Cap codes arising from duality

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Abstract

We describe a simple method to construct many different linear codes arising from caps in a projective space whose automorphism group is known in advance. This procedure begins with a fairly small point set in an appropriate finite projective space.

1 Introduction

An $[n,k,d]_q$ linear code is a k-dimensional subspace C of a vector space V_q^n, with $n > k$, over the finite field $\mathbb{F}_q$. The dual code $C^\perp$ of C is its orthogonal subspace in V_q^n, that is,

$$C^\perp = \left\{ \mathbf{v} \in V_q^n \mid \mathbf{v} \cdot \mathbf{c} = 0 \text{ for any } \mathbf{c} \in C \right\},$$

where '$\cdot$' denotes the usual inner product of two vectors $\mathbf{u} = (u_1, u_2, \ldots, u_n)$ and $\mathbf{v} = (v_1, v_2, \ldots, v_n)$ of V_q^n defined by

$$\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2 + \cdots + u_n v_n.$$

If the generating matrix of C is in the standard form

$$G = \left(I_k \mid A \right),$$

where I_k is the $k \times k$ identity matrix and A is a suitable $n \times (n-k)$ matrix, then the dual code $C^{\perp}$ is generated by the parity check matrix of C in the form

$$H = (-{}^{T}A \mid I_{n-k}) \, .$$

From a theoretical point of view, the concept of duality in coding theory can be derived from the more general concept of the Gale transform of sets of points in projective spaces; see for instance [6].

Given two positive integers r and s, let $\gamma = r + s + 2$. The Gale transform is an involution that takes a set Γ consisting of γ points of an r-dimensional projective space $\mathrm{PG}(r, \mathbb{F})$ onto a set Γ' consisting of γ points of an s-dimensional projective space $\mathrm{PG}(s, \mathbb{F})$. The most general—and purely geometric—definition of the Gale transform is as follows. Let $\mathbb{F}$ be a field and r, s positive integers greater than 1. Set $\gamma = r + s + 2$ and let $\Gamma \subseteq \mathrm{PG}(r, \mathbb{F})$, $\Gamma' \subseteq \mathrm{PG}(s, \mathbb{F})$ be two nondegenerate sets (i.e., not contained in any proper subspace), each consisting of γ points, whose projective coordinate vectors are the rows of a $\gamma \times (r+1)$ matrix M and a $\gamma \times (s+1)$ matrix M', respectively. Then, the set Γ' is said to be the Gale transform of Γ if and only if there exists a nonsingular diagonal matrix D such that

$${}^{T}MDM' = \mathbf{0}, \tag{1}$$

where $\mathbf{0}$ denotes the $(r+1) \times (s+1)$ zero matrix. The diagonal matrix in (1) is necessary to avoid the dependence on the choice of homogeneous coordinates.

This situation takes place when we consider the generating matrix G of an $[n, k, d]_q$ linear code and its parity check matrix H, that is, the generating matrix of its dual code: in this case we have ${}^{T}HDG = \mathbf{0}$, where $\mathbf{0}$ denotes the $k \times (n-k)$ zero matrix and $D = I_n$.

For a detailed treatise on the theory of the Gale transform from a geometric point of view, and an historical account on its development over the last two centuries, the interested reader is referred to [6]. A concise survey on the Gale transform from a finite geometry point of view, and recent results, can be found in [4].

Here we use some properties of the Gale transform applied to the points whose projective coordinates in $\mathrm{PG}(r, q)$ are the rows of the generating matrix of an $[n, k, d]_q$ linear code C in order to produce, besides the dual $[n, n-k, d]_q$ code $C^{\perp}$, many linear codes associated to caps in $\mathrm{PG}(n - r - 2, q)$, with different parameters and admitting the same automorphism group of C.

2 Preliminary results on caps

An n-cap in a finite projective space $\mathrm{PG}(r,q)$ is a set consisting of n points no three of which are collinear; see [9] for a complete treatise on this topic. When the Gale transform is applied to a point set $\Gamma \subseteq \mathrm{PG}(r,\mathbb{F})$, then the resulting set $\Gamma' \subseteq \mathrm{PG}(s,\mathbb{F})$ inherits some crucial properties from Γ; see [5, 3, 4].

