Chapter 72
Conoids and Hyperbolic Paraboloids in Le Corbusier's Philips Pavilion

Alessandra Capanna

The Expo 1958

The Philips Pavilion at the Brussels World's Fair (Fig. 72.1) is the first of Le Corbusier's architectural works to connect the evolution of his mathematical thought on harmonic series and modular coordination with the idea of three-dimensional continuity (Capanna 2000).[1] This propitious circumstance was the consequence of his collaboration with Iannis Xenakis, the famous contemporary musician working at that time as engineer in Rue de Sèvres.[2] Xenakis's very deep interest in mathematical structures was improved on his becoming acquainted with the Modulor, while at the same time Le Corbusier encountered double ruled quadric surfaces.

At the beginning of 1956, Louis Kalff, the art director of Philips Industries, proposed to Le Corbusier a new kind of participation in the World's Fair: their intention was not to expose their products, but rather they wanted a bold show of sound and light effects, to illustrate what Philips's technical progress was able to lead to. "I won't design a pavilion with façades, I'll give to you a *Poème Électronic* and the bottle containing it!", answered Le Corbusier (Xenakis 1976). He designed a building that represented a real synthesis of arts: coloured lights, contemporary

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[1] Precisely the use of ruled surfaces was yet foreseen for enveloping Chandighar's assembly hall, designed during approximately the same period as the Philips Pavilion with the contribution of Xenakis, as one can learn reading the letters conserved at the Fondation Le Corbusier, Paris.

[2] For more about Xenakis, see (Capanna 2001).

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Fig. 72.1 View of the pavilion. Photo: Wouter Hagens

music, the projection of enormous warped images in a space without architectonical quality. It could be, at the minimum, even a scaffold.

Minimal Surfaces in Three Space

The idea of a container without an aesthetic claim allowed Le Corbusier to think only about the show; in the meantime he entrusted Xenakis with a “mathematical translation” of his sketches, which represented the volume of a rounded bottle with a stomach-shaped plan (Xenakis 1976).

Synthetically, the form of the pavilion had to comply with the following principle: there was to be a maximum of free volume for a minimum of enclosing surface. The classical answer is the sphere, but the sphere, beautiful in itself, is bad for acoustics and is less perceptibly rich than some other double-curved, warped or skewed forms. In that large family of surfaces they had to single out those conforming self-loadbearing shells, easy to build in a common reinforced concrete yard.

Xenakis planned a first solution of the stomach-shaped volume defined by conoids and hyperbolic paraboloids (Fig. 72.2). The pavilion was enveloped by the conoid *E*, a composite surface formed by two conoids *A* and *D*, the hyperbolic paraboloids *K* and *G*, the connecting cone *L* and a couple of empty triangles for the entrance and the exit. Those warped surfaces culminated on three cusps, 17, 13 and 11 m high.

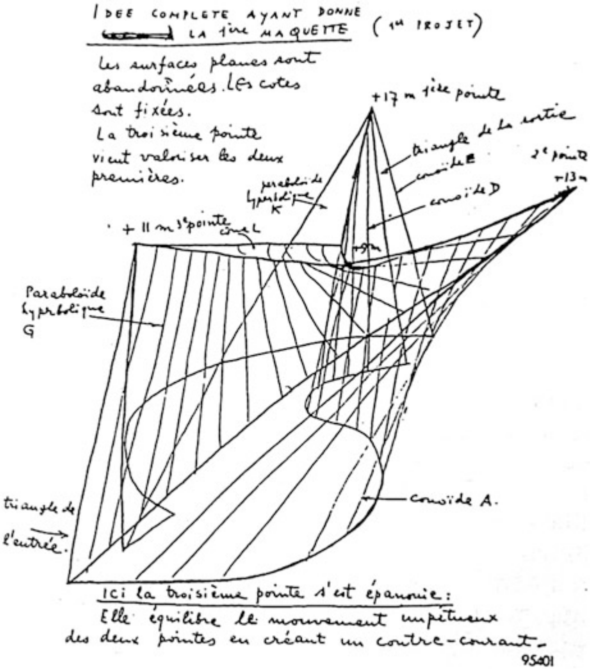

Fig. 72.2 First project. Sketch: I. Xenakis

Some constructive difficulties brought about work on a new project. First of all, it was necessary to change all conoids[3] into hyperbolic paraboloids, whose straight-lined generatrices made statics calculation easier. This second project was the fruit of a hybrid of analytic and descriptive geometry.

