generalized displacement components referred to a master node. Thus, the dynamic stiffness matrix of the constrained system may be written as

$$\left[\begin{array}{c|c} \mathbf{Z}_{FF} & \mathbf{Z}_{FE} \\ \hline \mathbf{Z}_{EF} & \mathbf{Z}_{EE} \end{array}\right] = \mathbf{A}^T\left(\mathbf{K}_P - \omega^2\mathbf{M}_P + \Re_P\right)\mathbf{A} \tag{2}$$

where the dynamic impedance matrix is suitably partitioned to highlight components relevant to the master node degree of freedom ($\mathbf{Z}_{FF}$) and $\mathbf{A}$ is the geometric matrix of the rigid constraint.

The derivation of the complex-valued foundation impedance matrix

$$\Im(\omega) = \left(\mathbf{Z}_{FF} - \mathbf{Z}_{FE}\mathbf{Z}_{EE}^{-1}\mathbf{Z}_{EF}\right) \tag{3}$$

is thus straightforward from the problem condensation. The impedance matrix $\Im$ expresses forces necessary to produce unit steady vibrations of the master node.

It is worth noting that matrix $\Im$ is generically fully populated; however, in the case of doubly symmetric pile layouts, locating the master node at the intersection of the two symmetry axes, the impedance matrix assumes the form

$$\Im(\omega) = \begin{bmatrix} \Im_x & 0 & 0 & 0 & \Im_{x-ry} & 0 \\ & \Im_y & 0 & -\Im_{y-rx} & 0 & 0 \\ & & \Im_z & 0 & 0 & 0 \\ & & & \Im_{rx} & 0 & 0 \\ & sym & & & \Im_{ry} & 0 \\ & & & & & \Im_{rz} \end{bmatrix} \tag{4}$$

For further details the reader may refer to Dezi et al. (2009).

3 LPM FOR TIME DOMAIN ANALYSIS

In the case of non-linear structural behaviour, the inertial interaction analysis must be performed in the time domain and the frequency-dependent impedances (4) of the soil-foundation system cannot be used directly. For this purpose LPMs constituted by frequency-independent parameters (springs, dashpots and masses), suitably assembled and calibrated in order to reproduce the dynamic behaviour of the soil-foundation system, can be introduced at the base of the superstructure (Wolf, 1988). Impedances of LPMs $\tilde{\Im}(\omega)$ must approximate those of the soil-foundation system within the frequency range in which the earthquake has the highest energy content and within which the fundamental structural vibration frequencies fall (typically 0÷10 Hz). The loss of precision in the solution is compensated by the advantage of using commercial structural analysis software in performing non-linear analyses.

In this work an LPM having the same degree of freedom of the foundation cap and able to approximate impedance matrix (4) is constructed (Figure 2) by assembling different uncoupled sub-models. It is characterized by 26 parameters: translational (m_x, m_y, m_z) and rotational (I_x, I_y, I_z) masses, lumped at the master node of the rigid cap, elastic (k_x, k_y k_z, k_{rx}, k_{ry}, k_{rz}) and viscous (c_x, c_y, c_z, c_{rx}, c_{ry}, c_{rz},) constants that define the relevant spring-dashpot elements and additional eccentric masses (m_{xh}, m_{yh}) connected to the master node by stiff links (with length h_x and h_y) and to the ground by spring-dashpot elements (k_{xh}, k_{yh}, c_{xh}, c_{yh}). These last components are introduced to catch the coupling between the rotation and the translation in x and y directions.

The impedance matrix of the proposed LPM is

$$\tilde{\Im}(\omega) = \left(\tilde{\mathbf{K}} - \omega^2\tilde{\mathbf{M}} + i\omega\tilde{\mathbf{C}}\right) \tag{5}$$

where

$$\tilde{\mathbf{K}} = \begin{bmatrix} k_x + k_{xh} & 0 & 0 & 0 & k_{yh}h_y & 0 \\ & k_y + k_{yh} & 0 & -k_{yh}h & 0 & 0 \\ & & k_z & 0 & 0 & 0 \\ & & & k_{rx} + k_{xh}h_x^2 & 0 & 0 \\ & sym & & & k_{ry} + k_{yh}h_y^2 & 0 \\ & & & & & k_{rz} \end{bmatrix} \tag{6}$$

$$\tilde{\mathbf{M}} = \begin{bmatrix} m_x + m_{xh} & 0 & 0 & 0 & m_{yh}h_y & 0 \\ & m_y + m_{yh} & 0 & -m_{yh}h & 0 & 0 \\ & & m_z & 0 & 0 & 0 \\ & & & I_x + m_{xh}h^2 & 0 & 0 \\ & sym & & & I_y + m_{yh}h^2 & 0 \\ & & & & & I_z \end{bmatrix} \tag{7}$$