Theorem 1 *For any set Γ consisting of k points in $\mathrm{PG}(r,q)$, with $r \geq 2$ and $n \geq 4$, the Gale transform Γ' of Γ is an n-cap in $\mathrm{PG}(n-r-2,q)$.*

For our purpose, it is convenient to gain some control over the automorphism groups of the geometrical objects we are dealing with. In this respect the Gale transform has a fairly nice behaviour, as it is shown by the following result [3].

Theorem 2 *Let Γ be an n-cap in $\mathrm{PG}(r,q)$ and Γ' its Gale transform. Then Γ and Γ' have isomorphic collineation groups.*

Basically, Theorem 2 allows us to transfer the action of the group G of a certain subset $\Gamma \subseteq \mathrm{PG}(r,q)$ to another projective space $\mathrm{PG}(s,q)$, thus providing a representation of G as a collineation group with a different support. It turns out that this new representation is reducible, i.e. the group G fixes some non-trivial subspace. The orbits of G on the points of $\mathrm{PG}(s,q)$ provide the key tool for the construction of many linear codes besides the code obtained from the Gale transform of the geometrical object we started off with.

3 Construction of cap codes

Caps in projective spaces are closely related to a broad class of linear codes. If $\mathscr{K}$ is an n-cap in a projective space $\mathrm{PG}(r,q)$, then the coordinate vectors of the points of $\mathscr{K}$ are the columns of the parity check matrix H of an $[n,n-r,d]_q$ linear code C with $d \geq 3$, that is, H is the generating matrix of an $[n,r+1,d']_q$ code $C^{\perp}$ which is the dual code of C; see [8, Chapter 14] for instance.

In [2] there is a first description of a cap code arising from a collineation group acting on a certain geometrical object. Here we are going to use a similar technique in a broader context.

In what follows we use a simplified representation of the Gale transform. Given a set $\Gamma \subseteq \mathrm{PG}(r,q)$ consisting of γ points, we choose homogeneous coordinates in such a way that the coordinates of the points of Γ are the rows of the block matrix

$$K = \left(\frac{I_{r+1}}{A} \right), \tag{2}$$

where A is an $(s+1) \times (r+1)$ matrix. Then, the homogeneous coordinates of the points of the Gale transform $\Gamma' \subseteq \mathrm{PG}(s,q)$ of Γ, with $\gamma = r+s+2$, are the rows of the block matrix

$$K' = \left(\frac{{}^{T}A}{I_{s+1}} \right). \tag{3}$$

3.1 Cap codes arising from a regular hyperoval in PG(2, 8)

In the projective plane PG(2, 8) over the finite field $\mathbb{F}_8$, let $\mathscr{C}$ be the conic of equation

$$XY + YZ + XZ = 0.$$

Since PG(2, 8) has even order, all the tangent lines of $\mathscr{C}$ are concurrent at a point N outside $\mathscr{C}$ called the nucleus of $\mathscr{C}$. Hence, the point set $\mathscr{C} \cup \{N\}$ is a 10-arc in PG(2, 8) which is called a regular hyperoval, and is preserved by the collineation group PGL(2, 8); see [10, Section 8.4]. It is easy to check that in this case N has projective coordinates $(1:1:1)$, thus the point set of $\mathscr{C} \cup \{N\}$ can be represented by a matrix K of the form (2) as follows:

$${}^{T}K = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & \omega^2 & \omega^3 & \omega^5 & \omega^4 & \omega & \omega^6 & 0 & 1 & 0 \\ 1 & \omega^3 & \omega^2 & \omega & \omega^6 & \omega^5 & \omega^4 & 0 & 0 & 1 \end{pmatrix}.$$

