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THE TILTED-COIL CONCEPT FOR ADVANCED TOKAMAK DEVICES

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F. BOMBARDA, R. GATTO

Abstract
The implications of the adoption of a tokamak's toroidal field coil characterized by tilting in the azimuthal direction are investigated. The major advantage introduced by tilted coils is that of a drastic reduction over most of the coil section of the electromagnetic forces. As a beneficial side-effect, the tilted coils generate a poloidal field, in addition to the toroidal field. The former advantage allows for a notable simplification of the machine layout, while the poloidal flux generated during the current rise, when used in conjunction with the conventional central solenoid, allows for discharges of much longer duration with respect to those obtainable in tokamaks with conventional (non-tilted legs) coils.

Key words: *tokamak, magnetic field coils, electromagnetic forces, poloidal magnetic flux.*

Riassunto
In questo rapporto si studiano gli effetti conseguenti all'inclinazione in direzione azimutale dei magneti toroidali di un tokamak. Il principale vantaggio consiste in una drastica riduzione delle forze elettromagnetiche. Un vantaggio ausiliario consiste nella generazione di campo poloidale, che si va ad aggiungere a quello toroidale. La riduzione delle forze permette una notevole semplificazione del progetto della macchina, mentre il flusso poloidale generato durante la crescita della corrente, se usato in serie con il flusso indotto dal solenoide centrale, permette di estendere la durata delle scariche rispetto al caso di un tokamak con magneti toroidali convenzionali.

Parole chiave: tokamak, magneti toroidali, forze elettromagnetiche, flusso magnetico poloidale.

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THE TILTED-COIL CONCEPT FOR ADVANCED TOKAMAK DEVICES

I. INTRODUCTION

Tokamaks have proven to be the most successful configuration for the magnetic confinement of plasmas. The design of viable fusion reactors, however, remains largely elusive at this time, since both physics and engineering issues limit device performances: at the present time, indeed, large volume, low field tokamaks cannot reach the high fusion gain that reactors require, while compact, high field devices have too low a duty cycle to be commercially attractive.

The fusion power P_α associated with the α-particles produced by D-T fusion reactions is proportional to $p^2 = n^2T^2$, where the pressure p in burning plasmas ranges between 20 and 35 atm. At the same time, since for stability reasons the parameter β_{pol} is limited, in practice, to a narrow range of values, the plasma pressure $p \propto B_\theta^2$. Hence, $P_\alpha \propto B^4$. Indeed, for all tokamaks the figure of merit is essentially the same: the strength of the average poloidal field $\bar{B}_\theta \equiv (\mu_0 I_p)/\oint dl$. In order to obtain the highest possible poloidal field, however, the toroidal field has to be suitably high: hence the interest for configurations that allow a reduction of the EM forces on the magnet coils.

The "Tilted-Coil" (TC) concept discussed here originates from an idea presented almost thirty years ago first in the form of a patent [1] and then of two papers [2,3] dealing with issues associated with the generation of large toroidal magnetic fields in compact tokamak devices. In Fig. 1 we reproduce the original figure of the tilted coil presented in Ref. [1]. The characteristics and mechanical properties of the "tilted-winding" coil have been further discussed in Ref. [2].

The basic idea is to give the Toroidal Field Coil (TFC) a tilt in the horizontal direction on the inboard side. The coil section shown in Fig. 1 starts on the outboard midplane, it wraps over the top of the torus cavity, then it is given an approximately 45° tilt as it follows down the inboard section, to eventually re-emerge on the outboard side.

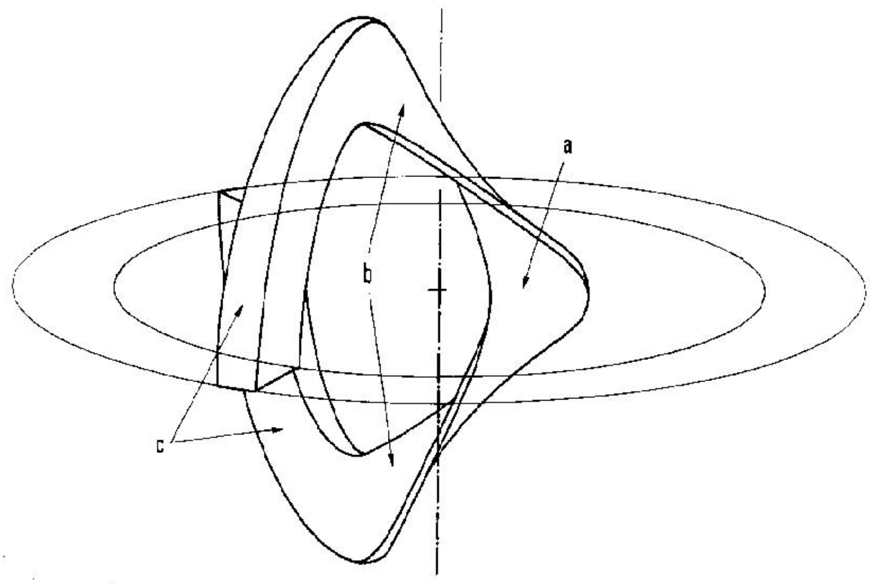

FIG. 1 - The "tilted-winding" coil as described in Ref. [2]. Legend: a: inner region; b: transition region; c: outer region.

The immediate advantage of this solution is that stresses in the inner leg of the toroidal field (TF) magnet are relieved, and much of the heavy steel structure presently needed to withstand the huge EM forces could be considerably reduced. In perspective, this type of coil could be made of ribbons of high-temperature superconductors, which are characterized by rather poor structural properties but can withstand high magnetic fields, combined with more robust intermediate temperature superconducting materials such as MgB2 [4]. At the same time, as the flux swing for generating the plasma (which represents most of the V-s consumption) is provided by the TFC, the discharge can be sustained for a longer time. Unloading mechanically the TFC makes it easier to generate the high fields required to approach ignition conditions in high density plasmas at relatively low temperatures. It is well known that at high densities the impurities are effectively screened from the main plasma region and that at lower edge temperatures the first wall sputtering is reduced, thus avoiding the need for a separate divertor region, which represents one of the most challenging element in the design of tokamaks.

Other than EM force reduction and poloidal flux generation, the tilted-coil system has additional advantages with respect to conventional TFCs [2]. Among these, we mention the ability of a tilted inner leg to withstand more easily the separating force (due to its "spring-like" configuration), and the possibility of inserting cooling channels in-between layers of the tilted inner leg.

