Optical single photons on-demand teleported from microwave cavities

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Abstract
We propose a scheme for entangling the optical and microwave output modes of the respective cavities by using a micro mechanical resonator. The micro mechanical resonator, on one side, is capacitively coupled to the microwave cavity and, on the other side, it is coupled to a high-finesses optical cavity. We then show how this continuous variable entanglement can be profitably used to teleport the non-Gaussian number state $|1\rangle$ and the superposition $(|0\rangle + |1\rangle)/\sqrt{2}$ from the microwave cavity output mode onto an output of the optical cavity mode with fidelity much larger than the no-cloning limit.

(Some figures may appear in colour only in the online journal)

1. Introduction

The on-demand generation of optical single photons on a chip is one of the most challenging and required results for the successful implementation of quantum information devices. Many proposals for the production of single optical photons have been described and realized in recent decades. Early experiments demonstrated photon generation from single ions [1], atoms [2–4] and molecules [5]. The challenges involved in overcoming the practical difficulties in isolating single particles make their use as single-photon sources very demanding. The first demonstration of a stable, triggered, room temperature single-photon source was done using a nickel nitrogen defect in micro-diamonds [6]. Another alternative is to use quantum dots [7], although radiation in all directions makes efficient collection difficult. It is also possible to use twin photons produced in parametric down-conversion to generate a 'heralded' source of single photons, 'heralded' meaning that the single-photon state is conditional on the detection of the other photon of the pair. The production of 1550 nm wavelength photons in this way was reported in [8–10]. Perhaps one of the main disadvantages of the single-photon sources described so far is that the emission is random. It is not possible to tell if a particular excitation pulse has generated a single-photon emission until that single photon is detected.

A true resource for quantum information on a chip is the mapping of qubit states onto microwave photon states. These photons are generated on-demand with a high repetition rate, high efficiency and good spectral purity [11, 12]. The recent production of single microwave photons, and superpositions of photon states into an LC resonator from a superconducting flux qubit [11, 13], and the creation of a microwave photon counter [14] are important steps towards on-chip quantum optics experiments.

In this work, we will show that the single microwave photon generated on-demand in superconducting cavities, and the superposition of number states, can be teleported with high fidelity into a single photon and superposition of number states at optical wavelength exploiting the entanglement between the output fields mediated by a mechanical resonator (MR).

Entanglement is the property possessed by a multipartite quantum system when it is in a state that cannot be factorized into a product of states or a mixture of such products. In an entangled state the various parties share non-classical and possibly non-local correlations, which are at the heart of the counter intuitive quantum phenomenon. In recent years,

a wide range of experimental and theoretical schemes have been proposed to observe entanglement also in macroscopic objects [15], for example, the proposed entanglement of two mirrors of a ring cavity by using the radiation pressure of the cavity mode [16]. Subsequently, different schemes have been proposed to entangle nano- and micro mechanical resonators with macroscopic and microscopic systems, such as the entanglement of a nanomechanical resonator with a Cooper pair box [17], or an optical mode [18], for entangling two charge qubits [19] or two Josephson junctions [20] via nanomechanical resonators, and for entangling two nanomechanical resonators via trapped ions [21] or Cooper pair boxes [22, 23]. One should also mention the recent proposal of a protocol for entangling mechanical micro-resonators with traveling wave light pulses, which is not subject to stability conditions like the schemes working in the steady state [24]. With this scheme it could be possible to entangle and verify the optomechanical entanglement by using two successive pulses; then continuous variable (CV) teleportation would also be possible.

Owing to recent improvements in nanofabrication techniques, a new scheme for entangling nanomechanical resonators with the microwave field of superconducting coplanar waveguide fields, without the mediation of Cooper pair boxes, was proposed [25]. In particular, this CV entanglement can be used to teleport an unknown quantum state. Quantum teleportation [26] is the transfer of an unknown quantum state from a sender (Alice) to a receiver (Bob) by means of the entanglement shared by the two parties and appropriate classical communication. The teleportation is perfect and Bob recovers an exact copy of the state teleported to him by Alice only if the quantum channel is an ideal maximally entangled state. If we deal with qubits represented by polarization states of photons, then we can employ a pair of polarization entangled photons generated by means of spontaneous parametric down-conversion, wherein the entanglement is almost perfect [27, 28]. However, in the case of continuous quantum variables [29, 30], an ideal channel is an unphysical infinitely squeezed state. In quantum optics, by considering the finite quantum correlations between the quadratures in a two-mode squeezed state, Braunstein and Kimble [30] proposed a realistic protocol employing a beam splitter and homodyne measurements, which approaches perfect teleportation in the limit of an infinite degree of squeezing. This teleportation was first realized in [31] using a Gaussian coherent state and then was successfully extended to a non-Gaussian state in [32].