where ω is a primitive element of $\mathbb{F}_8$ such that $\omega^3 + \omega + 1 = 0$. The matrix ${}^{T}K$ can be taken as the generating matrix of a $[10, 3, d]_8$ linear code. We checked with MAGMA [1] that this code is equivalent (see [8, Chapter 5]) to a $[10, 3, 8]_8$ MDS code with weight distribution

$$(0, 1), (8, 315), (10, 196).$$

The Gale transform of $\mathscr{C} \cup \{N\}$ is the set $\mathscr{C}' \subseteq \mathrm{PG}(6, 8)$ whose points can be represented by a matrix K' of the form (3) as follows:

$${}^{T}K' = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & \omega^2 & \omega^3 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & \omega^3 & \omega^2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & \omega^5 & \omega & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & \omega^4 & \omega^6 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & \omega & \omega^5 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & \omega^6 & \omega^4 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

By Theorem 1 the point set $\mathscr{C}'$ is a 10-cap in PG$(6,8)$, and by Theorem 2 $\mathscr{C}'$ admits a collineation group isomorphic to the group of $\mathscr{C}$ that we started with, namely PGL$(2,8)$. Hence, the matrix ${}^TK'$ provides the generating matrix of a $[10,7,4]_8$ cap code with the same automorphism group PGL$(2,8)$. This code turns out to be equivalent to an MDS code C' with weight distribution

$$(0,1),(4,1470),(5,7056),(6,49980),(7,191520),\\ (8,507465),(9,787920),(10,551740).$$

Note that starting off with the conic $\mathscr{C}$ only, without including its nucleus, the Gale transform $\mathscr{C}'$ is a 9-cap in PG$(5,8)$ producing a $[9,6,4]_8$ MDS code C'' with weight distribution

$$(0,1),(4,882),(5,3528),(6,19992),(7,57456),(8,101493),(9,78792).$$

The action of the group PGL$(2,8)$ as a permutation group on PG$(6,8)$ is reducible, thus the following list of orbits can be easily obtained.

- 1 fixed point N' coming from the nucleus N of the conic $\mathscr{C}$.
- 2 orbits of length 9, that is,
  - an orbit of length 9, corresponding the conic $\mathscr{C}$, which is a 9-cap producing a cap code equivalent to a $[9,7,3]_8$ MDS code;
  - another 9-cap producing a cap code equivalent to a $[9,3,7]_8$ MDS code.
- 10 orbits of length 63, none of which is a cap.
- 1 orbit of length 72 which is a cap producing a $[72,7,49]_8$ code.
- 8 orbits of length 84, 7 of which are caps producing cap codes equivalent to an $[84,7,56]_8$ code.
- 8 orbits of length 168, 7 of which are caps producing cap codes equivalent to $[168,7,d]_8$ codes with d equal to either 114, 120 or 129.
- 120 orbits of length 252, 64 of which are caps producing cap codes equivalent to $[252,7,d]_8$ codes with d equal to either 168, 180, 182, 186, 192, 196, 198, 200, 202, 204 or 206.
- 529 orbits of length 504, 141 of which are caps producing cap codes equivalent to $[504,7,d]_8$ codes with d equal to either 366, 392, 396, 399, 402, 404, 406, 408, 410, 411, 412, 414, 416, 419 or 420.

From Theorem 2, all the codes listed above admit the same group PGL$(2,8)$ as their automorphism group.

3.2 Cap codes arising from a conic in PG(2, 9)

In the projective plane $\pi = \mathrm{PG}(2,9)$, let $\mathscr{C}$ be the conic of equation

$$YZ - X^2 - \omega Z^2 = 0,$$

with ω a primitive element of $\mathbb{F}_9$ such that $\omega^2 + 2\omega + 2 = 0$. The point set of $\mathscr{C}$ can be represented by a matrix K of the form (2) as follows:

$${}^{T}K = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & \omega & \omega^7 & \omega^2 & \omega^6 & \omega^5 & \omega^3 & 0 & 1 & 0 \\ 1 & \omega^3 & \omega^2 & \omega^7 & \omega^5 & \omega^6 & \omega & 0 & 0 & 1 \end{pmatrix}.$$

