DEBT STABILISATION AND DYNAMIC INTERACTION BETWEEN MONETARY AND FISCAL POLICY:
IN MEDIO STAT VIRTUS

AUTHORS

Francesco Purificato
University of Naples Federico II, Department of Law

Mauro Sodini
University of Naples Federico II, Department of Law
Technical University of Ostrava, Ostrava, Czech Republic, Department of Finance, Faculty of Economics

ABSTRACT

Recent literature has focused on studying how fiscal and monetary authorities in a monetary union can interact during a debt stabilisation process. This literature shows that only exogenous shocks or differences among countries determine deviations from the target level of macroeconomic variables in the steady state equilibrium. This paper aims to reformulate such modelling in a time setting, assuming that policy authorities do not coordinate and cannot perfectly predict the decisions made by their counterpart. This paper shows that simple decision processes driven by the best response mechanism or adaptive expectations do not guarantee convergence to steady-state equilibrium. Instead, persistent fluctuations in the level of macroeconomic variables and policy instruments may emerge if the relative weight given by the fiscal authority to the components of its objective function, namely, debt stability and output stability, is too unbalanced.

JEL codes: E52, E58, E62, E63.

Keywords: Monetary Policy, Fiscal Policy, Monetary Union, Nonlinear Dynamics, Bifurcations.

1.0 INTRODUCTION

The sovereign debt crisis mainly affected the Economic and monetary union (EMU) from 2009 to 2012. To counteract the adverse effects of the financial crisis, EMU countries implemented expansionary fiscal policies, sustaining economic activity and supporting the financial sector; as a consequence of increased indebtedness, some countries faced increasing difficulties in accessing financial markets. The European institutions called on these countries to adopt an aggressive fiscal consolidation, the so-called austerity, but the crisis was only slowly resolved in the second half of 2012 when the European central bank (ECB) declared its readiness to support struggling countries with an expansionary monetary policy. The development of the sovereign debt crisis highlights the extent to which the interaction between monetary and fiscal policy is crucial when an economy goes through a debt stabilisation process (De Grauwe and Ji, 2012).

By using a reduced form of the New Keynesian macro model (Woodford, 2003) and describing the interaction among policymakers as a strategic policy game, a vein of literature analyses how the monetary policy and the national fiscal policies affect the steady state equilibrium within a monetary union, for both the union-wide economy and the single country economy (Bofinger et al., 2006; Bofinger and Mayer, 2007; Foresti, 2018; Chortareas and Mavrodimitrakis, 2021). Regarding the critical issue of debt stabilisation, this literature shows that if member countries of a monetary union are perfectly homogenous, particularly if they have the same level of accumulated debt and the same target for the debt level, then each fiscal authority can reach its debt target. Notice that the homogeneity condition also implies that countries share the same economic structure, the same target levels of the monetary authority for the macroeconomic variables, and that symmetric supply and demand shocks may only occur (Gatti e van Wijnbergen, 2002; Hughes Hallett, 2008; Foresti, 2015, 2018; Mavrodimitrakis, 2020). Thus, this literature underlines the critical finding that if the homogeneity condition holds, the fiscal authority can implement a successful debt stabilisation process.

The main contribution of this paper is to investigate if and how this key finding holds in a dynamic framework. The previous literature studies the decision problem of the agents through a policy game, where the monetary authority and the fiscal authority act independently and simultaneously, such that a Cournot-Nash equilibrium emerges. This equilibrium is always stable in the sense that, in correspondence with it, both the monetary and fiscal authorities obtain the highest possible benefit, minimising their loss function. Nevertheless, the policy game is simultaneous; therefore, we cannot learn anything about the convergence process toward equilibrium. This issue motivates our choice to extend the analysis in a dynamic framework.

As a first step, based on Mavrodimitrakis (2020), we present a static model that allows us to highlight the key finding of the literature; however, we assume that there are no asymmetries between individual economies; namely, we present a simple static model of a closed economy. A supply equation and a demand equation describe the structure of this economy while the monetary authority and the fiscal authority interact through a policy game, where both authorities act independently and simultaneously, such that a Cournot-Nash equilibrium defines the steady state equilibrium. The monetary authority pursues the objective of macroeconomic stability that implies both price and output stability; therefore, the determination of the policy rate is a function of the primary government balance implemented by the fiscal authority, and it targets long-run values of the inflation rate and output. The fiscal authority aims at output stability and debt stability; therefore, the setting of the primary government balance is a function of the policy rate set by the monetary authority, and it targets long-run values of the debt level and output. As a result, the Cournot-Nash equilibrium of the policy game is perfectly consistent with the optimal values of the inflation rate, output gap, and debt level.

As a second step, we analyse the policy game between fiscal and monetary authorities within a dynamic setting. The main assumption is that, at the end of the current period, the authorities independently and simultaneously set the level of their policy instruments for the next period. Due to the concomitance between the end of the previous period and the time when the fiscal policy adopts decisions for the following period, this assumption implies that when an authority adopts its decisions, it does not know the choice of the other authority for the subsequent period; namely, only knows the current level of the other policy instrument. In addition, it may also be the case that the fiscal authority does not precisely know the actual debt level and the actual interest rate to be paid on that debt. As the main result of our analysis, we find that the steady-state equilibrium of the static model can become unstable; specifically, if the fiscal authority's preferences are too unbalanced, namely, it is overly concerned with either debt stability or output stability, the dynamics do not converge to the stationary state of the model, but they are captured by a limit cycle, showing persistent oscillations. This key finding also holds under different extensions of the basic model. Specifically, we consider a lower bound for the policy rate, that is, a floor exists for the interest rate; in defining the decision-making process of the authorities, we consider both a mechanism of static expectations and a mechanism of adaptive expectations; finally, we also consider different degrees of accuracy in estimating the actual debt level and the actual interest rate to be paid on that debt by the fiscal authority.

Thus, the main result of this paper is that, regardless of the heterogeneity of the countries belonging to a monetary union and the presence of supply and demand shocks, if the interaction between the monetary authority and fiscal authority occurs in a dynamic context and the fiscal authority's preferences are unbalanced, then each authority fails to reach its goals; namely, they fail in leading the macroeconomic variable towards their target levels. Regarding the process of debt stabilisation, our finding also contributes to explaining the strong instability that characterised the sovereign debt crisis until 2012: probably, during the first phase of the crisis, the European institutions promoted an excessive fiscal consolidation, forcing national fiscal authorities to place too much emphasis on the objective of debt stability compared to the objective of output stability.

The remaining part of this paper is organised as follows. Section 2 presents and discusses the essential features of the static model. Section 3 analyses the interaction between the monetary and fiscal authorities within a dynamic framework, detecting the analytical conditions that imply the limit cycle as a possible steady-state equilibrium. Finally, Section 4 summarises the conclusions.

