Letters

Ground Transient Resistance of Underground Cables

Rodolfo Araneo and Salvatore Celozzi

Abstract—**During transients involving multiconductor lines, the importance of the ground finite conductivity is well known and various techniques and expressions have been presented in literature for the inclusion of its contribution into the per unit length parameters. The direct time domain approach based on the introduction of the transient parameters and on the numerical solution of the telegrapher's equations demonstrated to be accurate and efficient for the analysis of typical transients. In this letter, the expressions for the ground transient resistance for underground cables, based on the closed-form inverse Laplace transform of the classical Pollaczek expressions (valid for the low-frequency-range), are presented and discussed.**

Index Terms—**Pollaczeck integral, time-domain methods, transient resistance, underground cables.**

I. Introduction

SINCE its introduction, the method proposed in [1] for the inclusion of losses into a direct time domain (TD) formulation of telegrapher's equations demonstrated to be versatile and accurate for the transient analysis of transmission lines (TL) in various configurations. Without entering into the well-established issue concerning the validity of the TL approximation, in the following the telegrapher's equations are considered to be valid.

A number of extensions have been proposed, either concerning the expressions of the transient resistances and the numerical solution of the TD-TL equations [2]–[9], but always with reference to overhead lines.

II. Discussion on the Different Expressions for the Transient Resistance

For the underground cables buried in a homogeneous soil and shown in Fig. 1, the self and mutual ground impedance can be computed, in the low-frequency range, according to the

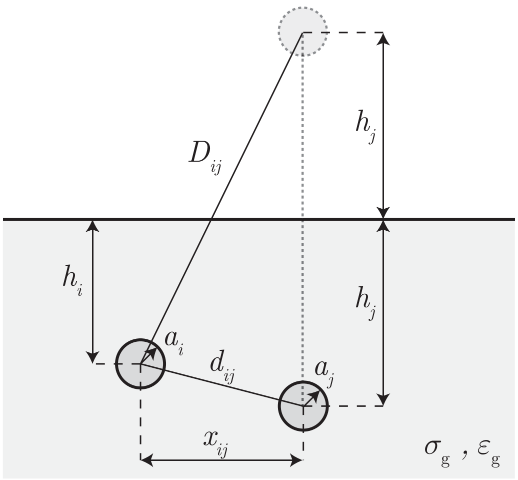

Fig. 1. Geometrical configuration of an underground multiconductor TL.

Pollaczek's [10], [11] expressions as

$$z_{\rm ii}(\omega) = \frac{j\omega\mu_0}{2\pi}\left[\mathrm{K}_0(\gamma_{\rm g} a_i) - \mathrm{K}_0\left(\gamma_{\rm g}\sqrt{a_i^2+4h_i^2}\right) + 2\int_0^\infty \frac{e^{-2h_i\sqrt{\lambda^2+\gamma_{\rm g}^2}}}{\lambda+\sqrt{\lambda^2+\gamma_{\rm g}^2}}\cos(a_i\lambda)\,\mathrm{d}\lambda\right] \quad (1a)$$

$$z_{\rm ij}(\omega) = \frac{j\omega\mu_0}{2\pi}\left[\mathrm{K}_0(\gamma_{\rm g} d_{\rm ij}) - \mathrm{K}_0(\gamma_{\rm g} D_{\rm ij}) + 2\int_0^\infty \frac{e^{-H_{\rm ij}\sqrt{\lambda^2+\gamma_{\rm g}^2}}}{\lambda+\sqrt{\lambda^2+\gamma_{\rm g}^2}}\cos(x_{\rm ij}\lambda)\,\mathrm{d}\lambda\right] \quad (1b)$$

where $j=\sqrt{-1}$, ω is the radian frequency, $\mathrm{K}_0(x)$ is the modified Bessel function of the second kind and order 0, and

$$H_{\rm ij} = h_i + h_j \quad (2a)$$

$$d_{\rm ij} = \sqrt{x_{\rm ij}^2 + (h_i - h_j)^2} \quad (2b)$$

$$D_{\rm ij} = \sqrt{x_{\rm ij}^2 + (h_i + h_j)^2} \quad (2c)$$

and a_i is the radius of the ith conductor (cable), $\mu_g = \mu_0$ and σ_g are, respectively, the ground magnetic permeability and conductivity, $\gamma_{\rm g} = \sqrt{j\omega\mu_g\sigma_g}$ is the propagation constant where a low-frequency approximation has been introduced. It should be observed that a general expression for the resistance of

