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An Uncertainty-Based Explanation of Symmetric Growth in Schumpeterian Growth Models*

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Abstract

We provide a re-foundation of the symmetric growth equilibrium characterizing the research sector of all vertical R&D-driven growth models. This result does not rely on the usual assumption of a symmetric expectation on the future per-sector R&D expenditure. Indeed, with this structure of expectations, returns in R&D are equalized, and agents turn out to be indifferent as to where targeting research: hence, the problem of the allocation of R&D investments across sectors is indeterminate. In line with the 'true' Schumpeterian perspective, we solve this indeterminacy by allowing for decision makers strictly uncertain about the future per-sector distribution of R&D efforts. By using the Gilboa-Schmeidler's MEU decision rule, we prove that the symmetric structure of R&D investment is the unique rational expectations (RE) equilibrium compatible with uncertainty-averse agents adopting a maximin strategy.

Keywords: R&D-Driven Growth Models, Multi-Prior Beliefs, Maxmin Strategy, Symmetric Equilibrium.

JEL Classification: 032, 041, D81.

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1 Introduction

Most vertical R&D-driven growth models (such as Grossman-Helpman (1991), Segerstrom (1998), Aghion-Howitt (1998, Ch.3)) focus on the *symmetric equilibrium* in the research sector, that is, on that path characterized by an equal size of R&D investments in each industry. In these models the engine of growth is technological progress, which stems from R&D investment decisions taken by profit-maximizing agents. By means of research, each product line can be improved an infinite number of times, and the firms manufacturing the most updated version of a product monopolize the relative market and thus earn positive profits. However, these profits have a temporary nature since any monopolistic producer is doomed to be displaced by successive improvements in her product line. The level of expected profits together with their expected duration, as compared with the cost of research, determines the profitability of undertaking R&D in each line.

The *plausibility* of the symmetric equilibrium requires that each R&D industry be equally profitable, so that the agents happen to be indifferent as to where targeting their investments. The profit-equality requirement implies two different conditions. First, the profit flows deriving from any innovation need to be the same for each industry: this is guaranteed by assuming that all the monopolistic industries share the same cost and demand conditions. Second, the monopolistic position acquired by innovating needs to be expected to last equally long across sectors: this requires that the agents *expect* the future amount of research to be equally distributed among the different sectors. As is well known to the reader familiar with the neo-Schumpeterian models of growth, future is allowed to affect current (investment) decisions via the forward-looking nature of the Schumpeterian 'creative destruction' effect.

Grossman and Helpman (1991, p.47) recognize the centrality of the assumption of symmetric expected R&D investments in order to justify the selection of the symmetric equilibrium: with the assumption that "the profit flows are the same for all industries [..] an entrepreneur will be indifferent as to the industry in which she devotes her R&D efforts provided that she expects her prospective leadership position to last equally long in each one. We focus hereafter on the symmetric equilibrium in which all products are targeted to the same aggregate extent. In such an equilibrium the individual entrepreneur indeed expects profit flows of equal duration in every industry and so is indifferent as to the choice of industry". Hence, in this framework it is crucial to assume that an equal amount of future R&D efforts in each industry is expected.

Expecting equal future profitability across sectors, however, does not constitute a sufficient condition for each agent to choose a symmetric allocation of R&D efforts: in fact, equal future profitability makes the investor indifferent as to where targeting research. As a result, when symmetric expectations are assumed the allocation problem of investments across product lines is indeterminate. Notice also that the way this allocation problem is solved is not always without consequence for this class of models, as recently pointed out by Cozzi (2003). For instance in a Segerstrom's (1998) framework, because of the 'increasing complexity hypothesis', the alternative prevalence of the symmetric or asymmetric equilibrium has powerful effects on the growth rate of the economy: if indifferent agents, for a whatever reason (a 'sunspot'), are induced to allocate their investment only in a small fraction of sectors, the dynamic decreasing returns to R&D investments will imply a lower aggregate growth rate, as compared to the one associated with a symmetric distribution of R&D efforts across all sectors. An equally relevant effect of sunspot-driven asymmetric R&D investments on steady-state growth rates reappears in the Howitt's (1999) extension to an ever expanding set of product lines (see Cozzi (2004)). Hence both solutions to the 'strong scale effect' problem (Jones (2004)) exhibit dependence of growth rates on the intersectoral distribution of R&D.