2.0 STATIC MODEL

The literature extensively used a reduced form of the New Keynesian macroeconomic model to study how asymmetries across countries affect the strategic fiscal-monetary interaction and the steady state equilibrium in a monetary union (Bofinger et al., 2006; Bofinger and Mayer, 2007; Foresti, 2018; Chortareas and Mavrodimitrakis, 2021). A recent line of research has focused the analysis on the case of a fiscal authority concerned with output stability and debt stability. This literature detects that the fiscal and monetary authorities succeed in fully stabilising the economy in the absence of heterogeneity among countries; notably, in the steady-state equilibrium, macroeconomic variable levels equal their optimal levels (Mavrodimitrakis, 2020; Foresti, 2015, 2018).

Using the basic model in Mavrodimitrakis (2020), this paper studies under which condition this result also holds in a dynamic context, regardless of the heterogeneity of the countries belonging to a monetary union. Assuming that there are no asymmetries across economies implies that the static model presented in this section basically describes a closed economy; nevertheless, this assumption allows for simplification of the analysis without affecting the soundness of the results. Concerning the basic model in Mavrodimitrakis (2020), the only significant departure is not to impose the usual simplifying assumption that the autonomous demand component is zero.

Section 2.1 describes the economy. Sections 2.2 and 2.3 investigate the optimisation problems of the monetary and fiscal authorities, while Section 2.4 focuses on the Nash equilibrium.

2.1 STRUCTURE OF THE ECONOMY

The model's non-policy block describes the economy's supply and demand sides. The supply equation reads as follows:

Equation [01] $$\pi = \pi^e + \gamma_y y + \varepsilon_s$$

The variable π defines the inflation rate while the variable π^e defines the inflation rate expected by private agents; the variable y defines the output gap, namely, the difference between the effective output and the potential output. The parameter γ_y is the sensitivity of the inflation rate to variations in the output gap y. Finally, the variable ε_s represents supply shocks, assumed to be an independent and identically distributed random variable with zero mean and constant variance. Notice that Equation [01] shows that if the output gap and supply shocks are equal to zero ($y = 0$; $\varepsilon_s = 0$), then the inflation rate is equal to the expected inflation rate ($\pi = \pi^e$); differently, if the effective output is greater than the potential output or supply shocks are positive ($y > 0$; $\varepsilon_s > 0$), then the inflation rate is more than the expected inflation rate ($\pi > \pi^e$).

The demand equation reads as follows:

Equation [02]
$$y = A - \delta_r(i - \pi^e) + \delta_g g + \varepsilon_d$$

The variable i defines the nominal interest rate, namely, the policy rate determined by the monetary authority; therefore, the term $i - \pi^e$ identifies the real interest rate. The variable g defines the primary government balance: $g < 0$ implies a primary government surplus, namely, a contractionary fiscal stance, while $g > 0$ indicates a primary government deficit, namely, an expansionary fiscal stance. Finally, the variable ε_d represents demand shocks, assumed to be an independent and identically distributed random variable with zero mean and constant variance. The parameter $A > 0$ represents an autonomous component of aggregate demand exclusively attributable to private agents, not government or demand shocks. The parameter $\delta_r > 0$ captures the sensitivity of the aggregate demand to the expected real interest rate; the parameter $\delta_g > 0$ identifies the fiscal multiplier, namely, the sensitivity of the aggregate demand to the primary government balance.

Equation [02] follows Mavrodimitrakis (2020), assuming that the aggregate demand depends on the real interest rate expected by private agents ($i - \pi^e$) rather than on the actual real interest rate ($i - \pi$), as in Foresti (2015). Notice that this assumption allows for simplifying algebraic manipulation without affecting analytical results.

Moreover, following Foresti (2015), Equation [02] assumes that the excess demand attributable to private agents may be greater than zero ($A \geq 0$); differently, following the literature, Mavrodimitrakis (2020) sets $A = 0$ because, in a static context, this assumption allows for simplifying the algebraic manipulation and the exposition. Nevertheless, Section 03 will explain that this seemingly harmless hypothesis is not neutral for the system's dynamic behaviour in a dynamic context.

2.2 Monetary Authority

The monetary authority pursues the objective of macroeconomic stability, which implies price and output stability, by targeting long-run values of the inflation rate and output gap. To simplify the algebraic manipulation, we use the standard assumptions that the inflation target is equal to zero ($\bar{\pi}_M = 0$) and the monetary authority is credible, such that the expected inflation rate is equal to the inflation target, in turn, assumed to be equal to zero ($\pi^e = \bar{\pi}_M = 0$). In addition, we also make another typical assumption that the target for the output gap is also equal to zero ($\bar{y}_M = 0$). The objective function of the monetary authority reads as follows:

Equation [03]
$$L_M = \frac{1}{2}[\pi^2 + \alpha_M y^2]$$

The parameter $\alpha_M \geq 0$ represents the relative weight of output stability with respect to price stability. The monetary authority embraces a regime of strict inflation targeting if it only cares about deviations of the inflation rate from its target value ($\alpha_M = 0$); differently, a regime of flexible inflation targeting holds if it is also concerned with deviations of the output gap from its target value ($\alpha_M > 0$).

The conduct of the monetary policy aims to minimise deviations of its concerned variables from target values, subject to the constraints of Equations [01] and [02]. Thus, the optimisation problem of the monetary authority with respect to the policy rate becomes:

$$\begin{cases} \min_i L_M \\ s.t. \\ \quad \pi = \gamma_y y + \varepsilon_s \\ y = A - \delta_r i + \delta_g g + \varepsilon_d \end{cases}$$

By substituting the constraints into the loss function and deriving with respect to the policy rate, we obtain the first-order condition; then, substituting into Equation [01], we determine the monetary authority's optimal choices concerning the output gap (y_{opt}) and the inflation rate (π_{opt}):

Equation [04] $$y_{opt} = -\frac{\gamma_y}{(\alpha_M+\gamma_y^2)}\varepsilon_s$$

Equation [05] $$\pi_{opt} = \frac{\alpha_M}{(\alpha_M+\gamma_y^2)}\varepsilon_s$$

The optimal values defined by Equations [04 – 05] can differ from zero only if supply shocks occur, meaning that the monetary authority succeeds in fully stabilising the economy after demand shocks. Following a supply shock, the monetary authority faces a trade-off between price stability and output stability, and it can only partially stabilise the economy according to the weight on output gap stabilisation ($\gamma_y/(\alpha_M + \gamma_y^2)$); otherwise, following a demand shock, the monetary authority benefits from the divine coincidence, and it can fully stabilise the economy offsetting the change in the aggregate demand.

Finally, by substituting Equation [04] into the demand equation, we obtain the reaction function of the monetary authority with respect to the fiscal stance.