In this work we use numerical computations to compare the EM forces acting on the various regions of a conventional (i.e., non-tilted) toroidal field coil with the forces acting on a coil that has some or all of its sections tilted. The effect of tilting depends on the location of the concerned sections: for example, tilting the inner and respectively the outer section of the coil leads to very different modifications of the EM forces acting on them. A clear understanding of the effect of tilting can be obtained by considering a TF coil composed of four distinct legs, which in the non-tilted case would form a rectangle in the meridian plane. The four legs will be referred to as the inner, lower, outer and upper leg (see Fig. 3). This simple modeling of the TF coil allows us to tilt separately one section at a time or simultaneously all sections together, to the end of comparing the results in the whole region of interest. In particular, we shall consider two main situations, one in which only the inner leg is tilted, and another one in which all legs are tilted. It must be pointed out that while most of the present tokamak devices have adopted D-shaped coils, nevertheless do exist machines, presently in operation, that have rectangular-shaped TFCs: like for example Alcator C-Mod at MIT.

The remaining of the paper is organized as follows. In the next section we present a brief overview of the forces acting on the coils of a tokamak device, and the problematics associated with them. In section III we introduce the tokamak geometry used for the numerical studies, present the code that has been written to compute the magnetic field generated by the coils, and use it to evaluate the electromagnetic (EM) forces acting on the coils in the "reference case" of a tokamak with conventional (i.e., not-tilted) toroidal coils. This preliminary study is carried out both for code validation, and to produce results that will serve as term of comparison for the subsequent studies of tilted configurations. In section IV we tilt only the inner leg of the coils and run the code to find the magnetic field, and study the reduction of the forces acting on the inner leg as a function of the tilting angle. In section V we develop further the idea of tilting by considering a coil that has all legs tilted, and optimize the tilting angle with the criterion of reducing as much as possible the EM forces. Section VI focuses on the flux savings consequent to the adoption of tilted coils. The paper ends with a Summary and Conclusion section.

II. FORCES IN TOROIDAL DEVICES FOR PLASMA MAGNETIC CONFINEMENT

In tokamaks, the interaction of the currents flowing in the coils with the magnetic field they generate gives rise to very high EM forces (tens of MN), to be compounded with the additional forces due to thermal heating (in copper magnets) and possibly neutron heating (in future devices). While the EM forces are only present during the active phase of the pulse, the forces due to thermal heating are present for much longer times. Therefore care must be taken to provide contrasting forces which are suitably time dependent. The plasma chamber, a ”doughnut” shaped metal vessel, is enclosed inside sets of TFCs that form a toroidally symmetric solenoid. The total current I flowing in the TFC generates a toroidal field $B_\phi = I/(5R)$ in the magnet cavity where the plasma vessel is located. The TFCs experience a centrifugal meridian force (which is highest for the inboard leg, where the toroidal field is strongest) but also out-of-plane forces arising from the interaction of the magnet current with the poloidal field B_θ. Another set of coils is housed in the bore of the doughnut, the Central Solenoid (CS). The flux generated by the magnetic field swing in the CS induces the current Ip in the plasma loop, which acts as the secondary coil of a transformer. The plasma current generates the poloidal field B_θ, which in tokamaks is weaker than the toroidal field. The resulting field lines are helically wound around the torus (an important parameter is the ratio q = number of toroidal turns for one poloidal turn), but the toroidal field and the poloidal field produced by the plasma current are not sufficient to ensure plasma equilibrium: external poloidal field coils are needed to provide the additional field required for the stabilization, positioning and shape control of the plasma column.

In large, superconducting devices such as ITER, forces associated with the sheer weight of the coils are also important. Moreover, the critical field of low temperature superconductors limits the maximum value of the toroidal field to about 12 T at the TFC-vessel interface, which for ITER corresponds to 6 T on axis. In turn all this limits the plasma density achievable in ITER to about $\lesssim 10^{20}$ m^{-3}. Note that so far the use of high temperature superconductors has been inhibited by their high cost as well as by their rather poor structural properties. Instead, at the other end of the tokamak spectrum, compact, high field devices of the Ignitor type need to contrast the EM forces in the toroidal magnet essentially by adding a lot of structural steel [5].

III. THE CODE AND ITS VALIDATION ON A REFERENCE CONFIGURATION

We have written a Mathematica [6] code to evaluate the magnetic field produced by a system of toroidal coils like the one used in tokamaks to generate the toroidal field required for plasma confinement. The core of the code is a subroutine that computes the magnetic field produced by a single finite straight wire. Modeling a single coil with a sequence of straight wires, the magnetic field produced by the entire coil system is obtained as a superposition of the magnetic fields produced by each single wire. To have the flexibility to reproduce with good approximation any desired coil geometry, we have segmented each one of the four legs of the coil into 8 wires, for a total of 32 wires per coil. Considering a system composed by 24 coils, the code evaluates, and adds, the field generated by 768 wires. The code is written in Cartesian coordinates, (x, y, z), and translates the results for the fields and the EM forces in cylindrical geometry, R,ϕ,Z, with $R = \sqrt{x^2 + y^2}$, $\phi = \arcsin(y/x)$ (for $x \geq 0$) and $Z = z$. The “reference” coil system is shown in Fig. 2, while the schematic of a single coil is presented in Fig. 3.

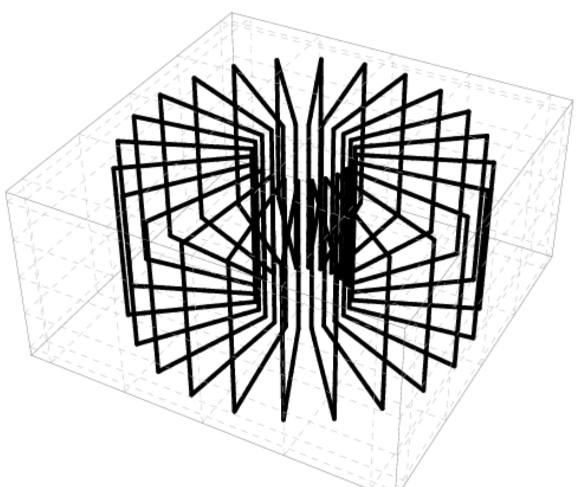

FIG. 2 – The “reference” coil system, with conventional non-tilted legs.

