Numerical and experimental assessment of the modal curvature method for damage detection in plate structures

Francesco Mosti, Giuseppe Quaranta, and Walter Lacarbonara

Abstract Use of modal curvatures obtained from modal displacement data for damage detection in isotropic and composite laminated plates is addressed through numerical examples and experimental tests. Numerical simulations are carried out employing COMSOL Multiphysics as finite element method-based solver of the equations governing the Mindlin-Reissner plate model. Damages are introduced as localized non-smooth variations of the bending stiffness of the baseline (healthy) configuration. Experiments are also performed on steel and aluminum plates using scanning laser vibrometry. The obtained results confirm that use of the central difference method to compute modal curvatures greatly amplifies the measurement errors and its application leads to unreliable predictions for damage detection, even after denoising. Therefore, specialized *ad hoc* numerical techniques must be suitably implemented to enable structural health monitoring via modal curvature changes. In this study, the Savitzky-Golay filter (also referred to as least-square smoothing filter) is considered for the numerical differentiation of noisy data. Numerical and experimental results show that this filter is effective for the reliable computation of modal curvature changes in plate structures due to defects and/or damages.

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1 Introduction

Vibration-based damage detection techniques have an advantage associated with their global approach by which faults within a mechanical system can be identified without *a priori* information about their location and regardless of their accessibility. Moreover, automatic real-time vibration-based structural health monitoring (SHM) systems can be effectively implemented. In this context, approaches based on changes of the modal characteristics of the structure induced by damages are widely used. While the validity of modal damping as damage index is still a controversial topic, variations of natural frequencies and mode shapes have been largely exploited to assess the occurrence of damage. Natural frequencies are particularly attractive for damage identification because they can be estimated from a few measurement points and are usually contaminated by small levels of noise, however, their sensitivity to damage is rather poor. Changes in mode shapes yield local information which turn out to be more suitable for damage localization than variations of the natural frequencies, provided that a sufficient number of measurement points is acquired. Notwithstanding the mentioned desirable features, several studies have shown that modal displacements, as expected, are not very sensitive to faults.

Conversely, changes in the modal curvatures with respect to those of the baseline (healthy) structure lead to a more effective definition and computation of damage indices. In [14] it was observed that the localized occurrence of a spike in the function obtained by subtracting the modal curvature of the undamaged structure from that of the damaged situation is an indicator of the damage location. Therefore, in view of practical applications, the main issue is concerned with the processing of the modal curvatures, a task that can be accomplished by means of direct measurements or via numerical methods. The possibility of directly measuring modal curvatures using optical fibre strain sensors was discussed in [6], but most of the current applications are based on the extraction of modal displacements from dynamic measurements. To this end, the use of accelerometers, electronic speckle pattern interferometry or scanning laser vibrometry is frequent. In this scenario, the whole reliability of the damage detection procedure largely depends on the accuracy of the numerical differentiation procedure. Attention has to be paid on the calculation of modal curvatures because differentiation of noisy data is well known to be an ill-posed problem. The central difference method is certainly the most popular tool for estimating the curvatures from modal displacements, see for instance [1, 2, 7, 12]. However, some studies have shown that errors amplification may become so large as to make the central difference method inappropriate for reliable damage localization [2]. As a consequence, some recent attempts have been made to circumvent the difficulties that can be found when implementing damage detection techniques based on numerically obtained modal curvatures. They include the use of wavelets [8, 4], Gaussian function derivatives [13], Laplace operator [5], smoothing spline and Savitzky-Golay filter [16].

The present contribution is part of a larger effort aimed at assessing the effectiveness of damage detection by modal curvatures as well as at improving their reliability for practical SHM applications. This chapter is concerned with the use of modal curvatures for damage detection in thin plate structures. Along these lines, damage detection in composite laminated plates based on modal curvatures has been discussed in [15]. Bending modal curvatures were calculated using the central difference method, and dynamic measurements were acquired by using a scanning laser vibrometer (SLV). Curvatures obtained from measured modal displacements were also recently considered to identify defects in composite T-stiffened panels [10] .