Equation [06] $$i = f(g) = \frac{1}{\delta_r}\left[A + \varepsilon_d + \frac{\gamma_y}{(\alpha_M+\gamma_y^2)}\varepsilon_s\right] + \frac{\delta_g}{\delta_r}g$$

Equation [06] describes how the monetary authority sets the policy rate as a positive function of the fiscal stance (g), supply and demand shocks ($\varepsilon_s;\ \varepsilon_d$) and the autonomous component of the aggregate demand (A). This equation displays that the policy rate and the primary government balance are perfect substitutes: if the fiscal authority implements an expansionary fiscal policy ($+\Delta g$), determining an increase in the output gap ($+\Delta y$) and, in turn, in the inflation rate ($+\Delta\pi$), then the monetary authority reacts through a contractionary monetary policy ($+\Delta i$) to minimise the loss defined by its objective function.

2.3 FISCAL AUTHORITY

The fiscal authority faces a short-run government budget constraint, which assumes the functional form of a debt accumulation equation.

Equation [07] $$D = (1 + i - \pi^e)D_0 + g$$

The endogenous variable D defines the country's debt level, while the exogenous variable D_0 defines the initial condition related to the accumulated level of debt. Equation [07] displays the outstanding level of debt as a positive function of the accumulated level of debt (D_0), the debt service ($(i - \pi^e)D_0$) and the fiscal stance (g).

The fiscal authority aims at the objectives of output and debt stability, adopting a fiscal policy that targets long-run values of the output gap and the debt level, which are assumed to be equal to zero to simplify the algebra ($\bar{y}_F = \bar{D}_F = 0$). The objective function of the fiscal authority reads as follows.

Equation [08]: $$L_F = \frac{1}{2}[D^2 + \alpha_F y^2]$$

The literature usually assumes that the fiscal authority pursues the goal of fiscal stance stability, being primarily concerned with stabilising the primary balance of the government budget. Over a medium-time horizon, though, it seems reasonable to assume that the solvency on issued debt rather than the primary balance variability is one of the main objectives of the government action (Foresti, 2015, 2018; Mavrodimitrakis, 2020). The parameter $\alpha_F \geq 0$ identifies the relative weight that the fiscal authority assigns to the output gap stabilisation with respect to the debt stabilisation: the greater this weight ($\alpha_F > 1$), the more fiscal policy is concerned with the stabilisation of the economy over the business cycle.

The fiscal authority aims to minimise deviations of its concerned variables from target values, subject to the budget constraint and the demand equation (Equations [07] and [02]). In this context, the optimisation problem of the fiscal authority with respect to the primary government balance becomes:

$$\begin{cases} \min_{g} L_F \\ s.t. \\ D = (1+i)D_0 + g \\ y = A - \delta_r(i) + \delta_g g + \varepsilon_d \end{cases}$$

Using the first-order condition and the optimal output gap (Equation [4]), we determine the fiscal authority's optimal choice for the debt level (D).

Equation [09] $$D_{opt} = \frac{(\alpha_F \delta_g)\gamma_y}{(\alpha_M + \gamma_y^2)} \varepsilon_s$$

Equation [09] shows that only in the presence of supply shocks the optimal debt level can differ from zero. Following supply shocks, the fiscal authority faces a trade-off between debt and output stability, and it can only partially stabilise the economy according to its preferences (α_F), the monetary authority's preferences (α_M) and the structure of the economy (δ_g, γ_y). Otherwise, the fiscal authority fully stabilises the economy after demand shocks.

By substituting Equations [02] and [07] into the first-order condition, we obtain the reaction function of the fiscal authority with respect to the policy rate.

Equation [10] $$g = f(i) = -\frac{1}{(1+\alpha_F \delta_g^2)} \left[\alpha_F \delta_g (A + \varepsilon_d) + D_0\right] + \frac{(\alpha_F \delta_g \delta_r - D_0)}{(1+\alpha_F \delta_g^2)} i$$

Equation [10] identifies a positive relationship between the primary government balance (g) and the policy rate (i) only if the condition $\alpha_F \delta_g \delta_r > D_0$ is satisfied. This condition implies that the policy tools are perfect substitutes only if a contractionary monetary policy ($+\Delta i$) determines an increase in the debt service (D_0) that is not large enough to offset the negative impact on the output gap (δ_r), as weighed by the fiscal authority's preferences (α_F), and if the fiscal multiplier is sufficiently large (δ_g). Equation [12] also shows that the primary government balance is a negative function of the autonomous component of the aggregate demand (A), demand shocks (ε_d) and the accumulated debt (D_0).

2.4 OPTIMAL ECONOMIC POLICY

To analyse the economic policy in this economy, we have represented the interaction between the monetary and fiscal authorities as a policy game where both authorities act independently and simultaneously, such that a Cournot-Nash equilibrium defines the steady state equilibrium. In this context, by considering the equation system consisting of the reaction functions (Equations [12] and [7]), we obtain the optimal policy rate and the optimal primary balance:

Equation [11] $$i_{opt} = \frac{(A+\varepsilon_d)}{(\delta_r + \delta_g D_0)} + \frac{(1+\alpha_F \delta_g^2)\gamma_y}{(\delta_r + \delta_g D_0)(\alpha_M + \gamma_y^2)} \varepsilon_s - \frac{\delta_g}{(\delta_r + \delta_g D_0)} D_0$$

Equation [12] $$g_{opt} = -\frac{(A+\delta_r+\varepsilon_d)}{(\delta_r + \delta_g D_0)} D_0 + \frac{(\alpha_F \delta_g \delta_r - D_0)\gamma_y}{(\delta_r + \delta_g D_0)(\alpha_M + \gamma_y^2)} \varepsilon_s$$

Equations [11] and [12] define the Cournot-Nash equilibrium of the policy game that is perfectly consistent with the optimal values of the inflation rate, output gap, and debt level (Equations [04], [05] and [09]); therefore, they also define the steady state equilibrium of the economic system.

3.0 DYNAMIC MODEL

Now, it is interesting to see if and how this problem changes in a dynamic framework. So far, we have considered the decision problem of the agents in a static setting: the monetary authority and the fiscal authority interact through a policy game, where both authorities act independently and simultaneously, such that a Cournot-Nash equilibrium emerges. This equilibrium is always stable in the sense that, in correspondence with it, both the monetary and fiscal authorities obtain the highest possible benefit, minimising their loss function. Nevertheless, the policy game is a simultaneous game; therefore, we cannot learn anything about the convergence process toward equilibrium, and this issue motivates our choice to extend the analysis in a dynamic framework.

Thus, the focus of the analysis is on studying the stability of the steady state equilibrium in a dynamic context, assuming that there are no supply and demand shocks ($\varepsilon_s = \varepsilon_d = 0$) and that the initial conditions of the variables of the model are close to their stationary states; particularly, we consider the initial value of debt close to zero ($D_0 \cong 0$). In pursuing their policies of getting closer to their targets, we assume that the monetary and fiscal authorities set the level of the policy rate and the primary government balance, respectively, at the end of a generic period $(t-1, t]$ for the period $(t, t+1]$.[1]

Assuming that the effects generated by the decision of the level of the policy rate on the output gap act immediately and that the inflation rate reacts without any delay to the output gap, the problem of the monetary authority can be written as follows:

$$\begin{cases} \min\limits_{i_{t+1}} \frac{1}{2}[\pi_{t+1}^2 + \alpha_M y_{t+1}^2] \\ s.t. \\ \pi_{t+1} = \gamma_y y_{t+1} \\ y_{t+1} = A - \delta_r i_{t+1} + \delta_g E[g_{t+1}] \end{cases}$$

where $E[g_{t+1}]$ is the monetary authority's expectation of the primary government balance, set by the fiscal authority at the time t for the time $t+1$.