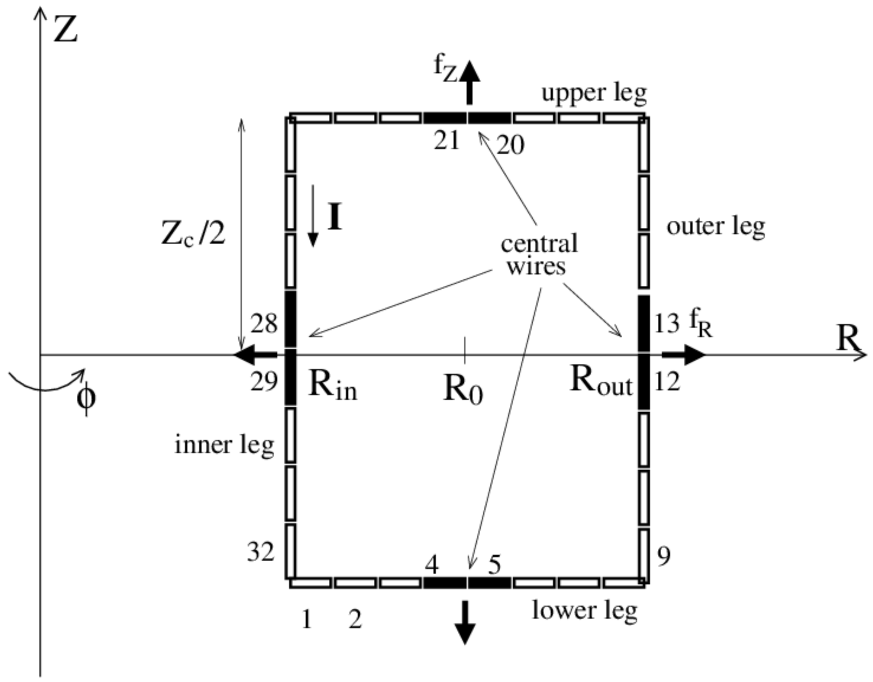

FIG. 3 - Schematic of a toroidal field coil as modeled in the code, composed of 32 straight wires. Wire n. 1 is the inner wire of the lower leg, and the remaining wires are numbered anti-clockwise. The coil current I flows anti-clockwise. The forces reported throughout the work are the in-plane forces (radial for the inner and outer legs, and axial for the lower and upper legs) averaged over the two central wires of each leg, shown in black in the figure (e.g., for the inner leg, is the radial force averaged over wires number 28 and 29). The cylindrical coordinates are also shown.

We take the major (R_0) and the minor (a) radii to be, respectively, 1.32 and 0.47 m, for an aspect ratio $A = R_0/a = 2.81$. The center-location of the inner and outer legs of the coils are located, respectively, at $R_{in} = 0.572$ m and $R_{out} = 2.068$ m, for a coil width $w = 1.496$ m and a coil aspect ratio $A_c = R_0/w = 0.88$. The height of a coil is $Z_c = 2.043$ m. These dimensions are similar to those adopted by the Ignitor machine [5], a compact high-field device designed to achieve ignition with a toroidal magnetic field of the order of 13 T on axis (with D-shaped toroidal coils). Due to the high value of the field, the problem of EM forces acting on the Ignitor coils have been studied in depth, and data on the stresses acting on the magnets are available, and can be used to compare the results of our studies. We stress that aspect ratio and elongation adopted in this work are not exactly equal to those indicated in the latest version of the Ignitor project, but have been modified to better fit the purpose of our study, a study that wants to remain qualitative in nature, and thus independent from the details of the adopted geometrical figures.

As a preliminary check of the code, and to create a reference scenario that will serve as a term of comparison for the following analyses dealing with tilted configurations, we consider a toroidal coil system made of 24 conventional (i.e., non-tilted) coils, and choose the magnet current in such a way as to have a toroidal field on axis equal to 13 T. This current turns out to be $I_M = 5RB_{\phi}/24 = 3.57$ MA. The toroidal field generated by this coil system is presented in Fig. 4.

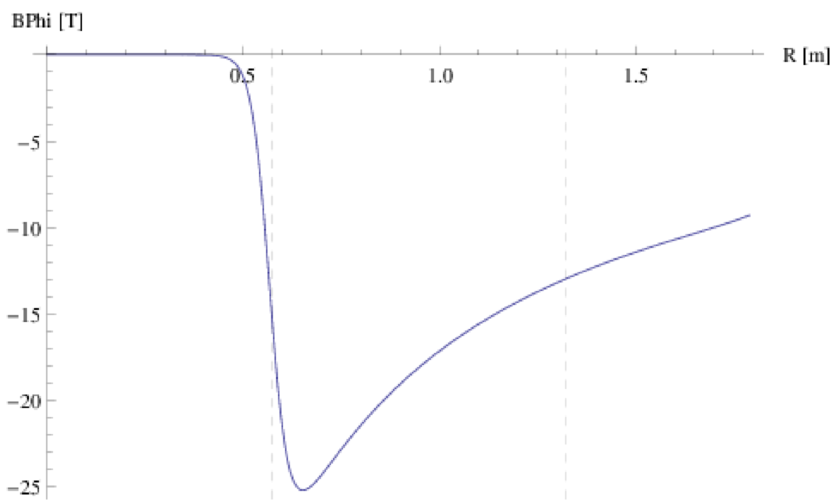

FIG. 4 - Toroidal field produced by the 24-coils of the reference system. The vertical lines locate the $R = R_{in}$ and the $R = R_0$ positions, and the R-axis is selected in-between two adjacent coils.

As mentioned before, the field is computed by adding up the contributions from all 768 straight wires in which the system has been discretized. Except for a narrow region around $R = R_{in}$, the total field shows a $1/R$ dependence for $R > R_{in}$, and vanishes for $R < R_{in}$, providing a positive feedback on the correct functioning of the code. We have also verified that the toroidal field is constant versus Z at $R = R_0$ inside the system's toroidal bore, and that the radial field is zero everywhere, as it should be due to toroidal symmetry.

