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**ITS Working Paper 425** 

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# THE OPTIMISATION OF INTEGRATED URBAN TRANSPORT STRATEGIES: TESTS BASED ON EDINBURGH

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# 1. INTRODUCTION

Even with relatively simple model packages and modern computers, it is not feasible to test <u>all</u> combinations of model input variables to see which combination gives the 'best' outcome. Neither is it usually possible to solve the models analytically for the optimum. Consequently, there is a potential role for a methodology which takes as input the results from a relatively small number of runs of the model package and then models the response surface in the region containing the optimum, in such a way that it can be analytically solved for the optimum. This paper will set out such a methodology together with a Case Study.

# 2. PURPOSE AND BACKGROUND

In the UK, local authorities bid for government funding for transport, in the form of 'Transport Supplementary Grants'. The UK Department of Transport is making it clear that, where appropriate, it would like to see such bids backed by modelling. Furthermore, in 1993 the UK DoT introduced what it called the 'Package Approach' for urban areas. Briefly, this requires a range of transport initiatives to be considered, not just highway construction, and a combination selected which complement one another in their pursuit of defined objectives. For large urban areas facing current and forecast transport problems, usually resulting from the inability of the current highway network to cater for growing automobile use, possible solutions other than highway construction might involve restraining automobile use (eg by direct prohibitions or charges for road use or parking) and the promotion of alternative modes (eg enhancing public transport frequencies, subsidising public transport fares, constructing light rapid transit systems or improving conditions for pedestrians and cyclists). Local politicians are now keen to investigate a wide range of such policies, with a view to incorporating them in an overall strategy to best tackle local transport needs.

In order rapidly to evaluate a range of strategy variants, a new type of 'strategic model' has been developed (see for example Bates et al., 1991). Compared to earlier 'four stage' models these are much quicker and cheaper to run, allowing considerable interrogation. The models are generally based on hierarchical logit structures with few zones and highly aggregated relationships (eg for speed/flow) and parameters based on relatively limited sampling or even totally imported. While such simplifications facilitate a larger number of model runs, it is still not feasible to run sufficient to be sure of having identified the optimum. The purpose of this paper is to present a method whereby a relatively small number of strategic model runs are used as the basis for the construction of a relatively simple regression model which can be analytically solved for its optimum. The method will be illustrated using the JATES model (Bates et al., 1991) for the Scottish capital, Edinburgh. The method expounded below was developed during a project funded by the UK Engineering and Physical Sciences Research The initial concept was first suggested to us by Mick Roberts of The MVA Council. Consultancy, who conducted the JATES study for Lothian Regional Council. We are grateful for the support of all the above but our conclusions do not necessarily reflect their policies.

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# 3. METHOD DEVELOPMENT

Initially, we developed our methods using a model of a hypothetical city. Full details can be found in Bristow et al (1994), an edited version in Bonsall et al (1994) and a brief description in May et al (1995). Briefly, we found that the regression approach offered no improvement (over previous methods) where the policies consisted solely of discrete options. The regression model required information on which pairs of projects were substitutes, such that they should never be combined in a strategy no matter how good they were individually. After careful consideration, we defined two projects as substitutes if the value of the target indicator for the pair together was less than that for either of the two projects taken separately. The only way we could operationalise this test in a systematic way, without making use of prior knowledge or expert opinion, was to run all pairs of projects as well as all projects singly. As we had set ourself a problem with 10 possible projects which might be implemented in any combination, we immediately required 55 runs (i.e. 10 projects singly and 45 pairs) of the strategic model in order to inform the regression model. Compared to that, it was clear to us that simple trial and error interrogation of the strategic model would locate the optimum combination with about 20 runs of the strategic model. Effectively, only those substitutes relevant to the optimum needed to be discovered. Even our attempts at trying to introduce prior or expert knowledge into our method did not suggest that a regression model could improve on the trial and error method, using the strategic model only, given the same prior or expert knowledge.

However, we had much more success with our regression method when we introduced continuous variables, such as road pricing charges, public transport fares and frequencies, and parking charges The problem we set ourselves this time was to find the highest social Net Present Value (NPV), which may be thought of as a measure of economic efficiency. We used as policy variables: road charges, at one of six cordon positions we could choose; bus fares (peak and off peak) and off peak bus frequency. After 26 runs of the strategic model we were satisfied that we were sufficiently close to the optimum. While it was extremely difficult for us to establish how this problem would have been tackled by analysts not using a regression model, we were satisfied that the interactions between the policy instruments were sufficiently complex to make it unlikely that such a high NPV value would have been discovered without a very large number of runs of the strategic model.

On the basis of our experience we drew up Figure 1. Although this looks very precise it must be emphasised that considerable discretion is necessary in its usage. In order to illustrate this we will later give a case study. Before going on to do that, we first describe the method in more detail.

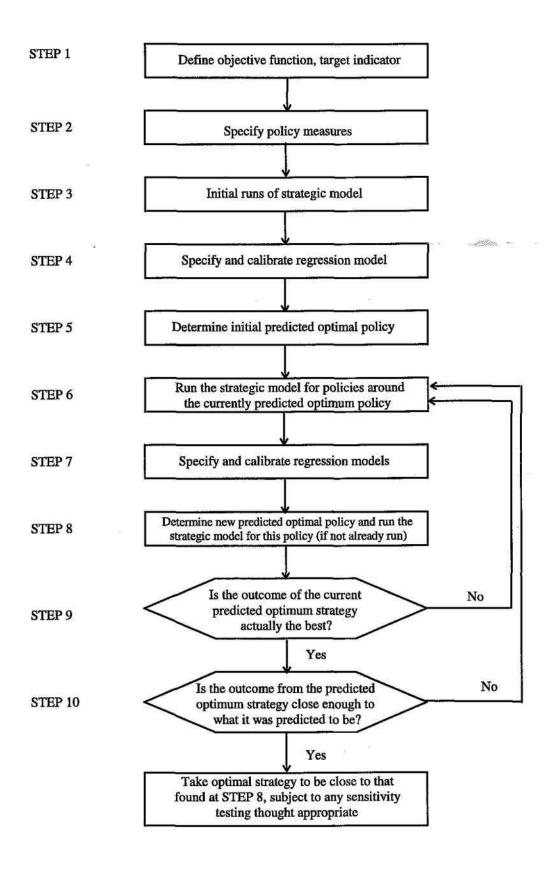


Fig 1: The optimisation process

## 4. THE METHOD

Even in the context of optimising over continuous policy variables (eg road charging fees, public transport fares) our view is that a fully systematic approach would be inordinately expensive. Such an approach would probably comprise some form of grid of, say, three levels of each policy variable arranged according to some statistical experimental design, probably orthogonal. In practice, such designs would be likely to throw up some very uninteresting and uninformative combinations, possibly contradicting assumptions built into the strategic model and so running the risk of producing misleading outputs from the model. For example, would we wish to consider charging higher bus fares in the off-peak than in the peak?