Concerning the fiscal authority, we assume that the effects of the primary government balance on the level of the output gap occur directly at time t and that the following law determines the dynamics of debt:

$$D_{t+1} = (1 + i_{t+1}^s)D_t + g_{t+1}$$

The variable i_{t+1}^s defines the interest rate actually paid on the debt issued at the end of time t for the time $t+1$; this rate applies to the entire amount of debt issued as we assume a complete rollover of the debt net of repayments by the government at the end of any period. If the fiscal authority issues government bonds at an indexed interest rate, then $i_{t+1}^s = i_{t+1}$; otherwise, if government bonds' yield is a non-indexed interest rate, then $i_{t+1}^s = i_t$. Notice that the specification of the debt dynamics implies that the accumulated debt at the time t will grow in the period $(t, t+1]$ according to the interest rate i_{t+1}^s.

Consequently, the problem for the fiscal authority becomes

$$\begin{cases} \min\limits_{g_{t+1}} \frac{1}{2}[D_{t+1}^2 + \alpha_F y_{t+1}^2] \\ s.t. \\ D_{t+1} = (1 + E[i_{t+1}^s])D_t^c + g_{t+1} \\ y_{t+1} = A - \delta_r E[i_{t+1}] + \delta_g g_{t+1} \end{cases}$$

where $E[i_{t+1}]$ is the fiscal authority's expectation of the policy rate level chosen by the monetary authority, at the time t for the time $t+1$, and $E[i_{t+1}^s]$ is the fiscal authority's expectation of the actual interest rate to be paid on government bonds. The variables D_t^c define the information available to the fiscal authority at the end of time t concerning the debt level (D_t). Indeed, due to the concomitance between the end of the previous period and the time when the fiscal policy adopts decisions for the following period, it may not be the case that the fiscal authority precisely knows the debt level.

Regarding the formation of the expectations introduced in the previous two problems, we assume that they are static, that is $E[g_{t+1}] = g_t$, $E[i_{t+1}^s] = i_t$ and $E[i_{t+1}] = i_t$, following the approach used by Puu (1991) to study duopoly dynamics in a goods market. In this context, except when the system is in the steady-state equilibrium,[2] both authorities are aware that their decisions will be made on predictions that will not come

[1] To simplify the presentation, in what follows we will identify the interval $(t-1, t]$ with time t. For a flow variable x, x_{t+1} denotes the flow in the interval $(t, t+1]$, while for a stock variable z, z_{t+1} denotes the level of the stock at the end of the period $(t, t+1]$.

[2] See next section.

true, but they react based on their existing information set, which does not include knowledge of g_{t+1} and i_{t+1}.

The following analysis will consider various hypotheses for determining the values of i^s_{t+1} and D^c_t. As a benchmark case, we will assume that the fiscal authority issues government bonds at an indexed interest rate ($i^s_{t+1} = i_{t+1}$). Furthermore, we will assume that there is an information lag relative to the knowledge of the debt level at the time t and that the fiscal authority estimates this level with that one of the previous period ($D^c_t = D_{t-1}$). At first glance, these assumptions might appear particularly restrictive; nevertheless, it is worth remembering that the sovereign debt crisis found its origin precisely in the Greek authorities' erroneous or fraudulent estimation of the actual debt level and the mistaken belief that sovereign bond yields would remain low to reflect a country's inability to leave the euro area.[3]

By solving the optimisation problems under the hypotheses $i^s_{t+1} = i_{t+1}$ and $D^c_t = D_{t-1}$, we obtain the dynamics of the variables g, i and D as described by the following map T:

$$T: \begin{cases} g_{t+1} = \dfrac{i_t \alpha_F \delta_g \delta_r - A\alpha_F \delta_g - D_{t-1}(1+i_t)}{1+\alpha_F \delta_g^2} \\ i_{t+1} = \dfrac{\delta_g g_t + A}{\delta r} \\ D_{t+1} = \left(1 + \dfrac{\delta_g g_t + A}{\delta r}\right) D_t + \dfrac{i_t \alpha_F \delta_g \delta_r - A\alpha_F \delta_g - D_{t-1}(1+i_t)}{1+\alpha_F \delta_g^2} \end{cases}$$

Using the equation that describes the time evolution of the debt ($D_{t+1} = (1+i_{t+1})D_t + g_{t+1}$), we obtain that

$$g_{t+1} = \frac{i_t \alpha_F \delta_g \delta_r - A\alpha_F \delta_g - (D_t - g_t)}{1+\alpha_F \delta_g^2}$$

and

$$D_{t+1} = \left(1 + \frac{\delta_g g_t + A}{\delta r}\right) D_t + \frac{i_t \alpha_F \delta_g \delta_r - A\alpha_F \delta_g - (D_t - g_t)}{1+\alpha_F \delta_g^2}$$

Notice that unlike classical papers studying the best response dynamics, as in Puu (1991), in this model, past decisions produce persistent changes through a third equation, namely, the debt accumulation equation, which changes the components of the index that is optimised by the fiscal authority at each t. Through simple calculations, we find a single fixed point for T, given by

$$x^* := (g^*, i^*, D^*) = (0, \frac{A}{\delta_r}, 0)$$

The expression $i^* = A/\delta_r$ can be interpreted as the Wicksellian natural interest rate, namely, the interest rate that is consistent with a stable price level (Blanchard, 2017). From the perspective of economic meaning, the fixed point of the map T implies that all macroeconomic variables equal their target values in the steady state equilibrium: the inflation rate is equal to zero, as the output gap and the debt level. Thus, in this dynamic framework, the monetary and fiscal authorities could achieve their objectives precisely as they would in the static framework in the absence of any supply and demand shocks ($\pi_{opt} = \pi^* = 0$, $y_{opt} = y^* = 0$, $D_{opt} = D^* = 0$).

To study the stability of the fixed point, we use the stability conditions in Brooks (2004). We recall that a fixed point for a three-dimensional discrete dynamical system is locally asymptotically stable if the following conditions are satisfied

[3] Notice that among the main conditions imposed by the European authorities on Greece for the disbursement of financial support, there was the obligation to make the national statistical office independent from the government. This body's lack of autonomy was considered one of the leading causes of the erroneous assessment of Greece's sovereign debt level.

