

Entropy Parameter Estimation in Large-Scale Roughness Open Channel

Michele Greco¹ and Domenica Mirauda²

Abstract: The entropy model allows estimating, in an expeditive way, both water discharge and flow velocity field in open channels. In fact, such a model presents an almost simple analytical structure based on the evaluation of a single parameter calculated through the ratio between the mean and maximum flow velocities in the cross section. Recent studies have demonstrated that for large-scale roughness, the evaluation of the entropy parameter seems to be affected by the local conditions near the bed. In order to investigate such influence, this paper proposes an explicit relationship between the entropy parameter and the relative submergence. This relation was validated using data collected in a rectangular tilting flume of laboratory in which the bed roughness was composed of elements of regular shape such as spheres. Several tests were performed in conditions of large-scale roughness ($1.9 < D/d < 6.4$) and for different values of slope ($0.05\% < i < 1\%$) and water discharge ($7 \text{ L/s} < Q < 76 \text{ L/s}$). The method shows a good agreement between the observed and calculated data for both the velocity profiles and water discharges. DOI: [10.1061/\(ASCE\)HE.1943-5584.0001009](https://doi.org/10.1061/(ASCE)HE.1943-5584.0001009). © 2014 American Society of Civil Engineers.

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Introduction

Knowledge of the velocity distribution is fundamental to estimate water discharge in a river cross section. Velocities are collected by way of current meter and processed obtaining the mean flow velocity. The velocity-area method is generally used to derive the expected value of the discharge to establish the stage-discharge relation. Sampling procedure of velocity measurements in a river cross section could be very expensive and very difficult when the site is remote and inaccessible. Other difficulties arise where the river discharges are highly variable. Therefore, the development of an expeditive methodology to estimate water discharge is beneficial for in situ operational activities because it both reduces time and cost and limits any difficulties that operators might face while taking the measurements.

Building upon the concepts of probability and entropy (Shannon 1948), Chiu (1987) obtained the mean flow velocity from the maximum velocity through a linear relationship defined by the entropy parameter, M . Since then, the entropy velocity profiles [one-dimensional (1D) and two-dimensional (2D)] have been applied to the hydraulics of steady and unsteady open channel flows (Chiu 1989; Chiu and Said 1995; Greco and Mirauda 2004; Moramarco et al. 2008; Marini et al. 2011; Mirauda et al. 2011a, b; Singh 2000).

Xia (1997), investigating some equipped sites along the Mississippi River, noted that the entropy parameter value was quite similar for sections located along straight branches, and was equal to 2.1; instead, for sites along bends the M value was different and

was equal to 4.8. Further studies (Ammari and Remini 2010; Ardiclioglu et al. 2005; Moramarco et al. 2004; Moramarco and Singh 2008) confirmed the values of M reported by Xia (1997). A recent study of Moramarco and Singh (2010) found that for low depths less than 1 m, M seems to be affected by the role of the riverbed and thus by the influence of the relative submergence, D/d (defined as the ratio between the maximum flow depth and the roughness height), on the flow field. Besides, the aspect ratio, B/D (B being the width of the cross section) impacts the position of the maximum velocity below the free surface (y_{\max}), and the flow velocity dip ($\delta h = 1 - y_{\max}/D$) can be assumed independent from the roughness concentration, but generally it decreases at the increase of the aspect ratio ($B/D > 6$) (Ferro 2003).

The objective of this paper is to investigate the influence of the relative submergence on the ratio between the mean flow velocity and the maximum velocity in open-channel flows over large-scale roughness ($D/d < 7$). Laboratory experiments were carried out on a tilting flume.

Discharge measurements for a bed surface with high roughness and low water depth condition were collected. Velocity profiles were acquired with micro-current meter in water flow over fixed bed concentration of wooden spheres, where the aspect ratio was less than 8. Subsequently, velocity measurements were processed to derive the entropy parameter for several slopes and water discharges.

Finally, the resulting relationship between the velocity ratio and the relative submergence was proposed and validated by comparing the percentage error between the observed and the computed water discharges.