In [33], we proposed a specific optomechanical system for the teleportation of Schrödinger's-cat states. In the present paper, we show how a CV quantum teleportation protocol can be implemented in the same optomechanical system for realizing the teleportation of a single-photon state and even of a coherent superposition of number states from microwave to optical frequencies. Combining this scheme with the demonstrated ability to generate on-demand single microwave photons, we could realise a deterministic source of single optical photons.

We consider a hybrid, strongly quantum-correlated system formed by a microwave cavity (MC) coupled to a high-finesse optical cavity (OC) via a vibrating micro
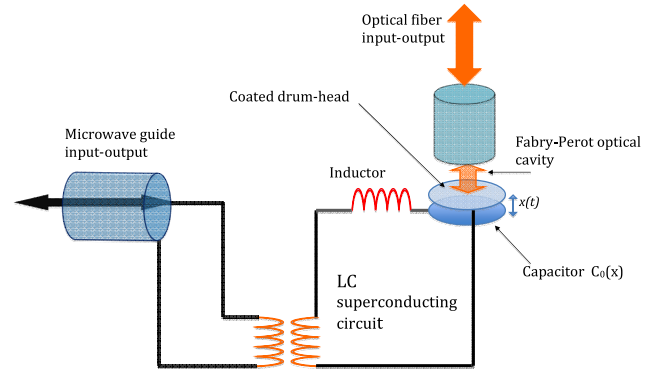

Figure 1. Schematic description of the device under study. A microwave transmission line source is coupled into a superconducting microwave resonator. The capacitance of this resonator is modulated by a bulk MR the motion of which modulates the frequency of an OC with a fibre output coupler.

cantilever. The MC mode is indirectly coupled to an OC mode via the common interaction with the vibrating micro mechanical resonator [34]. We show that with the current scheme, it is possible to generate a reversible stationary CV entanglement between the output fields of optical and microwave resonators, which gives a realistic device capable of CV quantum teleportation for non-Gaussian single-photon states and even the coherent superposition of two different photon number states.

This paper is organized as follows. In section 2, we briefly describe the proposed system [33] and derive the linearized quantum Langevin equations (QLEs). In section 3, we study the steady state of the system and quantify the entanglement between the outputs of optical and microwave fields by using the logarithmic negativity. In section 4, the fidelity of the teleportation is studied for particular non-Gaussian states, while the conclusions are summarized in section 5.

2. System dynamics

The system studied in this work is sketched in figure 1. We assume an MR which, on the one side, is capacitively coupled to a driven superconducting MC of resonant frequency ω_w and, on the other side, it is coupled to a driven OC with resonant frequency ω_c. Such a system might be possible using the lumped-element superconducting resonator with a free-standing drum-head capacitor recently developed in [36]. In fact, by adding an optical coating, the drum-head capacitor could also play the role of the reflecting micromirror of a Fabry–Perot OC formed by a second standard input mirror. The microwave and optical cavities are driven at the frequencies $\omega_{0w} = \omega_w - \Delta_{0w}$ and $\omega_{0c} = \omega_c - \Delta_{0c}$, respectively. The Hamiltonian of the coupled system reads [25, 34, 35, 37]

$$H = \frac{\hat{p}_x^2}{2m} + \frac{m\omega_m^2 \hat{x}^2}{2} + \frac{\hat{\Phi}^2}{2L} + \frac{\hat{Q}^2}{2[C + C_0(\hat{x})]} - e(t)\hat{Q}$$
$$+ \hbar\omega_c a^\dagger a - \hbar G_{0c} a^\dagger a \hat{x} + i\hbar E_c(a^\dagger e^{-i\omega_{0c}t} - a e^{i\omega_{0c}t}), \quad (1)$$

where $(\hat{x}, \hat{p}_x)$ are the canonical position and momentum of an MR with frequency ω_m, $(\hat{\Phi}, \hat{Q})$ are the canonical coordinates for the MC, describing the flux through an equivalent inductor