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underground cables can be derived by (1b) with the assumption $x_{\mathrm{ii}} = a_i$ in the case of the self-resistance. Hence, only this expression will be considered in the following without loss of generality. The last term of (1b) is the so-called Pollaczek's integral J_{ij}

$$J_{\mathrm{ij}} = \frac{j\omega\mu_0}{2\pi}\left[\int_0^\infty \frac{e^{-H_{\mathrm{ij}}\sqrt{\lambda^2+\gamma_{\mathrm{g}}^2}}}{\lambda+\sqrt{\lambda^2+\gamma_{\mathrm{g}}^2}}\cos(x_{\mathrm{ij}}\lambda)\,\mathrm{d}\lambda\right] \tag{3}$$

whose numerical integration is difficult because of the highly oscillating behavior. An accurate approximation in the high frequency range was proposed by Sunde [12] who did not neglect the displacement currents in the propagation constant $\gamma_{\mathrm{gex}} = \sqrt{j\omega\mu_g(\sigma_g + j\omega\varepsilon_{\mathrm{g}})}$. More complex ground impedance expressions were derived independently by Wait [13], Bridges [14], and Chen [15] based on a rigorous electromagnetic approach where the propagation wavenumbers of the modes supported by the buried cable are computed by solving a nonlinear frequency-dependent eigenvalue problem. Holding the TL approximation, it has been showed [16] that the Wait's ground impedance is equivalent to the Sunde's expression. Anyway, some comments about the suitability of the Sunde's model in a TL approach were raised in [17]: it has been observed that in the high-frequency region, especially for higher earth resistivities, the Sunde's model yields to propagation values higher than those of the propagation constant of the earth, a result that is not in full agreement with the TL approximation where only the fundamental quasi-TEM mode is assumed to propagate.

In the past, several closed-form approximations and analytical/numerical schemes have been proposed to compute the frequency-dependent ground impedance, from which the transient ground resistance can be derived by performing the inverse Fourier transform (IFT) numerically. A review of the literature reveals a large number of closed form approximations, namely, in the best knowledge of the authors, Vance [18], Semlyen [19], Saad [20], Wedepohl and Wilcox [21], Bridges [14], Wait [13], Alvarado [22], Petrache [23], and Theethayi [16]. Furthermore, several numerical approaches have been proposed to efficiently compute the Pollaczek's intregral by Uribe [24], [25] and Legrand [26]. It should be noted that the inclusion of the ground contribution in the cable model [27], [28] may yield to discontinuities in the p.u.l. impedances frequency trends [29], [30]. Furthermore, the presence of a multilayer soil has been considered in [31].

From (1b), the transient ground resistance $\zeta_{\mathrm{g,ij}}(t)$ can be derived as the IFT of $z_{\mathrm{ij}}(\omega)/j\omega$. Two approximations have been proposed in literature for the transient ground resistance $\zeta_{\mathrm{g,ij}}(t)$ directly in the TD, for a single-conductor configuration in a homogeneous soil.

The first approximation can be carried out neglecting γ_{g} in the exponential factor at the numerator of the Pollaczek's integral in (3) so that the integral becomes similar to the Carson integral for the ground resistance of an overhead line.