We checked with MAGMA [1] that this code is equivalent to a $[10,3,8]_9$ MDS code with weight distribution

$$(0,1),(8,360),(9,80),(10,288).$$

The Gale transform of $\mathscr{C}$ is the set $\mathscr{C}' \subseteq \mathrm{PG}(6,9)$ whose points can be represented by a matrix K' of the form (3) as follows:

$${}^{T}K' = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & \omega & \omega^3 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & \omega^7 & \omega^2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & \omega^2 & \omega^7 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & \omega^6 & \omega^5 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & \omega^5 & \omega^6 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & \omega^3 & \omega & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

By Theorem 1 the point set $\mathscr{C}'$ is a 10-cap in PG(6, 9), and by Theorem 2 $\mathscr{C}'$ admits a collineation group isomorphic to the group of $\mathscr{C}$ that we started with, namely PGL(2, 9). Hence, the matrix ${}^{T}K'$ provides the generating matrix of a $[10,7,4]_9$ cap code with the same automorphism group PGL(2, 9). This code turns out to be equivalent to an MDS code C' with weight distribution

$$(0,1),(4,1680),(5,10080),(6,77280),(7,343680),$$
$$(8,1036440),(9,1840880),(10,1472928).$$

The action of the group PGL(2, 9) as a permutation group on PG(6, 9) is reducible, thus the following list of orbits can be easily obtained.

- 2 orbits of length 10, which are both caps. One of them is the above mentioned 10-cap $\mathscr{C}'$, while the other one produces a $[10,4,6]_9$ code with weight distribution $(0,1),(6,240),(8,2160),(9,2000),(10,2160)$.

- 1 orbit of length 30, which is not a cap.
- 1 orbit of length 36, which is a cap producing a $[36, 7, 21]_9$-cap code.
- 1 orbit of length 45, which is a cap producing a $[45, 7, 28]_9$-cap code.
- 1 orbit of length 80, which is not a cap.
- 2 orbits of length 90, none of which is a cap.
- 8 orbits of length 120, none of which is a cap.
- 22 orbits of length 180, 4 of which are caps producing cap codes equivalent to $[180, 7, d]_9$ codes with d equal to either 139 or 141.
- 9 orbits of length 240, none of which is a cap.
- 164 orbits of length 360, 86 of which are caps producing cap codes equivalent to $[360, 7, d]_9$ codes with d equal to either 256, 264, 280, 284, 286, 288, 290, 292, 294, 296, 297 or 298.
- 738 orbits of length 720, 172 of which are caps producing cap codes equivalent to $[720, 7, d]_9$ codes with d equal to either 552, 570, 576, 582, 584, 588, 592, 594, 596, 600, 602, 603, 604, 606, 608, 609, 610, 612 or 614.

4 Conclusion

We described a method to construct many different linear codes arising from caps in a projective space whose automorphism group is known in advance. The steps of this construction can be summarized as follows:

1. Take a point set $\mathcal{O}$ consisting of γ points in a projective space $\mathrm{PG}(r, q)$, with its automorphism group G regarded as a (possibly irreducible) permutation group.
2. Apply the Gale transform to $\mathcal{O}$ in order to obtain another point set $\mathcal{O}' \subseteq \mathrm{PG}(s, q)$, containing the same number γ of points, with $\gamma = r + s + 2$.
3. Compute the orbits under the action of the group G regarded as a (possibly reducible) permutation group with support $\mathrm{PG}(s, q)$.
4. Determine which orbits from 3 are k-caps.
5. For each k-cap obtained at Step 4 take the $k \times (s+1)$ matrix K whose rows are the projective coordinate vectors of the points.

6. For each coordinate matrix K take the transpose ${}^T K$ as the generating matrix for the required $[k, s+1, d]_q$ code.