L and the charge on an equivalent capacitor C, respectively; $(a, a^\dagger)$ represent the annihilation and creation operators of the OC mode ($[a, a^\dagger] = 1$), $E_c = \sqrt{2P_c \kappa_c / \hbar \omega_{0c}}$ is related to the input driving laser, where P_c is the power of the input laser and κ_c describes the damping rate of the OC, $G_{0c} = (\omega_c / \mathcal{L})\sqrt{\hbar / m \omega_m}$ gives the optical radiation–pressure coupling, where m is the effective mass of MR, and $\mathcal{L}$ is an effective length, which depends upon the OC geometry. The coherent driving of the MC with damping rate κ_w is given by the electric potential $e(t) = -i\sqrt{2\hbar \omega_{0w} L} E_w (e^{i\omega_{0w}t} - e^{-i\omega_{0w}t})$, where $E_w = \sqrt{2P_w \kappa_w / \hbar \omega_{0w}}$ with P_w the power of the input microwave source. We finally stress that the optical and microwave cavities might support additional degenerate modes which we ignored in writing equation (1). This is valid as long as one assumes small cavities, in which the free spectral range (FSR) is much larger than the mechanical frequency ω_m. In this case, scattering of photons from the driven mode into other cavity modes is negligible. This guarantees that only one cavity mode participates in the optomechanical interaction and the neighbour modes are not excited by a single central frequency input laser.

The capacitive coupling between the MC and the MR as a function of the resonator displacement $\hat{x}$ is given by $C_0(\hat{x})$. We expand this function to the lowest order around the equilibrium position of the resonator corresponding to a separation d between the plates of the capacitor, with the corresponding bare capacitance C_0.

The Hamiltonian (1) in the interaction picture with respect to the frequencies of the two cavity pump fields, written in terms of the raising and lowering operators of the MC, i.e. $b = \sqrt{\frac{\omega_w L}{2\hbar}} \hat{Q} + \frac{i}{\sqrt{2\hbar \omega_w L}} \hat{\Phi}$, $b^\dagger ([b, b^\dagger] = 1)$, the dimensionless position and momentum operators $\hat{q} = \sqrt{\frac{m\omega_m}{\hbar}}\hat{x}$ and $\hat{p} = \frac{\hat{p}_x}{\sqrt{\hbar m \omega_m}}$ ($[\hat{x}, \hat{p}] = i\hbar$), and defining $G_{0w} = \frac{\mu \omega_w}{2d}\sqrt{\frac{\hbar}{m\omega_m}}$, reads [25, 34]

$$H = \hbar \Delta_w b^\dagger b + \hbar \Delta_c a^\dagger a + \frac{\hbar \omega_m}{2}(\hat{p}^2 + \hat{q}^2) - \hbar G_{0w}\hat{q}b^\dagger b$$
$$- \hbar G_{0c}\hat{q}a^\dagger a - i\hbar E_w(b - b^\dagger) + i\hbar E_c(a^\dagger - a). \quad (2)$$

However, the dynamics of the three modes is also affected by damping and noise processes, due to the fact that each of them interacts with its own environment. We can describe them adopting a QLE treatment in which the Heisenberg equations for the system operators associated with equation (2) are supplemented with damping and noise terms.

The resulting QLEs linearized with respect to the semiclassical steady state and written in terms of fluctuations of quadrature operators [34] can be compactly written as

$$\dot{\mathbf{u}}(t) = \mathbf{A}\mathbf{u}(t) + \mathbf{n}(t), \quad (3)$$

where

$$\mathbf{u}(t) = [\delta q(t), \delta p(t), \delta X_c(t), \delta Y_c(t), \delta X_w(t), \delta Y_w(t)]^T \quad (4)$$

and

$$\mathbf{n}(t) = [0, \xi(t), \sqrt{2\kappa_c}X_c^{in}, \sqrt{2\kappa_c}Y_c^{in}, \sqrt{2\kappa_w}X_w^{in}, \sqrt{2\kappa_w}Y_w^{in}]^T, \quad (5)$$

with the drift matrix defined by

$$\mathbf{A} = \begin{pmatrix} 0 & \omega_m & 0 & 0 & 0 & 0 \\ -\omega_m & -\gamma_m & G_c & 0 & G_w & 0 \\ 0 & 0 & -\kappa_c & \Delta_c & 0 & 0 \\ G_c & 0 & -\Delta_r & -\kappa_c & 0 & 0 \\ 0 & 0 & 0 & 0 & -\kappa_w & \Delta_w \\ G_w & 0 & 0 & 0 & -\Delta_w & -\kappa_w \end{pmatrix}, \quad (6)$$

and $G_c = \frac{2\omega_c}{L}\sqrt{\frac{P_c \kappa_c}{m\omega_m \omega_{0c}(\kappa_c^2 + \Delta_c^2)}}$, $G_w = \frac{\mu \omega_w}{d}\sqrt{\frac{P_w \kappa_w}{m\omega_m \omega_{0w}(\kappa_w^2 + \Delta_w^2)}}$, γ_m the damping rate of the mechanical mode and $\xi(t)$ the Brownian noise acting on the mechanical motion, and X_j^{in} and Y_j^{in} ($j = c, w$) the shot noise quadratures entering the respective cavities. All these noises can be considered as delta correlated white noises [37, 38].