Following the analytical steps reported in [9] for an overhead line, an approximated transient ground resistance may be derived as

$$\begin{aligned}\zeta_{\mathrm{g,ij}}(t) &= \frac{\mu_0}{2\pi}\left[\frac{\exp\left(-\frac{\tau_{\mathrm{d}}}{4t}\right)-\exp\left(-\frac{\tau_{\mathrm{D}}}{4t}\right)}{2t}\right]\\
&+\frac{1}{2}\left\{\frac{\mu_0}{\pi\tau_{\mathrm{ij}}^*}\left[\frac{1}{2\sqrt{\pi}}\sqrt{\frac{\tau_{\mathrm{ij}}^*}{t}}+\frac{1}{4}\exp\left(\frac{\tau_{\mathrm{ij}}^*}{t}\right)\mathrm{Erfc}\left(\frac{\tau_{\mathrm{ij}}^*}{t}\right)-\frac{1}{4}\right]\right.\\
&\left.+\frac{\mu_0}{\pi\tau_{\mathrm{ij}}}\left[\frac{1}{2\sqrt{\pi}}\sqrt{\frac{\tau_{\mathrm{ij}}}{t}}+\frac{1}{4}\exp\left(\frac{\tau_{\mathrm{ij}}}{t}\right)\mathrm{Erfc}\left(\frac{\tau_{\mathrm{ij}}}{t}\right)-\frac{1}{4}\right]\right\}\end{aligned} \tag{4}$$

where $\mathrm{Erfc(x)}$ is the complementary error function, and

$$\begin{aligned}\tau_{\mathrm{ij}} &= \mu_0\sigma_{\mathrm{g}}\left(\frac{H_{\mathrm{ij}}}{2}+j\frac{x_{\mathrm{ij}}}{2}\right)^2 \qquad \tau_{\mathrm{ij}}^* = \mathrm{conj}\left(\tau_{\mathrm{ij}}\right)\\
\tau_{\mathrm{d}} &= \mu_0\sigma_{\mathrm{g}}d_{\mathrm{ij}}^2 \qquad \tau_{\mathrm{D}} = \mu_0\sigma_{\mathrm{g}}D_{\mathrm{ij}}^2.\end{aligned} \tag{5}$$

The second approximation was proposed by Petrache *et al.* [23] and it is based on the analogy with overhead lines. Replacing the height h of the overhead conductor with the radius a_i of the underground cable, the transient ground resistance of the overhead line under low-frequency approximation can be adapted to the case of a single underground cable as

$$\zeta_{\mathrm{g}}(t) = \frac{\mu_0}{\pi\tau_{\mathrm{g}}}\left[\frac{1}{2\sqrt{\pi}}\sqrt{\frac{\tau_{\mathrm{g}}}{t}}+\frac{1}{4}\exp\left(\frac{\tau_{\mathrm{g}}}{t}\right)\mathrm{Erfc}\left(\frac{\tau_{\mathrm{g}}}{t}\right)-\frac{1}{4}\right] \tag{6}$$

where $\tau_{\mathrm{g}} = \mu_0\sigma_{\mathrm{g}}a_i^2$. The accuracy of the proposed approximation was investigated comparing its predictions with the IFT of the frequency domain data obtained by means of the logarithmic approximation proposed by Petrache *et al.* in [23] considering one set of ground and cable parameters.

III. Exact Solution and Numerical Results

Recently, Theodoulidis [32] proposed three different exact solutions of the Pollaczek's integral (3) in the form of converging series: The first two are given in terms of Bessel functions of half and odd integer order while the third one is given in terms of confluent hypergeometric functions. The third solution shows a better converging behavior for all the parameter values than the first two that exhibit fast convergence only for certain parameter ranges. After some algebraic manipulations, Theodoulidis showed that his third solution is based on an expression equivalent to the expression previously proposed by Wedepohl and Wilcox [21] which is here selected to provide the most convenient form to compute the Pollaczek's integral.