Experimental Apparatus and Measurements

The tests were carried out on a tilting flume [Fig. 1(a)] of the Hydraulics Laboratory at the Engineering School of Basilicata University. The flume was connected to a hydraulic circuit for stable water discharges. The channel bed tilts 0–1% (the end section of the flume had a grid installed to regulate the water depth for each discharge and yielded a small longitudinal variation in the

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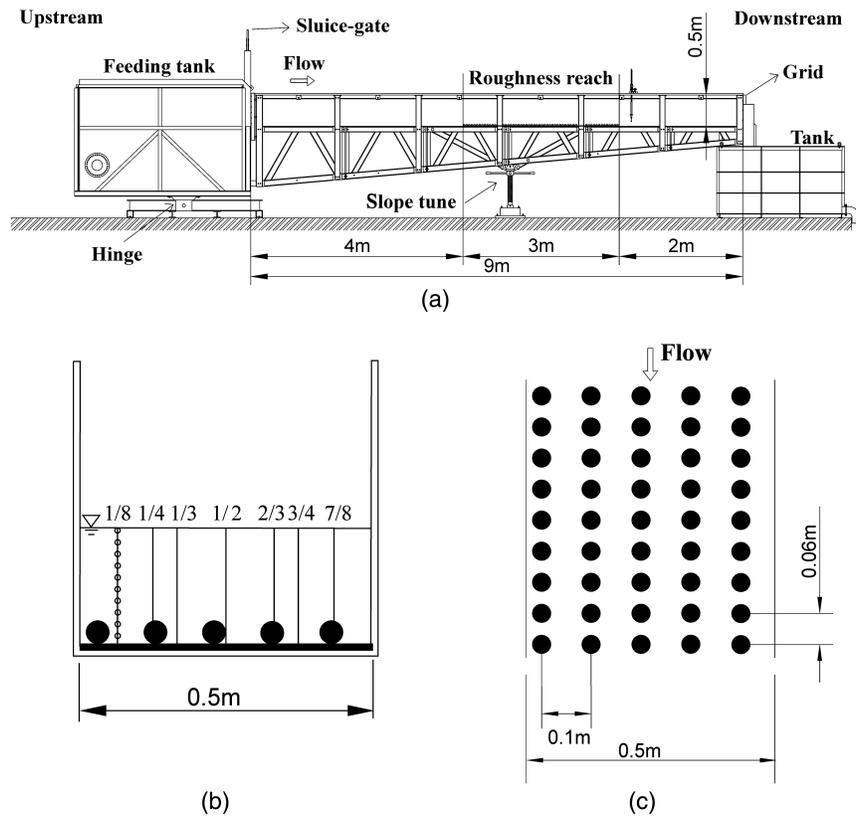


Fig. 1. (a) Experimental flume; (b) cross section; (c) 2D plain of roughness pattern

flow depth). The grid creates a quasi-constant flow depth condition in the measurement section.

The water discharge was measured with a concentric orifice plate installed in the feed pipe. The flow depth was measured by two hydrometers placed at both the beginning and the end of the measurement reach, and the water depth, y , was assumed as the average value.

A set of wooden spheres ($d = 0.035$ m) was arranged throughout the bed in the measurement reach [Figs. 1(b) and c)], reproducing fixed bed with high roughness according to Bayazit (1976) and Bathurst et al. (1981). For the bed arrangement, the number of spheres, N , in the measurement reach yield a surface concentration, λ , expressed as the ratio between the total projected area of the spheres and the measurement reach area, equal to 0.15, corresponding to the maximum flow resistance according to Rouse (1965).

The experiments were conducted in steady flow conditions for different values of water discharge ($Q = 7 - 76$ L/s) and bed slope ($i = 0.05 - 1\%$). Thus, the flow velocity measurements were carried out for different values of water discharge, slope, and corresponding ratios depth to roughness, y/D (relative submergence). The velocity profiles were measured in many points at seven verticals along a cross section located in the middle of the measurement reach at $1/8(V1)$, $1/4(V2)$, $1/3(V3)$, $1/2(V4)$, $2/3(V5)$, $3/4(V6)$, and $7/8(V7)$ of the flow width [Fig. 1(b)]. The local velocities were collected by a micro-current meter with a measuring head diameter of 0.01 m. Hence, the number of the measurement points per vertical was obtained imposing the criterion that the difference between two consecutive measured velocities must be less than 20% of the maximum measured value (ISO 1997). Depending on the velocity measurement points, the mean velocity for each vertical was calculated. Once the mean

velocity for each vertical was evaluated, the water discharge was computed by the way of the mean-section method (ISO 1997). In this method, the partial discharge is computed by multiplying the average value of mean velocities of two adjacent verticals for the area included in the respective verticals. This was repeated for each segment, and the total water discharge was obtained by adding the partial discharge of each segment.