In this paper we provide an alternative route to make the focus on the symmetric equilibrium compelling. Our basic idea is that the agent's beliefs on the future (per sector) distribution of R&D investments are characterized by uncertainty (or ambiguity), in the sense that information about that distribution is too imprecise to be represented by a (single additive) probability measure. The traditional distinction between 'risk' and 'uncertainty' traces back to Frank Knight (1921), and states that risk is associated with ventures in which an objective probability distribution of all possible events is known, while uncertainty characterizes choice settings in which that probability distribution is not available to the decision-maker. As is well known, the axiomatization of the subjective expected utility (SEU) model, provided among the others by Savage (1954), strongly contributed to undermine any meaningful distinction between risk and uncertainty. In recent years a number of attempts have been made to extend the SEU model in order to substantiate that distinction[1]. Here we will follow the maxmin expected utility (MMEU) theory axiomatized by Gilboa and Schmeidler (1989)[2]. In

[1] For instance Bewley (1986) has developed his theory of the 'status quo', by dropping the axiom of complete preferences inside the Anscombe-Aumann's (1963) version of the SEU model.

[2] Gilboa and Schmeidler (1989) provide an axiomatic foundation of the maxmin expected utility

representing subjective beliefs, it suggests to replace the standard single (additive) prior with a closed and convex set of (additive) priors. The choice among alternative acts is determined by a maximin strategy. For each act the agent first computes the expected utilities with respect to each single prior in the set and picks up the minimal value. Finally she compares all these values and singles out the act associated with the highest (minimal) expected utility. According to this model, the agent is said to be *uncertainty averse* if the given set of priors is not a singleton[3]. Hence, in our framework the decision maker will be assumed to maximize her expected pay-off with respect to the R&D investment decision, while singling out the worst choice scenario, that is, the minimizing probability distribution over the future configuration of R&D investments. Unlike in Epstein and Wang (1994), in this paper the maxmin decision rule eliminates indeterminacy and makes the symmetric - and growth maximizing - allocation of R&D investment emerge as the unique equilibrium.

Importantly, our assumption on the agents' ignorance does not regard any fundamental of the economy and is to be interpreted as a way of treating sector-specific 'extrinsic uncertainty'. Moreover, since uncertainty does not affect aggregate variables, in order to develop our argument we do not need to introduce either the optimal consumption problem solved by households, or the profit-maximizing problem solved by firms (for which the reader is referred to Segerstrom (1998)). Since the problem is the distribution of a given amount of R&D efforts across product lines, all we need is the description of the R&D sector.

Our result holds for a however small probability that a however small fraction of individual's portfolio be affected by strong uncertainty. Hence, a microscopic departure from the standard treatment of extrinsic uncertainty rules out the possibility of asymmetric equilibria and the potential macroscopic growth consequences associated with them.

The rest of the paper is organized as follows. In Section 2 we briefly describe the basic structure of the R&D sector, with particular reference to the Segerstrom's (1998)

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theory in the framework of Anscombe and Aumann (1963). Several applications of this theory have been elaborated over the last few years. We recall, among the others, Epstein and Wang (1994), and the book by Hansen and Sargent (2003). Notice that, still in the Anscombe-Aumann's (1963) framework, a 'taste for uncertainty' can alternatively be modeled via the Choquet expected utility (CEU) theory axiomatized by Schmeidler (1989). In it, expected utility is computed according to a capacity (that is, a not necessarily additive probability) via the Choquet integral. MMEU and CEU can bring the same results when the capacity is convex.

[3] Notice however that this definition of uncertainty aversion has been questioned, and some alternative definitions have been proposed (see for example Epstein (1999)).

formalization. In Section 3 we explain the core of our argument, enunciate and prove the proposition. In Section 4 we conclude with some remarks.

2 R&D Sector

In this Section we provide a description of the vertical innovation sector, which is basically common to most neo-Schumpeterian growth models. This sector is characterized by the efforts of R&D firms aimed at developing better versions of the existing products in order to displace the current monopolists[4]. We assume a continuum of industries indexed by ω over the interval $[0,1]$. There is free entry and perfect competition in each R&D race. Firms employ labor and produce, through a constant returns technology, a Poisson arrival rate of innovation in the product line they target. Adopting Segerstrom's (1998) notation, any firm j hiring l_j units of labor in industry ω at time t acquires the instantaneous probability of innovating $Al_j/X(\omega,t)$, where $X(\omega,t)$ is the industry-specific R&D difficulty index.