> As a matter of fact it is impossible to arrange all the surfaces of this kind only working with its algebrical functions. The refinement in curvature and surface dynamism cannot be imagined by studying its equation. The pavilion must be plastic, above all; for this reason in an infinity of possible curves we had to choose the best composition of warped surfaces (Xenakis 1958: 7).

Thus, the new height of the cusps and their consequent projections were established accordingly on the horizontal plane, so as to increase the dimension of the central cone *L*. The first cusp was fixed at 21 m from the ground, the second at 13 m and the third at 18. Afterwards, all paraboloids were modelled in accordance with the condition that the intersection with the horizontal plane conformed to the primitive scheme of a "stomach-shaped" plan.

Compared with the first one, this project introduced unchanged the hyperbolic paraboloids *G* and *K*, conforming the principal surfaces for filmed projections; it redefined and visibly widened cone *L*, and conoids *A*, E and *D* were transformed in five paraboloids: *A*, *E* and *B, N* and *D*. Moreover, the paraboloids *C* and *F* had been

[3] Conoids are those ruled surfaces defined by a curvilinear directrix and a couple of straight-lined directrices.

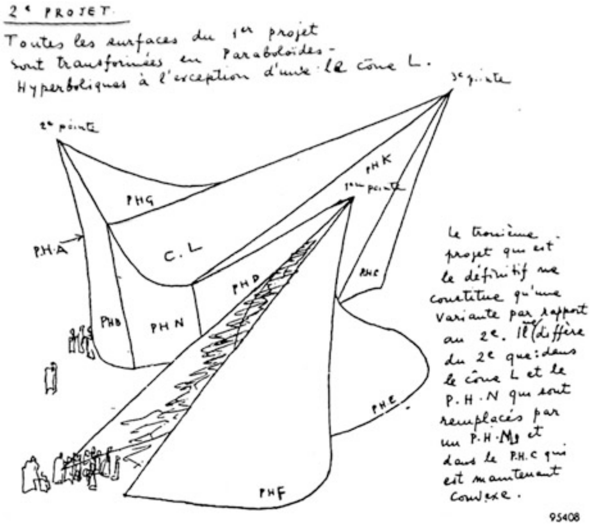

Fig. 72.3 Second project. Sketch: I. Xenakis

inserted. This last one, approached to the curved wall E, delimited the space necessary for the room of the automation of the show, technical spaces and services (Figs. 72.3 and 72.4).

This volume still required two pillars for the stability of the whole building. Xenakis later said that, with the assistance of Mr. Duyster, the Strabed engineer who suggested building the pavilion in pre-compressed concrete, he decided on some final modifications in order to abolish all the vertical supports:

> Mr. Duyster thought that the cone L and the hyperbolic paraboloid N formed the only hyperbolic paraboloid later on defined with the M. Geometrical clearness was so improved. . . . I suggested to him a further slight change to the hyperbolic paraboloids M and B. . . . Moreover I changed the concave paraboloid C into a convex one: the stability of the very steeply sloped third cuspid was so improved as well. Then I remoulded the hyperbolic paraboloids up to the edges of the exit-entrance triangles. The pavilion became completely self-loadbearing, lightweight and pillarless (Xenakis 1958: 10–11).

From this comes the configuration that makes use of hyperbolic paraboloids, thus shaping a kind of enveloping form, closed and opened to the world at the same time through the convergence of its geometrical construction (Fig. 72.5).