The mean velocity, U_m , of the cross-sectional area, A , was calculated, taking into account the presence of roughness elements placed on the channel bed, as follows:

$$U_m = \frac{Q}{A} = \frac{Q}{B(D - y_0)} \quad (1)$$

where B = flume width; and y_0 = reference level computed according to Ferro and Baiamonte (1994). The latter corresponds to the one obtained by replacing the bed roughness elements (wooden spheres) with an equivalent bed layer having the same volume and constant thickness

$$y_0 = \frac{N\pi d^3}{6BL} = 0.36 \text{ cm}$$

in which N = sphere number placed in the measurement reach with length L .

Table 1 reports the ranges of the measured parameters sorted by test number, with i (%) = bed slope, Q = water discharge, y/D = relative submergence, U_m = mean flow velocity, and U_{\max} = maximum flow velocity. As shown in Table 1, the Reynolds number, R , is very high and goes from 1.41×10^8 to 1.54×10^9 . Therefore, the experiments refer to fully developed turbulent flows as they normally occur in open channels and rivers in condition of large-scale roughness.

Table 1. Ranges of Measured Parameters in the Experiments

Test	i (%)	Q (L/s)	y/D	U_m (m/s)	U_{max} (m/s)	R
1–9	0.05	7–76	2.17–6.43	0.20–0.68	0.34–0.94	1.55×10^8 – 1.54×10^9
10–18	0.1	7–75	2.14–6.40	0.19–0.68	0.34–0.94	1.45×10^8 – 1.53×10^9
19–27	0.25	7–76	2.06–6.31	0.21–0.70	0.36–0.98	1.49×10^8 – 1.54×10^9
28–36	0.375	7–75	2.03–6.23	0.21–0.70	0.36–0.98	1.47×10^8 – 1.52×10^9
37–45	0.5	7–76	2.00–6.17	0.22–0.71	0.41–1.01	1.53×10^8 – 1.54×10^9
46–54	0.625	7–76	1.97–6.09	0.21–0.72	0.39–1.02	1.46×10^8 – 1.54×10^9
55–63	0.75	7–75	1.91–6.00	0.22–0.73	0.41–1.04	1.50×10^8 – 1.53×10^9
64–72	0.875	7–74	1.90–5.91	0.21–0.73	0.40–1.03	1.41×10^8 – 1.51×10^9
73–81	1	7–76	1.89–5.83	0.22–0.76	0.41–1.08	1.46×10^8 – 1.54×10^9

Analysis of the Experimental Entropy Parameter

Also, the known velocity entropy profile derived by Chiu (1987) is based on the probabilistic approach and represents a very useful operative and modeling tool for researchers and engineers because it was founded on a few synthetic parameters that are relatively easy to derive. The maximum velocity of the cross section plays a primary role in the entropy description of the velocity profile through the relation (Chiu 1987)

$$\frac{\xi - \xi_0}{\xi_{max} - \xi_0} = \int_0^u p(u)du \tag{2}$$

where u = local velocity measured in the flow field along a vertical line; ξ = dimensionless variable depending on the reference system employed for the local representation of the flow field; ξ_0 and ξ_{max} = values of the dimensionless variable corresponding to the minimum ($u = 0$) and the maximum ($u = U_{max}$) of the velocity, respectively; and $p(u)$ = probability density function derived by entropy maximization (Chiu 1989)

$$H(u) = - \int_0^{U_{max}} p(u) \ln[p(u)]du \tag{3}$$

and assumed as follows:

$$p(u) = e^{a_1 + a_2 u} \tag{4}$$

in which a_1 and a_2 are parameters (Lagrange multipliers)