In our view, in the light of our experimentation, we believe that our regression model can learn all it needs about such situations by inference from more sensible combinations. Furthermore, we may often wish to include with our continuous policy variables some discrete ones, such as the infrastructure projects discussed earlier. Some pairs of these may be alternative ways of achieving the same end, and should never be carried out together. Other combinations might incorporate synergy and give progressively better performance as each is implemented. An example of this would be a highway widening scheme, where the benefits of widening the stretch between junctions 12 and 13 is increased if the stretch between junctions 11 and 12 is also to be widened, and increased still further if the stretch between junctions 10 and 11 or 13 and 14 are also to be widened. We feel the complexities here implicit are not amenable to a rigid systematic statistical experimental design approach, and that subjective designs incorporating prior and expert opinion should be used instead.

Another, particularly potent, reason for wishing to avoid including in the experimental design (many) particularly poorly performing combinations is that we wish to avoid exactly modelling the actual function forms of the relationships involved (with respect to the continuous policy variables). An 'exact' model would use up many more degrees of freedom than we have. Following experimentation, we have settled on using quadratic forms as approximations to the true underlying forms, with some success in the locality of the optimum. In other words, if we have strategic model runs for three or four levels of a given continuous policy variable, and these all produce outcomes close to the optimum with all else held equal, then a simple regression model of constant, linear and squared terms in this variable will permit a good estimate of the optimum level of this variable (for other variables at their current settings) by elementary calculus.

Let Y be the target variable (STEP 1), and suppose that we wish to maximise it. With all else held equal we have a number (at least three!) of combinations  $(X_i, Y_i)$  resulting from setting policy variable X equal to level  $X_i$  and observing the outcome level of the target indicator,  $Y_i$ . Simple regression will fit an equation like  $Y = a + bX + cX^2$ 

 $\frac{dY}{dX} = b + 2cX = 0 \text{ for a turning point}$ 

$$\frac{d^2Y}{dX^2} = 2c \qquad < 0 \text{ for a maximum}$$

In order to have a maximum we must have c negative, in which case the optimum level of X is  $\underline{-b}$ 

2c . In this way it can be seen that our method involves inspection of the quadratic coefficients in the regression to ensure they are of the correct sign (negative for a maximisation) and then checking the linear coefficients to see if (once divided by twice the absolute value of the respective quadratic coefficients) they give sensible predicted optimal levels.

Our experimentation indicates that, to begin with, a regression with only slightly fewer variables than observations (ie strategic model runs) will provide worthwhile (but limited) help in determining what further strategic model runs should be conducted. If all the independent variables are so defined that they take the value zero in the do-minimum, and the dependent variable is defined as the change in the target indicator with respect to its value in the dominimum, then a constant term is not strictly needed. However, the time and effort of performing additional regressions with a constant in is more than repaid by the additional robustness this gives to the interpretation process, given that our quadratic relationships will be only a very crude approximation. A large constant is (in these circumstances), of course, an indication of problems. We suggest trying runs with and without constants.

Consider now the general case (STEP 2) where we wish to model n continuous policy variables (using quadratic approximations) and m discrete project levels (using dummy variables). Together with the constant, this takes up 2n + m + 1 degrees of freedom, without considering yet any interaction effects. As a rule of thumb, we would suggest conducting roughly 2n + m + 5 strategic model runs before trying some initial regression models (STEP 3). Furthermore, it will often be the case that some strategic model runs have already been carried out. These runs will very probably be of use to us, but should be 'balanced out' by careful choice of the rest of the experimental design, so that the regression model has a good spread of input data.

Once we have sufficient strategic model runs for the initial regression modelling (STEP 4), this should be carried out in a free-ranging way, without expecting any good fit. Variables with significant coefficients of the wrong sign are an immediate problem, and suggest experimentation with interaction terms, although for the time being this will be severely constrained by the few degrees of freedom. Wrong sign non-significant coefficients can often be induced to change their sign by minor adjustments to the (regression) model form. Any remaining wrong sign non-significant variables should be dropped from the equation, together with any variables whose coefficients are very small. In this way further degrees of freedom become available for experimentation.

All regression results should be kept for further perusal, not just the preferred ones. Particular attention should be given to the residuals for the target indicator (dependent variable). As we are interested in optimising this, we strongly suggest that a weighting be used in the regression,

giving greater weight to 'good' strategic model runs. Experimentation with different weightings is to be encouraged since if the weighting is too uneven there will be a deterioration in the regression model. With so few degrees of freedom we cannot afford to effectively throw away observations by being too fierce with our weighting and effectively giving some observations zero weighting.

Since we are using a quadratic approximation to, presumably, much more complex relationships, we must not expect it to fit well over all possible policies. The approximation only needs to be good in the vicinity of the optimum. Elsewhere we will wish to tolerate much larger errors. Technically, this raises the problem of heteroscedasticity, a violation of one of the usual assumptions of regression modelling. By weighting 'poor' runs lowly we are implicitly adjusting to overcome this problem, allowing the regression model to be not unduly distracted by these poor runs.

Here we should give a word of warning. Conventional goodness of fit statistics (such as the coefficient of determination,  $R^2$ ) are not very helpful, since the fewer degrees of freedom we have the better the model fits, until with no degrees of freedom (observations = coefficients) the model fits exactly. Such a model will have little, if any, ability to generalise, and so cannot be expected to tell us the location of the optimum.

After this initial regression modelling, we should have some idea of what additional strategic model runs will help us to better calibrate our regression model, given its purpose of finding the optimum. A predicted optimum set of policies (strategy) can be found (STEP 5) from the initial regression and the temptation to run this on the strategic model will probably be too great to resist. Nevertheless, the statistical precision of this prediction can be expected to be very poor indeed, and so the second tranche (STEP 6) of strategic model runs should contain a mix. Correlations in the data set should be considered. Particular difficulties encountered during the initial regression modelling should be addressed. Important variables having non significant regression coefficients should be favoured by being given precedence in the choice of their levels, probably being given a wider range than hitherto. Variables, especially dummy variables, where the message is already clear should no longer be experimented with - if a particular project is clearly worthwhile it should be included in all the following strategic models runs, and projects that are clearly no good should be excluded from all following runs. We can always go back to check at the end to see if any important interaction effect has thereby been missed. Likely interactions should be specifically allowed for. In all, we would suggest carrying out about five or six further strategic model runs (of which one will probably be close to the previously predicted optimum from the initial regression model) before returning to further regression work.