We now proceed to evaluate the EM force acting on the coils. For a current-carrying straight conductor, the expression of the force is given by $\mathbf{F} = L\mathbf{I} \times \mathbf{B}$ where L is the length of the conductor. We begin by considering the inner leg, and in particular its two central wires located just above and below the xy-plane (wires 28-29 shown black-filled in Fig. 3), and compute the force acting on them. Note that it is irrelevant which coil, out of the 24 composing the system, is selected for the computation of the forces, due to the toroidal symmetry of the system. In a non-tilted toroidal coil system, as the one of the reference configuration under study, the only relevant component of the EM force (per unit length) acting on the coil's inner legs is the radial component, $\boldsymbol{f} \equiv F/L = -\hat{R}\, I_z B_\phi$. Considering first the wire n. 28 in Fig. 3, we find for the components of the current (in MA) and the magnetic field (in T), respectively, $\mathbf{I} = (I_R, I_\phi, I_Z) = (0, 0, -3.6)$ and $\mathbf{B} = (B_R, B_\phi, B_Z) = (0, -14.4, 0)$, so that the force per unit length is $f_R = I_Z B_\phi \simeq -51.4$ (MN/m). Due to symmetry, the lower wire (n. 29 in Fig. 3) experiences the same force. This EM radial force on the inner leg is the largest one due to the $1/R$ dependence of the toroidal field, and it is in very good agreement with the force calculated by the FORTE code (-51.7 MN/m) for a single plate of the actual Ignitor TF magnet. We have also computed the forces acting on the remaining three legs of the coil. The force acting on the outer leg is radial and directed in the positive R-direction. The forces acting on the two central wires are equal (for symmetry reasons) and have been found to have the value +15.4 MN/m. The forces acting on the upper and lower legs are purely vertical, equal in magnitude to 23.3 MN/m, and with positive and negative sign, respectively.

In summary, the magnitude of the centrifugal forces (averaged over the two central wires) acting on the inner, lower and outer legs of the reference (non-tilted) configuration are, respectively, -51.4, -23.3, and 15.4 MN/m.

IV. THE TILTING OF THE INNER LEG

The EM forces acting on the inner leg of the TFCs can be reduced by an appropriate azimuthal tilting of the conductors. The idea underlying this stress reduction is simple. In cylindrical coordinates, the radial Lorentz force per unit length acting on the inner leg is given by

$$f_R \equiv \boldsymbol{F}/L|_r = I_\phi B_z - I_z B_\phi \equiv f_{R,1} + f_{R,2}. \quad (1)$$

This expression shows that the radial component could be reduced or even eliminated if the current flowing in the magnet acquires a toroidal component while the field acquires a vertical component. When this happens, the component $f_{R,2}$ is reduced due to the decrease of the current, and a new component $f_{R,1}$, which opposes $f_{R,2}$, is introduced.

In our initial study of the tilted-coil concept, the tilting angle of the inner leg has been set equal to 45°, the angle which leads to a zero force in the limit of infinitely large aspect ratio. The tilting is in the azimuthal direction, as in all cases considered in this work. A sketch of the code modelization for the case of a tilted inner leg is shown in Fig. 5.

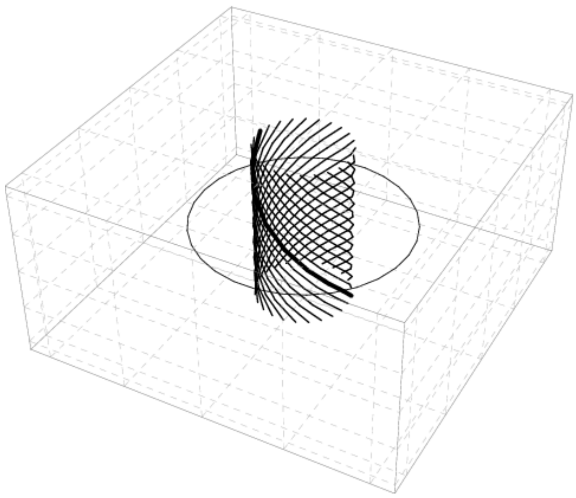

FIG. 5 - The central solenoid generated by the tilted inner legs (the circle denotes the $R = R_0$ location). One inner leg is shown in bold. While descending, the tilted leg goes around the torus by an angle of approximately 205°. The poloidal flux generated by the tilted inner legs inside the plasma region is equal to 23.32 [Wb]

The toroidal field due to the entire coil system is very similar to the one relative to the standard coil system shown in Fig. 4, with a shape and magnitude that is slightly different only in the region closest to the inner leg. The additional vertical field generated by the tilted system is shown in Fig. 6.

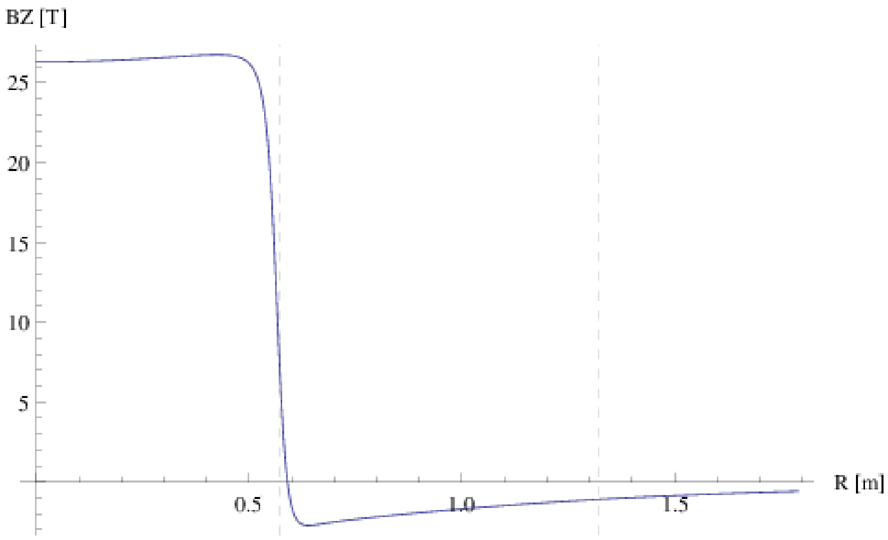

FIG. 6 - Vertical field produced by the coil system with tilted inner leg.

It is large and constant inside most of the doughnut hole, and is small and decaying slowly to zero outside it. The poloidal flux generated inside the plasma region equal to 23.32 [Wb]. The ratio of the radial EM force acting on the inner leg, averaged over the two central wires, to the force found in the non-tilted case is $f_R/f_R^0 = 0.09$. The force has been reduced by more than one order of magnitude, but has not vanished. This is to be expected, since we are dealing here with a finite-aspect-ratio configuration. To show the decreasing trend of the radial force as the tilting angle is increased, we have run the code using tilting angles of 22.5°, 46°, 47°, 48°, and 49° as well. The results are shown in Fig. 7, where the radial force averaged over the central two wires, and the force averaged over the entire inner leg are reported. Using a linear interpolation we find that in the former case the force goes to zero for the tilting angle $\Gamma \simeq 46.8°$, while in the latter case for $\Gamma \simeq 48.2°$.

