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Natural convection of water near 4°C in a bottom-cooled enclosure

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Abstract

A study of natural convection in water-filled square enclosures whose bottom wall is cooled at 0°C, whereas the top wall is partially or entirely heated at a temperature ranging between 10°C and 30°C is performed numerically through a computational code based on the SIMPLE-C algorithm, assuming temperature-dependent physical properties, for cavity widths in the range 1 cm–10 cm, with the main aim to point out the basic heat and momentum transfer features.

Keywords: Natural convection; Enclosure; Water; Density-inversion temperature; Cooling from below; Numerical analysis.

1. Introduction

Natural convection in a horizontal water layer differentially heated at the bottom and top boundaries with density inversion in the bulk leads to the formation of an upper stably stratified fluid region, and a lower convectively unstable region from which motion can propagate upwards. This phenomenon is often referred to as penetrative convection.

Nomenclature

c	specific heat at constant pressure, J/(kg K)
g	acceleration of gravity, m/s^2

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H	distance of the 4°C isotherm from the bottom wall in the conductive motionless field
h_c	average coefficient of convection at cooled surface, W/(m^2 K)
k	thermal conductivity, W/(m K)
L	length of the heated portion of the top wall, m
Nu	Nusselt number
Ra	Rayleigh number
t	temperature, °C
W	width of the enclosure, m
Greek symbols	
μ	dynamic viscosity, kg/(m s)
θ	dimensionless temperature
ρ	density, kg/m^3
Subscripts	
c	cooled surface, at the temperature of the cooled surface
h	heated surface, at the temperature of the heated surface

Besides the theoretical papers related to infinite horizontal layers with either stress-free or rigid top and bottom boundaries [1-3], the studies carried out on penetrative convection of water in enclosures laterally confined by solid walls, which can be of interest for cooling energy storage applications, are relatively few. The first of them was a demonstrative experimental work conducted by Townsend [4] who used a tank having an ice-covered square bottom of 30 cm × 30 cm, filled with water to a depth of 15 cm, whose upper surface was maintained at about 25°C. Later investigations were performed numerically on square enclosures by Zubkov and Kalabin [5], on cubic cavities by Zubkov et al. [6], and on shallow cylindrical containers by Li et al. [7], in all of which the heating-from-below condition was implemented assuming constant physical properties, except for the density in the buoyancy force term of the momentum equation. Primary aim of all these works was to determine the existence of multiple stable steady-state or periodically-oscillating flow pattern solutions, whereas much less attention was addressed to the heat transfer performance of the enclosure.

In this general framework of extremely limited availability of data for the cooling-from-below configuration, a study of transient natural convection in water-filled square enclosures whose bottom wall is cooled at 0°C, whereas the top wall is partially or entirely heated at a temperature above 4°C., is proposed with the main scope to analyze the propagation of convective motion across the cavity up to the achievement of a steady-state or a periodically-oscillating asymptotic solution, and to discuss the basic heat and momentum transfer features.

2. Mathematical formulation and computational procedure

A water-filled square enclosure of width W is considered. The enclosure is cooled at the bottom wall, which is kept at a uniform temperature t_c = 0°C, and partially heated at the opposite wall. The discrete source, of length L, which is located at the center of the top wall, is maintained at a uniform temperature $t_h \geq 10$°C. The remaining right and left parts of the heated wall, as well as both sidewalls of the enclosure, are assumed to be perfectly insulated. A sketch of the model geometry is shown in Fig. 1. The buoyancy-induced flow is considered to be two-dimensional, laminar and incompressible, with negligible viscous dissipation and pressure work. The dependence of any physical property on temperature is approximated as a fourth-order polynomial function obtained by the best fit of 1001 equispaced reference data in the range 0°C–100°C extracted from the NIST Chemistry WebBook.