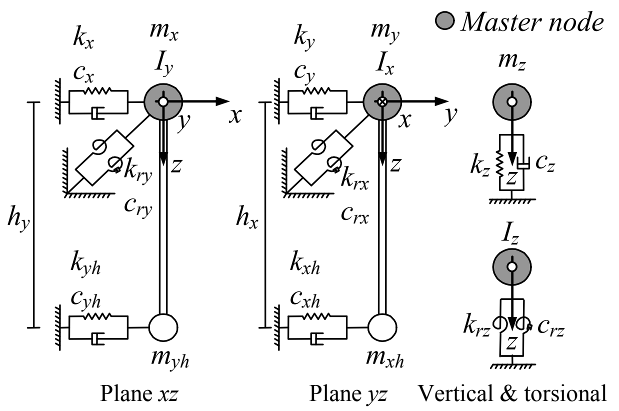


Figure 2. Assemblage of uncoupled LPMs.

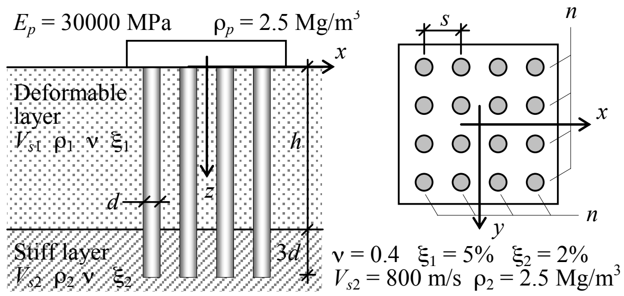


Figure 3. Scheme of investigated soil-foundation systems

Table 1. Parameters variability.

n	$\frac{s}{d}$	$\frac{h}{d}$	$\frac{E_p}{\rho_1 V_{s1}^2}\left(\frac{\rho_p}{\rho_1}\right)$
2	2	6	2000 (1.67)
3	3	12	442 (1.47)
4	4	18	94 (1.25)
5		24	

$$\widetilde{\mathbf{C}} = \begin{bmatrix} c_x + c_{xh} & 0 & 0 & 0 & c_{yh} h_y & 0 \\ & c_y + c_{yh} & 0 & -c_{yh} h & 0 & 0 \\ & & c_z & 0 & 0 & 0 \\ & & & c_{rx} + c_{xh} h_x^2 & 0 & 0 \\ & sym & & & c_{ry} + c_{yh} h_y^2 & 0 \\ & & & & & c_{rz} \end{bmatrix} \quad (8)$$

4 NON-DIMENSIONAL PARAMETRIC ANALYSIS

LPMs have been derived for a wide number of end-bearing pile groups with square patterns suitably parameterised. The scheme of the soil-foundation systems considered and the parameters defining both the geometry and the mechanical properties are reported in Figure 3. The Poisson's ratio ν, the soil hysteretic damping ratio ξ, the shear wave velocity V_{s2} and density ρ_2 of the bedrock and the pile elastic modulus E_p and density ρ_p are considered to be constant.

According to the Buckingham's theorem, the i-th component Π_i of the non-dimensional impedance matrix depends on the following quantities:

$$\Pi_i = f\left(n, \frac{s}{d}, \frac{h}{d}, \frac{E_p}{\rho_1 V_{s1}}, \frac{\rho_p}{\rho_1}, \frac{\omega d}{V_{s1}}\right) \quad (9)$$

which include geometric and mechanical parameters, as well as, the non-dimensional frequency $\omega d/V_{s1}$. Due to the square pile layout, the non-dimensional impedance matrix has only five independent components expressed as

$$\Pi_1 = \frac{\Im_i}{\rho_1 V_{s1}^2 d} \qquad \Pi_2 = \frac{\Im_{ri}}{\rho_1 V_{s1}^2 d^3} \quad (10a, b)$$

$$\Pi_3 = \frac{\Im_{i-rj}}{\rho_1 V_{s1}^2 d^2} \qquad \Pi_4 = \frac{\Im_z}{\rho_1 V_{s1}^2 d} \quad (11a, b)$$

$$\Pi_5 = \frac{\Im_{rz}}{\rho_1 V_{s1}^2 d^3} \quad (12)$$

All the combinations of parameters reported in Table 1 are considered for a total of 144 analysis cases. Analyse are performed by means of the numerical model of Dezi et al. (2009), implemented in the Matlab programming environment.