$$a_1 = \ln \left[\frac{M}{(e^M - 1)U_{max}} \right] \tag{5}$$

$$a_2 = \frac{M}{U_{max}} \tag{6}$$

The variable $p(u)$ in Eq. (4) satisfies the constraints that the integration of $p(u)$ within the interval $(0, U_{max})$ should be unity, and that $p(u)$ should also be such that the mean velocity, U_m , in the cross-sectional area, A , is Q/A . With $p(u)$ represented by Eq. (4), Eq. (2) can be integrated to yield the simplest form of entropy velocity profile equations (Chiu 1989)

$$\frac{u}{U_{max}} = \frac{1}{M} \ln \left[1 + (e^M - 1) \cdot \frac{\xi - \xi_0}{\xi_{max} - \xi_0} \right] \tag{7}$$

where M = dimensionless entropy parameter that, together with the maximum velocity, U_{max} , plays an important role for the analytical problem closure.

For the vertical where U_{max} occurs, identified as the y -axis, ξ can be expressed through the relationship (Chiu 1988)

$$\xi = \frac{y}{D-h} \exp \left(1 - \frac{y}{D-h} \right) \tag{8}$$

where h = depth below the water surface in which the maximum of the velocity is observed; and y = vertical distance from the channel bed.

Under the probabilistic formulation used to derive Eq. (7), the mean velocity in a channel cross section, regardless of its geometrical shape, can be obtained as the mathematical expectation of u (Chiu and Said 1995)

$$U_m = \int_0^{U_{max}} u p(u) du = \Phi(M) U_{max} \tag{9}$$

in which

$$\Phi(M) = e^M \cdot (e^M - 1)^{-1} - \frac{1}{M} \tag{10}$$

where the coefficient $\Phi(M)$ depends on the flow regime, flow field type (pipe or open channel), flow morphology (straight or meandering) (Xia 1997), and, thus, on the flow stage.

Eq. (9) shows that U_m and U_{max} together can determine $\Phi(M)$ and thus the entropy parameter M through Eq. (10). For each measurement set, u and U_{max} were measured along the vertical depth and U_m was calculated according to Eq. (1). In particular, it should be pointed out that U_{max} represents the maximum value in the data set of velocity points sampled in the flow area during velocity measurements (Chiu and Said 1995). Therefore, plotting the observed pairs (U_m, U_{max}) of each measurement set in Fig. 2, sorted by classes of uniform relative submergence, the best fit line can be estimated, thus providing the values of $\Phi(M)$ through its

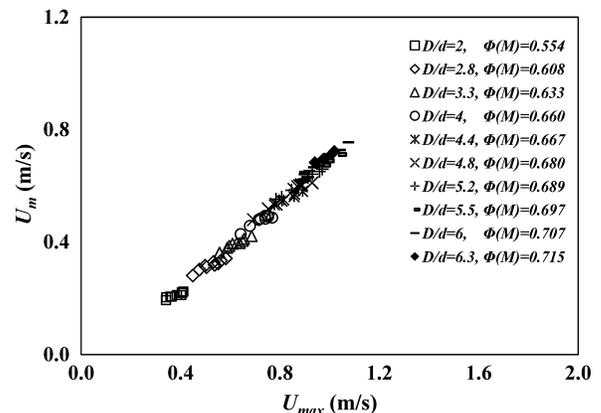


Fig. 2. Relation between mean, U_m , and maximum, U_{max} , velocities observed during the experiments and sorted by the relative submergence D/d

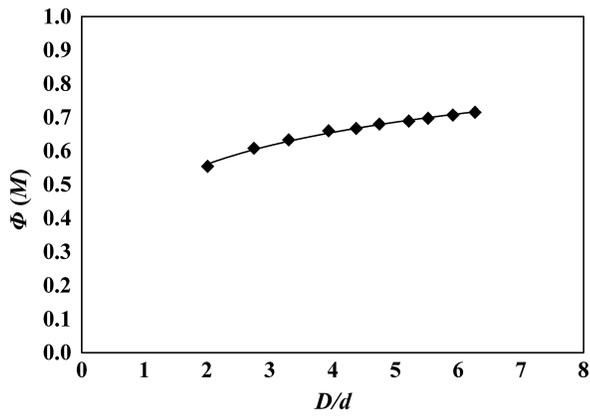


Fig. 3. Distribution of the observed $\Phi(M)$ versus the relative submergence, D/d

slope, and hence M for each class of D/d through Eq. (10). As shown in Fig. 2, the parameter $\Phi(M)$ has different values according to the variation of D/d , going from 0.554 to 0.715.