In the present work, an extensive numerical study is conducted for isotropic and composite laminated plates. Simulations are performed using COMSOL Multiphysics. The Mindlin-Reissner plate model is implemented and defects are introduced as localized non-smooth variations of the bending stiffness of the baseline configuration. Experiments are also performed on steel and aluminum plates using SLV. Although the central difference method is often preferred to calculate the modal curvatures, our study confirms that it greatly amplifies measurement errors. Its final outcomes are ineffective for damage detection, even if measurement errors are as low as possible and denoising is performed. On the contrary, numerical and experimental investigations demonstrate that the Savitzky-Golay filter yields reliable predictions of modal curvatures changes for practical SHM applications.

2 Damage detection by modal curvatures

We assume that noisy modal displacements ϕ_i are available at positions (x_i, y_j), and the measurement points along a given y coordinate are taken to be spaced by the constant sampling distance Δx. The problem is concerned with the determination of the second derivative with respect to x denoted by $\phi''(x_i)$, i.e., a suitable estimate of the exact modal curvature at x_i about direction $y = y_j$ (note that this is the exact linear bending curvature within the Kirchhoff-Love plate theory whereby the transverse shear strains are neglected). Therefore, the 2D numerical differentiation problem is reduced to multiple 1D problems, as it was done in previous works [15]. In the present work, two numerical differentiation techniques are considered, namely, the standard central difference approximation and the Savitzky-Golay filter. The methods described below apply to all targeted mode shapes and plate configurations (undamaged or damaged). Any reference to the mode number and to the structural configuration is, therefore, omitted for sake of conciseness.

The most popular approach for obtaining curvatures via numerical differentiation from (displacement) mode shapes ϕ is based on the central difference approximation. For the ith point, the central difference method yields the following expression:

$$\phi''(x_i) = \frac{\phi_{i-1} - 2\phi_i + \phi_{i+1}}{\Delta x^2}. \quad (1)$$

The original Savitzky-Golay filter [17] (also referred to as least-square smoothing filter) and its variants represent an important class of local methods for numerical differentiation of noisy data [3]. In their seminal paper, Savitzky and Golay demonstrated that fitting a polynomial to a set of input samples and then evaluating the obtained polynomial at a single point within the approximation interval is equivalent to discrete convolution with a fixed impulse response. The key derivation steps are summarized next for a better appreciation of the proposed application to damage detection. A set of $(2m + 1)$ consecutive samples is considered together with a local coordinate system, i.e., $q \in \{-m, \ldots, 0, \ldots, +m\}$. The lth-order least-square polynomial is represented by

$$f(q) = \sum_{r=0}^{l} b_r q^r. \tag{2}$$

The Savitzky-Golay approach applies Eq. 2 at the midpoint only ($q = 0$) whereas the value of the output at the next sample is obtained by shifting the analysis interval to the right by one sample and repeating the procedure at the new midpoint. The sth-order derivative of $f(q)$ in Eq. 2 evaluated at $q = 0$ only requires the expression for b_s. The central sth-order derivative of the polynomial form in Eq. 2 can also be expressed as

$$\phi^s(0) = \sum_{q=-m}^{m} h_q^s \phi_q \tag{3}$$

in which h_q^s is the convolution weight of the qth filter point. Instead of considering a power series, the approach developed in [9] is based on use of discrete orthogonal polynomials, whereby the Gram polynomials turn out to be particularly suitable to this case. In doing so, the sth-order derivative at any point ξ is obtained by using the following expression:

$$\phi^s(\xi) = \frac{1}{(\varDelta x)^s} \sum_{q=-m}^{m} h_q^{\xi,s} \phi_q \tag{4}$$

where the scale factor is needed when the original coordinate system is considered (i.e., the points are separated by $\varDelta x \neq 1$). The convolution weight for ϕ_q with $-m \leq q \leq m$ has the form:

$$h_q^{\xi,s} = \sum_{r=0}^{l} \frac{(2r+1)(2m)^{(r)}}{(2m+r+1)^{(r+1)}} P_r^m(q) P_r^{m,s}(\xi), \tag{5}$$

where

$$P_r^m(\xi) = \sum_{a=0}^{r} \frac{(-1)^{a+r}(a+r)^{(2a)}(m+\xi)^{(a)}}{(a!)^2(2m)^{(a)}} \tag{6}$$

and

$$P_r^{m,s}(q) = \frac{2(2r-1)}{r(2m-r+1)} \left[q P_{r-1}^m(q) + s P_{r-1}^{m,s-1}(q) \right] - \frac{(r-1)(2m+r)}{r(2m-r+1)} P_{r-2}^{m,s}(q) \tag{7}$$