The second round of regression modelling (STEP 7) should be more systematic than the first. Rather than looking for a path of continual improvement from the starting model, various alternatives should be specified and tried in turn, eg dropping non significant variables one by one. It will be particularly instructive now to study the effect on the residuals of the 'good' runs as this is done. A major goal should be to find a regression model whose predictions for the, say, five best (i.e. the five most optimum) observations are in the correct order. Again, the weighting scheme can be helpful, but inclusion of the right variables and interaction terms is paramount. For each candidate regression model the predicted optimum variable levels can be determined as shown above, and then input into the regression model to get a prediction for the target variable. By definition this will be better than the predicted values for any of the runs in the data set (ie, theY s), but may well not be better than the actual best value from the strategic model runs (ie the Ys). For poor regression models, however, the predicted optimum may be vastly better than any actual run conducted so far. This is merely a characteristic of the method and does not necessarily indicate that such an improvement is actually achievable. In all probability it will not. As the predicted optimum indicator value comes closer to previously achieved values (ie gets worse) we can have greater confidence in our regression model.

We should stress here, as indicated in Figure 1, that there may be a protracted iterative process. At whatever stage in such an iteration, if we have calibrated a regression model at STEP 7, we should estimate the predicted optimal policy (STEP 8) and consider its adequacy (STEP 9 and STEP 10). Firstly (STEP 9) we should consider if the strategic model run for this predicted optimal policy is actually any better than existing model runs - if it isn't, we should return to STEP 6. Secondly (STEP 10) we should consider how close the modelled (Y) and predicted (Y) target values are for the predicted optimum policy. If the strategic model value is even better than the optimum policy, then it would seem safe to stop, although large disparities might suggest some lack of understanding which might beneficially be remedied by further investigation. If the prediction is greatly above the strategic model value then there is a clear case for continuing the search (i.e. returning to STEP 6).

# 5. APPLICATION TO EDINBURGH

This section applies our method to a strategic model for Edinburgh. A study, called JATES, was carried out for Lothian Regional Council by the MVA Consultancy (May et al, 1992). The JATES model was constructed on a hierarchical logit basis (Bates et al., 1991). Many model runs were conducted, but because a full cost-benefit analysis was not always conducted, and because the model was ever evolving, only 32 compatible runs with complete output were supplied to us, and there was no possibility of performing further compatible runs. Therefore our case study of Edinburgh consists of two distinct parts.

The 32 runs on which information was available were subjected to statistical analysis, reported below in section 5.1. The second part of our work involving new runs carried out on a revised version of JATES is reported in section 5.2.

#### 5.1 STATISTICAL MODELLING OF EXISTING JATES RUNS

The 32 runs available were based on combined strategy tests run by The MVA Consultancy during the JATES study. The dependent or target variable to be optimised is NPV. The independent or policy variables are described below. They are all expressed in terms of differences from the do-minimum run.

Fares F1 F1 = 0 where fares are the same as in the expressed as a proportionate change fr F1 = -0.5 expresses a 50% reduction minimum.	om the do-minimum, e.g.
F1SQ F1 x F1	
Road Pricing RRAT There is no charge in the do-minimum as a ratio of the charge in the $T/C1/1.0$ in $T/C1/1.0 = 1$ and the charge in the actual charges in this strategy run are peak direction in the peak surcharge.	) run, such that the charge $e$ do-minimum = 0. The
The ratio of the all day charge to the p cases.	eak surcharge is 1:3 in all
RSQ RRAT x RRAT	
Infrastructure Investment NS North/South metro, dummy variable otherwise 0.	e equal to 1 if present,
EW East/West metro, dummy variable equa otherwise 0.	l to 1 if present,
WRR Western Relief Road, dummy variab otherwise 0.	ble equal to 1 if present
PARK Additional parking space dummy variate otherwise 0.	able equal to 1 if present
Road CapacityReductionsRC1Smallest reduction in road capacity, du otherwise 0.	mmy variable 1 if present
RC2 Reduction, > RC1, < RC3, dummy otherwise 0.	y variable, 1 if present
RC3 Pedestrianisation in city centre and exter leading to the most significant reduction variable 1 - if present, otherwise 0.	

#### Interaction terms

- RC1R A dummy variable equal to 1 where RC1 and road pricing are present otherwise 0.
- PARR A dummy variable equal to PARK where road pricing is present, otherwise 0.

FARR Equal to F1 if road pricing is present otherwise 0.

A number of further interaction terms were specified and experimented with, e.g. road capacity and road pricing, infrastructure investment and road pricing, and between the different infrastructure investment projects and road pricing.

After some experimentation the model that we found to perform best is shown in Table 5.1.

Variable	Coefficient	Standard erro		
RC1	-92.7	8.97		
RC2	-146.7	8.34		
RC3	-359.3	10.84		
RC1R	-13.4	7.70		
NS	+143.7	5.58		
EW	+15.2	2.29		
WRR	+18.4	2.42		
PARK	+104.7	5.91		
PARR	-29.7	7.02		
F1	-177.8	23.81		
F1SQ	-25.9	13.36		
FARR	+21.4	7.25		
RRAT	+69.1	17.03		
RSQ	-19.8	10.33		

Table 5.1: Regression Model of Existing Data Set.

We then interpret the model to give a prediction of the optimum policy combination, this gives us:-

Predicted NPV=	-92.7	RC1	the minimum possible road capacity reduction
	-13.4	RC1R	interaction between road capacity reduction and road pricing
	+143.7	NS	north/south metro
	+15.2	EW	east/west metro
	+18.4	WRR	Western relief road
	+104.7	PARK	parking investment
	-29.7	PARR	interaction between parking investment and road pricing
	+21.4	FARR	interaction fares and road pricing
	-177.8	<b>F</b> 1	Fare
	-25.9	F1SQ	Fare squared
	+69.1	RRAT	Road pricing
	-19.8	RSQ	Road pricing squared

It is necessary to solve the quadratic terms in this equation, in order to identify the best fare and road pricing levels. Solving for fare:-

(i) NPV = -177.8 F1 + 21.4 FARR - 25.9 F1SQ dNPV = -177.8 F1 + 21.4 RDUM - 51.8 F1 dF1  $F1 = -\frac{153.4}{51.8}$  if RDUM = 1 = -2.96

Some interpretation of this result is necessary, the fare is specified as a ratio where -1 is the equivalent of zero fares. Thus, it is reasonable to interpret -2.96 as a recommendation to set the fare level as low as possible, in this case zero, so F1 = -1.