This result, reported in Fig. 3, shows how $\Phi(M)$ is affected by the local conditions near the bed, and thus by the relative submergence, D/d . This influence is represented by the following relation:

$$\Phi(M) = 0.136 \ln(D/d) + 0.466 \quad (11)$$

Therefore, knowing the value of $\Phi(M)$ from Fig. 3, it is possible to obtain the entropy parameter value M through Eq. (10),

according to the relative submergence class, and reconstruct the velocity profile through Eq. (7). Fig. 4 plots two examples of the observed velocity distribution and the computed entropy profile at the vertical where the maximum velocity occurs for two different slopes and aspect ratios.

Inspecting Figs. 4(a and b) further, it can be noted that the maximum velocity occurs below the free surface at the position y_{\max} , allowing the existence of the flow velocity dip ($\delta h = 1 - y_{\max}/D$) that decreases slightly at the increase of the aspect ratio (Ferro 2003). As shown by the error bands of $\pm 15\%$ in Figs. 4(a and b), the velocity profile reconstructed through the entropy parameter well represents the observed velocities. This demonstrates how the entropy model is able to reproduce the shape of the velocity profile even in open-channel flow with large-scale roughness.

The obtained results represent important issues in order to develop an expeditive methodology to estimate the water discharge in large roughness and narrow open-channel flows. Performing a few measurements of velocity, depth, and roughness [i.e., Wolman (1954) sampling method], the water discharge can be estimated reducing times and costs, and limiting the difficulties operators might face during the measurements. The robustness of such results can be outlined by the sensitivity analysis of the percentage error [$\varepsilon(Q) = 1 - Q_c/Q$] between the observed discharges (Q) and the computed ones (Q_c) through the evaluation of the coefficient $\Phi(M)$ by Eq. (11). The comparison between the observed and computed discharges is shown in Fig. 5(a) for all the tests. It highlights that the discharge values are reproduced by the proposed methodology

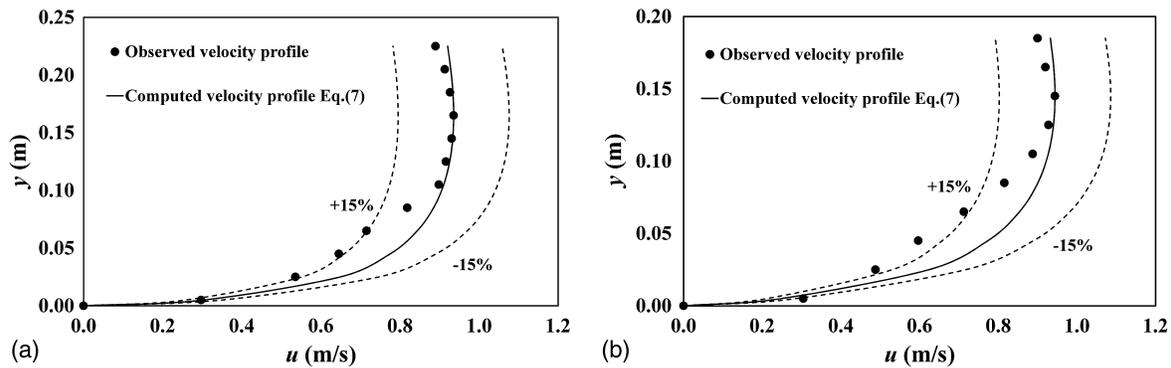


Fig. 4. Observed and calculated velocity profiles along the vertical V4 (at 1/2 of the cross section width) corresponding to the slopes: (a) 0.05% (aspect ratio 2.2); (b) 0.75% (aspect ratio 5.3)