Correspondence of Geometrical and Mechanical Properties in Hyperbolic Paraboloid Shells

> In my opinion the fascination of forms derived by the minimal surfaces in design is based on several properties:

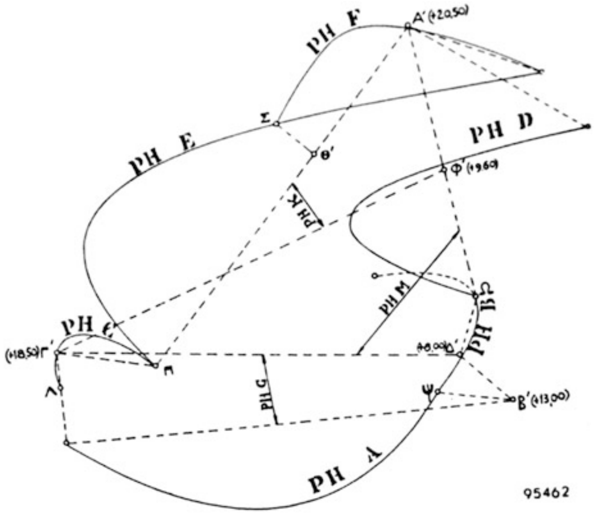

Fig. 72.4 General plan of the pavilion

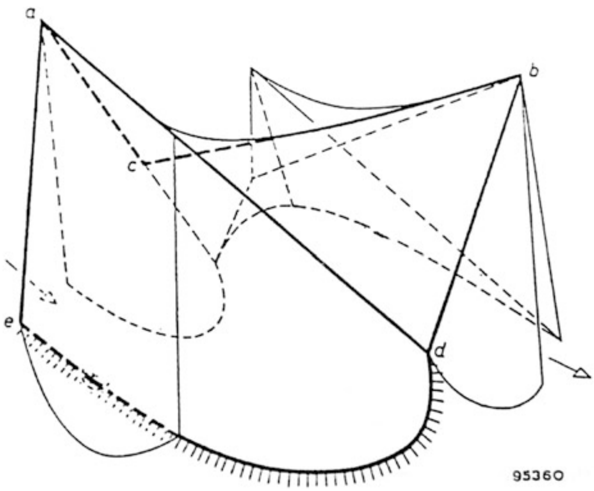

Fig. 72.5 Volume

1. the shapes of minimal surfaces can be astonishing from the aesthetical point of view;
2. the shapes of minimal surfaces allow the optimal use of materials;
3. the structural surfaces with a saddle shape are very stable and resistant;
4. the structures of minimal surfaces have natural geometric rigidity (Almgren 1982).

In 1935, B. Lafaille and E Aimond published their research on the distribution of forces in thin curved walls, vaults and shells defined by simple ruled quadric surfaces such as hyperbolic paraboloids and conoids. Mathematicians had long known those kinds of surfaces, but architects only began designing with this geometry after the accepted use of reinforced concrete to build modelled roofings. Frei Otto pointed out the influence of Bernard Lafaille in the

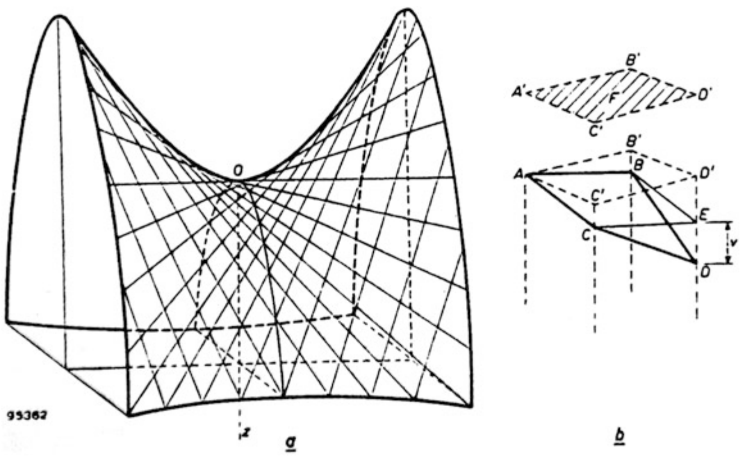

Fig. 72.6 (**a**, *left*) The double-system of generatrices of a hyperbolic paraboloid; (**b**, *right*) Part of the surface delimited by the intersection of four generatrices: AB, CD, AC, BD

development of lightweight roofing and tensile structures; he also remarked that at the Brussels World Fair about 20 pavilions made use of saddle surfaces positively corresponding to the aims of temporary buildings.

The coincidence of saddle surfaces with minimal surfaces was later denied.[4] However, C.G.J. Vreedenburgh, from the University of Technology of Deft, confirmed all the positive characteristics of such structures. He mathematically verified that hyperbolic paraboloids, rigidly fixed to the ribs, fulfilled the requirements of building[5]; he then provided some charge tests.

