High-Reynolds-number effects on turbulent scalings in compressible channel flow

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The effect of the Reynolds number in a supersonic isothermal channel flow is studied using a direct numerical simulation (DNS). The bulk Mach number based on the wall temperature is 1.5, and the bulk Reynolds number is increased up to $Re_\tau \approx 1000$. The use of van Driest velocity transformation in the presence of heated walls has been questioned due to the poor accuracy at low Reynolds number. For this reason alternative transformations of the velocity profile and turbulence statistics have been proposed, as, for instance, semi-local scalings. We show that the van Driest transformation recovers its accuracy as the Reynolds number is increased. The Reynolds stresses collapse on the incompressible ones, when properly scaled with density, and very good agreement with the incompressible stresses is found in the outer layer.

1 Introduction

The effect of the compressibility on turbulence has been classified into two types. The first effect consist in the obvious mean density and temperature variations. The second is due to the density and temperature fluctuations. Morkovin [1] postulated that in non-hypersonic flows (Mach < 5) density and temperature fluctuations are negligible and for this reason the mean profiles are expected to scale to the incompressible ones when mean density variations are properly taken into account. The channel flow is a good candidate to study wall turbulence, because it is homogeneous in two direction, thus avoiding the uncertainties of the boundary conditions. Many studies of compressible turbulent channel flows have appeared [2]- [3], revealing that, even if density and temperature fluctuations are small, the turbulent statistics do not collapse to their incompressible counterparts also when density variations are taken into account. For this reason other scalings have been proposed [2]- [4]. Huang [2] proposed to use $y^* = \rho u_\tau^* y/\overline{\mu}$, with $u_\tau^* = \sqrt{\tau_w/\overline{\rho}}$ instead of $y^+ = \rho_w u_\tau y/\mu_w$, with $u_\tau = \sqrt{\tau_w/\rho_w}$, to scale the turbulent stresses. An integral Reynolds number, $Re_\tau^c = y^{c+} = \int_0^{y^+} \mu_w/\mu \mathrm{d}y^+$, has been also proposed by Brun [4], with a corresponding transformation for the velocity and the Reynolds stresses. To get some additional insight, we will go through all the different transformations, and understand the effect of the Reynolds number on their accuracy, if any. For that purpose a series of DNS (see table 1) is performed at bulk Mach number $M_b = u_b/a_w = 1.5$, where u_b is the bulk channel velocity and a_w the speed of sound at the wall.

Table 1: Test cases set-up. $M_b = 1.5$. Isothermal walls. Box dimensions $6\pi H \times 2H \times 2\pi H$. N_x, N_y and N_z indicate the number of points in the streamwise, wall normal and spanwise direction respectively. Δx^+, Δy^+ and Δz^+ are the mesh spacings in inner units.

Case	Re_b	Re_τ	${Re_\tau}^*$	${Re_\tau}^c$	N_x	N_y	N_z	Δx^+	Δy^+	${\Delta_z}^+$
CH1	3000	215	141	169	512	128	256	8	0.4-5.6	5.2
CH2	7667	500	395	500	1024	256	512	9.2	0.6-6	6.1
CH3	17000	1015	677	802	2048	512	1024	9.3	0.77-5.7	6.2

2 Compressible Scalings

The van Driest transformation for velocity can be derived from the mean momentum balance using the mixing length hypothesis, thus yielding

$$u_{vd} = \int_0^{\overline{u}} \left(\frac{\overline{\rho}}{\overline{\rho}_w}\right)^{1/2} \mathrm{d}\overline{u}. \tag{1}$$

In the presence of strong wall-cooling, viscosity variations must be taken into account [4] and reasoning on the viscous sublayer it was obtained,

$$u^c = \int_0^{\overline{u}} \frac{y^+}{y^c} \frac{\overline{\mu_w}}{\overline{\mu}} \sqrt{\frac{\overline{\rho}}{\overline{\rho}_w}} d\overline{u}, \tag{2}$$

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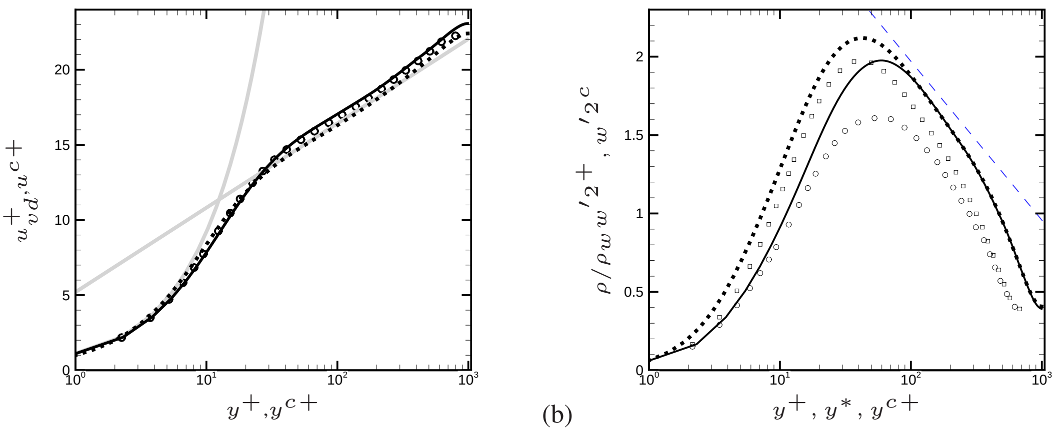
Figure 1a shows a comparison between van Driest and Brun transformation for velocity compared to the incompressible dataset of [5], for flow case CH3 (see table 1). At sufficiently high Reynolds number the van Driest transformation recovers its accuracy. Only minor differences are observed with respect to the transformation of Brun [4] for flow case CH3. Figure 1b shows the variances of w, spanwise velocity component. The variance transformed according to Huang Brun and van Driest is compared to the incompressible profile. A very good agreement is recovered for the van Driest (density based) transformation. The formation of a universal logarithmic region for the density scaled fluctuations of w is also evident in this case. Good agreement with the incompressible variance in the viscous region is found when using the semi-local scaling, while Brun's transformation shows similar results as the density based one in this region.



(a) (b)

Fig. 1: Transformed velocity profiles(a) and spanwise velocity variance(b) for flow case CH3. van Driest transformation(solid), Brun's scaling(circles), Huang semi-local scaling(squares), compared to the incompressible dataset [5]. Gray thick solid line in panel (a) indicates the log-law $u^+ = 1/0.41 \log(y^+) + 5.2$, and the blue dashed line in panel(b) the logarithmic layer.

3 Conclusion

The effect of the Reynolds number on the turbulent scalings for a compressible isothermal channel has been studied. The van Driest transformation stems from the mean momentum balance, specified for the outer layer and it becomes accurate only if the Reynolds number is sufficiently high. The van Driest transformation has a more solid physical background with respect to Brun's, because it is derived from the flow equations. The introduction of a semi-local scaling y^*, instead of y^+, for the Reynolds stresses permits an improvement of the collapsing in the viscous region, in accordance to what has already been observed at lower Reynolds number [2], even if its derivation remains unclear, and its use is mostly based on empiricism. The application of Brun's scaling to the turbulent stresses does not show any improvement in the viscous region with respect to the standard mean density scaling.

References

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