Once the points of each cap are determined, it is fairly easy to compute—using MAGMA [1] for instance—the parameters and the generating matrices for all the codes mentioned in Sections 3.1 and 3.2. In Tables 1 and 2 each set of parameters is associated with a starting point of an orbit under the action of the inherited collineation group which yields a cap providing a code with those parameters. In each table ω denotes a primitive element of the base field. We observe that the lengths of some of these codes are well beyond the maximum length available in the most popular on-line public repositories for linear codes over $\mathbb{F}_8$ and $\mathbb{F}_9$; see [7] for instance.

parameters	starting point	parameters	starting point
$[9,7,3]_8$	$(0:0:1:0:0:0:0:)$	$[252,7,206]_8$	$(1:\omega:\omega^2:\omega^6:0:\omega^6:\omega^3)$
$[9,3,7]_8$	$(1:\omega^6:\omega:0:\omega^5:\omega:\omega^6)$	$[504,7,366]_8$	$(0:1:0:0:0:\omega^2:\omega^5)$
$[72,7,49]_8$	$(1:\omega:\omega^2:\omega^3:\omega^3:\omega:\omega^4)$	$[504,7,392]_8$	$(1:\omega^6:1:\omega^2:1:0:1)$
$[84,7,56]_8$	$(1:1:1:\omega^6:\omega^5:1:0)$	$[504,7,396]_8$	$(1:1:\omega^6:\omega^3:1:\omega^3:0)$
$[168,7,114]_8$	$(1:\omega^3:\omega:\omega^2:\omega^2:1:\omega^6)$	$[504,7,399]_8$	$(0:1:\omega^2:\omega^5:0:\omega^3:\omega^2)$
$[168,7,120]_8$	$(0:1:\omega^4:\omega^3:1:\omega^2:\omega^5)$	$[504,7,402]_8$	$(1:1:\omega^5:0:1:1:\omega^5)$
$[168,7,129]_8$	$(1:\omega^5:\omega^2:\omega^2:1:1:\omega^4)$	$[504,7,404]_8$	$(1:\omega^6:0:\omega^5:\omega^4:\omega^5:1)$
$[252,7,168]_8$	$(1:\omega:0:\omega^2:\omega^6:1:\omega)$	$[504,7,406]_8$	$(1:0:0:\omega:1:0:\omega^4)$
$[252,7,180]_8$	$(1:\omega^4:0:1:\omega:\omega^6:\omega^4)$	$[504,7,408]_8$	$(1:\omega^5:\omega^4:\omega:\omega^3:1:0)$
$[252,7,182]_8$	$(1:\omega^2:\omega^6:\omega^6:1:\omega^3:\omega^5)$	$[504,7,410]_8$	$(1:\omega:\omega:1:1:\omega^3:\omega)$
$[252,7,186]_8$	$(1:\omega^6:\omega^5:1:\omega^4:\omega^6:\omega^3)$	$[504,7,411]_8$	$(1:1:\omega^5:\omega^6:\omega^5:\omega^2:\omega^2)$
$[252,7,192]_8$	$(1:0:\omega^2:0:1:0:\omega^2)$	$[504,7,412]_8$	$(1:\omega^2:\omega^5:\omega^6:\omega:1:0)$
$[252,7,196]_8$	$(1:0:\omega^5:\omega^4:\omega^2:\omega^4:1)$	$[504,7,414]_8$	$(1:\omega^6:0:\omega^6\omega^6:\omega^6:\omega)$
$[252,7,198]_8$	$(1:0:\omega^3:\omega^4:\omega^2:1:\omega^3)$	$[504,7,416]_8$	$(1;\omega^5:0:\omega^3:\omega^3:1:\omega^4)$
$[252,7,200]_8$	$(1:\omega^4:0:1:\omega^4:\omega^2:\omega^3)$	$[504,7,419]_8$	$(1:\omega^2:\omega^3:\omega^3:\omega:1:\omega)$
$[252,7,202]_8$	$(1:\omega^4:\omega^4:1:\omega:\omega^2:\omega^5)$	$[504,7,420]_8$	$(1:1:0:\omega^2:\omega^4:\omega^6:\omega)$
$[252,7,204]_8$	$(1:\omega^2:1:\omega^3:\omega:\omega^2:\omega^5)$		