Since independent Poisson processes are additive, the specification of the innovation process implies that the industry-wide instantaneous probability of innovation is $AL_I(\omega,t)/X(\omega,t) \equiv I(\omega,t)$, where $L_I(\omega,t)=\sum_j l_j(\omega,t)$. The function $X(\omega,t)$ describes the evolution of technology; as in Segerstrom (1998), we assume it to evolve in accordance with:

$$\frac{\dot{X}(\omega,t)}{X(\omega,t)} = \mu I(\omega,t),$$

where μ is a positive constant. Then, by substituting for $I(\omega,t)$ into the expression above and solving the differential equation for $X(\omega,t)$ we get:

$$X(\omega,t) = X(\omega,t_0) + \mu A \int_{t_0}^{t} L_I(\omega,z)dz$$

Whenever a firm succeeds in innovating, it acquires the uncertain profit flow that accrues to a monopolist, that is, the stock market valuation of the firm: let us denote it with $v(\omega,t)$. Thus, the problem faced by an R&D firm is that of choosing the amount of labor input in order to maximize its expected profits[5]:

$$\max_{l_j}[v(\omega,t)Al_j/X(\omega,t) - l_j]$$

[4] It seems irrelevant to our purpose to distinguish whether the monopolistic sector is that of the final goods - as in Segerstrom (1998) - or that of the intermediate ones - as in Aghion and Howitt (1998, Ch.3) and Howitt (1999).

[5] We consider labor as the numerarie.

which provides a finite, positive solution for l_j only when the arbitrage equation $v(\omega,t)A/X(\omega,t)=1$ is satisfied. Notice that in this case, though finite, the size of the firm is indeterminate because of the constant return research technology.

The firm's market valuation at a given instant t, $v(\omega,t)$, is the expected discounted value of its profit flows from t to $+\infty$:

$$v(\omega,t)=\int_t^{+\infty}\pi(s)\exp\left[-\int_t^s[r(\tau)+I(\omega,\tau)]\,d\tau\right]ds,$$

where

$$I(\omega,\tau)=\frac{AL_I(\omega,\tau)}{X(\omega,\tau)}=\frac{AL_I(\omega,\tau)}{X(\omega,t_0)+\mu A\int_{t_0}^{\tau}L_I(\omega,z)dz}.$$

By plugging $I(\omega,\tau)$ into $v(\omega,t)$, we finally obtain the following expression for $v(\omega,t)$:

$$v(\omega,t)=\int_t^{+\infty}\pi(s)\exp\left\{-\int_t^s\left[r(\tau)+\frac{AL_I(\omega,\tau)}{X(\omega,t_0)+\mu A\int_{t_0}^{\tau}L_I(\omega,z)dz}\right]d\tau\right\}ds \tag{1}$$

The usual focus on the symmetric growth equilibrium is based on the assumption that the R&D intensity $I(\omega,\tau)$ is the same in all industries ω and strictly positive. The suggestion of a new rationale for this symmetric behavior is the topic of the next Section.

3 The Re-Foundation of the Symmetric Equilibrium

We assume that the agent has a fuzzy perception of the future configuration of R&D efforts, and formalize this 'ambiguity' via the MMEU approach: this agent is then provided with a set of prior beliefs over this configuration, and evaluates her expected pay-off with respect to the minimizing prior inside this set.

Before proceeding with the analysis, let us clarify two important aspects of the model's structure. In the previous Section we have referred to the R&D firm as the one choosing the size and the distribution among sectors of R&D investment. However, R&D firms are financed by consumers' savings, which are channeled to them through the financial market. Thus, since the consumer is allowed to choose the R&D sectors where to employ her savings, she ends up with being our fundamental unit of analysis. The role of the R&D firms merely becomes that of transforming these savings into research activity.

Notice also that in the basic set-up by which our paper is inspired (Grossman and Helpman (1991) and Segerstrom (1998)), the agent is assumed to be risk-averse. Yet she is able to completely diversify her portfolio - by means of the intermediation of costless financial institutions - and, hence, to care only about deterministic mean returns. This assumption is retained in our set-up - which allows for a whatever asymmetric configuration of investments - since, in order to carry out this diversification, it is sufficient to allocate investments in a non-zero measure interval of R&D sectors (and not necessarily in the whole of them), according to a measure that is absolutely continuous with respect to the Lebesgue measure of the sector space. The crucial difference with respect to the standard framework is then concerned with the assumption of multiple prior beliefs in the face of uncertainty, where uncertainty only affects the mean return of the R&D investment and not its volatility, against which the agent has already completely hedged.