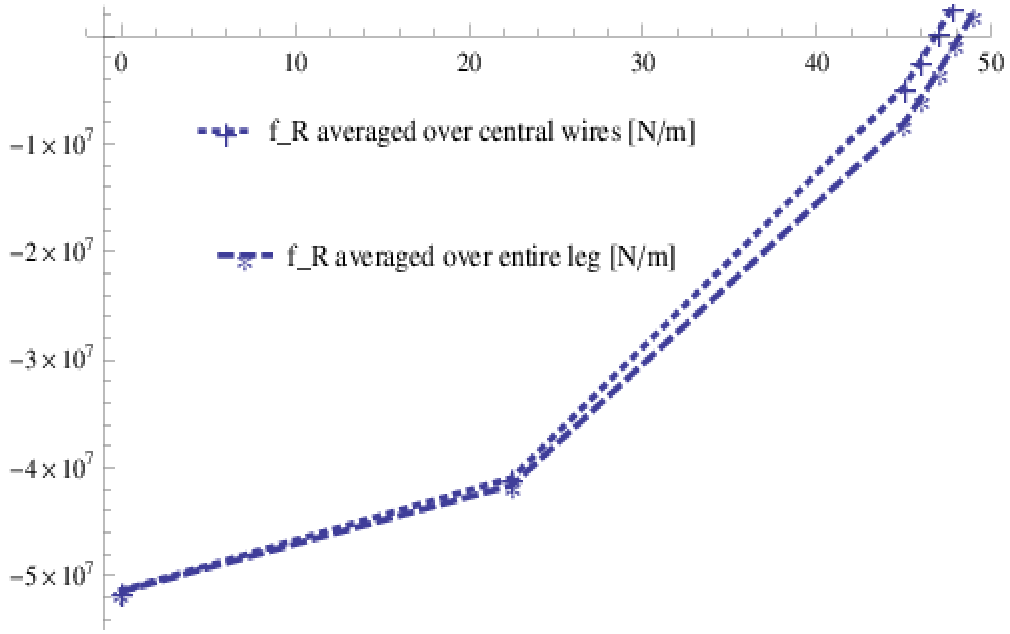

FIG. 7 - Radial force acting on the inner leg of the coils versus the tilting angle.

V. THE TOTALLY TILTED MAGNET

So far we have studied the effect of tilting the inner leg of the coil, showing that for an angle close to 45° the radial force acting on the central region of this leg vanishes. Next we pose the question whether it is advantageous to tilt also some, or all, of the remaining legs of the coil, and in particular whether is it possible to find a toroidal configuration that experiences zero centrifugal EM forces (at least in the central region of its legs). Our considerations will be based on the generalization of Eq. (1) to include all cylindrical components:

$$\boldsymbol{f} \equiv \boldsymbol{F}/L = \widehat{\boldsymbol{R}}\left(I_{\boldsymbol{\phi}} B_Z - I_Z B_\phi\right) + \widehat{\boldsymbol{\phi}}\left(I_Z B_R - I_R B_{\phi Z}\right) + \widehat{\boldsymbol{Z}}\left(I_{\boldsymbol{R}} B_{\boldsymbol{\phi}} - I_{\boldsymbol{\phi}} B_R\right) \qquad (2)$$

$$\equiv \widehat{\boldsymbol{R}}\left(f_{R,1} + f_{R,2}\right) + \widehat{\boldsymbol{\phi}}\left(f_{\phi,1} + f_{\phi,2}\right) + \widehat{\boldsymbol{Z}}\left(f_{z,1} + f_{z,1}\right).$$

In the remaining of the paper we shall tilt the various legs of the coil and compute the force acting on the central region of each leg. We have chosen to define the latter as the average force between the two central straight wires just below and above the equatorial plane for the inner and outer legs, and just before and after the R = R_0 location for the lower and upper legs.

These central wires are shown black-filled in Fig. 3. In some cases the average force will be zero. This means that we have been able to find tilting angles of the two central wires that eliminate the force acting on them. These two tilting angles might be equal or different. The latter situation is always true when considering the lower and the upper legs due to the $1/R$ variation of the field.

A. The tilting of each individual leg

Before looking for a coil with all its legs tilted in such a way as to reduce as much as possible the EM forces acting on them, it is instructive to examine the effect of tilting each individual leg of the coil separately (i.e., extending what we have already done with the inner leg).

Consider first the lower leg. In the conventional configuration the only component of the coil current is radial and the force is in the negative vertical direction and equal to $f_{Z,1} = I_R B_\phi$ (with $I_R > 0$ and $B_\phi < 0$): call it the "stretching" force. When the leg is tilted in the $\pm\phi$ direction, the acquired I_ϕ and B_R components generate the additional force $f_{Z,2} = -I_\phi B_R$. The latter is a "compressing" force because I_ϕ and B_R turn out to have opposite signs. For an appropriate tilting angle, the compressing and the stretching forces might cancel each other, leaving a zero vertical force. The same argument applies to the upper leg and its vertical force acting on it. The situation with respect to the outer leg is however different. In the reference coil the in-plane force is radially outward and given by $f_{R,2} = -I_Z B_\phi > 0$. This is of the same kind as the force acting on the inner leg, and is expected to have the opposite direction (i.e.., outward) due to the opposite sign of I_Z, and to be of smaller magnitude due to the smaller value of B_ϕ. When the leg is tilted, a new force component arises, $f_{R,1} = I_\phi B_Z$. The current I_ϕ has the same direction and magnitude as for the inner leg (positive). The vertical field B_Z, however, is now positive, and this leads to a $f_{R,1} > 0$ which adds to $f_{R,2}$. The combination of these two effects (a reduction of $f_{R,2}$ and the arising of $f_{R,1}$) can lead to three different situations depending on the geometry of the torus: (1) any tilting increases f_R, (2) an initial tilting decreases f_R but further tilting leads to its increase, and (3) a tilting angle can be found that eliminates f_R. We now quantify these qualitative considerations.

First we consider the outer leg. In the reference configuration, we have found that this leg is subject to the positive radial force $f_{R,2} = -I_Z B_\phi$, and that its average over the two central wires of the leg was f_R^0= 15. 4 MN/m. We have run an optimization procedure to look for the tilting angle that minimizes this average force, finding however that this reduction is minimal, i.e., $f_R/f_R^0 \lesssim 1$. We conclude that for the geometry under consideration the centrifugal force acting on the outer leg cannot vanish by a tilting of the leg, but can only be slightly decreased. Let's consider now the lower leg. We have found that the axial force, averaged on the central two wires, decreases continuously as the tilting angle increases, but it is still greater than zero for an angle of 89. 5° (reduced by a factor f_{ZR}/f_Z^0 = 0. 30). The only way to have a zero axial force on the lower leg (and upper, for symmetry) would be to tilt at least another leg of the coil. This would introduce additional components of the magnetic field that would ease the task, and thus perhaps a zero axial force for the lower leg could be found for a tilting angle less than 90°. This is indeed the case, as it will be shown in the next subsection.