Road pricing gives a similar equation:-

d <u>NPV</u> = dRRAT	69.1	RRAT - 19.8 RSQ
0	=	69.1 - 39.6 RRAT
RRAT	=	<u>69.1</u> 39.6
		1.74
	dRRAT 0	dRRAT 0 = RRAT =

This implies a road pricing level 1.74 times that in C1/T/1.0, giving a charge of 87p all day, with a peak direction in the peak surcharge of  $\pounds 2.61$ .

Substituting into the main equation we have

Predicted = -92.7(1) - 13.4(1) + 143.7(1) + 15.2(1) + 18.4(1) + 104.7(1) - 29.7NPV (1) - 177.8(-1) - 25.9(-1)<sup>2</sup> + 21.4(1) + 69.12(1.74) - 19.8(1.74)<sup>2</sup> = £337M

The model tells us to choose the smallest possible reduction in road capacity, to set fares equal to zero, invest in both metros, the ring road and parking capacity and impose road pricing at a ratio of 1.74 of the C1/T1/1.0 fee. This gives an all day fee of £0.87 and a peak period peak direction surcharge of £2.61.

The above prediction for NPV is derived taking the smallest of the road capacity reductions. Because of the way the regression model was set up, it is legitimate to infer that without any road capacity reduction NPV would be 92.7 higher, at around £430M. There are several significant interaction terms. Firstly, between the investment in parking in the central zones and road pricing; in the presence of road pricing the NPV of the parking investment is reduced. This is as one would expect, as road pricing reduces the number of cars travelling to the city centre so one would expect the demand for parking space to be reduced. Secondly, there is an interaction between fares and road pricing, the model suggests that a zero fare is the best, in the presence of road pricing the benefits of a zero fare are reduced. Again this is the expected result, in that the role of zero fares in attracting car users onto public transport is partly fulfilled by the presence of road pricing.

We found certain interactions to be insignificant and most notably between the metro investments and the WRR, and between road pricing and the metro and WRR. It is not immediately obvious why this should be. The interaction term between RC3 and road pricing was not valid as there was only one observation. Similarly, the interaction term between the two metro lines could not be defined by the model.

This model suggests that further JATES tests should focus on certain variables in an effort to improve the model.

- (1) Test the predicted <u>optimum</u> combination from this model.
- (2) <u>Fares</u> Test zero fares and a number of other variants to introduce a wider range into the data. Fares may be better expressed as an average fare in the peak and off-peak separately if such information is available. It would then be interesting to test variations in the relative levels of peak and off-peak fares, which proved illuminating in the PLUTO tests.
- (3) <u>Road pricing</u> The model suggests a fee higher than any tested, so we need to introduce a greater range of values into the tests again. In addition, it would be interesting to test different ratio of the charges in the peak direction and all other times.

Unfortunately, the version of the model at ITS could not produce output wholly compatible with that in these runs, as the model had evolved over time. It was therefore necessary to treat this modelling exercise as complete in itself.

#### 5.2 APPLICATION OF THE OPTIMISATION METHOD USING JATES

#### 5.2.1 Policies to be tested

The next stage of our work with JATES was to carry out new model runs, following the guidelines developed using PLUTO. We selected as our starting point the runs identified in the JATES final report (MVA, 1992) as C1 and C6. Their main characteristics are described in Table 5.2.

Table 5.2: Final Report Tests C1 and C6

Policy	C1	C6
Infrastructure	NS EW	NS
	WRR	WRR
Capacity reduction (%)	10%	1 <b>0%</b>
Fares level (%)	-50%	+25%
Road Pricing	Yes	No
NPV (£M)	410	110

where:	NS	=	North - South Light Rapid Transit
	EW	=	East - West Light Rapid Transit
	WRR	=	Western Relief Road

The changes are all relative to the do-minimum run. The capacity reduction is a decrease in road capacity in the city centre. The change in fare level is relative to the expected levels in the do-minimum in the year 2010. Road pricing is a charge of  $\pm 1.50$  to enter or leave the city centre and is payable at all times of the day. Our runs will be tested against the original do-minimum run (see Appendix 1) and the trend scenario.

As a result of certain differences in the evaluation package we obtained slightly different estimates of NPV when rerunning these tests, C1 NPV =  $\pm 453.3M$  and C6 NPV =  $\pm 129.9M$ .

#### 5.2.2 Road pricing

Before progressing to tests based on changes in a number of variables, we wished to examine a variable that could be defined as continuous. This was intended to test for the quadratic type of relationship between NPV and prices, that we identified in the PLUTO tests. Due to the number and variety of fare levels and peak: off-peak differentials in the do-minimum run, this is done most simply with regard to road pricing. The first step was to look back at the various road pricing strategies in tests run by MVA, these are listed in Table 5.3.

Run Code	Road Pricing Strategy			
T/M/1.0	£1.50	am peak	charge to cross	
	£0.50	off peak	boundary either way	
	£1.00	pm peak		
T/M/1.3	£0.50	all day		
	£1.00	peak directi	on surcharge	
T/M/1.4	£0.50	all day	2	
	£2.00	peak directi	on surcharge	
T/C1/1.10	£0.25	all day		
	£0.50	peak directi	on surcharge	
	(T/M/1	.3 x 0.5)	c	
T/C1/1.17	£1.50	at all times		
T/C1/1.18	£0.75	all day		
	£1.50		on surcharge	
	(T/M/1	.3 x 1.5)	es constante de la response de la Sala.	

Table 5.3: Road Pricing Tests Extant (in chronological order)

The strategy in T/M/1.0 appears to be overcomplex for our requirements and so T/M/1.3 was chosen as the starting point for our road pricing tests. The premise was that 'expert' opinion contributed to the initial values for the road pricing tests and that therefore they were a good place to start. In keeping with our method, where do-minimum values do not exist (eg road pricing, new modes) or are deemed inappropriate, an expert opinion should be sought to establish starting values for the variables in question.

Our matrix of three tests has the MVA road pricing strategy of run T/M/1.3 as the mid-point for the tests. We tested around this by halving and doubling the fees, if the second test had shown NPV increasing, then the third test would have been a further move in that direction, to ensure that the highest NPV is enclosed by our tests. (See Bristow et al 1994, for further details on testing quadratic functions). The results are shown in Table 5.4.