Table 1: Parameters for codes from caps in PG(6, 8)

parameters	starting point	parameters	starting point
$[10,7,4]_9$	$(1:1:1:1:1:1:1)$	$[720,7,570]_9$	$XXXXXXX$
$[10,4,6]_9$	$(0:1:\omega:\omega^3:\omega:\omega^3:1)$	$[720,7,576]_9$	$(1:0:\omega^3:\omega^6:\omega^6:\omega^7:\omega^3)$
$[36,7,21]_9$	$(1:0:\omega:\omega^7:0:\omega^7:1)$	$[720,7,582]_9$	$(1:\omega:\omega:\omega^3:\omega^7:\omega:\omega)$
$[45,7,28]_9$	$(0:0:1:1:2:2:0)$	$[720,7,584]_9$	$(1:\omega^3:1:0:\omega^2:\omega^6:0)$
$[180,7,139]_9$	$(1:\omega:1:\omega^3:2:\omega^6:\omega^2)$	$[720,7,588]_9$	$(1:\omega^3:\omega^2:\omega^3:\omega:1:1)$
$[180,7,141]_9$	$(1:\omega^7:\omega^2:\omega^7:0:\omega^3:\omega)$	$[720,7,592]_9$	$(1:\omega^7:\omega^2:2:2:\omega^6:0)$
$[360,7,256]_9$	$(1:1:\omega^3:\omega^5:\omega^2:\omega^3:\omega)$	$[720,7,594]_9$	$(1:1:\omega^6:\omega 2:\omega^6:0)$
$[360,7,264]_9$	$(0:0:0:1:0:2:\omega^3)$	$[720,7,596]_9$	$(1:\omega^5:2:\omega^7:\omega^3:2:\omega)$
$[360,7,280]_9$	$(1:\omega^2:\omega:\omega^6:2:\omega^5:\omega)$	$[720,7,600]_9$	$(1:2:2:0:\omega^6:\omega^5:2)$
$[360,7,284]_9$	$(1:\omega^6:\omega^7:1:2:\omega^2:\omega^2)$	$[720,7,602]_9$	$(1:0:2:\omega^5:\omega^6:\omega^3:\omega^3)$
$[360,7,286]_9$	$(1:0:\omega^5:\omega:\omega^5:\omega^7:\omega^7)$	$[720,7,603]_9$	$(1:\omega^3:\omega^5:\omega^6:\omega^6:\omega^6:\omega)$
$[360,7,288]_9$	$(1:2:2:\omega^6:0:2:\omega^6)$	$[720,7,604]_9$	$(1:1:\omega^3:\omega^2:\omega^5:\omega^6:\omega^6)$
$[360,7,290]_9$	$(1:\omega:2:0:\omega^6:\omega^7:\omega^2)$	$[720,7,606]_9$	$(1:2:1:0:\omega^6:\omega^3:2)$
$[360,7,292]_9$	$(0:1:0:1:\omega^7:0:\omega^5)$	$[720,7,608]_9$	$(1:\omega^6:\omega^6:0:\omega^3:1:2)$
$[360,7,294]_9$	$(1:1:0:2:\omega:2:0)$	$[720,7,609]_9$	$(1:2:0:2:1:2:0)$
$[360,7,296]_9$	$(1:2:\omega^3:\omega^2:\omega^6:\omega^7:\omega^3)$	$[720,7,610]_9$	$(1:\omega^5:\omega:\omega^5:1:\omega:\omega^5)$
$[360,7,297]_9$	$(1:2:\omega^2:\omega^2:\omega:1:\omega^3)$	$[720,7,612]_9$	$(1:\omega^3:\omega^6:\omega^3:\omega:\omega^2:1)$
$[360,7,298]_9$	$(1:\omega^7:\omega^2:0:\omega^7:0:\omega^7)$	$[720,7,614]_9$	$(1:2:0:1:\omega:1:\omega^3)$
$[720,7,552]_9$	$(0:1:2:1:\omega^7:\omega^2:2)$		

Table 2: Parameters for codes from caps in PG$(6,9)$

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