The system of the governing equations of mass, momentum and energy conservation, in conjunction with the related boundary conditions, is solved numerically through a control-volume formulation of the finite-difference method, starting from the initial condition of motionless fluid at the uniform temperature of the cooled bottom wall, for different cavity widths up to 0.1 m. The pressure-velocity coupling is handled through the SIMPLE-C algorithm. Convective terms are approximated using the QUICK discretization scheme, while a second-order backward scheme is applied for time integration. Both space discretization and time stepping are chosen uniform. Starting from the assigned initial fields of the dependent variables across the enclosure, at each time-step the system of discretized algebraic equations is solved iteratively by way of a line-by-line application of the Thomas algorithm. A standard under-relaxation technique is enforced in all steps of the computational procedure to ensure adequate convergence. Details on the numerical code and its validation can be found in Corcione et al. [8, 9]. At each time-step, after the spatial convergence is achieved, the heat transfer rates at the heated and cooled surfaces are calculated. Time-integration is stopped once an asymptotic solution, either stationary or periodic, is reached. In the latter case, the heat transfer rates are averaged across one period of oscillation.

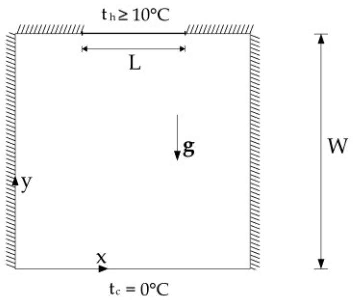

Fig. 1. Sketch of the model geometry and coordinate system.

3. Results and discussion

A compilation of asymptotic local results is presented in Fig. 2 and 3, where selected isotherm and streamline contour plots are reported for W = 0.03 m – 0.07 m, t_h = 10°C – 30°C and L / W = 0.5, and for W = 0.05 m, t_h = 20°C and L / W = 0.2 – 0.8, respectively. The 4°C isotherm, represented using a dashed line, is superimposed on any streamline pattern, so as to give an indication of the effect of the maximum water density on the flow field.

First of all, it is worth mentioning that, having fixed the temperature of the cooled wall, penetrative convection takes place as long as the temperature of the upper heated surface is lower than a limit value that increases with increasing the cavity width. In fact, penetrative convection arises only if the distance H of the 4°C isotherm from the cooled bottom wall is such that motion can take place in the fluid layer underneath, otherwise heat transfer across the enclosure occurs by pure conduction. This means that in all those cases a motionless conductive solution is obtained, which is e.g. the case of W = 0.03 m, t_h = 30°C and L / W = 0.5, H is such that the corresponding Rayleigh number Ra_H calculated using the difference between the water densities at 4°C and 0°C is lower than the critical value for the onset of motion, whose order of magnitude is that typical for a horizontal fluid layer with one rigid and one stress-free boundary.

As regards the penetration depth, d_p, which approximately coincides with the average distance of the 4°C isotherm from the cooled bottom wall, it may be seen that its ratio to the cavity width increases as the cavity size increases, and the heated fraction of the top wall and its temperature decrease.