Table 5.4: Road Pricing -	First	test matrix
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Run Code	Roa	NPV (£m)	
CRP1	£0.50 £1.00	all day peak surcharge	476.5
CRP2	£0.25 £0.50	all day peak surcharge	475.6
CRP3	£1.00 £2.00	all day peak surcharge	440.4
<b>C</b> 1	£1.50	all day	453.4

These tests were all run using C1 (final report) as the base model, this is included in Table 5.4 for comparison, the only changes made were to the road pricing levels. At this stage we are maintaining the same ratio between the all day charge and the peak direction surcharge, although this could be altered at a later stage. The NPVs obtained are very interesting as those for CRP1 and CRP2 are very similar, one might expect to find the optimum value in between. Therefore a fourth test was run using values 0.8 times those in T/M/1.3. The NPV obtained from this run was 478.5M, which is higher than the two values surrounding it as expected.

In this case with four model runs we have reached a point at or near a local optimum. Although we cannot be certain that we have reached a global optimum, however, we have tested a fairly wide range of road pricing fee levels and used very few model runs. Therefore, the assumption of quadratic form for continuous variables is appropriate.

#### 5.2.3 Combined policy tests

This section reports the application of the optimisation procedure developed using PLUTO (see figure 1), to the JATES model. As our target (Y) variable, or key indicator (STEP 1), we chose the Net Present Value of benefits over costs (NPV), as produced by JATES.

We tested a number of policy variables (STEP 2), to give the optimisation process a variety of different variables to deal with at the same time. These were:

Road Pricing	<ul> <li>all day charge (pence) to cross cordon around city centre (zero in do-minimum)</li> </ul>
	<ul> <li>peak direction surcharge (pence) (zero in do- minimum)</li> </ul>
Bus fares	- peak (as a proportion of the do minimum bus fares)

off peak (as a proportion of current peak fares; there was no off-peak reduction in the dominimum)

Rail fares-as a proportion of the do minimum rail faresLRT fares-as a proportion of the do minimum bus

Infrastructure Investment

as a proportion of the do minimum <u>bus</u> fares (since there was no LRT in the do-minimum)

two levels, medium and high (medium consists of a Western Relief Road and a North South Light Rapid Transit Line, high consists of medium plus an East-West Light Rapid Transit Line, none of which were in the do-minimum)

We selected as our starting point the runs identified as successful by the consultants (MVA, 1991) as C1 and C6. Their main characteristics are described in Table 5.5, where they appear as runs 2 and 3 respectively.

The first task was to establish precisely how physically to set the variables of interest within JATES. One or two, especially LRT fares and off-peak discounts, are slightly awkward and outside the model menu. The next task was to define the required minimum number of test runs and then devise an appropriate matrix of tests. Firstly, we considered the independent variables (STEP 2) that we would wish to include in our regression model:

- road pricing all day charge (to cross cordon around the city centre)
- road pricing all day charge squared
- road pricing peak direction peak surcharge
- road pricing peak direction peak surcharge squared
- bus fare peak
- bus fare peak squared
- bus fare off-peak
- bus fare off-peak squared
- rail fare
- rail fare squared
- LRT fare
- LRT fare squared

Infrastructure level: dummy 1 = high, 0 = medium

Constant

As a simplification, we are investigating the impact of differing peak/off-peak fares for buses but not for rail or LRT. Rail is of less importance as a mode, and the do-minimum off-peak discounts vary according to the origin and destination. It is therefore very difficult to define logical changes in the overall balance of peak and off-peak rail fares that can be defined relative to the do-minimum.

Hence we have 13 variables plus a constant term, before we even consider the possibility of inter-action terms. Consequently, the minimum number of runs to begin with is about 18. This is the (2n + m + 5) discussed in section 4, where n = 6 continuous variables, and m = 1 dummy variable. A matrix of tests was then devised (STEP 3) to contain a wide range of variation of values for all the pricing variables. Table 5.5 shows these, together with catturn NPV, expressed as a change from the do-minimum case.