In order to make the focus on the symmetric equilibrium compelling we start by assuming that a symmetric future configuration of R&D investment is expected to occur with probability $1-p$, while p stands for the aggregate probability of all possible configurations; the interval $[0, p]$ represents the unrestricted set of priors assigned to each of them. Following the MMEU approach, if there exists a configuration, among all possible ones, which minimizes the expected returns in R&D investment, then the minimization of the agent's pay-off with respect to the unrestricted set of priors implies the assignment of probability p to the minimizing configuration and of probability 0 to all the others. Since the minimizing configuration is a function of the agent's investment choice, this choice can then be formalized as the result of a 'two-player zero-sum game' characterized by:

- the *minimizing* behavior of a 'malevolent Nature', which selects the worst possible configuration of future R&D efforts and
- the *maximizing* behavior of the agent, whose optimal choice must take into account the worst-case strategy implemented by Nature.

We start our analysis at the beginning of time $t = t_0$, and assume that, at this time, all industries share the same difficulty index $X(\omega, t_0) = X(t_0)$ $\forall \omega \in [0, 1]$ in order to focus on the role of expectations on the kind of equilibrium that will prevail. Our problem can then be stated as follows. At time $t = t_0$, the agent is asked to allocate a given amount of R&D investment among all the existing industries: in maximizing her expected pay-off she will take into account the minimizing strategy that a 'malevolent Nature' will be carrying out in choosing the composition of future R&D efforts. We denote with $l_m(\omega, t_0) \equiv l_m(t_0)[1+\alpha(\omega)]$ the agent's investment in sector ω at time t_0, and with $L_I(\omega, t) \equiv L_I(t)[1+\varepsilon(\omega)]$ the agent's expectations about the aggregate research in sector ω at a generic point in time t. $l_m(t_0)$ and $L_I(t)$ are, respectively, the agent's average investment per sector at t_0 and the expected average research per sector at a generic t. $\varepsilon(\cdot)$ and $\alpha(\cdot)$ represent relative deviations from these averages satisfying:

$$\int_0^1 \varepsilon(\omega)d\omega = 0 \qquad \int_0^1 \alpha(\omega)d\omega = 0 \qquad \text{and}$$

$$\varepsilon(\omega) \geq -1 \qquad \alpha(\omega) \geq -1.$$

The presence of the two functions $\alpha(\cdot)$ and $\varepsilon(\cdot)$ is intended to allow for asymmetry both in the agent's investment and in expected research[6]. Note that $\alpha(\cdot)$ and $\varepsilon(\cdot)$ are unbounded above because the zero-measure of each sector allows the investment in any of them to be however big, without violating the constraint on the total R&D investment.

From now on we will drop the argument t_0 in the expression for[7] $l_m(\omega, t_0)$ and enunciate the following:

Proposition 1 *For a however small probability* (p) *of deviation* $(\varepsilon(\omega))$ *from symmetric expectations on future R&D investment, decision makers adopting a maxmin strategy to solve their investment allocation problem, choose a symmetric investment strategy, i.e.* $l_m[1+\alpha(\omega)] = l_m$ $\forall\omega \in [0,1]$. *The associated distribution of expected R&D efforts among sectors is:* $L_I(t)[1+\varepsilon(\omega)] = L_I(t)$ $\forall\omega \in [0,1]$.

Proof. Our problem can be stated as:

$$\max_{\alpha(\cdot)}\left[\int_0^1 (1-p)l_m[1+\alpha(\omega)]\frac{A}{X(t_0)}v(t_0)d\omega + p\min_{\varepsilon(\cdot)}\left[\int_0^1 l_m[1+\alpha(\omega)]\frac{A}{X(t_0)}v(\omega, t_0)d\omega\right]\right]$$

[6] These definitions imply:

$$\int_0^1 L_I(t)[1+\varepsilon(\omega)]d\omega = L_I(t) = L(t)\int_0^1 l_m(t)[1+\alpha(\omega)]d\omega = L(t)l_m(t)$$

where $L(t)$ denotes the mass of agents in the economy at time t. With reference to Section 2 the following relation between l_j and l_m holds:

$$\int_0^1 \sum_j l_j(\omega, t)\, d\omega = L(t)l_m(t).$$

[7] As we show below, this does not imply any loss of generality.

s.t. $\int_0^1 \varepsilon(\omega)d\omega = 0; \qquad \int_0^1 \alpha(\omega)d\omega = 0;$