B. The tilting of all legs

Here we consider the original Ignitor-like geometry (R_0 = 1.32 m, A = 2.81, Z_c = 2.043 m and A_c = 0.88), and run the code to find the optimal tilting angles for all four legs which lead to the reduction (or even the elimination, if possible) of the centrifugal forces averaged over the central two wires of each leg. In this optimization process we shall divide each leg into two equal sections, and shall assign the same tilting angle to all elements belonging to one or the other of such sections. [These two tilting angles associated to each leg are denoted with: Γ_f , relative to the first four wires (e.g., referring to the lower leg in Fig. 3, wires n. 1-4), and Γ_l, relative to the last four wires (e.g., wires n. 5-8).] Running the optimization code we have found that the centrifugal forces acting on the inner, lower and upper legs can be totally eliminated, while the EM force on the outer leg remains unchanged relative to the non-tilted case, because any tilting leads to its increase. The results are reported in Table I, which shows the values of the reduction factors (*RF*), i.e., the ratios of the forces in the optimized configuration over the forces in the reference (non-tilted) configuration. We observe that in our lexicon "a force equal to zero" actually means "a force less than 150 N/m", as we have adopted this criterion to stop the code iterations.

	Γ_f	Γ_l	RF_R	RF_ϕ	RF_Z
lower leg	86.6°	83.2°	$f_R/f_Z^0 = -0.83$	$f_\phi/f_Z^0 = 0.28$	$\boldsymbol{f_Z/f_Z^0 = 0}$
outer leg	0°	0°	$\boldsymbol{f_R/f_R^0 = 1}$	$f_\phi/f_R^0 \simeq 0$	$f_Z/f_R^0 = 0$
upper leg	83.2°	86.6°	$f_R/f_Z^0 = 0.83$	$f_\phi/f_Z^0 = 0.28$	$\boldsymbol{f_Z/f_Z^0 = 0}$
inner leg	30.1°	30.1°	$\boldsymbol{f_R/f_R^0 = 0}$	$f_\phi/f_R^0 \simeq 0$	$f_Z/f_R^0 \simeq 0$

Table I. Optimized tilting angles (Γ) and force reduction factors (RFs) for the A = 2.8 and Ac = 0.88 system with tilted legs. RFs relative to centrifugal forces are in bold. The notation ≃ 0 means less than 10^{-5}.

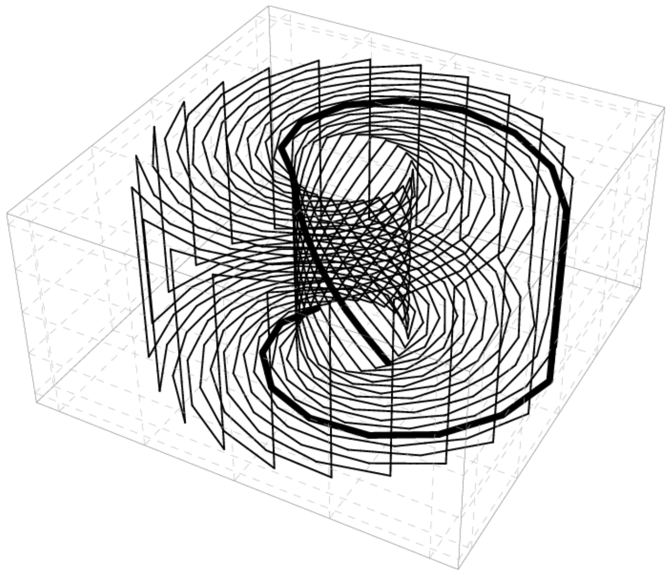

FIG. 8 - Optimized coil system with Ac = 2.81*. The outer leg is straight because any tilting of it increases the radial force acting on its central region. The poloidal flux generated by the tilted legs inside the plasma region is equal to* 53.57 *[Wb].*

The optimized system is shown in Fig. 8. We notice the large values of the tilting angles of the lower and upper legs. At this large inclination, the toroidal component of the coil current is increased while the radial component is decreased. By looking at Eq. (2), we see that this leads to a reduction of the (conventional) vertical component $f_{Z,1} = I_R B_\phi$ and an increase of the new component $f_{Z,2} = -I_\phi B_R$ that opposes the previous one. Table I also shows that a negative side-effect of the tilting in the azimuthal direction is that of introducing off-plane forces. For the inner leg these new forces are several orders of magnitude smaller than the conventional force f_R^0. For the lower and upper legs, however, these new forces are smaller but of the same order of magnitude than f_Z^0. In the present work we do not address these forces, but it is important to observe that these forces could be reduced or even eliminated by introducing a tilting also in the vertical direction. We shall follow this direction in a future work.

There are two ways to reduce further the radial force on the outer leg: namely, increasing the aspect ratio (i.e., increasing R_0), or shortening the coil height (decrease Z_c). We shall consider these two situations in the following subsections.

C. The tilting of all legs for larger aspect ratios

A way to reduce the force on the outer leg (while maintaining stress-free the remaining legs) is to increase the aspect ratio. As a first case we have shifted outward the coils by increasing the major radius (which is also the center location of the coils) five times, from 1.32 m to 6.60 m, giving an aspect ratio equal to A = 14.04, all other geometric quantities being left unchanged. Differently from the previous case, the larger aspect ratio allows a partial reduction of the force on the central region of the outer leg, while maintaining equal to zero the forces on the remaining legs. In particular, for a tilting angle of 70.7° the three components of the reduction factors relative to the outer leg are ($\boldsymbol{f_R/f_R^0}$, f_ϕ/f_R^0, f_Z/f_R^0) = (**0.470**, 0.002, 0.005). We report in bold the radial RF, as the radial component is the only one for the force in the reference (non-tilted) configuration. The other relevant difference from the previous cases is that the optimal tilting angles of all legs are now smaller, the average values for the lower and inner leg being, respectively, 38.9° and 27.5°.