Equation [13]:

$$\begin{cases} 1-|det(J^*)|<0 \\ M(J^*)-tr(J^*)\,det(J^*)+(det(J^*))^2-1<0 \\ tr(J^*)+det(J^*)-M(J^*)-1<0 \\ tr(J^*)+det(J^*)+M(J^*)+1>0 \end{cases}$$

where J^* is the Jacobian Matrix evaluated at the fixed point and M is the sum of the three principal minors of the Jacobian matrix J^*.

The following result holds:

Proposition 1: *The fixed point* p^* *is locally asymptotically stable if*

$$\left(2\alpha_F^2\delta_g^4+\alpha_F\delta_g^4\right)A^2+\left(4\alpha_F^2\delta_g^4\delta_r+3\alpha_F\delta_g^2\delta_r+\delta_r\right)A-\alpha_F\delta_g^2\delta_r^2<0$$

Proof. The Jacobian Matrix of T is given by

$$J(g_t,i_t,D_t)=\begin{pmatrix} \frac{1}{1+\alpha_F\delta_g^2} & -\frac{1}{1+\alpha_F\delta_g^2} & \frac{\alpha_F\delta_g\delta_r}{1+\alpha_F\delta_g^2} \\ \frac{\delta_g D_t}{\delta_r}+\frac{1}{1+\alpha_F\delta_g^2} & 1+\frac{\delta_g g_t+A}{\delta_r}-\frac{1}{1+\alpha_F\delta_g^2} & \frac{\alpha_F\delta_g\delta_r}{1+\alpha_F\delta_g^2} \\ \frac{\delta_g}{\delta_r} & 0 & 0 \end{pmatrix}$$

which evaluated at p^* gives

$$J^*=J(g^*,i^*,D^*)=\begin{pmatrix} \frac{1}{1+\alpha_F\delta_g^2} & -\frac{1}{1+\alpha_F\delta_g^2} & \frac{\alpha_F\delta_g\delta_r}{1+\alpha_F\delta_g^2} \\ \frac{1}{1+\alpha_F\delta_g^2} & \frac{\delta_g^2(A+\delta_r)\alpha_F+A}{\delta_r\left(1+\alpha_F\delta_g^2\right)} & \frac{\alpha_F\delta_g\delta_r}{1+\alpha_F\delta_g^2} \\ \frac{\delta_g}{\delta_r} & 0 & 0 \end{pmatrix}$$

We have that

$$det(J^*)=-\frac{\delta_g^2(A+\delta_r)\alpha_F}{\delta_r\left(1+\alpha_F\delta_g^2\right)}<0$$
$$tr(J^*)=1+\frac{A}{\delta_r}$$
$$M(J^*)=\frac{A+\delta_r-\alpha_F\delta_g^2\delta_r}{\delta_r\left(1+\alpha_F\delta_g^2\right)}$$

Referring to Equation [13], we have that the first condition boils down in $det\,J^*>-1$ and reads as

Equation [14]

$$\frac{\delta_r-A\alpha_F\delta_g^2}{\delta_r\left(1+\alpha_F\delta_g^2\right)}>0$$

that solved for A gives $A<\overline{A}:=\frac{\delta_r}{\alpha_F\delta_g^2}$.

The second one becomes

Equation [15]:

$$\frac{F\cdot A^2+G\cdot A+H}{\delta_r^2(1+\alpha_F\delta_g^2)^2}<0$$

where

$$F = 2\delta_g^4\alpha_F^2 + \delta_g^2\alpha_F > 0$$
$$G = 4\delta_g^4\delta_r\alpha_F^2 + 3\delta_g^2\delta_r\alpha_F + \delta_r > 0$$
$$H = -\delta_g^2\delta_r^2\alpha_F < 0$$

The third one reads as

$$-\frac{1}{1 + \alpha_F\delta_g^2} < 0$$

which is always satisfied as well as the fourth

$$\frac{3\delta_r + 2A}{\delta_r\left(1 + \alpha_F\delta_g^2\right)} > 0$$

By focusing on Equation [15], we have that, for all the feasible parameter values, the equation $F \cdot A^2 + G \cdot A + H = 0$ has two solutions in A with opposite sign A^-, A^+. Thus, by considering the nonnegativity of A, we have that the second condition is satisfied for $0 \leq A < A^+$. Tedious calculations show that

$$A^+ < \overline{A}$$

therefore, the only condition related to the stability of p^* is described by Equation [15].

■

In order to clarify the stability results, Figure 01 shows the stability region in the parameter space (α_F, A). Interestingly, we note that only a 0 level of A makes p^* stable, at least locally, for any α_F level. In contrast, a positive value of A requires that α_F ranges in an interval (α_F^1, α_F^2), with the black curve bounding this area that decreases asymptotically towards the $\alpha_F - axis$.

For α_F values outside such an interval, the point p^* is unstable. Figure 02, Panel (A) and Panel (B), shows the occurrence of quasiperiodic dynamics for $\alpha_F < \alpha_F^1$ and $\alpha_F > \alpha_F^2$. In other words, varying α_F, if α_F crosses the boundary of the grey region, then the point p^* undergoes a (supercritical) Neimark-Sacker bifurcation, as illustrated in Panel (B).

From the point of view of the economic meaning, the parameter $A > 0$ determines a positive value for the natural interest rate ($i^* > 0$), which defines the policy rate value in the steady state equilibrium; therefore, any deviation of the debt level from its steady state level implies that debt level changes autonomously over time, independent of fiscal authority's decisions, as bonds' yield is greater than zero. In addition, in a dynamic framework, the parameter α_F also assumes a specific economic significance: it defines the fiscal policy responsiveness to the difference between the policy rate and the natural interest rate. In determining the primary government balance, if α_F is excessively low, then the fiscal authority attaches relatively too much weight to the impact of the monetary policy on the debt burden rather than to its effects on the aggregate demand; on the contrary, if α_F is excessively high, then attaches relatively too much weight to the impact of the monetary policy on the aggregate demand rather than to its effects on the debt burden. In any case, the response of the fiscal authority becomes disproportionate, leading to the instability of the steady-state equilibrium.

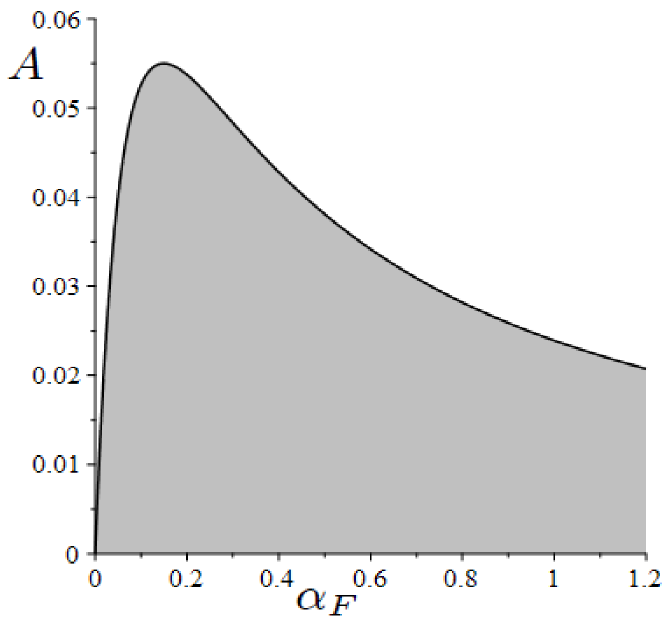

FIGURE 01. Stability region of the fixed point p^* in grey.
Parameter values: δ_r = *0.4*, δ_g = *1.8*.