$\alpha(\omega) \in [-1, \infty); \qquad \varepsilon(\omega) \in [-1, \infty);$

with:

$$v(t_0) = \int_{t_0}^{+\infty} \pi(s) \exp\left\{ -\int_{t_0}^{s} \left[r(\tau) + \frac{AL_I(\tau)}{X(t_0) + \mu A \int_{t_0}^{\tau} L_I(z)dz} \right] d\tau \right\} ds$$

$$v(\omega, t_0) = \int_{t_0}^{+\infty} \pi(s) \exp\left\{ -\int_{t_0}^{s} \left[r(\tau) + \frac{AL_I(\tau)[1+\varepsilon(\omega)]}{X(t_0) + \mu A \int_{t_0}^{\tau} L_I(z)[1+\varepsilon(\omega)]dz} \right] d\tau \right\} ds$$

where we have substituted for $L_I(\omega, t) \equiv L_I(t)[1+\varepsilon(\omega)]$ into (1).

By plugging the expressions for $v(t_0)$ and $v(\omega, t_0)$ into the maxmin problem, and by using the condition $\int_0^1 \alpha(\omega)d\omega = 0$, that problem can be restated as:

$$\max_{\alpha(\cdot)} \left\{ (1-p)l_m \frac{A}{X(t_0)} \int_{t_0}^{+\infty} \pi(s) \exp\left[-\int_{t_0}^{s} \left(r(\tau) + \frac{AL_I(\tau)}{X(t_0)+\mu A \int_{t_0}^{\tau} L_I(z)dz} \right) d\tau \right] ds + \right.$$

$$\left. + p \min_{\varepsilon(\cdot)} \int_0^1 l_m[1+\alpha(\omega)] \frac{A}{X(t_0)} \left\{ \int_{t_0}^{+\infty} \pi(s) \exp\left[-\int_{t_0}^{s} \left(r(\tau) + \frac{AL_I(\tau)[1+\varepsilon(\omega)]}{X(t_0)+\mu A \int_{t_0}^{\tau} L_I(z)[1+\varepsilon(\omega)]dz} \right) d\tau \right] ds \right\} d\omega \right\}$$

Notice that the first addend of the maximand is constant with respect to $\alpha(\cdot)$ and $\varepsilon(\cdot)$. Then our problem can ultimately be stated as:

$$p\frac{A}{X(t_0)} \max_{\alpha(\cdot)} \left\{ \min_{\varepsilon(\cdot)} \int_0^1 l_m[1+\alpha(\omega)] \left\{ \int_{t_0}^{+\infty} \pi(s) \exp\left[-\int_{t_0}^{s} \left(r(\tau) + \frac{AL_I(\tau)[1+\varepsilon(\omega)]}{X(t_0)+\mu A \int_{t_0}^{\tau} L_I(z)[1+\varepsilon(\omega)]dz} \right) d\tau \right] ds \right\} d\omega \right\}$$

s.t. $\int_0^1 \varepsilon(\omega)d\omega = 0 \ ; \qquad \int_0^1 \alpha(\omega)d\omega = 0;$

$\alpha(\omega) \in [-1, +\infty); \qquad \varepsilon(\omega) \in [-1, \infty).$

Notice that this problem admits the same solution for a however small probability p.

In order to prove that the unique equilibrium is provided by $\alpha(\omega) = \varepsilon(\omega) = 0$ $\forall \omega \in [0, 1]$, we will proceed through the following steps (the reader can refer to Figure 1, where c_1,c_2,c_3,c_4 represent the agent's pay-offs).

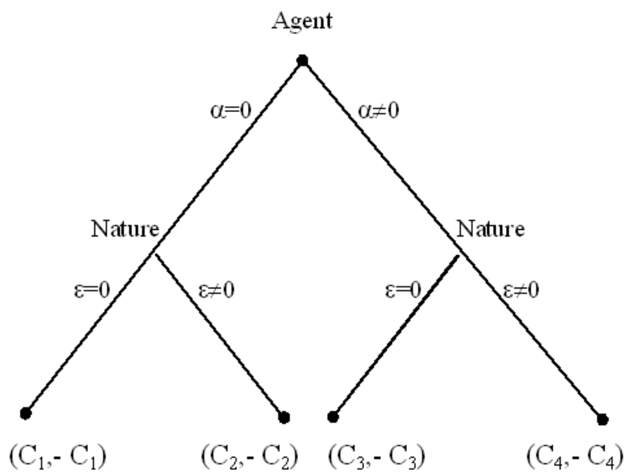

Figure 1: The Game between the Agent and Nature

1. We will first prove that, if the agent plays a symmetric strategy, $\alpha(\omega) = 0$ $\forall\omega \in [0,1]$, then the worst harm Nature can inflict to the agent is also associated with a symmetric strategy: $\varepsilon(\omega) = 0$ $\forall\omega \in [0,1]$ (that is, with reference to Figure 1: $c_1 < c_2$)