To verify that the force on the outer leg can be eliminated by further increasing the aspect ratio, we have considered the extreme case with $R_0 = 20 \times 1.32 = 26.4$ m, i.e.. an aspect ratio $A = 56.17$. We have found that indeed the meridian forces averaged on the central region of all legs are now zero, for optimized tilting angles of the lower, outer and inner legs of 33.0°, 72.7°, and 26.5° degrees, respectively.

As a next step, we have reduced the height of the coil so as to obtain a square section: $Z_c = 1.496$ m. We have found that the average force on the two central wires of the outer coil vanishes for a minimum value of R_0 equal to 9.9 m (aspect ratio $A = 21.06$): i.e., seven-and-a-half times larger than in the original configuration.

Even though a square coil leads to the vanishing of the force on the legs for a major radius that is "only" 7.5 times the reference one, this situation still seems to be non-interesting for a realistic device. For this reason we have also considered the case of a square coil with R_0 equal to the value it had in the original geometry, namely $R_0 = 1.32$ m ($A = 2.81$). The results are reported in Table II, and the corresponding illustrative picture is presented in Fig. 9.

	Γ_f	Γ_l	RF_R	$RF\phi$	RF_Z
lower leg	73.6°	67.0°	$f_R/f_Z^0 = -1.04$	$f_\phi/f_Z^0 = 0.55$	$\boldsymbol{f_Z/f_Z^0 = 0}$
outer leg	51.3°	48.8°	$\boldsymbol{f_R/f_R^0 = 0.83}$	$f_\phi/f_R^0 = 0.04$	$f_Z/f_R^0 = -0.05$
upper leg	67.0°	73.5°	$f_R/f_Z^0 = 1.03$	$f_\phi/f_Z^0 = 0.54$	$\boldsymbol{f_Z/f_Z^0 = 0}$
inner leg	29.3°	29.3°	$\boldsymbol{f_R/f_R^0 = 0}$	$f_\phi/f_R^0 \simeq 0$	$f_Z/f_R^0 \simeq 0$

Table II. Optimized tilting angles (Γ) and force reduction factors (RFs) for squared coils with A = 2.81. RFs relative to centrifugal forces are in bold. The notation ≃ 0 means less than 10−3.

As Table II shows, this configuration leads to an inner leg with zero radial force, and azimuthal and axial forces that are about 3 orders of magnitude smaller than f_R^0. This is achieved for a tilting angle of only ~ 30°. The inclination of the outer leg by ~ 50° has led to a 20% reduction of the radial force, while it has introduced ϕ and Z components two orders of magnitude smaller than f_R^0. Finally, a tilting angle of about 70° has led to an axial force on the lower and upper leg that is zero. However, in this case the newly generated radial and azimuthal forces are of the same order of magnitude than f_Z^0. Again, introducing a tilting angle also in the vertical direction (in addition to the tilting in the azimuthal direction) could help in ameliorating the situation.

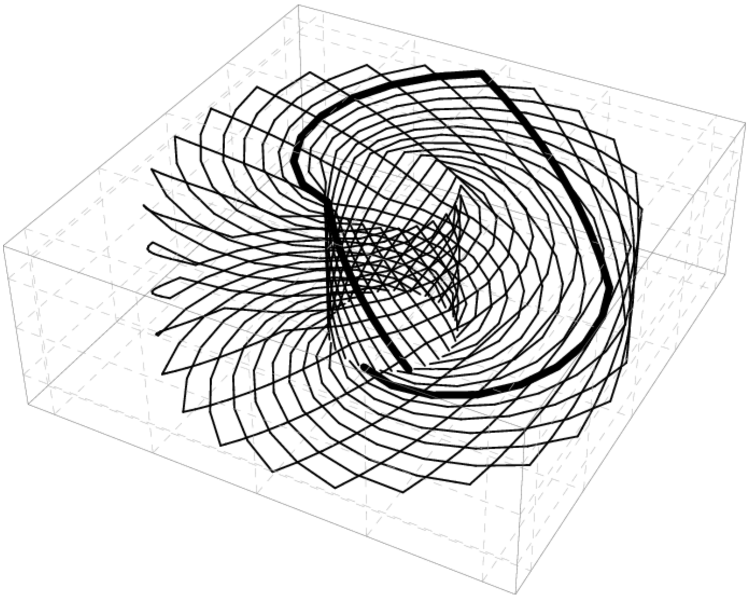

FIG. 9 - Optimized squared coil system with $A_c = 0.88$ *and* $Z_c = 1.5$ *m. The poloidal flux generated inside the plasma region is equal to* 55.72 *[Wb].*

D. The tilting of all legs for reduced coil's height

An alternative path to facilitate the vanishing of the EM forces by proper tilting angles is that of reducing the height of the coil (Z_c). The resulting favourable effect is due to the increased attraction between the outer leg and the upper and lower legs. We have found that radial force on the outer leg can be totally eliminated when the height is reduced by a factor 1/5. The optimal tilting angles for the lower, outer and inner legs have been found to be, respectively, 57.1°, 48.9° and 26.0°.

VI. FLUX SAVINGS WITH THE TILTED COIL

The other potential advantage of the tilted coil solution is that it opens up the opportunity of having the toroidal magnet provide most of the magnetic flux swing required for generating the plasma current. Simple equations can be written to link the magnetic flux stored in the toroidal magnet (with a tilted legs), to the magnetic field on axis $B_\phi^0 = (\mu_0 I_M)/(2\pi R_0)$. The value of the vertical field generated in the bore of the toroidal solenoid of radius R_{in} by tilting the inner leg of the toroidal magnet by an angle Γ, in the limit of a continuous ideal solenoid of height $2Z_c >> 2\ R_{in}$, is the following:

$$B_\theta^0 = \mu_0 \frac{I_M^\phi}{2Z_c} = \mu_0 \frac{I_M}{2Z_c} \sin\Gamma = B_\phi^0 \frac{\pi R_0}{Z_c} \cos\Gamma \quad (3)$$

where I_M [MA·turn] is the total current flowing in the toroidal magnet coils and the magnetic field B is expressed in T.

In the simplistic approximation of an infinitely thin magnet, the stored flux is $\psi_{CS} = B_\theta^0 S$, where $S = \pi R_{in}^2$ is the area of the central hole. Therefore

$$\psi_{CS} \cong \pi^2 B_\phi^0 \frac{R_0 R_{in}^2}{Z_c} \sin\Gamma \quad (4)$$

In order to sustain the plasma, $\psi_{CS} > LI_p$, with I_p the plasma current and L the plasma total inductance. Thus:

$$L = \mu_0 R_0 \left[\ln\frac{16R_0}{a(1+\kappa)} - 2 + \frac{\ell_i}{2}\right] \cong 1.5R_0\ \mu\text{H} \quad (5)$$

whereby one may consider typical values $R_0/a \cong 3$ and plasma elongation $\kappa \cong 1.8$, and where $\ell_i/2 \cong 0.4$ is the internal plasma inductance per unit length (essentially related to the peaking of the plasma current profile).