are the rth-order Gram polynomial and its sth-order derivative, respectively. The calculation of the modal curvature through Eq. 4 is best performed by constructing the table $H^s(\xi, q) = \{h_q^{\xi,s} : -m \leq \xi \leq +m, -m \leq q \leq +m, s = 2\}$ using Eq. 5. This strategy is computationally efficient because the convolution weights do not change, provided that the measurement points as well as m and l do not vary with the mode shape number in both undamaged and damaged situations. Note that the classical Savitzky-Golay approach [17] does not allow the calculation of the modal curvatures at the first m points and at the last m points. By using the Gram polynomial-based strategy developed in [9], the curvatures at the first m points and at the last m points are calculated by using the coefficients of $H^s(\xi, q)$ for ξ from $-m$ to -1 and for ξ from 1 to m, respectively. As in the classical Savitzky-Golay approach, the rest of the n samples uses the center point weighting, which is obtained by setting $\xi = 0$.

Once the experimental modal displacements have been determined, the modal curvatures are numerically obtained and the damage can be identified by comparison. To this end, the modal curvature-based damage index $I(x_i|y = y_j)$ proposed in [1] is

$$I(x_i|y = y_j) = \sum_{k=1}^{n_t} \left| {\phi^0}''_{ik} - \phi''_{ik} \right| \tag{8}$$

where n_t is the number of target modes, ${\phi^0}''_{ik}$ is a numerical estimate of the modal curvature of the kth undamaged mode shape at the ith measurement point x_i about the direction $y = y_j$, and ϕ''_{ik} is the corresponding curvature for the damaged structure. Ho and Ewins [11] (see also [2]) presented the mode shape curvature squared method based on the following proposition of the damage index:

$$I(x_i|y = y_j) = \sum_{k=1}^{n_t} \left| {{\phi^0}''_{ik}}^2 - {\phi''_{ik}}^2 \right| . \tag{9}$$

On the other hand, the following different formulation of the damage index is proposed in [16]:

$$I(x_i|y = y_j) = \left(\sum_{k=1}^{n_t} \left({\phi^0}''_{ik} - \phi''_{ik} \right)^2 \right)^p . \tag{10}$$

Numerical analyses presented in [16] showed that if the numerical estimation of the modal curvatures is sufficiently accurate, then the damage index given by Eq. 10 with $p > 1$ (e.g., $p = 2$) magnifies the distance between peaks due to damage and those that appear in other positions even when noiseless data are considered. This turns out to be beneficial in reducing false positives when noisy data are considered.

3 Numerical and experimental applications

The Mindlin-Reissner plate model is considered wherein damage or defects are introduced as localized non-smooth variations of the bending stiffness of the undamaged original configuration. The finite element software COMSOL Multiphysics is used to perform FE discretization of the equations of motion provided in PDE form. The considered composite plate has four layers with lay-up 0°/45°/90°/-45°.

The elastic constants are: E_1=137.137 GPa, E_2=9.308 GPa, ν_{12}=0.304, ν_{21}=0.017, G_{12}=4.551 GPa, G_{23}=4.206 GPa, and the mass per unit volume is ρ=1568 kg/m^3. A specific damage is introduced by halving the stiffness of the second layer within a region having an extension equal to 4% of the plate side length. A white Gaussian noise was added to the modal displacements so that the final error (about 1%) is able to simulate experimental data after denoising. The damage index is computed according to Eq. 10 with $n_t = 5$ and $p = 1$. The damage index function is then normalized by dividing it by its absolute maximum value. Figure 1 shows a sample of the obtained numerical results for the composite laminated plate.