This dynamical system is obtained by linearization around the semiclassical fixed point and we need to choose the parameters of the system in such a way as to have stability.

3. Entanglement between output modes

The intracavity optical field (microwave field) fluctuation $\delta a(t)$ ($\delta b(t)$) and its output are related by the usual input–output relation [38] which is characterized by $\delta a^{out}(t) = \sqrt{2\kappa_c}\delta a(t) - a_{in}(t)$ ($\delta b^{out}(t) = \sqrt{2\kappa_w}\delta b(t) - b_{in}(t)$). The output optical field $\delta a^{out}(t)$ satisfy the same commutation relation as the input optical field $a^{in}(t)$, i.e. the only nonzero commutator is $[\delta a^{out}(t), \delta a^{out}(t)^\dagger] = \delta(t - t')$ as well as for microwave operators $\delta b^{out}(t)$. From the continuous output field $\delta a^{out}(t)$ ($\delta b^{out}(t)$), one can extract many independent optical modes (microwave modes), by selecting different time intervals or, equivalently, different frequency intervals [39] depending on the details of the measurements made on the output. One can define a generic set of N output modes by means of the corresponding annihilation operators

$$\delta a_k^{out}(t) = \int_{-\infty}^{t} ds\, g_k(t - s)\delta a^{out}(s), \quad k = 1, 2, \ldots, N, \quad (7)$$

where $g_k(t)$ is the causal filter function. In a typical experimental setup, one of these modes is selected and mixed in a beam splitter with a strong local oscillator prior to detection on a photodetector resulting in a homodyne current. This current can then be integrated over some appropriate time window. Thus, we can define the measurement in terms of filtered output modes when the temporal mode function of the local oscillator is suitably chosen. The two output modes of interest originate from two different cavities and consequently describe two independent modes. Therefore, we can assume the following filter functions in terms of the Heaviside step function $\theta(t)$ as

$$g_j(t) = \sqrt{\frac{2}{\tau_j}}\theta(t)\,e^{-(1/\tau_j + i\Omega_j)t}, \quad (j = c, w), \quad (8)$$

characterized by bandwidths $1/\tau_j$ and central frequencies, Ω_j.

Because equation (3) is linear and the noises are Gaussian, the covariance matrix (CM) completely describes

all output moments of this system. The entanglement between the optical and microwave output modes is then fully determined by the CM $\mathbf{V}^{\mathrm{out}}$, whose matrix elements are

$$V_{ij}^{\mathrm{out}}(t) = \tfrac{1}{2}\langle u_i^{\mathrm{out}}(t)\, u_j^{\mathrm{out}}(t) + u_j^{\mathrm{out}}(t)\, u_i^{\mathrm{out}}(t)\rangle. \qquad (9)$$

Once the system's parameters are chosen satisfying the stability condition, by using the output cavity modes equations (7) and (8), one can derive the following general expression for the stationary output CM [37]:

$$\mathbf{V}^{\mathrm{out}} = \int \mathrm{d}\omega\, \tilde{\mathbf{T}}(\omega)(\tilde{\mathbf{M}}^{\mathrm{ext}}(\omega) + \mathbf{P}_{\mathrm{out}})$$
$$\times \mathbf{D}_{\mathrm{ext}}(\tilde{\mathbf{M}}^{\mathrm{ext}}(\omega)^{\dagger} + \mathbf{P}_{\mathrm{out}})\tilde{\mathbf{T}}^{\dagger}(\omega), \qquad (10)$$

where $\tilde{\mathbf{T}}(\omega)$ is the Fourier transforms of

$$\mathbf{T}(t) = \begin{pmatrix} \delta(t) & 0 & 0 & 0 & 0 & 0 \\ 0 & \delta(t) & 0 & 0 & 0 & 0 \\ 0 & 0 & R_c & -I_c & 0 & 0 \\ 0 & 0 & I_c & R_c & 0 & 0 \\ 0 & 0 & 0 & 0 & R_w & -I_w \\ 0 & 0 & 0 & 0 & I_w & R_w \end{pmatrix}, \quad (11)$$

and $\tilde{\mathbf{M}}^{\mathrm{ext}}(\omega) = (\mathrm{i}\omega\mathbf{I} + \mathbf{A})^{-1}$ with $\mathbf{I}$ the identity matrix, $\mathbf{P}_{\mathrm{out}} = \mathrm{Diag}[0, 0, 1/2k_c, 1/2k_c, 1/2k_w, 1/2k_w]$, the drift matrix $\mathbf{A}$ is given by equation (6), $\mathbf{D}_{\mathrm{ext}} = \mathrm{Diag}[0, \gamma_m(2\bar{n}_b+1), 2\kappa_c, 2\kappa_c, 2\kappa_w(2N(\omega_w)+1), 2\kappa_w(2N(\omega_w)+1)]$ is the diffusion matrix due to existence of noise terms in the linearized QLEs (3), $R_j = \sqrt{2\kappa_j}\, Re[g_j(t)]$, $I_j = \sqrt{2\kappa_j}\, Im[g_j(t)]$ ($j = $ c, w).