Using transformation of variables and Cauchy's theorem, the ground impedance of the underground cable can be rewritten according to [21] in the frequency domain as

$$\begin{aligned}z_{\mathrm{ij}}(\omega) &= \frac{j\omega\mu_0}{2\pi}\left\{\mathrm{K}_0\left(\gamma_{\mathrm{g}}d_{\mathrm{ij}}\right)+\left[2\chi_{\mathrm{H}}^2-1\right]\mathrm{K}_0\left(\gamma_{\mathrm{g}}D_{\mathrm{ij}}\right)\right.\\
&+\frac{2}{\gamma_{\mathrm{g}}D_{\mathrm{ij}}}\left[2\chi_{\mathrm{H}}^2-1\right]\mathrm{K}_1\left(\gamma_{\mathrm{g}}D_{\mathrm{ij}}\right)+\\
&-\frac{2}{\gamma_{\mathrm{g}}^2}\frac{H_{\mathrm{ij}}^2-x_{\mathrm{ij}}^2}{D_{\mathrm{ij}}^4}\exp\left(-\gamma_{\mathrm{g}}H_{\mathrm{ij}}\right)\left(1+\gamma_{\mathrm{g}}H_{\mathrm{ij}}\right)+\\
&\left.-\frac{2x_{\mathrm{ij}}H_{\mathrm{ij}}}{D_{\mathrm{ij}}^2}\int_{\chi_{\mathrm{H}}}^1\left(2\sqrt{1-\lambda^2}-\frac{1}{\sqrt{1-\lambda^2}}\right)\mathrm{e}^{-\lambda\gamma_{\mathrm{g}}D_{\mathrm{ij}}}\,\mathrm{d}\lambda\right\}\end{aligned} \tag{7}$$

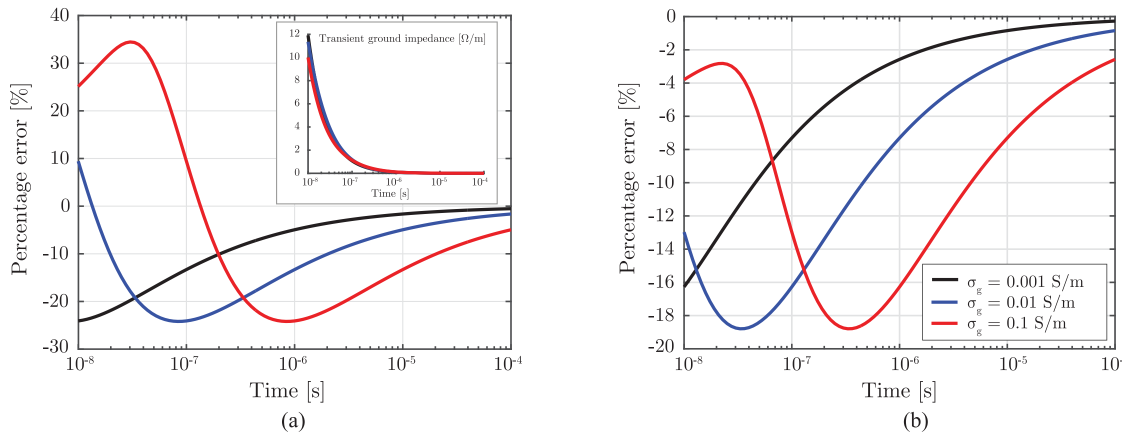

Fig. 2. Percentage error of the Carson-like approximation equation. (a) and Petrache *et al.* equation. (b) with respect to the exact one for different values of the ground conductivity ($h_1 = 1$ m, $a_1 = 1.5$ cm, $\varepsilon_g = 10$). The inset of figure (a) shows the trend of the transient ground resistance ζ_{11} versus time t for the three ground conductivities.