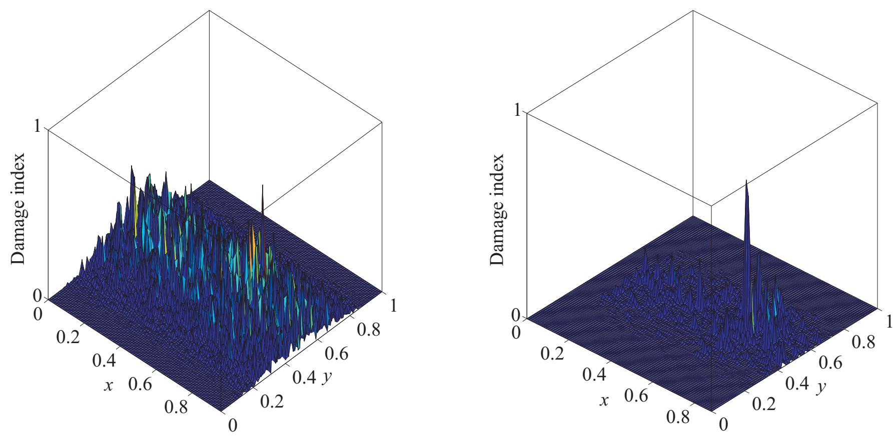

Fig. 1 Damage detection in composite laminated plate based on numerical data (the coordinates of the midpoint of the narrow damaged region are $x = 0.67$ and $y = 0.59$): comparison between central difference method (left) and Savitzky-Golay filter (right).

These results demonstrate that the central difference method does not lead to the proper damage identification because of countless false positives. On the other hand, use of the Savitzky-Golay filter gives a clear definite spike at the exact damage location while reducing the number of peaks due to error amplifications induced by the numerical differentiation procedure.

Experiments were also performed on isotropic plate structures using SLV (see Fig. 2 for an overview of the experimental layout). The results shown in Fig. 3 further confirm that use of the Savitzky-Golay filter for the computation of the modal curvatures provides a satisfactory identification of the damage in thin plate-like structures (the damage index is computed according to Eq. 10 with $p = 1$). The purposefully introduced damage is well identified in both cases, i.e., the maximum values of the damage index (red-colored zone) lie within the region of the plate where the defect is introduced whereas very low values can be found elsewhere. The center of the identified damaged region and that of the defect are roughly coincident for the steel plate. For the aluminum plate, the location of the center is slightly off from the exact one.

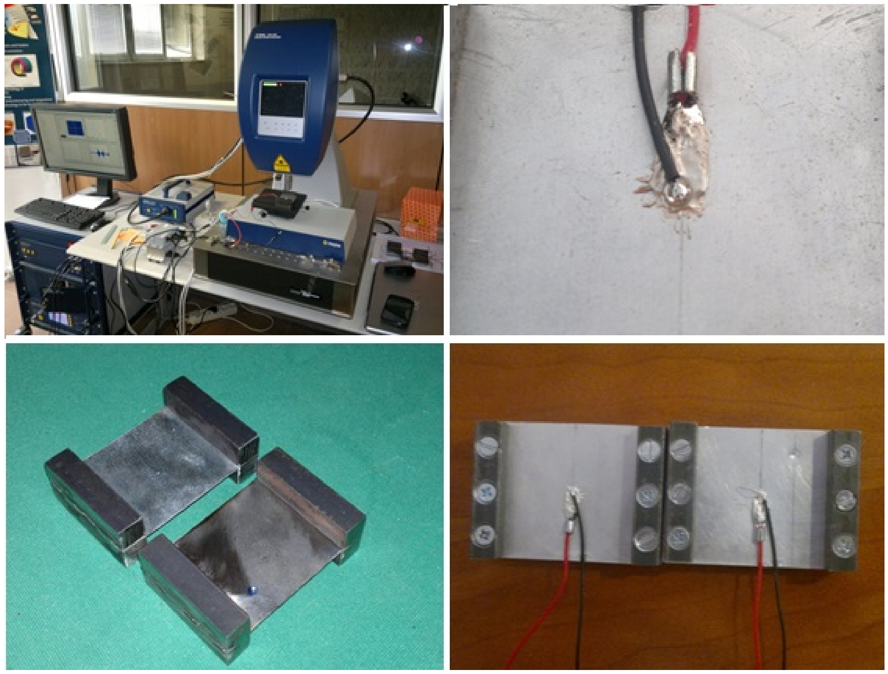

Fig. 2 Experimental layout (clockwise from the upper left corner): Polytec scanning laser vibrometer, piezoelectric actuator, undamaged and damaged 50 mm × 50 mm aluminum and steel plates (thickness equal to 0.5 mm). The plates have clamped boundary conditions on two sides and free conditions on the other two sides. Damage is introduced by halving the plate thickness within a circle having 2 mm diameter and the center about 14 mm far from one corner.