All Day 2.00 1.50	Peak Surcharge 1.00	General Change	Off-				
1.50	1.00		peak				
annual area		-25	-50	-25	-25	H	+220.0
	0	-50	0	-50	-50	H	+453.4
0	0	+25	0	+25	+25	М	+129.9
0.25	1.00	-50	-20	-20	0	М	+163.7
1.50	0	+25	-50	+25	0	н	+230.7
2.50	0.50	+100	-100	+25	0	M	-383.0
0.25	0	0	-20	-60	+50	M	-92.5
0	2.00	a and a start of the	-50	-40	and the second sec	12277822	-111.3
1.00	0	and the second second	-100	+50			-217.3
0	1.00	+100					+243.6
2.00	0	0		000000000000000000000000000000000000000	1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1	and the second second	+136.1
0		(() (2) () () () () () () () () () () () () ()	-100	10000	1.000		+114.1
0.50			0		0		+476.5
0.25					9	2000 LOCA	+475.6
1.00	2.00	-50	0	-50	-50	H	+440.4
0.50	0	+25	-20	+50	-80	Μ	+102.8
1.50	2.00	-50	-50	+100	+25	M	-555.3
2.50	0	0	-100	-40	-40	M	-130.2
	2.50 0.25 0 .00 0 2.00 0 0.50 0.50 1.50 2.50 hange	2.50         0.50           0.25         0           0.25         0           0.00         0           0.00         0           0.00         0           0.00         0           0.00         0           0.00         0           0.00         0.50           0.50         1.00           0.25         0.50           0.00         2.00           0.50         0           1.50         2.00           2.50         0	2.50 $0.50$ $+100$ $0.25$ $0$ $0$ $0$ $2.00$ $+100$ $.00$ $0$ $-50$ $0$ $1.00$ $+100$ $2.00$ $0$ $0$ $0$ $0.50$ $+25$ $0.50$ $1.00$ $-50$ $0.25$ $0.50$ $-50$ $0.25$ $0.50$ $-50$ $0.50$ $0$ $+25$ $1.50$ $2.00$ $-50$ $2.50$ $0$ $0$ hanges are with respect to the second	2.50 $0.50$ $+100$ $-100$ $0.25$ $0$ $0$ $-20$ $0$ $2.00$ $+100$ $-50$ $.00$ $0$ $-50$ $-100$ $0$ $1.00$ $+100$ $-20$ $0$ $0$ $-50$ $-100$ $0$ $1.00$ $+100$ $-20$ $2.00$ $0$ $0$ $-50$ $0$ $0.50$ $+25$ $-100$ $0.50$ $1.00$ $-50$ $0$ $0.50$ $1.00$ $-50$ $0$ $0.50$ $0.50$ $-50$ $0$ $0.50$ $0$ $+25$ $-20$ $.50$ $0$ $-50$ $-50$ $2.50$ $0$ $0$ $-100$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	2.50 $0.50$ $+100$ $-100$ $+25$ $0$ $0.25$ $0$ $0$ $-20$ $-60$ $+50$ $0$ $2.00$ $+100$ $-50$ $-40$ $-20$ $.00$ $0$ $-50$ $-100$ $+50$ $+75$ $0$ $1.00$ $+100$ $-20$ $-60$ $-60$ $2.00$ $0$ $0$ $-50$ $+100$ $0$ $0$ $0.50$ $+25$ $-100$ $-20$ $+100$ $0.50$ $1.00$ $-50$ $0$ $-50$ $-50$ $0.50$ $1.00$ $-50$ $0$ $-50$ $-50$ $0.25$ $0.50$ $-50$ $0$ $-50$ $-50$ $0.50$ $0$ $-50$ $0$ $-50$ $-50$ $0.50$ $0$ $-50$ $-50$ $-50$ $0.50$ $0$ $-50$ $-50$ $-80$ $0.50$ $0$ $-50$ $-50$ $+100$ $+25$ $0.50$ $0$ $-100$ $-40$ $-40$	2.50 $0.50$ $+100$ $-100$ $+25$ $0$ $M$ $0.25$ $0$ $0$ $-20$ $-60$ $+50$ $M$ $0$ $2.00$ $+100$ $-50$ $-40$ $-20$ $M$ $.00$ $0$ $-50$ $-100$ $+50$ $+75$ $M$ $0$ $1.00$ $+100$ $-20$ $-60$ $-60$ $H$ $2.00$ $0$ $0$ $-50$ $+100$ $0$ $H$ $0$ $0.50$ $+25$ $-100$ $-20$ $+100$ $H$ $0.50$ $1.00$ $-50$ $0$ $-50$ $-50$ $H$ $0.25$ $0.50$ $-50$ $0$ $-50$ $-50$ $H$ $0.50$ $0$ $-50$ $0$ $-50$ $H$ $0.50$ $0$ $-50$ $0$ $-50$ $H$ $0.50$ $0$ $-50$ $0$ $-50$ $H$ $0.50$ $0$ $-50$ $-40$ $H$ $0.50$ $0$ $-100$ $-40$ $-40$ $0$ $0$ $-100$ $-40$ $-40$

Table 5.5: Edinburgh Combined Policy Tests - Matrix of Initial 18 Runs

The matrix includes the MVA tests C1 and C6 (as runs 2 and 3) and also the three road pricing tests (runs 13 to 15) described in section 5.2.2. We are attempting to utilise existing information in order to minimise the total number of additional runs required. The 18 runs carried out show

a wide range of NPV outcomes, from -£555.3M to £476.5M. With 18 data points and an initial 14 independent variables, the regression modelling was never expected to yield a very good model, but merely a first indication as to levels for the policy variables that would yield a high NPV.

The first stage in constructing a useful regression model (STEP 4) was to enter all the variables and squared terms specified above.

The second stage was to introduce a weighting on the NPV dependent variable. We have already mentioned the wide range of NPV values in the data set; we wish to concentrate the explanatory power of the regression model on the cases with high NPV's rather than on the negative outcomes. Therefore, we introduced a weight (W2) as follows,

 $W2 = (NPV + 600)^2$ 

We also tried

W1 = (NPV + 600)W3 = (NPV + 600)<sup>3</sup> and W4 = (NPV + 600)<sup>4</sup>

We found W2 to give the most satisfactory performance and that weighting is used throughout this paper. We chose the constant '600' as being just sufficient to give all the runs a non-trivial positive weight (the smallest NPV being -£555.3M as stated above).

The model produced was unsatisfactory, with rail fares and both road pricing variables insignificant. For the sake of completeness and to allow our modelling process to be followed through, models are shown in Appendix 2; this one is model 1.

The next stage was to remove the unhelpful variables, both linear and squared terms, in sequence, to see if the model was improved.

- (i) Remove all day road pricing (and its square) this model is a slight improvement giving a positive peak period peak direction surcharge of 97p. It also suggests free bus fares, a reduction of 72% on LRT fares, an increase in rail fares of 17.4%, and the high level of infrastructure investment, (model 2). The increase in rail fares, at the same time as large reductions in fares on other modes, was counter intuitive but was not significant.
- (ii) Remove peak direction peak period road pricing surcharge this model (model 3) was unsuccessful; the sign on all day road pricing remained negative. The model suggests free fares on bus and LRT, an increase in rail fares of 18%, and the high level of infrastructure investment. Again, lack of significance of variables leads us to investigate further.

(iii) Remove rail fare variables - this yields our most rational model so far, although many variables have very large standard errors. This model (model 4) suggests;

Road pricing	-	all day 43p
	-	peak direction peak period surcharge 89p
Bus fares	-	decrease of 22%
	20	no off-peak discount
LRT fares	-	decrease of 56%
Rail fares	<del>.</del>	no change
Infrastructure		high

We also experimented with interaction terms to a limited extent, but no improvements to the model were noted.

As model 4 provides a rational picture with logical signs and no inconsistencies, we used this to predict our initial optimum run (STEP 5):-

169.7	constant

- +5.23 bus fares down 22%
- +32.35 LRT fares down 56%
- +22.73 Road pricing, peak direction peak period surcharge 89p
- +8.83 Road pricing, all day 43p
- +266.0 High infrastructure investment
- £504.84M Predicted NPV increase

This policy combination was then run through JATES (Run 19, Table 5.6) and produced an actual NPV of £435.9M, which is towards the top end of the NPVs identified so far, but clearly not an optimum. However, the regression is starting to point us in the right direction, which is the most we can expect with so few runs and so many variables. The next step was to identify five more JATES runs (STEP 6) intended to provide the regression model with more information. Values were chosen to reflect what the model was telling us and to try to aid it where obvious problems were present. The issues which we considered here were:-

- (i) Road pricing the model is having problems disentangling the two types of charge (all day and peak surcharge). We therefore tried three runs where only one was implemented and two with different balances between the two.
- (ii) Bus fares all the models suggest a fares reduction; model 4 actually involves the smallest reduction. We can be confident about the direction of change and so can try larger reductions, including two runs with zero bus fares. The models are also unanimous in rejecting an off-peak discount, so we kept off-peak fares at the same level as peak fares for these runs.
- (iii) Rail fares the models have trouble with rail fares; this is not too worrying as rail is a minor mode. We tried three runs with no change in rail fares and three with reductions.
- (iv) LRT fares all the models suggest fares should be reduced; we tried a range of reductions.
- (v) Infrastructure all the models suggested investing in the high level of infrastructure.