4.1 LPM optimization

The non-dimensional impedance matrix of each investigated soil-foundation system is approximated with expression (5), namely the real components are approximated with second order parabolas while imaginary components with straight lines passing for the axes origin. Due to the square pile layout, the 26 parameters defining the LPM reduce to only 16 as the coupled rotational-translational behaviour in x and y directions are the same. In particular the non-dimensional parameters of the LPM are

$$\Omega_1 = \frac{k_i}{\rho_1 V_{s1}^2 d} \qquad \Omega_2 = \frac{m_i}{\rho_1 d^3} \quad (13a, b)$$

$$\Omega_3 = \frac{c_i}{\rho_1 V_{s1} d^2} \qquad \Omega_4 = \frac{k_{ih}}{\rho_1 V_{s1}^2 d} \quad (14a, b)$$

$$\Omega_5 = \frac{m_{ih}}{\rho_1 d^3} \qquad \Omega_6 = \frac{c_{ih}}{\rho_1 V_{s1} d^2} \quad (15a, b)$$

$$\Omega_7 = \frac{k_{ri}}{\rho_1 V_{s1}^2 d^3} \qquad \Omega_8 = \frac{I_i}{\rho_1 d^5} \quad (16a, b)$$

$$\Omega_9 = \frac{c_{ri}}{\rho_1 V_{s1} d^4} \qquad \Omega_{10} = \frac{h_i}{d} \quad (17a, b)$$

$$\Omega_{11} = \frac{k_z}{\rho_1 V_{s1}^2 d} \qquad \Omega_{12} = \frac{m_z}{\rho_1 d^3} \quad (18a, b)$$

$$\Omega_{13} = \frac{c_z}{\rho_1 V_{s1} d^2} \qquad \Omega_{14} = \frac{k_{rz}}{\rho_1 V_{s1}^2 d^3} \quad (19a, b)$$

$$\Omega_{15} = \frac{I_z}{\rho_1 d^5} \qquad \Omega_{16} = \frac{c_{rz}}{\rho_1 V_{s1} d^4} \quad (20a, b)$$

in which $i = x, y$. Such parameters are identified according to a least square procedure within the non-dimensional frequency range ($0 \div a_{0,\max}$) with

$$a_{0,\max} = \frac{20\pi d}{V_{s1}} \tag{21}$$

To avoid thermodynamic inconsistencies the optimization problem has been constrained by imposing LPM stiffness, damping and mass matrices to be positive definite. Furthermore, many commercial computer programs for structural analysis do not support the implementation of negative masses so that a further constraint has been imposed to the mass sign; in some cases this enforces solutions with null masses. Impedances of the soil-foundation system are thus overall well approximated in the desired frequency range even if some inaccuracies may arise in same cases as a consequence of the LPM limits in capturing impedances fluctuations.

Table 2. Parameters of Equation (22).

Ω_i	α	β	χ	δ	ε	ϕ
Ω_1	0.002	1.379	1.028	0.107	1.294	-9.670
Ω_2	0.737	2.641	0.963	-0.041	0.076	0.532
Ω_3	0.021	2.158	1.481	-0.003	1.086	-7.836
Ω_4	1.568	1.004	0.157	-0.219	0.421	-1.597
Ω_5	2229.4	3.503	0.545	-2.817	-3.546	44.915
Ω_6	0.884	1.774	0.819	0.143	0.141	0.444
Ω_7	0.020	3.782	2.035	-0.552	1.577	-8.485
Ω_8	0.278	5.969	3.225	0.161	-0.906	8.521
Ω_9	5.6e-4	4.957	3.908	0.571	0.386	0.772
Ω_{10}	0.155	0.394	0.017	-2.8e-4	0.423	-1.572
Ω_{11}	0.117	1.246	0.201	-0.343	1.607	-8.556
Ω_{12}	823.41	3.325	-0.542	0.344	-4.475	55.078
Ω_{13}	0.161	2.516	1.337	0.060	0.720	-3.129
Ω_{14}	1.282	3.170	1.938	-0.080	0.165	0.177
Ω_{15}	12.712	4.284	2.517	-0.027	-0.197	-2.243
Ω_{16}	0.021	4.500	3.210	0.099	-0.084	5.849

Table 3. Parameters of Equation (23).