2. We will then prove that, if Nature chooses $\varepsilon(\omega) = 0$ $\forall\omega \in [0,1]$, the pay-off the agent will obtain is independent of her investment strategy (that is, $c_1 = c_3$).

3. Then the problem is to exclude that, if $\alpha(\omega) \neq 0$ in a non-zero measure set, the configuration $\varepsilon(\omega) = 0$, $\forall\omega \in [0,1]$ represents a minimizing strategy for Nature (which would leave the problem indeterminate). We will be able to exclude all asymmetric configurations of the agent's investment by proving that, for all of them, Nature can cause a worse damage (with respect to the one associated with $\varepsilon(\omega) = 0$ $\forall\omega \in [0,1]$) to the agent by playing an asymmetric strategy (that is, we will prove $c_4 < c_3$). Then the configuration given by $\alpha(\omega) = 0$ and $\varepsilon(\omega) = 0$ $\forall\omega \in [0,1]$ will emerge as the unique equilbrium (since $c_1 = c_3 > c_4$). Let us proceed step by step.

1. ($c_1 < c_2$). If $\alpha(\omega) = 0$ $\forall\omega \in [0,1]$, we first show that the function:

$$\Phi \equiv \int_0^1 l_m \left\{ \int_{t_0}^{+\infty} \pi(s) \exp\left[-\int_{t_0}^{s} \left(r(\tau) + \frac{AL_I(\tau)[1+\varepsilon(\omega)]}{X(t_0)+\mu A \int_{t_0}^{\tau} L_I(z)[1+\varepsilon(\omega)]dz} \right) d\tau \right] ds \right\} d\omega$$

is a sum over ω of strictly convex functions in $\varepsilon(\omega)$. In fact, set:

$$f(\varepsilon,\tau)\equiv -\left(r(\tau)+\frac{AL_I(\tau)[1+\varepsilon(\omega)]}{X(t_0)+\mu A\int\limits_{t_0}^{\tau}L_I(z)[1+\varepsilon(\omega)]dz}\right)$$

Since $\dfrac{\partial^2 f(\varepsilon,\tau)}{\partial\varepsilon^2}>0$, then $f(\varepsilon,\tau)$ is strictly convex[8]. As a result, the function $F(\varepsilon,s)\equiv\int\limits_{t_0}^{s}f(\varepsilon,\tau)d\tau$, as a sum of strictly convex functions, is also strictly convex, that is: $\dfrac{\partial^2 F(\varepsilon,s)}{\partial\varepsilon^2}>0$. Now, for each $s\in[t_0,+\infty]$, we can define:

$$H(\varepsilon,s)\equiv l_m\pi(s)\exp\left[F(\varepsilon,s)\right].$$

If we compute the second derivative of this function we obtain:

$$\frac{\partial^2 H(\varepsilon,s)}{\partial\varepsilon^2}=l_m\pi(s)\left\{\exp\left[F(\varepsilon,s)\right]\left[\frac{\partial F(\varepsilon,s)}{\partial\varepsilon}\right]^2+\exp\left[F(\varepsilon,s)\right]\frac{\partial^2 F(\varepsilon,s)}{\partial\varepsilon^2}\right\},$$

which is always strictly positive since, as we have shown above, $\dfrac{\partial^2 F(\varepsilon,s)}{\partial\varepsilon^2}>0$. Then $H(\varepsilon,s)$ is also strictly convex and so it is the sum of all $H(\varepsilon,s)$ over $t\in[t_0,+\infty)$.

Finally, given that Φ is a sum over $\omega\in[0,1]$ of strictly convex functions then, by Jensen inequality, the minimum is only reached when $\varepsilon(\omega)=0$ $\forall\omega\in[0,1]$.