Before proceeding to the final construction in the assigned exposition area, two models were required: one at a scale of 1:25 made with plaster, which served for studying deformations due to accidental loads, proper weight and the eventuality of fire; another at a scale of 1:10, built to simulate the reality of setting up the thin shells; this one was also used to individualize the lines along which to put in tension the cables, that coincided with the generating geometry. Again, the spirit of geometry and mathematical theory adhere perfectly with constructive practice.

By testing the elementary deformations, it was possible to verify that in the enclosing hyperbolic paraboloid shapes, a uniform system of solicitations on the whole surface takes place: they were all funicular strains up to the edges, along which the loads are transferred to the foundations in the form of normal efforts.

Owing to the large dimension and to the steep slope of the surfaces, it was impossible to built them in one piece, therefore experiments with a sort of assembly

[4] See Emmer (2015 ('Periodical Minimal Surfaces')); compare with the unconditional statement taken from Otto (1973).

[5] For the detailed exposition of the demonstration coming down from the equation of hyperbolic paraboloids, see Vreedenburgh (1958).

system were undertaken. The big paraboloids were divided in portions of around a square metre, cut according to the irregular network constituted by the intersection of the generatrices (Figs. 72.6 and 72.7) and then temporarily fixed to a framework (Fig. 72.8).

Mathematics, Music, Architecture: The *Poème Électronic*

In any project such as this, scientific thought is a means with which to realize ideas that have been born of intuition or of some kind of vision, even if they are not of scientific origin.

In 1954 Xenakis was composing *Metastasis*.[6] He declared that he had some visual fantasy: straight lines representing the geometrical form of glissandi, which transformed themselves into an auditory fantasy.

> I started from a very simple serial problem: that is how to reach through the 12 sounds a different formulation of chords. 1 sketched some lines connecting every other single sound deriving from a chord of 12. Suddenly I thought of those lines as if they were glissando; this effect of sonorous continuity was linked together with my remote experiences with mass internal transformations (Restagno 1988).

Metastasis was the starting point for his compositional research in which science and mathematics are applied. Paradoxically, he declared that this work was not inspired by music itself or by some logical principle, but by the impressions gained during the Nazi occupation of Greece. He listened to the noise of the masses marching towards the centre of Athens, to the shouting of slogans, to the intermittent shooting of guns, superposing each other in a chaos. He never forgot the unforeseeable transformation of the regular, rhythmic noise of a hundred thousand people into some fantastic disorder.

From the beginning of the 1950s, Xenakis was interested in two compositional themes. First of all, he wanted to write a kind of dodecaphonic music with the help of computations, which builds the macroforms out of a few basic principles. In *Metastasis* he made computations based on the permutation of intervals, with the help of the axiomatic approach known in mathematics. In the second place, he was interested in the continuous change of chords. Let us take, for instance, 6 of 12 notes. We get a harmonic colour. Then let us take the complementary pairs of those six notes. Once again, we get a harmonic colour. The change between the two occurs without any transition, abruptly.

[6] The orchestra opus in which the Modulor defines a strict relationship between tempo and sound. The name derives from "meta" that means beyond, after and "stasis" that means immobility. The problem fascinated the ancient philosophers, beginning with Parmenide and Zenone Their paradox about Achille and the turtle illustrate the very problem about the contrast between movement and immobility. At the same time the whole word metastasis recalls the medical way to speak about the proliferation of carcinogenic cells, as if one can indicate that they are similar to the improvement of the mass density.