	Ω_2	Ω_3	Ω_4	Ω_6	Ω_7	Ω_9	Ω_{11}	Ω_{13}	Ω_{14}	Ω_{15}	Ω_{16}
a	0	1.318	0	1.575	0	0.966	1.206	2.848	0.010	0	2.050
b_1	0	0.439	0	0	0.271	2.196	0	0.604	0.043	1.048	0
b_2	0	0	0	0	0	0	0.191	0	0	0.385	0.028
b_3	0	0	0	0	0	0	0	0	0.009	0.089	0
b_4	0	0	0	0	0	0	0	0	0.007	0	0
b_5	0	0	0	0	0	0	0	0	0.002	0.191	0
c_1	2.883	0.086	0	0	0.932	0	0	0.026	0.238	0.049	0.220
c_2	0	0	0	0.116	0.265	0.419	0	0	0.714	0	0.442
c_3	0	0	1.067	0	0	0.011	0.073	0.014	0.058	0	0
c_4	0	0.003	0.201	0	0.089	0	0.087	0.008	0.074	0	0
c_5	9.627	0	0.156	0	0	0	0	0	0	0.073	0
d_{12}	1.975	0.429	2.539	0.929	1.426	1.133	0.239	0.630	1.435	0	1.461
d_{13}	0	0	3.360	0.555	0.947	0.095	0.564	0.099	0.911	0.041	0.039
d_{14}	0	0.130	0.278	0	0.390	0	0.115	0.160	1.500	0.057	0.315
d_{15}	0.625	0	0	2.781	0	1.849	0	0	0.031	0	0.200
d_{23}	0	0	2.135	0	0.459	0.108	0	0	0.204	0.070	0
d_{24}	0	0.083	0	0.018	0.076	0	0	0	0.498	0.021	0
d_{25}	0	0	0	0.711	0	0.378	0	1.321	0.190	0	0
d_{34}	0	0	0	0.100	0	0	0	0	0.067	0	0
d_{35}	0	0	3.985	0.966	0	0	0	0	0.007	0.255	0
d_{45}	0	0	0	0	0	0	0	0	0.005	0.743	0
e_1	0	0	0	0	0	0	0	0	0	0.267	0
e_2	0	0	0	0	0	0	0	0.044	0	0.088	0
e_3	0	0.005	3.210	0	0.225	0	0.088	0.021	0.219	0	0
e_4	0	0	0	0	0	0	0	0	0.208	0	0
e_5	0	0	0	1.465	0	0	0	0	0	0.190	0.098
f_1	0.347	0	0.035	0	0.078	0	0.036	0	0.028	0	0
f_2	0	0	0.213	0	0	0	0.006	0	0.007	0	0
f_3	0	0	0	0.061	0	0	0.033	0	0.001	0	0
f_4	0.003	0	0	0	0	0	0	0	0.008	0	0
f_5	0	0	0	0	0	0	0	0	0.115	0.070	0
h_{12}	0	0	0	0	0.029	0	0	0	0	0	0
h_{13}	0	0	0.064	0	0	0	0	0	0	0	0
h_{14}	0	0	0	0	0	0	0	0	0.016	0	0
h_{15}	0	0	0	0	0	0	0	0	0.164	0.139	0
h_{23}	0	0	0.557	0.024	0	0	0.010	0	0	0.003	0
h_{24}	0.029	0	0	0	0	0	0	0	0	0	0
h_{25}	0	0	0	0	0	0	0	0	0	0.193	0
h_{34}	0.005	0	0.077	0	0	0	0	0	0.009	0.022	0
h_{35}	0	0	0	0	0.150	0	0	0.021	0.081	0	0
h_{45}	0	0	0	0	0	0	0	0	0	0.713	0

5 FORMULAS FOR LPM DEFINITION

Regression formulas to define the LPM previously introduced are proposed in order to overcome the demanding dynamic pile-soil-pile interaction analysis necessary for the evaluation of the soil-foundation impedances and the subsequent calibration of the frequency independent parameters.