In order to establish the conditions under which the output of optical and microwave modes are entangled, we consider the logarithmic negativity E_N, which can be defined as [40]

$$E_N = \mathrm{Max}[0, -\ln(2\eta^-)], \qquad (12)$$

where $\eta^- \equiv 2^{-1/2}(\Sigma(\mathbf{V}') - \sqrt{\Sigma(\mathbf{V}')^2 - 4\det \mathbf{V}'})^{1/2}$ is the minimum symplectic eigenvalue of the partially transpose reduced CM, and we have used the 2×2 block form of the reduced CM equation (10) as

$$\mathbf{V}' = \begin{pmatrix} \mathbf{B} & \mathbf{C} \\ \mathbf{C}^{\mathrm{T}} & \mathbf{B}' \end{pmatrix}. \qquad (13)$$

Then

$$\Sigma(\mathbf{V}') \equiv \det \mathbf{B} + \det \mathbf{B}' - 2\det \mathbf{C}, \qquad (14)$$

and

$$\mathbf{B} = \begin{pmatrix} V_{33}^{\mathrm{out}} & V_{34}^{\mathrm{out}} \\ V_{34}^{\mathrm{out}} & V_{44}^{\mathrm{out}} \end{pmatrix}, \quad \mathbf{B}' = \begin{pmatrix} V_{55}^{\mathrm{out}} & V_{56}^{\mathrm{out}} \\ V_{56}^{\mathrm{out}} & V_{66}^{\mathrm{out}} \end{pmatrix},$$
$$\mathbf{C} = \begin{pmatrix} V_{35}^{\mathrm{out}} & V_{36}^{\mathrm{out}} \\ V_{45}^{\mathrm{out}} & V_{46}^{\mathrm{out}} \end{pmatrix}. \qquad (15)$$

To determine the best entanglement between the output of optical–microwave modes, we have plotted the logarithmic negativity versus the normalized central frequency Ω_c/ω_m at four different values of the normalized inverse bandwidth
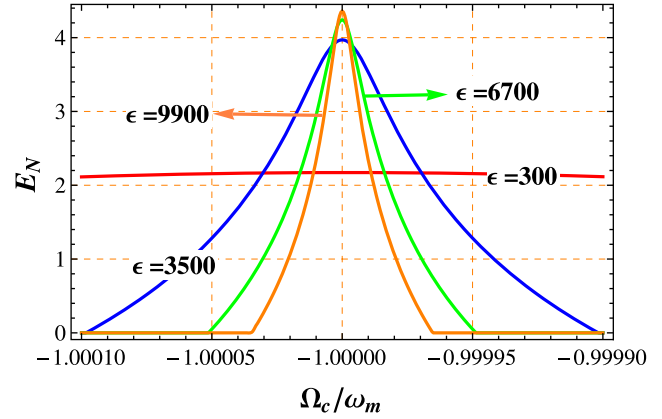

Figure 2. E_N at four different values of the normalized inverse bandwidth $\epsilon = \tau\omega_m$ versus the normalized frequency Ω_c/ω_m, at fixed central frequency of the microwave output mode $\Omega_w = \omega_m$. $\lambda_{0c} = 810$ nm and power $P_c = 3.4$ mW, with $\omega_{0w}/2\pi = 10$ GHz and microwave input power $P_w = 42$ mW.