The following pay-off, obtained by setting $\varepsilon(\omega)=\alpha(\omega)=0$ $\forall\omega\in[0,1]$ in Φ:

$$\int\limits_0^1 l_m\left\{\int\limits_{t_0}^{+\infty}\pi(s)\exp\left[-\int\limits_{t_0}^{s}\left(r(\tau)+\frac{AL_I(\tau)}{X(t_0)+\mu A\int\limits_{t_0}^{\tau}L_I(z)dz}\right)d\tau\right]ds\right\}d\omega$$

is then the one that the agent can surely obtain if she plays a symmetric strategy.

2. ($c_1=c_3$). If $\varepsilon(\omega)=0$, $\forall\omega\in[0,1]$, then the agent would be totally indifferent in the allocation of her R&D efforts. In fact, the maximum problem obtained by setting $\varepsilon(\omega)=0$ $\forall\omega\in[0,1]$ is:

$$\max_{\alpha(\cdot)}\int\limits_0^1 l_m[1+\alpha(\omega)]\int\limits_{t_0}^{+\infty}\pi(s)\exp\left[-\int\limits_{t_0}^{s}\left(r(\tau)+\frac{AL_I(\tau)}{X(t_0)+\mu A\int\limits_{t_0}^{\tau}L_I(z)dz}\right)d\tau\right]dsd\omega,$$

which, since $\int\limits_0^1\alpha(\omega)d\omega=0$, always gives the same constant value:

$$l_m\int\limits_{t_0}^{+\infty}\pi(s)\exp\left[-\int\limits_{t_0}^{s}\left(r(\tau)+\frac{AL_I(\tau)}{X(t_0)+\mu A\int\limits_{t_0}^{\tau}L_I(z)dz}\right)d\tau\right]ds.$$

[8] It is $\dfrac{\partial^2 f(\varepsilon,\tau)}{\partial\varepsilon^2}=\dfrac{2A^2L_I(\tau)\mu X(t_0)\int_{t_0}^{\tau}L_I(z)dz}{\left(A\mu(1+\varepsilon(\omega))\int_{t_0}^{\tau}L_I(z)dz+X(t_0)\right)^3}>0$ since $\varepsilon(\omega)\geq -1$.

3. ($c_4 < c_3$). Assume $\alpha(\omega) \neq 0$ for some non zero measure set of $\omega \in [0,1]$. Then the Nature's minimum problem with respect to $\varepsilon(\cdot)$ can be stated as follows:

$$\min_{\varepsilon(\cdot)} \int_0^1 l_m[1+\alpha(\omega)] \left\{ \int_{t_0}^{+\infty} \pi(s) \exp\left[-\int_{t_0}^{s} \left(r(\tau) + \frac{AL_I(\tau)[1+\varepsilon(\omega)]}{X(t_0)+\mu A \int_{t_0}^{\tau} L_I(z)[1+\varepsilon(\omega)]dz} \right) d\tau \right] ds \right\} d\omega$$

$$\text{s.t.} \quad \int_0^1 \varepsilon(\omega) d\omega = 0$$

The solution to this problem is $\varepsilon[\alpha(\omega)]$, which is the reaction function of Nature, that is, her optimal (minimizing) response to any possible value of $\alpha(\omega)$. We do not need, however, to find it explicitely since our conclusion will follow straightforwardly. We can build the Lagrangian and then derive the first-order conditions (f.o.c.):

$$L = \int_0^1 l_m[1+\alpha(\omega)] \left\{ \int_{t_0}^{+\infty} \pi(s) \exp\left[-\int_{t_0}^{s} \left(r(\tau) + \frac{AL_I(\tau)[1+\varepsilon(\omega)]}{X(t_0)+\mu A \int_{t_0}^{\tau} L_I(z)[1+\varepsilon(\omega)]dz} \right) d\tau \right] ds \right\} d\omega +$$

$$\zeta \int_0^1 \varepsilon(\omega) d\omega$$

For every $\omega \in [0,1]$, the f.o.c. with respect to ε are:

$$l_m[1+\alpha(\omega)] \int_{t_0}^{+\infty} \pi(s) \exp\left[-\int_{t_0}^{s} \left(r(\tau) + \frac{AL_I(\tau)[1+\varepsilon(\omega)]}{X(t_0)+\mu A \int_{t_0}^{\tau} L_I(z)[1+\varepsilon(\omega)]dz} \right) d\tau \right] ds \left[-\int_{t_0}^{s} \frac{AL_I(\tau)X(t_0)}{\left(X(t_0)+\mu A(1+\varepsilon(\omega)) \int_{t_0}^{\tau} L_I(z)dz\right)^2}\right.$$

$$= -\zeta$$

It results that, if $\alpha(\omega) \neq 0$ for some $\omega \in [0,1]$, and if the constraint $\int_0^1 \alpha(\omega) d\omega = 0$ holds, the necessary conditions for a minimum can never be satisfied if $\varepsilon[\alpha(\omega)] = 0$ $\forall \omega \in [0,1]$[9].