The following monomial expression is first assumed for the generic non-dimensional parameter Ω_i:

$$\Omega_i = \alpha n^{\beta} \left(\frac{s}{d}\right)^{\chi} \left(\frac{h}{d}\right)^{\delta} \left(\frac{E_p}{\rho_1 V_{s1}^2}\right)^{\varepsilon} \left(\frac{\rho_p}{\rho_1}\right)^{\phi} \tag{22}$$

Coefficient α and exponents β, χ, δ, ε and ϕ, reported in Table 2, have been calculated by a multiple regression analysis.

Figure 4 (black dots) compares the optimized non-dimensional parameters of the LPM with values obtained from the proposed formulas. Maximum errors are around 30% with the exception of non-dimensional masses (Ω_5, Ω_8 and Ω_{12}) for which errors are higher as a consequence of the constrained optimization that, for a significant number of cases, provide null values (grey dots) that cannot be captured with equation (22).

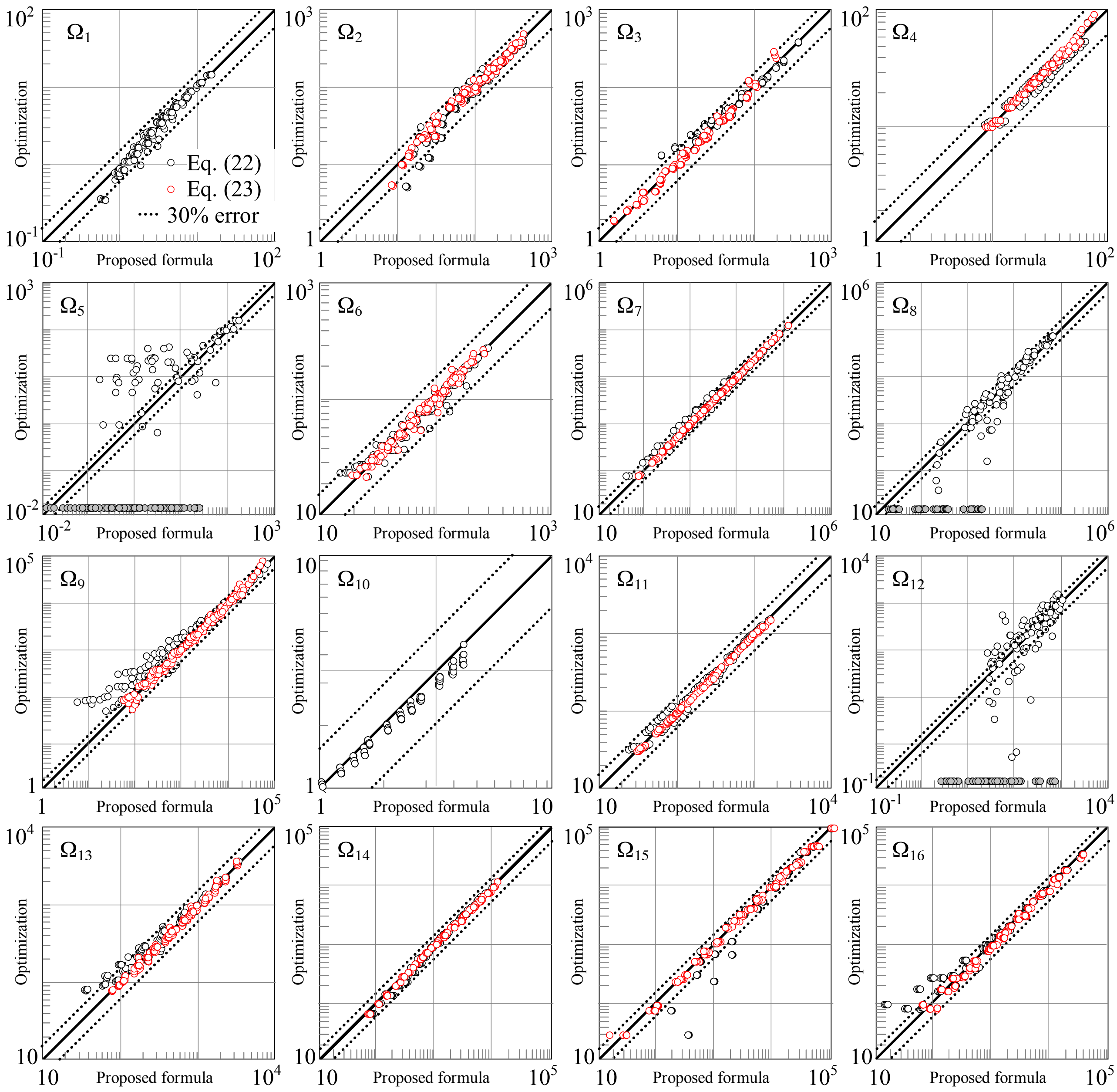


Figure 4. Comparisons between the optimized non-dimensional parameters of the LPM and the proposed formulas.

In an attempt to improve the accuracy of the LPM estimations, more complex formulas have been considered as an alternative to equations (22) for some of the parameters. In particular, the following expression has been considered for the *i*-th parameter

$$\Omega_i = e^{g_i(x_1, x_2, x_3, x_4, x_5)} \tag{23}$$

in which

$$x_1 = \ln(n) \quad x_2 = \ln\left(\frac{s}{d}\right) \quad x_3 = \ln\left(\frac{h}{d}\right) \tag{24a,b,c}$$

$$x_4 = \ln\left(\frac{E_p}{\rho_1 V_{s1}^2}\right) \qquad x_5 = \ln\left(\frac{\rho_p}{\rho_1}\right) \tag{25}$$

and

$$g_i = \frac{a^i + \sum_j \left(b_j^i x_j + c_j^i x_j^2\right) + \sum_{k,l} d_{kl}^i x_k x_l}{1 + \sum_j \left(e_j^i x_j + f_j^i x_j^2\right) + \sum_{k,l} h_{kl}^i x_k x_l} \tag{26}$$

for j, k, $l = 1\ldots5$ and $k \neq l$. Coefficients appearing in Equation (26) have been determined, with reference to improved parameters, by a multiple regression analysis and are reported in Table 3. As can be observed from Figure 4 (red dots), most of the LPM parameters are now better approximated by Equation (23). It is worth nothing that despite this last formulation is rather complex, it may be easily implemented in common spreadsheets.