$\epsilon = \epsilon_w = \epsilon_c = \tau\omega_m$ at $\Delta_w = \omega_m$, $\Delta_c = -\omega_m$ and $\Omega_w = \omega_m$ in figure 2, where we have assumed an experimental situation representing a feasible extension of the scheme of [36], i.e. we have assumed a lumped-element superconducting circuit with a free-standing drum-head capacitor, which is then optically coated to form a micromirror of an additional optical Fabry–Perot cavity. We have taken parameters similar to that of [36] for the MC and MR, that is, an MR with $\omega_m/2\pi = 10$ MHz, $Q = 15 \times 10^4$, and an MC with $\omega_w/2\pi = 10$ GHz, $\kappa_w = 0.04\omega_m$, driven by a microwave source with power $P_w = 42$ mW. The coupling between the two is determined by the parameters $d = 100$ nm, $\mu = 0.013$. We have considered a lower mechanical quality factor, and resonator higher mass $m = 10$ ng than that of [36], in order to take into account the presence of the coating, which typically worsens the mechanical properties. We have then assumed an OC of length $\mathcal{L} = 1$ mm and damping rate $\kappa_c = 0.04\omega_m$ driven by a laser with wavelength $\lambda_{0c} = 810$ nm and power $P_c = 3.4$ mW. The choice of these parameters satisfy the stability condition. As is shown in figure 2, the higher the ϵ (small bandwidths) the larger the stationary entanglement appears around the blue detuned sideband at $\Omega_c = -\omega_m$. Thus, it could be possible to control the entanglement of the microwave–optical modes by varying the detection bandwidth τ^{-1}. From the experimental point of view this means that one can obtain an effective entanglement distillation by appropriately filtering the output fields. Similar results have also been obtained in the case of entanglement of the output of optical modes and the movable mirror [37].

4. Continuous variable teleportation between the optical–microwave output modes

4.1. Single-photon state

We have seen that the vibrational mode of the MR realizes an effective entanglement between optical and microwave output modes. Since optical (microwave) traveling wave fields (output modes) are typically used for CV quantum information applications, this fact suggests the possibility

of using the output microwave fields (optical fields) in the manipulation and storage of CV quantum information.

The first experimental demonstration of a CV quantum information protocol was the quantum teleportation of an unknown coherent state of an optical mode onto another optical mode illustrated in [31]. Quantum teleportation requires the use of shared entanglement between two distant stations (the quantum channel), Alice and Bob, and of a classical channel for the transmission of the results of the Bell measurement from Alice to Bob. The coupling between the microwave and optical output modes establishes the required quantum channel, i.e. the shared entangled state between the output of optical mode in Alice's hand and that of microwave mode at Bob's station. The teleportation scheme in the current setting is the same as that proposed in [30] but exchanging the role of Alice and Bob. In the scheme proposed in this paper a single photon number state of the radiation field in the MC is prepared by a verifier (Victor) in a source cavity and then emitted towards Bob's station. To implement a teleportation protocol Bob needs to make a joint measurement on the output of the source cavity and his part of the entangled microwave–optical modes.

He can do this by mixing the filtered output of the source cavity on a balanced beam splitter with the output of the microwave mode. The two outputs of the beam splitter are then subject to a homodyne measurement, using two IQ mixers, with a pulsed local oscillator mode matched to the single-photon source cavity. The integrated homodyne current then produces two measurement results, X_+ and P_-. Currently, the quantum efficiency of the homodyne measurement is not high due to the need to amplify the signal prior to using the IQ mixer; however, the use of phase-dependent amplifiers, such as Josephson parametric amplifiers [42], and the recent development of single microwave-photon detectors [14] indicate that there should not be many obstacles in improving the quantum efficiency.

These results are then passed through a classical channel to Alice, who completes the protocol by implementing conditional displacements of her component of the shared entangled beams. These displacements will need to be done using a pulsed local oscillator synchronous with and phase-locked to that used for the microwave measurements at Bob's station. Upon receiving this information, Alice displaces her part of the entangled state (the output of optical mode) as follows: $\hat{X}_c^{out} \rightarrow \hat{X}_c^{out} + \sqrt{2}X_+$ and $\hat{P}_c^{out} \rightarrow \hat{P}_c^{out} - \sqrt{2}P_-$. We emphasize that Alice and Bob do not assume any prior knowledge of the input state and adhere to unity-gain teleportation, so that the teleporter does not have any restriction regarding the specific family of quantum states it can faithfully teleport.

To quantify the quality of the teleportation protocol in the system under study, one can use the fidelity that in the case of a pure state $|\psi_{in}\rangle$, it is given by $F = \langle\psi_{in}|\rho_{out}|\psi_{in}\rangle$, where ρ_{out} is the output state of the protocol. In our case, the non-Gaussian single-photon state can only be teleported and retrieved at the output port when $F > F_{th}$ [41], a threshold bound $F_{th} = 2/3$ known as the no-cloning limit. Thus, we have a practical criterion to determine the successful transfer of single-photon number state.