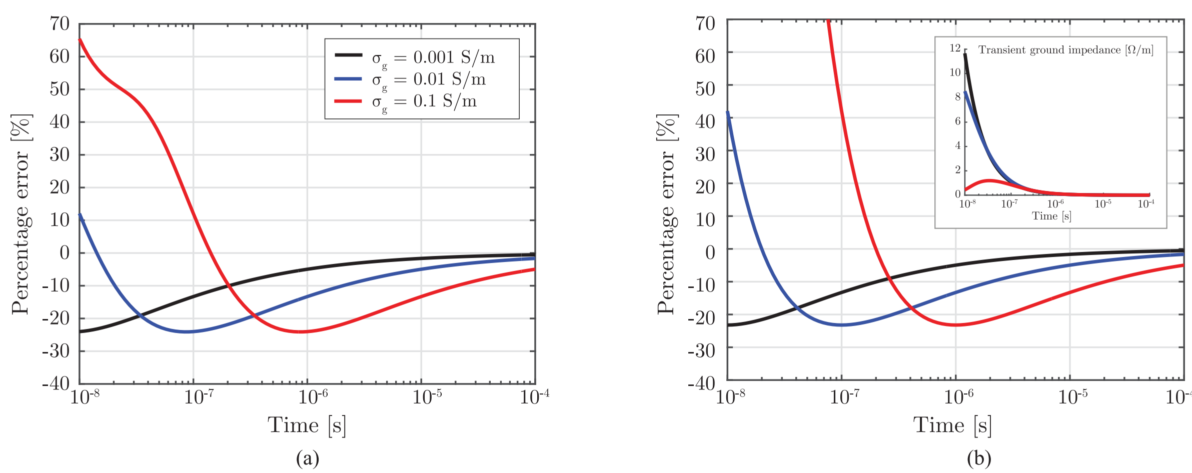

Fig. 3. Percentage error of the Carson-like approximation equation with respect to the exact one, for different values of the ground conductivity and mutual distance between conductors ($h_1 = h_2 = 1$ m, $a_1 = a_2 = 1.5$ cm, $\varepsilon_g = 10$). The inset of figure (b) shows the trend of the transient ground resistance ζ_{12} versus time t for the three ground conductivities in the case of $x_{12} = 1$ m.

where $\chi_{\mathrm{H}} = H_{\mathrm{ij}}/D_{\mathrm{ij}}$ and $\mathrm{K}_1(x)$ is the modified Bessel function of the second kind and order 1. Starting from this expression, we apply the Laplace transform to pass from the frequency domain to the TD. After some algebraic manipulations, the final exact expression of the transient ground resistance under a low-frequency approximation reads

$$\begin{aligned}&\zeta_{\mathrm{g,ij}}(t) = \\ &\frac{\mu_0}{2\pi}\left\{\frac{\exp\left(-\frac{\tau_{\mathrm{d}}}{4t}\right)}{2t} + \left[2\chi_{\mathrm{H}}^2 - 1\right]\exp\left(-\frac{\tau_{\mathrm{D}}}{4t}\right)\left(\frac{1}{2t} + \frac{2}{\tau_{\mathrm{D}}}\right) + \right.\\ &-\frac{4\left(\chi_{\mathrm{H}}^2 - \chi_{\mathrm{x}}^2\right)}{\sqrt{\pi}\tau_{\mathrm{D}}}\left[\sqrt{\frac{\tau_{\mathrm{H}}}{4t}}\exp\left(-\frac{\tau_{\mathrm{H}}}{4t}\right) + \frac{\sqrt{\pi}}{2}\mathrm{Erfc}\left(\sqrt{\frac{\tau_{\mathrm{H}}}{4t}}\right)\right] + \\ &-\frac{8x_{\mathrm{ij}}H_{\mathrm{ij}}}{\sqrt{\pi}D_{\mathrm{ij}}^2}\frac{1}{\tau_{\mathrm{D}}}\left[\sqrt{\frac{\tau_{\mathrm{x}}}{4t}}\exp\left(-\frac{\tau_{\mathrm{H}}}{4t}\right) + \right.\\ &\left.\left.-\frac{\sqrt{\pi}}{2}\left(1 + \frac{\tau_{\mathrm{D}}}{4t}\right)\exp\left(-\frac{\tau_{\mathrm{D}}}{4t}\right)\mathrm{Erfi}\left(\sqrt{\frac{\tau_{\mathrm{x}}}{4t}}\right)\right]\right\}\end{aligned} \tag{8}$$