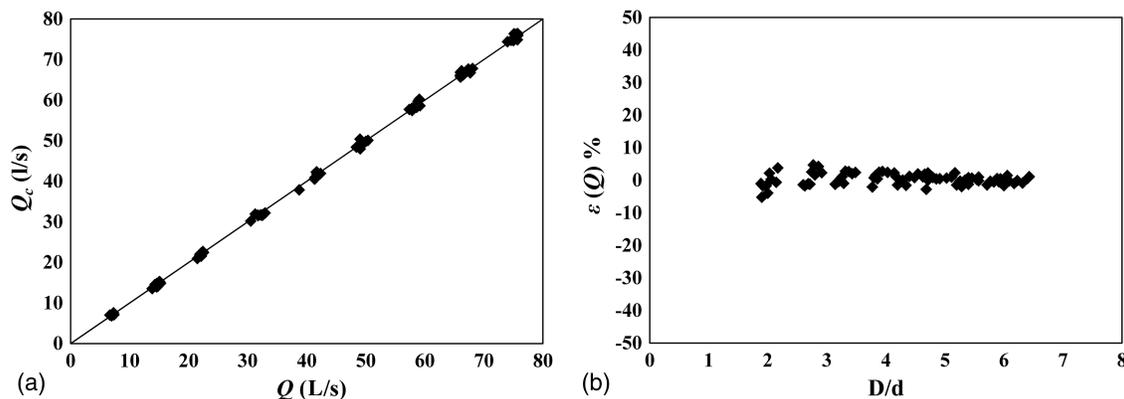


Fig. 5. (a) Comparison between computed, Q_c , and observed, Q , discharges; (b) percentage error for the discharge, $\varepsilon(Q)\%$, versus the relative submergence, D/d

fairly well for the investigated range of relative submergences. Inspecting Fig. 5(b), which refers to the distribution of the percentage errors in function of the relative submergence, higher errors in the evaluation of water discharge occur for the relative submergence less than 3, while for increasing values of D/d the error percentage tends to decrease.

Conclusion

This paper introduces a suitable relation between the entropy parameter and relative submergence in open-channel flows with large-scale roughness ($1.9 < y/D < 6.4$). This dependence was tested on a rectangular tilting flume of laboratory in which the bed roughness was composed of elements of regular shape such as spheres, with slope between 0.05 and 1%, and the water discharge ranged in the interval 7–76 L/s.

The reliability of such a formulation was validated by comparing the observed and computed velocity profiles and by calculating the percentage error between the observed and computed water discharges. In the first case, the velocity profile reconstructed through the entropy approach well represents the observed velocities. In fact, the data fall within the error bands of $\pm 15\%$. In the second case, the error between the two (observed and computed) water discharges is very low and, in particular, ranges between $\pm 5\%$ for relative submergences less than 3, decreasing instead at the increase of the flow depth.

The obtained result, which addresses the measures in situ only to the relative submergence and maximum flow velocity of the cross section, allows evaluating the water discharge and reconstructing the solid of the velocity in an expeditive way, so reducing data acquisition times and costs.

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Notation

The following symbols are used in this paper:

- A = cross sectional area (m^2);
- a_1, a_2 = parameters (Lagrange multipliers);
- B = cross-section width (m);
- B/D = aspect ratio;
- D = maximum flow depth (m);
- d = roughness elements height (m);
- i = bed slope;
- M = entropy parameter;
- N = number of roughness elements;
- $p(u)$ = probability density function of u ;
- Q = observed water discharge (m^3/s);
- Q_c = computed water discharge (m^3/s);
- U_m = mean flow velocity (m/s);
- U_{max} = maximum flow velocity (m/s);
- u = flow velocity (m/s);
- y = vertical distance from channel bed (m);
- y_{max} = location of the maximum flow velocity (m);
- y_0 = location where the log velocity profile predicts the zero velocity (m);
- y/d = relative submergence;
- δh = flow velocity dip;

- $\varepsilon(Q)\%$ = percentage error computing the discharge Q ;
- λ = roughness concentration;
- ξ = dimensionless variable depending on the reference system;
- ξ_{max} = dimensionless variable at which corresponds the maximum flow velocity ($u = U_{max}$);
- ξ_0 = dimensionless variable at which corresponds the minimum flow velocity ($u = 0$); and
- $\Phi(M)$ = ratio between the mean flow velocity and the maximum velocity.

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