Formulas (22) and (23) are very useful tools for practical applications because once determined the few non-dimensional parameters that characterise a specific pile group, namely number of piles in a row (n), spacing (s/d), depth (h/d), modulus of elasticity ($E_p/\rho_1 V_{s1}^2$) and density (ρ_p/ρ_1), they readily provide the 16 non-dimensional parameters of the LPM. The parameters of the LPM to be introduced in the structural analysis program are thus calculated by inverting relationships (13-20) without the need of carrying out a specific analysis of the soil-foundation system.

6 APPLICATION

A single 10 m high circular bridge pier, extracted from a multi-span bridge, is considered with the relevant mass.

The geometry of the structure and the soil-foundation system is reported in Figure 5 with the mechanical properties of the soil. The concrete of both pier and piles has Young's modulus $E_p = 30000$ MPa and density $\rho_p = 2.5$ t/m^3. Furthermore, a structural damping $\xi_s = 5\%$ is assumed for the pier. For the sake of brevity, analysis is performed only with reference to the transverse direction of the bridge. The response obtained by adopting the actual soil-foundation impedances, resulting from the numerical procedure of Dezi et al. (2009), is compared with those achieved by considering an LPM to simulate the foundation behaviour. Parameters of the LPM are obtained by (*i*) an optimization procedure (within the range 0÷10 Hz), (*ii*) by adopting Equation (22) and finally (*iii*) by considering the refined Equation (23).

Figure 6 compares the translational, rotational and roto-translational impedances of the soil-foundation system obtained with the procedure of Dezi et al.(2009), with those of the LPMs defined with the three methods. Equation (22) behaves generally quite well with exception of the rotational impedance for which a significant error is evident. On the other hand, the use of Equation (23) guarantees a greater level of accuracy also in this case.