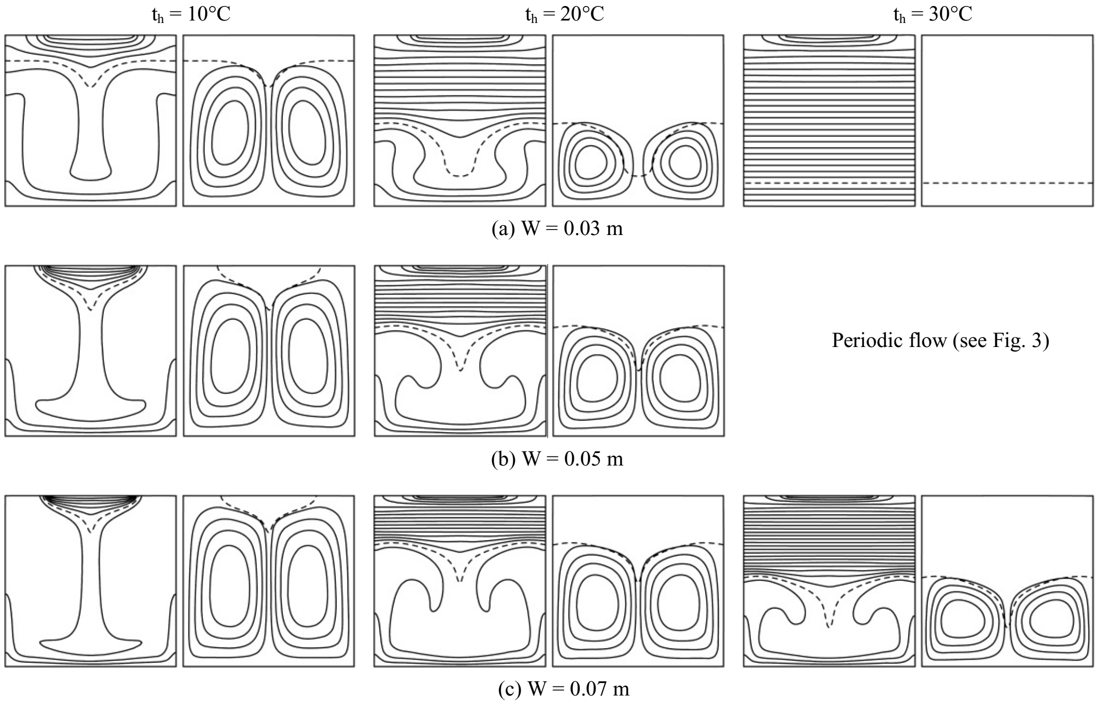

Fig. 2. Asymptotic isotherms and streamlines for t_h = 10°C – 30°C, L / W = 0.5, and (a) W = 0.03 m, (b) W = 0.05, (c) W = 0.07 m.

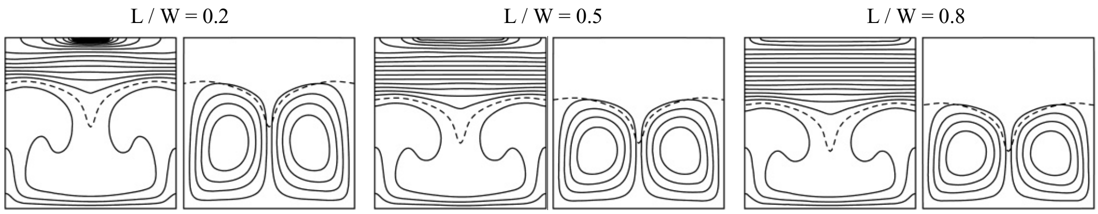

Fig. 3. Asymptotic isotherms and streamlines for t_h = 20°C, W = 0.05 m and L / W = 0.2 – 0.8.

Indeed, as expected, the major dependence of d_p / W is on the temperature of the heated surface. In particular, once the dimensionless temperature of the heated top wall, θ_h, and the Rayleigh number of the cavity, Ra_W, are defined as

$$\theta_h = \frac{t_h - 4°C}{t_h - t_c} \quad , \quad Ra_W = \frac{c\rho g \Delta\rho W^3}{\mu k} \quad , \tag{1}$$

where c, ρ, μ and k are the specific heat at constant pressure, the mass density, the dynamic viscosity and the thermal conductivity of water at 4°C, and, as for Ra_H, Δρ is the difference between the water densities at 4°C and 0°C, the following correlation can be established with a 4.2% standard deviation of error:

$$d_p / W = 0.125\, Ra_W^{0.08} \theta_h^{-1.45} (L/W)^{-0.22}$$

for $5 \times 10^4 \leq Ra_W \leq 5 \times 10^6$, $0.6 \leq \theta_h \leq 0.87$, $0.2 \leq L/W \leq 1$. (2)

Another point to enlighten is that periodicity is typical of the configurations whose Rayleigh number Ra_H is that corresponding to the onset of motion or just after. Actually, once convection starts, the flow pattern in the bottom fluid layer initially consists of four cells, which are symmetric about the vertical midplane of the enclosure. As long as the 4°C isotherm moves upward, the buoyancy force increases and, consequently, such a four-cell flow pattern becomes unstable, bringing to the formation of a two-cell flow pattern. On the other hand, at values of Ra_H of the order of 1000 the buoyancy force is not strong enough to allow the penetration of motion at a distance from the bottom wall such that the two-cell configuration can remain stable. This results in a periodic flow configuration consisting of an alternate succession of a two-cell flow pattern and a four-cell flow pattern, as documented in Fig. 4 through five snapshots of the isotherms and streamlines plotted at subsequent times spanning one period of oscillation.