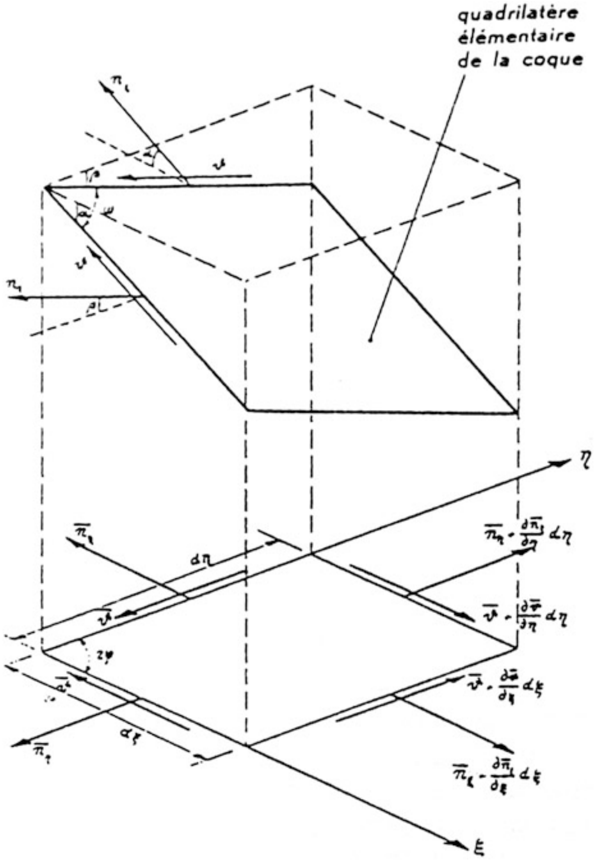

Fig. 72.7 Elementary quadrilateral

Coming from the theory of geometrical transformation and from the study upon sonorous masses—and its regular and irregular variations—the question is whether it was possible to get from one point to another without breaking the continuity.

In music, if we remain in the same scale, the only solution is a *glissando*. From the first six-note group toward the second one we can start in the direction of any note and in each case we get a different perceptive result. Moreover, if we have talked about the idea of continuous variation ("continuity charge"), in *Metastasis* the parallel development of the idea of discontinuous variation ("discontinuity charge"), represented by the permutation of intervals and the organisation of time based on the Golden Section, is very interesting.

In Le Corbusier's studio, Xenakis ran into fertile ground for his compositive obsessions. Le Corbusier's lessons on the mathematical spirit of the Modulor—about the opposition between the harmony of nature and the intellectualism of rules—joined forces with his researches on the Golden Section applied to dimensional changes of scales. Music becomes the image of a continuous movement produced by its own geometrical structure: a held note becomes a particular case of sonorous

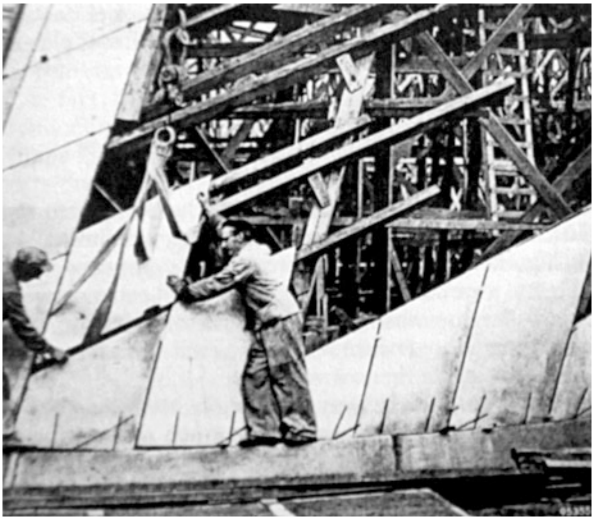

Fig. 72.8 Shell's assemblage

curve (that is, the straight line) and, on the contrary, we find that the *glissando* makes perceptible a perfect continuity that can go on to infinity.

In *Metastasis* and in the Philips pavilion, Le Corbusier and Xenakis proved that the forms used in music and in architecture are closely linked: in the musical work, this problem led to *glissandi*, while in the pavilion, it resulted in the hyperbolic paraboloid shapes. The difference between physical and musical space is that the former is homogeneous: both dimensions are lengths and distances. In music the natures of two dimensions (pitch and time) are alien to one another, only connected by their ordering structure.

In the *Poème Électronic* the correspondence between music and architecture is not only a matter of geometry. It was projected by Le Corbusier as if it were an orchestral work in which lights, loudspeakers, film projections on curved surfaces, spectators' shadows and their expression of wonder, objects hanging from the ceiling and the containing space itself were all virtual instruments. Architecture played, at the same time, the role of orchestral instrument and of sound box, container and contents.