Figure 7 shows the Frequency Response Function (FRF) of the system, namely the displacement of the mass subjected to a unit harmonic force, evaluated by means of a steady state analysis. The fixed base model is reported as well to show significance of SSI effects. As expected, a frequency shift and a displacement increase is evident for FRFs relevant to compliant base models. From their comparisons it can be observed that the use of Equation (23) leads to a very high level of precision, even if Equation (22) furnishes satisfactory results and inaccuracies are largely compensated by the expression simplicity and rapidity of use.

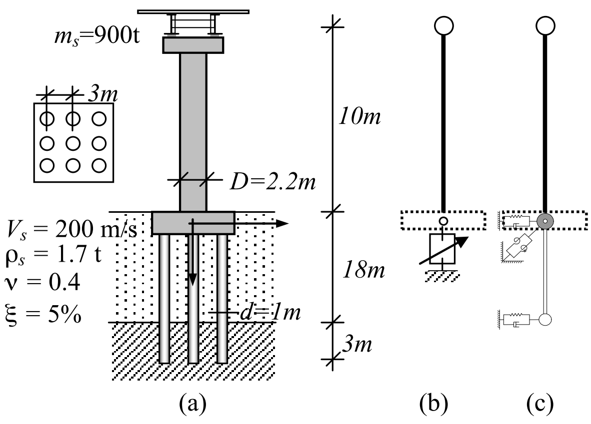


Figure 5. Case study: (a) geometry, (b) model with frequency dependent impedances and (c) model with LPM.

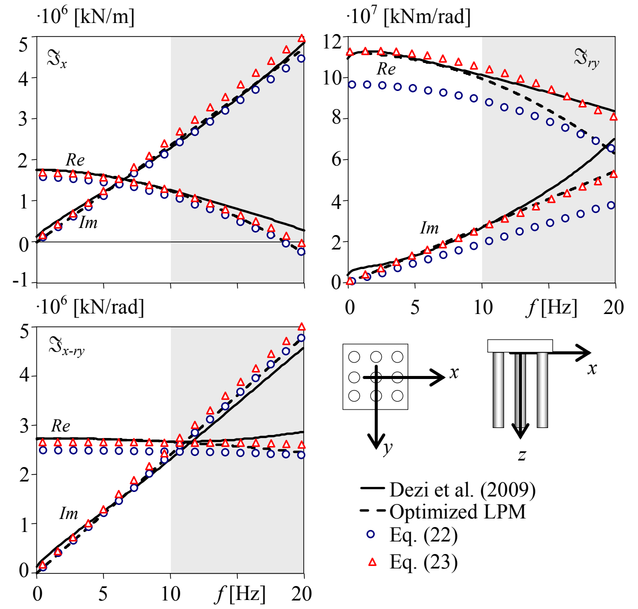


Figure 6. Soil foundation impedances.

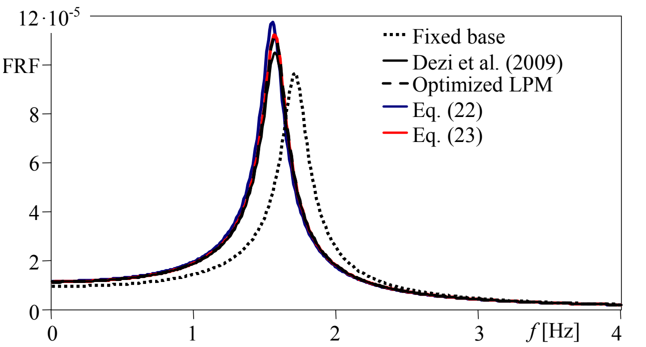


Figure 7. Frequency Response Function of the system

Finally, the response of the bridge subjected to the Strofades Earthquake (Greece - 18.11.97 $M_w = 6.6$) is shown in Figure 8 where the time histories of the relative displacement of the fixed and compliant base models are shown.

It can be observed that the response obtained with the optimized LPM is practically coincident with that resulting from the use of the proposed formulas with just some slight differences using Equation (22).