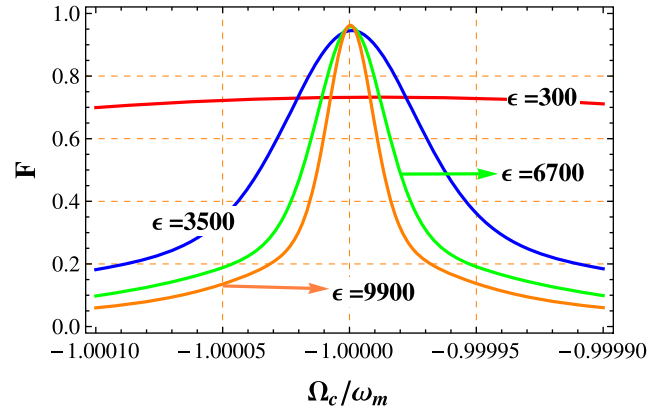**Figure 3.** Teleportation fidelity F at four different values of $\epsilon = \tau\omega_m$ versus Ω_c/ω_m and for the single-photon state $|1\rangle$. The other parameters are the same as in figure 2.

We thus restrict the discussion to the case when the input state is a single photon number state $|1\rangle$ with the characteristic function

$$\phi_{n=1}^{in}(\lambda) = L_{n=1}(|\lambda|^2)\exp(-|\lambda|^2/2)$$

$$= (1 - |\lambda|^2)\exp(-|\lambda|^2/2), \qquad (16)$$

where $L_n(x)$ is the Laguerre polynomial of degree n. Our quantum channel is a Gaussian channel with the corresponding characteristic function $\Phi^{ch}(\vec{\xi}) = \exp(-\vec{\xi}^T\mathbf{V}^{ch}\vec{\xi}/4 + i\vec{d}^T\vec{\xi})$ (where $\vec{\xi}^T = (X_c^{out}, Y_c^{out}, X_w^{out}, Y_w^{out})$ is the vector in the phase space of variables and $\mathbf{V}^{ch}$ is the reduced CM $\mathbf{V}'$). We also assume that Alice and Bob share a zero-displacement state, implying that $\vec{d} = \vec{0}$. Thus, the fidelity of the teleportation can be written in terms of the channel and the input state as [43]

$$F = \pi^{-1}\int d^2\eta|\phi^{in}(\eta)|^2[\phi^{ch}(\eta^*, \eta)]^*. \qquad (17)$$

By plugging equation (16) into (17) and after some algebraic rearrangement, we obtain

$$F = \pi^{-1}\int d^2\eta(1 - |\eta|^2)^2\exp(-\vec{\mu}^T\boldsymbol{\Gamma}\vec{\mu}), \qquad (18)$$

where $\vec{\mu}^T = [\eta_I, -\eta_R]$, ($\eta = \eta_R + i\eta_I$) and $\boldsymbol{\Gamma} = 2\mathbf{V}_{coh} + \mathbf{Z}\mathbf{B}\mathbf{Z} + \mathbf{Z}\mathbf{C} + \mathbf{C}^T\mathbf{Z} + \mathbf{B}'$, $\mathbf{Z} = \text{Diag}(1, -1)$, $\mathbf{V}_{coh} = \text{Diag}(1/2, 1/2)$.

The fidelity of teleportation after performing the integral is given by

$$F = \frac{1}{\sqrt{\det\boldsymbol{\Gamma}}}\left(1 + \frac{1}{\det\boldsymbol{\Gamma}}\left[\tfrac{1}{2} - (\Gamma_{11} + \Gamma_{22})\right]\right.$$

$$\left. + \frac{3[\Gamma_{11}^2 + \Gamma_{22}^2 + 2\Gamma_{12}^2]}{4(\det\boldsymbol{\Gamma})^2}\right). \qquad (19)$$

Figure 3 shows the fidelity of the teleportation protocol between the microwave and optical output modes versus normalized central frequency Ω_c/ω_m in the case of four different values of ϵ with the same data of figure 2. Clearly, at $\Delta_c = \omega_m$, $\Delta_w = -\omega_m$ and $\Omega_w = \omega_m$ the fidelity is highly peaked around $\Omega_c = -\omega_m$, where it is higher than the
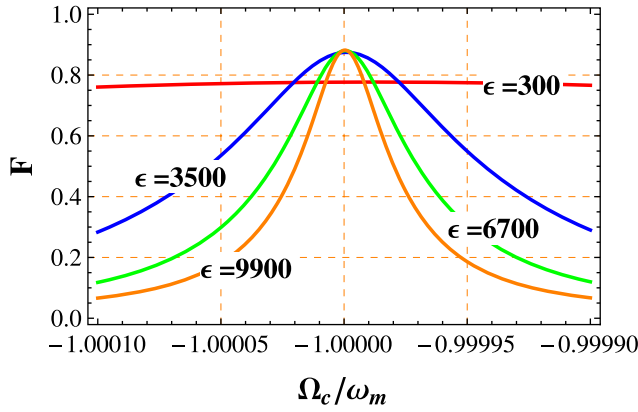
Figure 4. Teleportation fidelity F at four different values of $\epsilon = \tau\omega_{\mathrm{m}}$ versus $\Omega_{\mathrm{c}}/\omega_{\mathrm{m}}$ for the coherent superposition of number states $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. The other parameters are the same as in figure 2.

no-cloning limit. Furthermore, similar to the logarithmic negativity, the fidelity of the protocol can be controlled by varying the frequency bandwidth τ^{-1}. It is interesting that for large enough values of frequency bandwidth τ^{-1} the teleportation fidelity is always greater than the no-cloning limit of 2/3 very near $\Omega_{\mathrm{c}} = -\omega_{\mathrm{m}}$, which shows the great practical potential of this system to teleport a single-photon state.