The run specifications are given in Table 5.6 along with outturn NPV values. Note that we have tried to avoid introducing correlation between variables by randomising the levels set for each run. In case of indecision as to what specifications are sensible to choose, the predicted NPV's of candidate specifications might be inspected. We did this sometimes, but not generally as we were more interested in getting a good spread of data for our regression models. In the event, as Table 5.6 shows, none of the additional 5 runs (20 to 24) were particularly poor in terms of NPV.

Run	Road Pricing (£)		Bus Fare Change (%)		Rail Fare Change (%)	LRT Fare Change (%)	Infrastructure (High = H)	NPV Change for Run (£M)
	All day	Peak Surcharge	General Charge	Off peak			n	
19	0.43	0.89	-22	0	0	-57	н	+435.9
20	0	1.50	-40	0	-20	-40	н	+410.1
21	0.75	0	-60	0	0	-80	H	+536.2
22	0.90	0.40	-80	0	-10	-20	Н	+502.3
23	0.60	1.20	-100	0	0	-100	H	+428.2
24	0	2.50	-100	0	-50	0	Н	+397.6

Table 5.6: Edinburgh Combined Policy Tests - An Additional 6 runs

It can be seen from the above table that we have obtained two NPV values above those in the 18 run data set by testing around the initial predicted 'optimum'. This data was then combined with the previous 18 runs and regression analysis undertaken (STEP 7), again using weighting W2.

The initial model including all the variables was imperfect, having a negative coefficient on the peak road pricing surcharge (model 5 see Appendix 2). The model suggested an all day road pricing level of 51p, zero bus fares, a 50% reduction in LRT fares, a 15% reduction in rail fares and high infrastructure investment. The 24 run model is now telling us that all public transport fares should move in the same direction.

Again our strategy was to remove problematic variables in sequence:-

 Remove peak direction peak period road pricing surcharge, non significant and negative. Model 6 results and is a reasonable one:-

Road pricing		all day 79p
	-	no peak surcharge
Bus fares	9 <b>44</b>	reduce by 52%
	-	no further off-peak discount
Rail fares	-	reduce by 2%
LRT fares	-	reduce by 53%
Infrastructure	( <b>H</b> )	high
The predicted	optim	um NPV is £492.4M, consisting of:-

156.9	constant
+40.0	road pricing
+13.1	bus fare reduction
+0	rail fare reduction
+20.8	LRT fare reduction
+261.6	High infrastructure
£492.4M	Predicted NPV increase

The contribution from rail fares is negligible and this variable is highly unreliable having a standard error some 10 times higher than the coefficient.

Remove rail fare variables as well as peak road pricing surcharge variables (model 7). This model's recommendations are:-

-	all day 75p
-	no peak surcharge
-	reduce by 58%
-	no further off-peak discount
-	no change
	reduce by 56%
-	high
	-

Predicted NPV using these values:-

143.7	constant
+16.71	Bus fare
+26.23	LRT fare
+41.99	Road pricing
+256.6	Infrastructure
£485.23M	Predicted NPV increase

Further extensive experimentation failed to improve the regression model, eg trying a combined road pricing variable, or introducing interaction terms.

Models 6 and 7 give very similar recommended input values for the key pricing variables, see Table 5.7 shows these, together with their predicted NPV values. Also shown is Run 21 (from Table 5.6) which is rather similar but would now be predicted to have a slightly lower NPV than Models 6 and 7. For Run 21 we actually know its NPV to be £536.2M (rather than £480M as would be predicted from our regression model using Runs 1 to 24).

(ii)

Variable	Model 6	Model 7	Run 21
Road pricing - all day	79p	75p	75p
Bus fare change - all day	-52%	-58%	-60%
LRT fare change	-53%	-56%	-80%
NPV increase (£M) - Predicted from Runs 1-24	492.4	485.2	480
- Actual			536.2
			5

Table 5.7: 24 Data Point Model Predictions Compared to Run 21

It appears that we are now very close to an optimum; the model prediction suggests that a slight improvement in NPV may be achieved with further runs. It is a question of judgement and resources how far marginal improvements should be pursued. STEP 8 was therefore dispensed with and Run 21 adopted as our optimal run. This is our best run (STEP 9), and its performance is better than any prediction (STEP 10). In practice, continuous policy variable levels will have to be rounded to convenient numbers. Here we would suggest 80p all day road pricing, halve bus fares, implement the high level of infrastructure investment, and set LRT fares equal to bus fares.

#### 6. CONCLUSIONS

In this paper we have presented a new methodology whereby difficult decisions can be made much easier. The context is that of strategic transport models, which are themselves not sufficiently exact to be worth solving analytically (were that possible) or of investigating by a grid search in order to find their 'optimum', as defined by a user. These models allow a wide range of transport policy variables to be set. Typically, many runs will be carried out to cover the range of policies that were in mind when the study was set up and the decision to build the strategic model taken. Once these are done, it is realised that there are an infinite number of other policies that could have been tested, consisting of various levels for each of several continuous and discrete variables. Even if the users are fairly happy with the best run so far from the strategic model, how can they be sure that it really is anywhere near optimal, given their own objective.

In this paper we have shown that it is possible to take any existing strategic model runs, do some more to provide a suitably rich data set, and then perform regression modelling to derive a satisfactorily accurate representation of the response surface of the strategic model in the vicinity of its optimum. We have found that, for continuous policy variables, quadratic forms are sufficient, so that we need only two parameters for each continuous policy variable to be included. Discrete policy variables (such as infrastructure projects) require one parameter for each level. Each parameter requires a degree of freedom, and that means another run of the strategic model. Consequently we are not interested in accurately representing the strategic model. Because the quadratic will not be correct, it will give poor performance over large ranges. We have taken the position that we are interested only in the vicinity of the optimum, and so have used a weighting whereby runs are given less weight the further they are away from the current best run. In extreme cases, a run may be so poor that it will be best dropped from the regression rather than compromise our method.

Our method clearly works. We have found (in Run 21, Table 3) a combination of policies which gives an NPV (relative to the do minimum) of £536.2M, compared to the £453.4M of the best run identified by the consultants, i.e. an 18% increase. This was achieved with just 24 runs of the strategic model - a grid search method would have hardly got started.