The effects of the cavity width, as well as the temperature and the heated fraction of the top wall, on the asymptotic value of the average coefficient of convection of the cooled bottom wall, h_c, when penetrative convection occurs are pointed out in Fig. 5, showing that h_c decreases with increasing W and t_h, whereas it depends quite marginally on L / W. Specifically, the heat transfer performance of the cavity expressed in terms of the average Nusselt number of the cooled bottom wall, defined as $Nu_c = h_c W/k_c$, with the average coefficient of convection of the cooled surface quantified as $h_c = q_c/[W(t_h - t_c)]$ with $q_c = -k_c \int_0^W \left.\frac{\partial t}{\partial y}\right|_{y=0} dx$, can be evaluated by way of the following empirical correlation with a 4.5% standard deviation of error:

$$Nu_c = 0.12\, Ra_W^{0.26} \theta_h^{-0.85} (L/W)^{0.01} (\mu_h / \mu_c)^{1.44}$$

for $5 \times 10^4 \leq Ra_W \leq 5 \times 10^6$, $0.6 \leq \theta_h \leq 0.87$, $0.2 \leq L/W \leq 1$, (3)

in which μ_h and μ_c are the values of the dynamic viscosity of water at temperatures t_h and t_c, respectively. Of course, since once an asymptotic solution is reached the heat transfer rates at the bottom and top walls are the same, i.e., $h_c W(t_h - t_c) = h_h L(t_h - t_c)$, the average coefficient of convection of the heated surface can be calculated as $h_h = h_c W / L$.

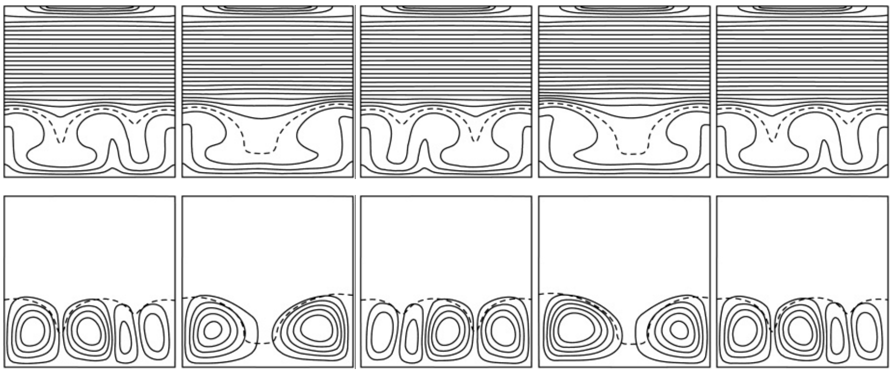

Fig. 4. Time evolution of the isotherms and streamlines along one period of oscillation for W = 0.05 m, t_h = 30°C and L / W = 0.5.

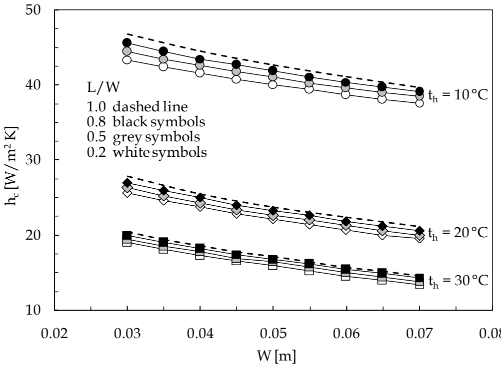

Fig. 5. Distributions of h_c versus W using t_h and L / W as parameters.

4. Conclusions

A study of natural convection in water-filled square enclosures having the bottom wall cooled at 0°C, and the top wall partially or entirely heated at a temperature in the range 10°C–30°C has been executed numerically using a code based on the SIMPLE-C algorithm, in the hypothesis of temperature-dependent physical properties, for cavity widths in the range 1 cm–10 cm. The basic heat and momentum transfer features have been pointed out, and a pair of correlations have been proposed for the calculation of the dimensionless penetration depth and the Nusselt number of the cooled bottom wall.

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Biography

Alessandro Quintino is a post-doctoral student at "Sapienza" University of Rome, Italy. He has a MS degree in Aeronautical Engineering and a PhD in Thermal Sciences.