where

$$\begin{aligned}\chi_{\mathrm{x}} &= \frac{x}{D} \quad \tau_{\mathrm{H}} = \mu_0\sigma_{\mathrm{g}}H_{\mathrm{ij}}^2 \quad \tau_{\mathrm{d}} = \mu_0\sigma_{\mathrm{g}}d_{\mathrm{ij}}^2 \\ \tau_{\mathrm{D}} &= \mu_0\sigma_{\mathrm{g}}D_{\mathrm{ij}}^2 \quad \tau_{\mathrm{x}} = \mu_0\sigma_{\mathrm{g}}x_{\mathrm{ij}}^2.\end{aligned} \tag{9}$$

It should be observed that the imaginary error function $\mathrm{Erfci}(x)$, despite its name, is a real function when the argument is real. Expression (8) can be recast in a form more suitable for numerical computation, as

$$\begin{aligned}&\zeta_{\mathrm{g,ij}}(t) = \\ &\frac{\mu_0}{2\pi}\left\{\frac{\exp\left(-\frac{\tau_{\mathrm{d}}}{4t}\right)}{2t} + \left[2\chi_{\mathrm{H}}^2 - 1\right]\mathrm{e}^{-\frac{\tau_{\mathrm{D}}}{4t}}\left(\frac{1}{2t} + \frac{2}{\tau_{\mathrm{D}}}\right) + \right.\\ &-\frac{4\left(\chi_{\mathrm{H}}^2 - \chi_{\mathrm{x}}^2\right)}{\sqrt{\pi}\tau_{\mathrm{D}}}\mathrm{e}^{-\frac{\tau_{\mathrm{H}}}{4t}}\left[\sqrt{\frac{\tau_{\mathrm{H}}}{4t}} + \frac{\sqrt{\pi}}{2}\mathrm{e}^{\frac{\tau_{\mathrm{H}}}{4t}}\mathrm{Erfc}\left(\sqrt{\frac{\tau_{\mathrm{H}}}{4t}}\right)\right] + \\ &\left.-\frac{8x_{\mathrm{ij}}H_{\mathrm{ij}}}{\sqrt{\pi}D_{\mathrm{ij}}^2}\frac{1}{\tau_{\mathrm{D}}}\mathrm{e}^{-\frac{\tau_{\mathrm{H}}}{4t}}\left[\sqrt{\frac{\tau_{\mathrm{x}}}{4t}} - \left(1 + \frac{\tau_{\mathrm{D}}}{4t}\right)\mathrm{D}_+\left(\sqrt{\frac{\tau_{\mathrm{x}}}{4t}}\right)\right]\right\}\end{aligned} \tag{10}$$

where $\mathrm{D}_+(x)$ is the Dawson's function, implemented in the most popular mathematical packages and available in the IMSL Numerical Libraries for C or Fortran programming. In addition, it is convenient—from a numerical point of view—to compute the exponentially scaled complementary error function $\mathrm{e}^{x^2}\,\mathrm{Erfc}(x)$.

Figs. 2 and 3 show the percentage relative error of the Carson-like approximation (4) and Petrache *et al.* approximation (6) with respect to the exact expression (10). It should be noted that the curves of the errors of the two approximate expressions considered in literature exhibit worse results in the early time and completely miss the value $\zeta(0)$ that, however, is very important when a recursive convolution algorithm is implemented. Thus, the availability of the analytical expressions results in a dramatic improvement of the overall accuracy on the transient waveforms.

IV. Conclusion

The ground transient resistance of underground cables has been computed analytically by means of an inverse Laplace transform of the Pollaczek low-frequency impedances, proper and mutual terms. In this way, the TD-TL equations in [1] may be considered without the various approximations proposed in literature and, furthermore, multiconductor configurations may be analyzed as well.

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