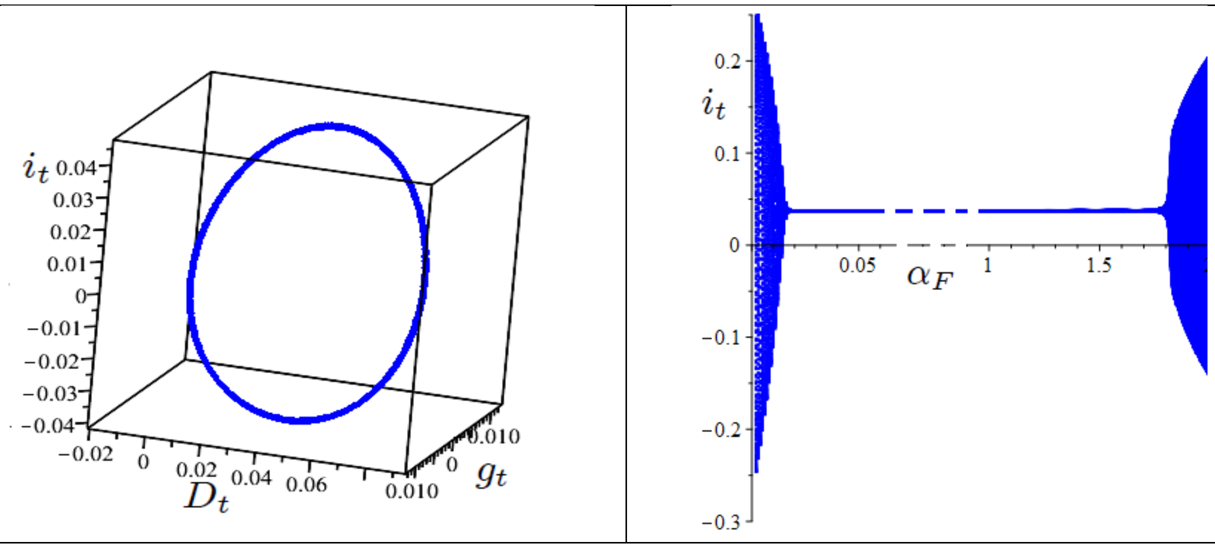

FIGURE 02. Panel (A): The birth of a limit cycle in the plane (g_t, i_t, D_t). Panel (B): Bifurcation diagram concerning α_F.
Parameter values: δ_r = *0.4*, δ_g = *1.8*. The parameter A is set to 0.0115, such that the stationary rate of interest is 0.02875 (2.875 per cent), to better illustrate the double bifurcations that occur for low and high values of the bifurcation parameter. Similar results, from a qualitative point of view, can also be obtained for lower values of A, generating a lower stationary interest rate.

Until now, we have considered the possibility for the interest rate to assume any positive and negative levels. While it is clear that i can show negative values, it seems very unlikely that the level of i can be set too low. In other words, the problem for the monetary authority should be formulated as follows

$$\begin{cases} \min_{i_{t+1} \geq i_{low}} \frac{1}{2}[\pi_{t+1}^2 + \alpha_M y_{t+1}^2] \\ s.t \\ \pi_{t+1} = \gamma_y y_{t+1} \\ y_{t+1} = A - \delta_r i_{t+1} + \delta_g E[g_{t+1}] \end{cases}$$

where i_{low} is the lower bound for the policy rate. In this case, the dynamics of the map T should be replaced with the dynamics generated by the map $T^{'}$, which is identical to the map T, except for the second equation that must be rewritten as

$$i_{t+1} = max\left\{i_{low}, \frac{\delta_g g_t + A}{\delta r}\right\}$$

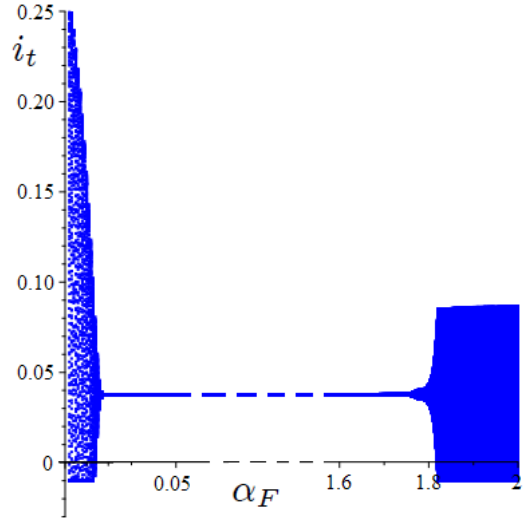

FIGURE 03. Bifurcation diagram concerning α_F.
Parameter values: $\delta_r = 0.4$, $\delta_g = 1.8$, $A = 0.015$.

Notice that as long as the levels assumed by i in the oscillations are above this level, the dynamics described by T also hold in this context. Anyway, for α_F values sufficiently far from the bifurcation values α_F^1 and α_F^2, the dynamics are modified by the presence of the lower bound, as shown in Figure 03.

3.1 ROBUSTNESS CHECKS

Regarding the stability of the system's Nash equilibrium, to understand the generality of the results just illustrated, it is now worth investigating what would happen whether the decisions of the fiscal authority were based on more accurate knowledge of the debt level and the actual interest rate to be paid on that debt. In addition, another element that is interesting to investigate concerns the presence of decision-making processes influenced by past decisions; in particular, the analysis will explore how the introduction of adaptive expectations could affect the stability of the system's Nash equilibrium.

As a first case, we will assume that the fiscal authority issues government bonds at a non-indexed interest rate ($i_t^s = i_t$). In addition, we will assume that there is no longer any information lag relative to the knowledge of the debt level at the time t and that the fiscal authority precisely estimates this level with that one at the end of the current period ($D_t^c = D_t$). Under these hypotheses, the following map (T_1) describes the dynamics of the variables g, i and D.

$$T_1: \begin{cases} g_{t+1} = \dfrac{i_t \alpha_F \delta_g \delta_r - A\alpha_F \delta_g - D_t(1+i_t)}{1+\alpha_F \delta_g^2} \\ i_{t+1} = \dfrac{\delta_g g_t + A}{\delta r} \\ D_{t+1} = (1+i_t)D_t + \dfrac{i_t \alpha_F \delta_g \delta_r - A\alpha_F \delta_g - D_t(1+i_t)}{1+\alpha_F \delta_g^2} \end{cases}$$

Figure 04 shows the stability region of the map T_1 in the parameter space (α_F, A). Despite the hypothesis of knowledge as accurate as possible concerning the values of the variable D_t^c, the system's stability is, anyway, not assured.[4] Nevertheless, notice that the Nash equilibrium of the system exhibits a larger area of stability in the parameter space (α_F, A); specifically, a noteworthy difference is that, given A, if the system is stable for a particular value of α_F, then it is stable for every lower value of α_F. From the point of view of economic meaning, this result implies that the system becomes unstable only if α_F is excessively high; namely, the fiscal authority attaches relatively too much weight to the impact of the monetary policy on the aggregate demand rather than to its effects on the debt burden. The same result also persists if the fiscal authority issues government bonds at an indexed interest rate $(i_t^s = i_{t+1})$.