4.2. Superposition state

Let us now consider the special case in which the input state is a coherent superposition of number states $|\psi^{\mathrm{in}}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. The Wigner characteristic function of this state is given by

$$\phi^{\mathrm{in}}(\eta) = \mathrm{Tr}(\rho e^{\eta a^\dagger - \eta^* a}) = \tfrac{1}{2}(2 - |\eta|^2 + 2\eta_{\mathrm{R}})e^{-|\eta|^2/2}. \quad (20)$$

The fidelity of teleportation can be obtained by substituting ϕ^{in} into equation (17), which reads as

$$F = \frac{1}{4\pi} \int \mathrm{d}^2\eta (4 + |\eta|^4 + 4\eta_{\mathrm{R}}^2 - 4|\eta|^2 + 8\eta_{\mathrm{R}} - 4|\eta|^2\eta_{\mathrm{R}})$$
$$\times \exp(-\vec{\mu}^{\mathrm{T}}\mathbf{\Gamma}\vec{\mu}). \quad (21)$$

Performing the integral, we obtain the fidelity of teleportation as follows:

$$F = \frac{1}{4\sqrt{\det \mathbf{\Gamma}}} \left(4 + \frac{1}{\det \mathbf{\Gamma}} \left[\frac{1}{2} - 2\Gamma_{22} \right] + \frac{3[\Gamma_{11}^2 + \Gamma_{22}^2 + 2\Gamma_{12}^2]}{4(\det \mathbf{\Gamma})^2} \right). \quad (22)$$

The teleportation fidelity for the superposition states equation (22) is shown in figure 4. We have chosen a particular superposition with fixed relative phase; however, a similar result would be obtained considering a more general superposition of number states. In this case as well, we have a small region around $\Omega_{\mathrm{c}} = -\omega_{\mathrm{m}}$ where the fidelity is higher than the no-cloning threshold, showing the possibility of teleporting qubit states from microwave to optical frequency on demand. Of course, also the reverse teleportation would be possible due to the symmetry of the device.

5. Conclusion

We have proposed a scheme for the realization of the CV teleportation of a single-photon state and the superposition of two number states of radiation between the outputs of optical and microwave modes by means of a micro mechanical resonator. As we have shown, the MR leads to the entanglement between an output of optical mode and an output of microwave mode. This entanglement can be used as a realistic Gaussian quantum channel to approach the CV quantum teleportation. We have shown that for experimentally feasible parameters and at optical and microwave frequencies the protocol is identical to the standard Braunstein–Kimble protocol [30], and the proposed scheme is able to teleport non-Gaussian number states and its superpositions with fidelity well above the no-cloning limit.

A similar result was obtained in [44], where a model to interconvert stationary and photonic qubits mediated by an MR was proposed. In our present scheme, the state transfer is obtained by using a standard CV teleportation protocol between the two outputs of the device, which are strongly entangled, while the work [44] uses a different setting where the excitations from the qubit are transferred to the MR and finally mapped onto a traveling photon.

From the experimental point of view the realization of our teleportation experiment lies on the possibility of making homodyne measurements at microwave frequency, as we have discussed above. At the moment the single photon and the superposition of number states are generated within the MC [11] interposing a Josephson phase qubit between the superconducting MC and a classical signal. The measurement of the Wigner function of the prepared states is also obtained by measuring the final state of the qubit, repeating a number of times the states to be measured. Therefore, even the single-microwave source should be implemented in such a way that the single-microwave photon and the number states superposition exit the microwave source cavity with a well-defined frequency. This could be obtained by using a selective filter function giving an output signal at the same frequency of the entangled output in Bob's hands. By repeating several times the preparation of the state and synchronizing the homodyne measurements at the Bob site with the Alice receiver, one should be able to reconstruct the Wigner function by a tomographic apparatus such as the one in [45] used for reconstructing the single photon Fock state at optical wavelength.

It is worth mentioning here that the same result could be obtained if, instead of an MC, the second cavity were another OC at a different frequency. In that case the device would be able to convert single photons and number states superpositions at different frequencies, potentially, at will.

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