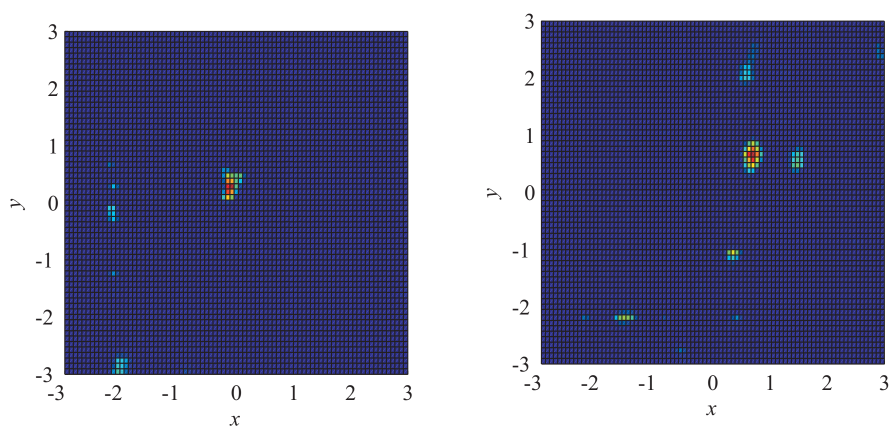

Fig. 3 Damage detection in isotropic plates based on experimental data (the curvatures are computed by the Savitzky-Golay filter over a 50 × 50 grid of points centered at the damaged region): steel plate (left) and aluminum plate (right).

The effectiveness of the Savitzky-Golay filter with respect to the central difference method was also evaluated for defects having different shapes. The experimental results shown in Fig. 4, for instance, are referred to a stretched narrow damaged region (the damage index is here computed according to Eq. 10 with $p = 2$). Results

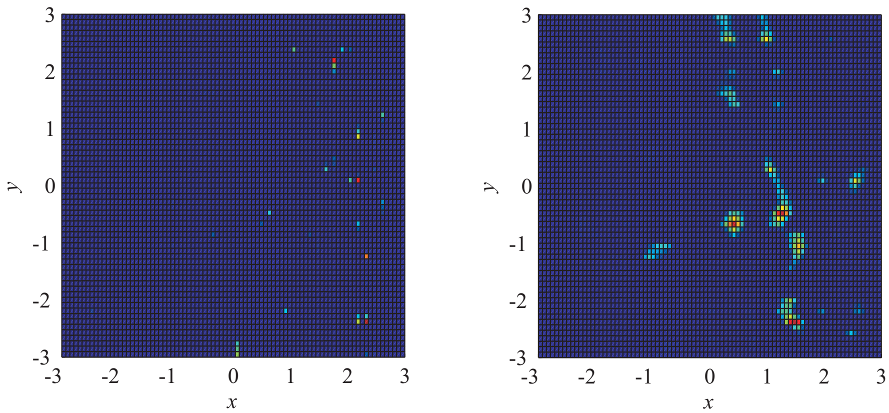

Fig. 4 Detection of a stretched damaged region in isotropic plate based on experimental data (the vertical axis of the damaged region is $x = 1$ and the curvatures are computed over a 50×50 grid): results obtained using the central difference method (left) and the Savitzky-Golay filter (right).

in Fig. 4 corroborate the fact that the central difference method does not provide a clear evidence about the existence of damage whereas the defect is well identified by means of the Savitzky-Golay filter. Specifically, high values of the damage index calculated through the Savitzky-Golay filter can be easily recognized at the bounds of the stretched damaged region, thus highlighting the location and extension of the defect.

4 Conclusions

This chapter provides an overview of a computational procedure for damage detection in plate structures based on modal curvatures estimated via numerical differentiation of modal displacement data. The ill-posedness of such inverse problem can cause abnormal amplifications in the calculation of the numerical derivatives, especially when considering relatively dense arrays of measurement points. Although the central difference method is extensively employed, this study – in consonance with previous recent works – confirmed that it is not effective for damage detection. Keeping measurement errors as low as possible and reducing the noise level may result to be beneficial to some extent for damage identification but does not fix the problem. Therefore, different numerical procedures have to be explored and, in this perspective, the present work illustrates the application of the least-square smoothing filter, also known as Savitzky-Golay filter. Despite its simplicity, this technique proved to be a reliable, rapid tool for a satisfactory numerical estimation of the modal curvatures from noisy modal displacements. Numerical and experimental results discussed herein demonstrated that the implementation of the Savitzky-Golay filter leads to a more effective identification of relatively small damages in thin plate structures together with a remarkable reduction of the number of false alarms.

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