The *Son et Lumière* Show

The *Poème Électronic* is an alchemy of 10 min length. Going into the pavilion, one's sight was lost in a neutral and disquieting space delimited by screen-walls largely displaying their geometrical construction. Loudspeakers, electro-acoustics and lighting sets were combined with the strained cable network in a curved interior where architecture was losing its character to become an allegorical and apocalyptical show. The *Poème* was composed therefore from seven sequences, projected according to a rigorous script.

This subdivision was not, however, in some way perceptible: the show flowed instead without solution of continuity with the exception of two pauses during which obscurity and the total silence went down, followed by the apparition of the *object mathematique* and of a mannequin that were lit up with ultraviolet light. In the place of the synthesis of the arts, the organic world of the living subject and the rational, concerning the intellect, were represented separately. The 8 min of Le Corbusier's sequence of images was accompanied by the music of Edgar Varèse and preceded by a brief interlude, composed by Xenakis: 2 min of concrete music destined to accompany the projection of an introductory text in English, French and Dutch. The most intense moments of this brief composition are those in which the most rapid rhythm succeeds to mix a feeling of granularity and powderines of the sound with that of perceptive continuity. The organization of time and rhythm are introduced here as continuity in the discontinuous, as a mass of single discreet elements. The result is the presentation, also in this very inner detail of the poem, of the general theme of the work, that of continuity. The introduction of the acoustic phenomenon as the element of dominion of the forms confirms the theory according to which the geometric-mathematic rule constitutes the common base of the architectural and musical compositions.

The novelty of the twentieth century, expressed in the Philips Pavilion, is the overcoming of a geometry that governs the simple repetition of crystallographical elements, in order to adopt more complex systems with independent and not homogeneous variables in comparison to the unit of measure. The introduction of the fourth dimension in the built space, after having discovered it in the abstract but equally real realm of mathematics, is not an automatic operation. To the three dimensions of Euclidean geometry is generally added the unit of measure of the time to make perceptible the effect of movement. A pictorial representation, or at least a sculptural one, of architecture, such as in a work of Boccioni, cannot exhaust the search of new spatial quality in architecture.

For Le Corbusier, the expression of beauty would still seem to derive from exactness, from a strong resistance to discord and from an iron will to oppose randomness—as is apparent from his reflections on the form of houses and cities; but the creation of the Philips Pavilion sketches the advent of sceneries in which the rule can be also that of the random or that of light and sound diffusion. In fact, the *Poème Électronic* predicted the deconstruction of music and of architecture: the same necessity, more than 10 years after the demolition of Philips Pavilion, was demonstrated, for example by John Cage, in the refusal of both dodecaphonical tyranny and the rigidity of the roles inside the places appointed for the listening. In the same way the space of the pavilion was flexible and the public could assume different positions to follow the show. Music and noise were always on the point of trespassing the one into the other. Deformity not only represented an ugly alteration of the form, but also constituted a first idea in the studies on the modification of architectural geometry. Double ruled quadric surfaces had freed the plastics of the volume, and with it the sonorous materials, conducting composition to the limit of disorder, without yet going beyond it.

"The splendid result was the natural gift of numbers. The implaccable and magnificent play of mathematics" (Le Corbusier 1954: 55). Music and architecture are thus inserted once again in the historical-scientific tradition that from pre-Socratic philosophy leads to contemporary mathematics, assuming eternal forms and innovating subject matters.

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Biography Alessandra Capanna is an Italian architect living and working in Rome. She received a degree and a PhD in Architecture from University of Rome "La Sapienza". Among her published articles on mathematical principles both in music and in architecture are: "Una struttura matematica della composizione", remarking the idea of self-similarity in composition; "Music and Architecture. A cross between inspiration and method", about three architectures by Steven Holl, Peter Cook and Daniel Libeskind (*Nexus Network Journal* 11, 2 (2009); "Iannis Xenakis. Combinazioni compositive senza limiti", "Limited, Unlimited, Uncompleted. Towards The Space Of 4d-Architecture" and "Tesseract Houses", about the topic of conceiving higher dimension architectures. She is a *Ricercatore* at the Faculty of Architecture of Rome "La Sapienza". She is the author of *Le Corbusier. Padiglione Philips, Bruxelles* (Universale di Architettura 67, 2000), on the correspondence between the geometry of hyperbolic paraboloids and technical and acoustic needs, and its aesthetics consequences.

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