7 CONCLUSIONS

Two formulas for the definition of an LPM able to reproduce the translational, rotational as

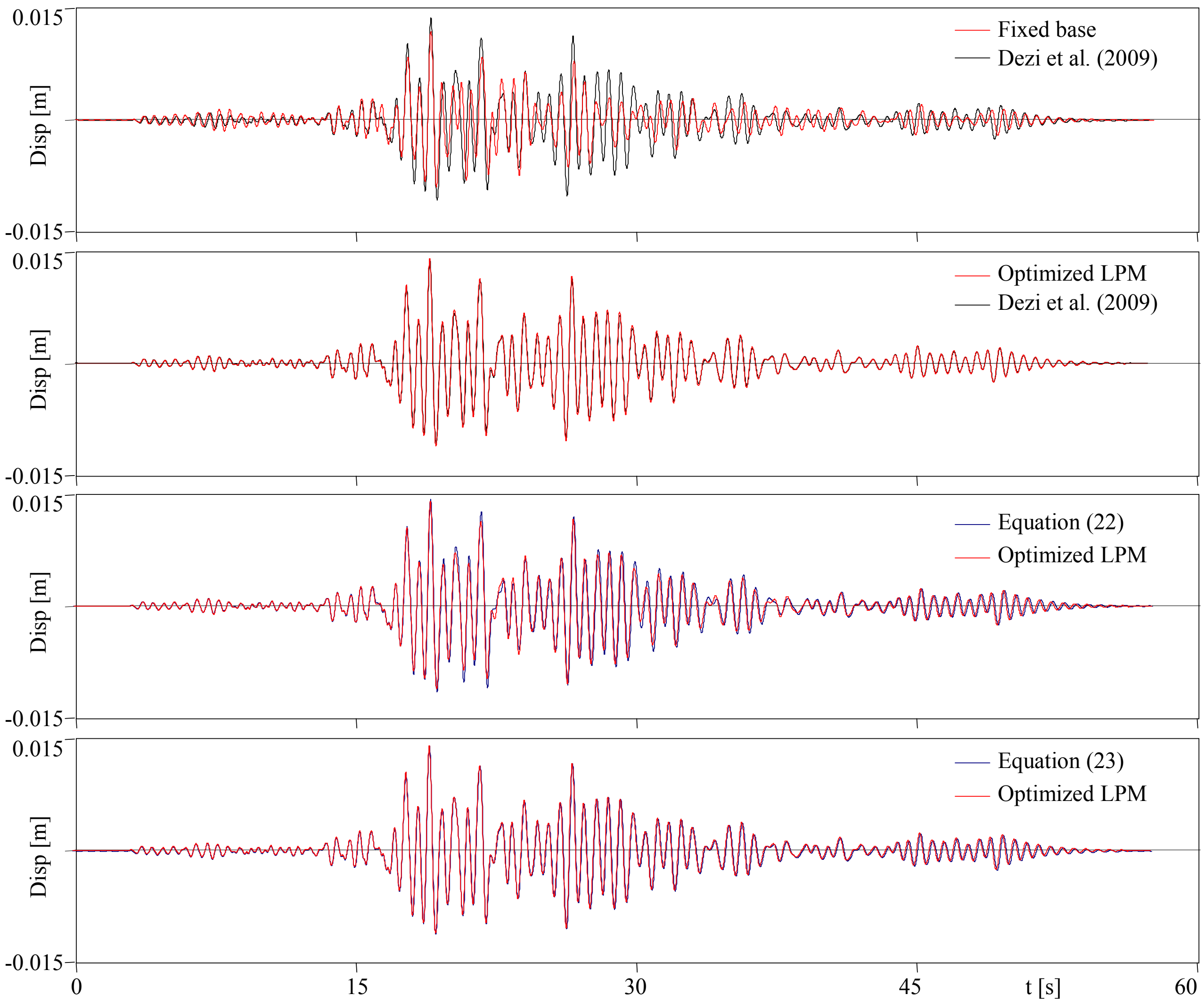


Figure 8. Case study: (a) geometry, (b) model with frequency dependent impedances and (c) model with LPM.

well as the roto-traslational coupling behaviour of pile group foundations have been presented. The proposed expressions have been calibrated with the results of an extensive non-dimensional parametric analysis. Head-bearing pile groups are considered and the analyses are performed by means of the numerical model developed by Dezi et al. (2009). Formulas may be readily adopted by engineers to define a restraint system that may be implemented in common structural analysis software to account for the soil-foundation compliance. One of the proposed expressions is very simple and may be used by means of a pocket calculator; the second one, which improve precisions of the parameter estimates, is more complex and may need the use of a spreadsheet. An application to a case study demonstrates that, despite some inaccuracies in the estimates of the soil-foundation impedances, the use of the simple formula provides a reliable prediction of the actual structural response.

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