To sum up, the agent perfectly knows the pay-off she will gain while playing a symmetric strategy (c_1). She also knows that, for a whatever asymmetric strategy she plays, Nature has a 'punishment power' associated with an asymmetric strategy, which renders her pay-off strictly lower than the one associated with symmetry. The agent will then choose $\alpha(\omega) = 0$ $\forall \omega \in [0,1]$ and, consequently Nature will select $\varepsilon(\omega) = 0$ $\forall \omega \in [0,1]$. ■

We now show that our result also holds true when the punishment power of Nature ($\varepsilon(\omega)$) is restricted to be however small. Accordingly, we impose the constraint $\varepsilon(\omega) \in$

[9] In fact, consider an economy with only two sectors, ω_1,ω_2. If it were $\varepsilon(\omega_1) = \varepsilon(\omega_2) = 0$, the satisfaction of the f.o.c. and the constraint would require $\alpha(\omega_1) = \alpha(\omega_2)$ and $\alpha(\omega_1) + \alpha(\omega_2) = 0$, which proves that there cannot exist ω where $\alpha(\omega) \neq 0$.

$[-\eta, \eta]$ $\forall \eta \in (0, 1)$.

Corollary 2 *For a however small probability* (p) *of deviation* $(\varepsilon(\omega))$*, and for a however small deviation* $(\varepsilon(\omega))$ *from symmetric expectations on future R&D investment, decision makers adopting a maxmin strategy to solve their investment allocation problem, choose a symmetric investment strategy, i.e.* $l_m[1 + \alpha(\omega)] = l_m$ $\forall \omega \in [0, 1]$*. The associated distribution of expected R&D efforts among sectors is:* $L_I(t)[1 + \varepsilon(\omega)] = L_I(t)$ $\forall \omega \in [0, 1]$.

Proof. The same proof as for the proposition holds true under the restriction $\varepsilon(\omega) \in [-\eta, \eta]$, since $\varepsilon(\omega) = 0 \in [-\eta, \eta]$ for a however small η. In fact, since $\varepsilon(\omega) = 0$ is always an inner point of the domain, the non-fulfillment of the f.o.c. guarantees that it is not a minimum. ■

We have shown that, even under $\varepsilon(\cdot)$ and p however small, the symmetric equilibrium emerges as the unique optimal investment allocation. That is to say, even though the agent is 'almost sure' $(p \to 0)$ of facing a symmetric configuration of future investments (which would leave her in a position of indifference in her current allocation problem), the mere possibility of a slightly different configuration $(\varepsilon \to 0)$ makes her strictly prefer to equally allocate her investments across sectors. This occurs because, whenever the agent evaluates an asymmetric allocation of her current investments, she will always be induced to expect the worst configuration of future investments inside the ε-generated set. Furthermore, the fact that the symmetric equilibrium is being derived at the beginning of time $t = t_0$ does not result in any loss of generality. In fact, this equilibrium guarantees that the difficulty index $X(\omega, t)$ starts growing at the same rate - and is therefore always equal - across sectors. This condition in turn assures that, at any point in time t, the agent continuosly faces a decision problem equivalent to the one we have analyzed and, hence, continuosly finds the same optimal (symmetric) solution.

4 Concluding Remarks

In the neo-Schumpeterian growth models the existence of the creative destruction effect implies that expectations on future R&D investments affect the allocation of the current ones. Therefore the usual focus on the symmetric equilibrium in the vertical research sector relies on the assumption of an expected symmetric per-sector distribution of

R&D expenditure. However, in making the agents indifferent as to where targeting their investments, this assumption is not sufficient to pin down univocally the symmetric structure of R&D efforts: actually, symmetric expectations on future R&D leave the current composition of R&D investments indeterminate, with potentially large effects on growth rates.

We have shown that a possible solution to this indeterminacy consists of assuming uncertainty on the future configuration of R&D investments and max-minimizing agents in the face of this uncertainty. Under this assumption, indeterminacy vanishes and the symmetric allocation of the vertical research expenditures emerges as the unique optimal choice.

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