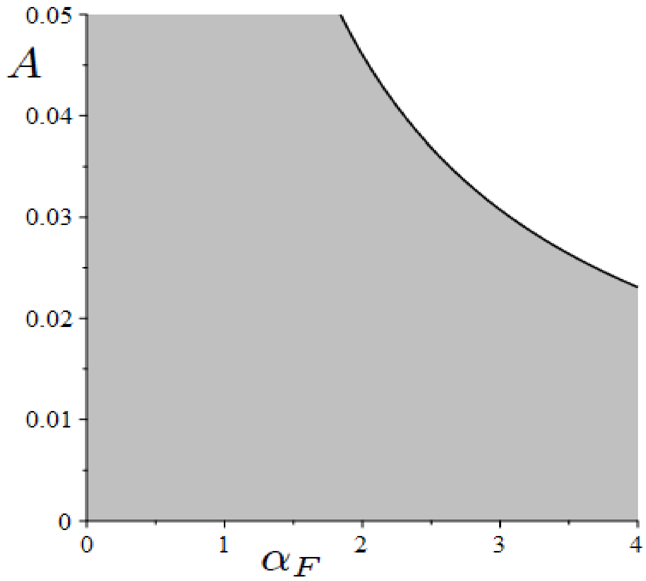

FIGURE 04. Stability region of the fixed point p^* in grey.
Parameter values: $\delta_r = 0.4$, $\delta_g = 1.8$.

As a second case, let us now consider the hypothesis in which there is again an information lag relative to the knowledge of the debt level at the time t and that the fiscal authority estimates this level with that one of the previous period $(D_t^c = D_{t-1})$. In this case, the budget constraint for the fiscal authority reads as

Equation [16] $$D_{t+1} = D_{t-1}(1 + i_t) + g_{t+1}$$

and due to the lag 2 in the Equation [16], the system becomes four-dimensional.

Introducing the new variable $z_{t+1} = D_t$, if the fiscal authority issues government bonds at a non-indexed interest rate $(i_{t+1}^s = i_t)$, then the dynamics of the system can be expressed as

$$T_2: \begin{cases} g_{t+1} = \dfrac{i_t \alpha_f \delta_g \delta_r - A \alpha_f \delta_g - z_t(1 + i_t)}{1 + \alpha_f \delta_g^2} \\ i_{t+1} = \dfrac{\delta g_t + A}{\delta r} \\ D_{t+1} = (1 + i_t) D_t + \dfrac{i_t \alpha_f \delta_g \delta_r - A \alpha_f \delta_g - z_t(1 + i_t)}{1 + \alpha_f \delta_g^2} \\ z_{t+1} = D_t \end{cases}$$

The characteristic polynomial of the Jacobian matrix evaluated in the steady state is

$$p(x) := x \left[x^3 - (A + \delta_r) \frac{x^2}{\delta_r} + \frac{A + \delta_r - \alpha_f \delta_g^2 \delta_r}{(1 + \alpha_f \delta_g^2) \delta_r} x + \frac{\delta_g^2 \alpha_f (A + \delta_r)}{(1 + \alpha_f \delta_g^2) \delta_r} \right]$$

[4] For reasons of space, we do not present the details of the analytical analysis of the model. These details, however, are available upon request to the authors.

Notice that one eigenvalue is 0, while the part inside the square brackets coincides with the characteristic polynomial associated to the Jacobian matrix of the map T evaluated at x^*. Indeed, considering the stability properties of the stationary point, the same result stated in Proposition 1 applies. Summing up, regardless of the specific assumptions, the analysis developed so far shows that in the case in which authorities behave as dictated by the optimal response rule, with static expectations on the choice of the competing authority, the equilibrium can be unstable. This means that even if the Nash equilibrium exists, the authorities do not co-ordinate on it. In addition, excluding the case in which the debt is known, given a combination of A and α_f for which the equilibrium is stable we have that there exists a sufficiently low level of α that destabilises the system.

As a third case, we will again consider the benchmark case but relax the hypothesis that authorities can immediately change the level of policy instruments at the desired level; expressly, we will assume an adaptive process whereby the monetary and fiscal authorities only modify their previous choices in the direction of the new optimal level of the policy tools rather than instantaneously jumping to it at each date. The new system reads as follows.

$$T_3: \begin{cases} g_{t+1} = \dfrac{i_t \alpha_f \delta_g \delta_r - A\alpha_f \delta_g - (D_t - g_t)}{1+\alpha_f \delta_g^2}\lambda + (1-\lambda) g_t \\ i_{t+1} = \dfrac{\delta g_t + A}{\delta r}\lambda + (1-\lambda) i_t \\ D_{t+1} = (1+i_t)D_t + \dfrac{i_t \alpha_f \delta_g \delta_r - A\alpha_f \delta_g - (D_t - g_t)}{1+\alpha_f \delta_g^2}\lambda + (1-\lambda) g_t \end{cases}$$

where $\lambda \in [0,1]$ defines the weight of the authorities' past decisions in affecting their current choices; if $\lambda = 1$, we obtain the benchmark case, and past decisions do not play any role. Introducing adaptive expectations, the expressions that describe the stability of the fixed point x^* become quite cumbersome, such that we avoid reporting them for reasons of space.[5] The following figure illustrates the role of λ for the stability of x^*.