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#### **APPENDIX 1: JATES DO MINIMUM DEFINITION AND SCENARIOS**

#### **JATES - DO-MINIMUM DEFINITION**

The "Joint Authorities Transportation and Environmental Study" or JATES has at its centre a strategic model designed to predict and evaluate the impacts of transport policies. The evaluation produces outputs on accessibility, environmental impacts, fuel consumption and accidents, as well as the more conventional assessment of Net Present Value (NPV).

The specification of the do-minimum run with regard to policies in place and assumptions on economic growth and distribution is the first issue to be examined.

#### JATES do-minimum policies

The do-minimum strategy is defined here with reference to the JATES Final Report and Information Note 13, and included the following changes from the base year;

- (a) M8 extension to the City bypass.
- (b) SCOOT UTC in the City centre, zones 1,2 and 12, assumed to give an increase in capacity of 5%. This is done by adjusting the base flows by +5% at the base speeds.
- (c) A real increase in public transport fares of +29% between 1990 and 2010, reflecting recent trends in fare levels.
- (d) Bus service provision, the final report has bus levels adjusted such that a "constant load factor" is maintained.
- (e) Private and public parking supply remain constant overall.
- (f) Parking charges, increase by 50% between 1990 and 2010.

#### 2.2 Scenarios

Four scenarios were developed, each making different assumptions on land use and economic growth in the future. The base reflects the situation in 1990. The four scenarios are as follows:-

- (a) **Trend** involves moderate economic growth, the majority of the growth in employment takes place outside the city, especially in West Lothian. There is some population transfer from the city to the external zones.
- (b) **High** involves a higher level of economic growth than that posited in the Trend scenario. Growth is concentrated in the West of the city, leading to higher employment in zones 2 and 9.
- (c) **Low** involves a lower level of economic growth than that in the trend scenario. There is a greater retention of jobs and people within the city.

(d) Balanced - involves the same level of growth as in the Trend scenario, but with a different distribution, "with both population and employment located so as to achieve a better balance within each strategic zone with the objective of reducing the need to travel".

The vast majority of the JATES model tests carried out by MVA use the Trend scenario. The others are used to carry out sensitivity testing on strategies.

Given that the bulk of existing output is based on the Trend scenario, it is rational that our tests also take the trend scenario as a basis for strategy testing.

#### **APPENDIX 2: REGRESSION MODELS**

This appendix contains the regression models described in section 5.2.3

#### Variable Definitions

RPA	2	Road pricing all day charge
RPP	$\simeq 1^{-1}$	Road pricing peak direction peak surcharge
BG	=	Bus fares, change
BOP	-	Off peak bus fares discount
RF	<u>1</u> 20	Rail fare change
LRT	€G	LRT fare change
INF		Infrastructure investment dummy
		(1 = high, 0 = medium)
RPAS	-	RPA <sup>2</sup>
RPPS		RPP <sup>2</sup>
BGS	÷	BG <sup>2</sup>
BOS	-	$BOP^2$
RFS	-	RF <sup>2</sup>
LRS		LRT <sup>2</sup>
Weigh	ts	$W2 = (NPV + 600)^2$
		$W3 = (NPV + 600)^3$
		$W4 = (NPV + 600)^4$

18 point data set, weight = W2

Variable	Model 1	Model 2	Model 3	Model 4
Constant	194.3	168.9	187.9	169.7
ŝ	(44.08)	(40.76)	(35.95)	(48.98)
RPA	-1.214		-0.7033	0.4100
	(1.452)		(0.7336)	(1.363)
RPAS	0.002891		0.001085	-0.004762
an an the second second	(0.00642)		(0.003804)	(0.005752)
RPP	-0.5919	0.4963	- 1	- 0.5114
	(1.105)	(0.6495)		(1.105)
RPPS	0.002539	-0.002564		-0.002876
province of the second	(0.005342)	(0.003011)		(0.005335)
BG	-1.382	-0.8068	-1.051	-0.4702
	(0.8428)	(0.5054)	(0.4710)	(0.7848)
BGS	0.001001	0.0009898	-0.001992	-0.01055
	(0.01104)	(0.009153)	(0.008252)	(0.009578)
BOP	3.221	5.287	4.154	4.974
	(2.394)	(1.773)	(1.414)	(2.532)
BOS	0.01064	0.02124	0.1886	0.03594
Chille:	(0.02574)	(0.01877)	(0.01677)	(0.02591)
RF	0.6034	0.5739	0.5792	6 S
	(0.5697)	(0.4980)	(0.4603)	
RFS	-0.01830	-0.01653	-0.01624	
	(0.009264)	(0.007361)	(0.006868)	4
LRT	-1.063	-0.5455	-0.9894	-1.150
	(0.5497)	(0.3945)	(0.4559)	(0.6307)
LRS	-0.001546	0.003798	-0.002109	-0.01022
	(0.01071)	(0.006673)	(0.008710)	(0.01104)
INF	314.0	272.4	298.7	266.0
	(45.64)	(31.65)	(28.83)	(44.25)

Variable	Model 5	Model 6	Model 7
Constant	172.1	156.9	143.7
	(35.59)	(37.45)	(35.09)
RPA	0.5859	1.006	1.121
	(0.6073)	(0.6162)	(0.6056)
RPAS	-0.005715	-0.0006331	-0.007482
	(0.003152)	(0.003384)	(0.003272)
RPP	-0.5668	100 0	
	(0.4502)		
RPPS	0.0009553		
	(0.001891)	17	54).
BG	-0.7665	-0.5041	-0.5768
<u>1</u>	(0.3245)	(0.3193)	(0.3123)
BGS	-0.003711	-0.004845	-0.004977
	(0.003495)	(0.003530)	(0.003444)
BOP	3.437	4.583	4.780
	(1.731)	(1.736)	(1.664)
BOS	0.02250	0.02831	0.03405
	(0.01712)	(0.1815)	(0.01689)
RF	-0.2437	-0.03374	And the second sec
	(0.3291)	(0.3366)	
RFS	-0.007939	-0.008258	
	(0.005736)	(0.005991)	
LRT	-1.073	-0.7845	-0.9365
	(0.5009)	(0.5039)	(0.4894)
LRS	-0.01079	-0.007387	-0.008359
	(0.006066)	(0.006293)	(0.006050)
INF	277.5	261.6	256.6
	(31.56)	(32.41)	(31.43)

24 Point data set, weight = W2