In most cases, the best performances in tokamaks are obtained by operating with values of the safety factor $q \cong 3.5 - 4$. The actual computation of this parameter requires complex calculations, but the simple so-called cylindrical approximation can be used to link the plasma current to the toroidal field:

$$q_{cyl} \cong \frac{5a^2 B_\phi}{R_0 I_p}\left[\frac{1+\kappa^2(1+\delta^2-1.2\delta^3)}{2}\right] \cong \frac{5a^2 B_\phi}{R_0 I_p}\left[\frac{1+\kappa^2}{2}\right] \approx 2-2.5 \quad (6)$$

where δ is the plasma triangularity that we can neglect here. By extracting the plasma current from the above expression, the flux requirement can be expressed as:

$$LI_p \cong 7.5\frac{a^2 B_\phi^0}{q_{cyl}} \tag{7}$$

The condition to be satisfied then becomes:

$$\psi_{CS} \cong \pi^2 B_\phi^0 \frac{R_0 R_{in}^2}{Z_c}\sin\Gamma > LI_p \cong 7.5\frac{a^2 B_\phi^0}{q_{cyl}} \tag{8}$$

$$\frac{R_0}{a^2}\frac{R_{in}^2}{Z_c} > \frac{7.5}{(\pi^2 \sin\Gamma)} \approx 1/q_{cyl} \tag{9}$$

This condition is largely verified for tokamaks, which means that most of the flux needed to create the plasma can be provided by the toroidal field coils with tilted inner legs. Clearly, to sustain the discharge once the target value of the toroidal field is reached, an auxiliary central solenoid or a current drive system has to take over.

		Ignitor	FTU	C-Mod	JET	ITER
B_ϕ (T)	Toroidal Field	13.	8.	8.1	3.45	5.3
I_p (MA)	Plasma Current	11.	1.6	2.	4.8	15.
R_0 (m)	Major Radius	1.32	0.935	0.67	2.96	6.2
a_p (m)	Minor Radius	0.47	0.31	0.21	1.25	2.
κ	Elongation	1.83	1.	1.85	1.68	1.7
$V{\cdot}s$	Available Flux	37.	6.4	7.5	34.	311.
B_{cs0} (T)	Poloidal Field at $R = 0$	35.8	37.2	13.5	8.55	11.4
ψ_{cs0} (Wb)	Stored Flux	49.2	27.2	1.7	48.	272.
ψ_{cs} (Wb)	Idem (finite thickness)	49.8	27.4	1.9	48.3	273.
ψ_p (Wb)	Plasma Ind + Int Flux	21.2	3.0	2.1	18.7	153.
ψ_r (Wb)	Plasma Resistive Flux	7.3	0.75	0.67	7.1	46.7
ψ_{tot} (Wb)	Plasma Total Flux	29.0	4.2	3.3	26.4	200.
L_{tot} (μH)	Plasma Total Inductance	1.9	1.9	1.1	3.9	10.
q_{cyl}	Geometrical Safety Factor	2.15	2.6	2.9	3.6	2.2

Table III. Nominal design parameters of five machines.

In Table III the nominal design parameters of five machines, in use or planned, are listed in the top section of the table. In the lower part of the table, values of various parameters are reported in the assumption that these devices had the inner leg of their toroidal magnets tilted by 45°. In every case but C-Mod the flux originated by the toroidal magnet is more than adequate to create the plasma, leaving the Central Solenoid to sustain the discharge for about twice the available Vs (at 0.5 Vs/s in hot plasmas), neglecting possible issues with the magnet heating. The C-Mod case is easily understood by noting that in this machine the inner legs of the toroidal magnet are located *inside* the CS coils.

VII. SUMMARY AND CONCLUSIONS

The azimuthal tilting of the inner leg of a toroidal magnet (previously conceived and investigated by other authors) is extended by considering the effects of tilting also the upper, lower and outer leg of the magnet, in order to identify a current pattern that simultaneously minimize, or even eliminates, the centrifugal forces acting on the central region of the coil's legs. The conceptual study we have carried out shows that a significant reduction of the forces can be obtained on all but the outer portion of a TF coil. For the coil system in which all legs are tilted except the outer leg, shown in Fig. 8 and Table I, the inner leg can be entirely unloaded for a tilting angle of around 30°, while the vertical force is eliminated in the upper and lower legs at an angle of about 85°, being replaced by toroidal and radial components. The latter angle is reduced to 70° if the outer leg is tilted by an angle of 50° (see Fig. 9 and Table II). Whether this tilting can be considered an advantage it remains to be seen, nevertheless it indicates that a flexible, ribbon-like conductor wrapped around the torus with such a geometry could, in principle, reduce the requirements of structural reinforcing materials. It should be pointed out that this study does not include the effects of the plasma and of the other equilibrium poloidal field coils. Strictly speaking, we have not optimized a "tokamak" configuration, but only reduced the forces acting on the toroidal field coils due to the fields they themselves generate. Both the poloidal field B_{pol} associated with the plasma current and the equilibrium vertical field B_v are of the order of few Tesla, that is, much weaker than B_ϕ. In the reference non-tilted case, the interaction of the former with the current flowing in the coil can be small if the plasma shape is sufficiently conformal to the coil, whereas the latter gives rise to out-of-plane forces on the upper and lower sections that are to a certain extent contrasted by circumferential symmetry. In the “optimal” tilted case, these sections develop radial forces to be compounded with the others. Then it is likely that, in full tokamak devices, the optimal tilting angles will be different from those found in the present study, but the fundamental idea should remain valid. A second beneficial effect of the tilting of the inner leg consists in a flux saving that can extend the duration of the discharge. For the coil system represented in Fig. 8, the poloidal flux generated inside the plasma region is equal to 53.57 [Wb]. Without arriving at the steady state reactor, a less stressed, long pulse, high field device can be conceived for operation at or near ignition. The engineering feasibility of such coils needs to be investigated, with the expectation that the ”optimal” configuration may turn out not to be the one that minimize the centrifugal forces in all directions. Another important issue to be verified is the compliance of the plasma to the simultaneous rise of the poloidal and toroidal magnetic fields, while increasing the plasma cross section. This point however, could be tested in existing experiments.

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