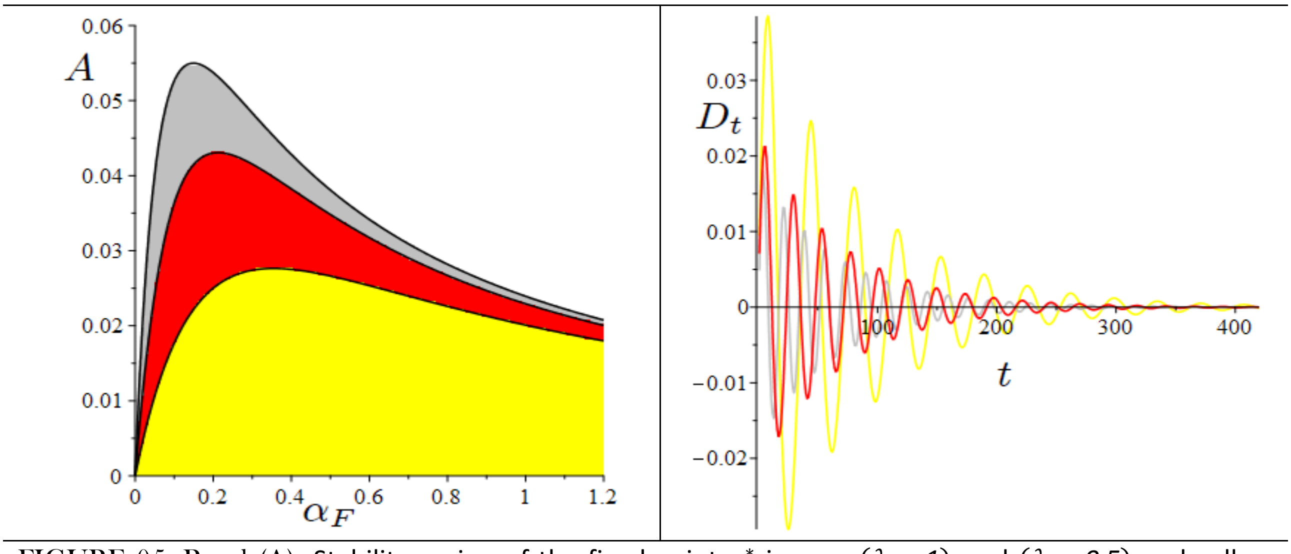

FIGURE 05. Panel (A): Stability region of the fixed point p^* in grey ($\lambda = 1$), red ($\lambda = 0.5$) and yellow ($\lambda = 0.2$). Panel (B): Debt stabilisation process in grey ($\lambda = 1$), red ($\lambda = 0.5$) and yellow ($\lambda = 0.2$). Parameter values in Panel (A): $\delta_r = 0.4$, $\delta_g = 1.8$, $\alpha_F = 1$. Parameter values in Panel (B): $A = 0.0115$, $\delta_r = 0.4$, $\delta_g = 1.8$, $\alpha_F = 1$, initial conditions $g_0 = 0.01$, $i_0 = 0.0284625$ and $D_0 = 0.001$.

[5] These details, however, are available upon request to the authors.

In this context, the role played by adaptive expectations seems quite surprising. Usually, the value of λ is inversely related to the stability of the stationary point; nevertheless, Figure 05 shows that a decrease in λ determines a reduction in the size of the stability region in the parameter space (A, α_F), Panel A, and an increase in the length of the debt stabilisation process following an initial perturbation of the system, Panel B. Basically, due to the presence of the stock variable D_t, if the weight $1 - \lambda$ is positive, then past decisions can affect current decisions through an additional channel represented by the process of debt accumulation, making either the convergence to equilibrium longer or the system unstable.

4.0 CONCLUSIONS

The evolution of the sovereign debt crisis highlights how the interaction between monetary and fiscal policies is crucial when an economy goes through a debt stabilisation process. The literature has highlighted that if member countries of a monetary union are perfectly homogenous, particularly if they have the same level of accumulated debt and the same target for the debt level, then each fiscal authority can reach its debt target.

The current paper has extended the analysis to a dynamic setting, finding that the steady state equilibrium of the static model, that is, the equilibrium non affected by external shocks, can become unstable; specifically, if the fiscal authority's preferences are too unbalanced, namely, it is overly concerned with either debt stability or output stability, the dynamics do not converge to the stationary state of the model, but they are captured by a limit cycle, showing persistent oscillations. Most interestingly, this result does not depend on the monetary authority's preferences as we assumed that there are no supply shocks and initial conditions only depart from the steady state values for the level of accumulated debt; therefore, the trade-off between debt stability and output stability faced by the fiscal authority is the only crucial element in determining the dynamics of the macroeconomic variables and policy instruments. The paper should be understood as a first attempt to use tools to analyse nonlinear dynamic systems in the field of interactions between economic policy authorities. In fact, the paper can be extended in several directions, concerning the number of authorities involved, the heterogeneity of their objectives and the possibility of collusion or coordination among them; in addition, the analysis should investigate the role played by the heterogeneity among economies belonging to a monetary union and the implications of different expectation formation methods.

Acknowledgements

The authors gratefully acknowledge the anonymous reviewers and participants at the MDEF 2022 conference held in Urbino for their valuable comments on an earlier draft of the manuscript, which allowed a substantial improvement in the quality of the article. Mauro Sodini acknowledges the support of the Czech Science Foundation (GACR) under project 20-16701S and VŠB-TU Ostrava under the SGS project SP2022/4.

REFERENCES

Blanchard, Olivier. 2017. *Macroeconomics.* Edinburgh: Pearson Education Limited.

Bofinger, Peter, and Eric Mayer. 2007. "Monetary and fiscal policy interaction in the Euro area with different assumptions on the Phillips curve." *Open Economies Review* (Springer) 18 (3): 291-305.

Bofinger, Peter, Eric Mayer, and Timo Wollmershäuser. 2006. "The BMW model: a new framework for teaching monetary economics." *The Journal of Economic Education* (Routledge) 37 (1): 98-117.

Brooks, Bernard P. 2004. "Linear stability conditions for a first-order three-dimensional discrete dynamic." *Applied Mathematics Letters* (Elsevier) 17 (4): 463-466.

Chortareas, Georgios, and Christos Mavrodimitrakis. 2021. "Policy conflict, coordination, and leadership in a monetary union under imperfect instrument substitutability." *Journal of Economic Behavior & Organization* (Elsevier) 183: 342-361.

De Grauwe, Paul, and Yuemei Ji. 2012. "Mispricing of sovereign risk and macroeconomic stability in the Eurozone." *JCMS: Journal of Common Market Studies* (Wiley) 50 (6): 866-880.

Foresti, Pasquale. 2015. "Monetary and debt-concerned fiscal policies interaction in monetary unions." *International Economics and Economic Policy* (Springer) 12 (4): 541-552.

Foresti, Pasquale. 2018. "Monetary and fiscal policies interaction in monetary unions." *Journal of Economic Surveys* (Wiley) 32 (1): 226-248.

Gatti, Donatella, and Christa van Wijnbergen. 2002. "Co-ordinating fiscal authorities in the euro-zone: a key role for the ECB." *Oxford Economic Papers* (Oxford Academic) 54 (1): 56-71.

Hughes Hallett, Andrew. 2008. "Debt targets and fiscal sustainability in an era of monetary independence." *International Economics and Economic Policy* (Springer) 5 (1): 165-187.

Mavrodimitrakis, Christos. 2020. "Debt stabilization and financial stability in a monetary union: Market versus authority-based preventive solutions." *International Journal of Finance & Economics* 1-18.

Puu, Tönu. 1991. "Chaos in duopoly pricing." *Chaos, Solitons & Fractals* (Elsevier) 1 (6): 573-581.

Woodford, Michael. 2003. *Interest and prices. Foundations of a theory of monetary policy.* Princeton: Princeton University Press.