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Hybrid Approach for Travel Time Seismic Tomography in Elliptical Anisotropic Media

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SUMMARY

Hybrid approach for seismic travel time tomography is proposed in the case of elliptical anisotropic media. A sequential scheme is presented that combines simulating annealing with linear least square inversion. Simulated annealing was implemented to obtain a velocity model that can be used as initial guess for linear least square inversion. Linear Traveltime Interpolation raytracing approximation is used to trace rays and calculate traveltimes. The procedure is tested both for a synthetic model and a real case. The real case comes from a previous study, that was solved by linear least square inversion. In both synthetic and real cases comparison is performed between the linear least square inversion results and hybrid approach; in the field study we compare the old results with those inferred by sequential approach.

Introduction

Seismic travel time tomography is needed in a large number of applications ranging from engineering and environmental problem until archeological prospection and characterization of building structures in both cross hole and surface configurations to map fields velocity of the studied area. Anisotropic seismic inversion is also at full used when characteristics of the materials show elastic parameters that depend on the direction of the investigation (Michelena R. J. et al.), in this case initially is necessary to individuate the main directions of the anisotropy and consequently the components of the velocity vector, it means that the unknowns are double with respect no anisotropic model.

Generally the linear inversion approach is the most used with respect the global optimization this is because less computational costs is need, by on the other side for the global optimization any initial guess is necessary, furthermore if the problem is well conditioned local minima are successful avoided. Sequential approach has the advantage to individuate by Simulated Annealing (SA) a good model near the global minimum that can be used as initial model for Linear Least Square Inversion (LLI), we tested the method in the case of anisotropic media that never was used for such model. For raytracing we used Linear Travel Time Interpolation (LTI) of Asakawa and Kawanaka (1993) who assumed that traveltime at any point along the cell boundary can be interpolated between the traveltime ad adjacent discrete points on the same cell boundary and modified by Cardarelli E. and Cerreto A. (2002) for elliptical anisotropic cases.

Simulated annealing

The SA algorithm follows the procedure proposed by Gokturkler (2011). SA is a stochastic optimization algorithm emulating the physical process whereby a melt is slowly cooled, and when its structure is "frozen" it means that minimum energy configuration is reached. In the SA algorithm, the values of the cost function correspond to the energy levels. In this approach the data misfit corresponds to the energy level of the melt, while the best model (optimum slowness cell values) that gives back the minimum error is searched by a randomize algorithm.

As initial guess, generally a homogeneous area is considered, in which the average of the velocities is calculated by using the straight ray path approximation, the model is characterized from a traveltime error E_0. Starting from this model, a new one is obtained by a random walk. If the new model has an error $E < E_0$ the new data set replaces the old one and a new set is selected; but if the error $E \geq E_0$ the new data set is conditionally accepted with the probability given by:

$$p(\Delta E) = \exp\left(\frac{-\Delta E}{kT}\right) \qquad (1)$$

where $\Delta E = E - E_0$, k is the Boltzmann constant and T is the temperature. A sequential use of this process individuate a set of parameters with a minimum traveltime error. During the solution the coefficient kT is lowered to take into account the cooling process. In this frame a sequential set of steps is performed.

Linearized least-square inversion

The least square inversion method is more popular because the easy calculation and the low computational cost, by the other hand to reach a good model near to the physical model some a priori information is need and this is a severe drawback in all those cases where any a priori information exist.

For the forward problem the equation proposed from Michelena Muir and Harris (1993) was used:

$$t_i(s) = \sum_{j=1}^{N} \sqrt{\Delta x_{ij}^2 S_j^2 + \Delta y_{ij}^2 S_{j+N}^2} \qquad (2)$$

where S_j are the slowness in the main direction of anisotropy S_{j+N} are the slowness in the perpendicular direction, and Δx and Δy are the components of the ray paths in the corresponding directions. The formula can be linearized by first-order Taylor series expansion centered on an initial model s_0. We used Coniugate-Gradient (CG) algorithm to invert data. The system was preconditioned using variable damping factors. The damping values were defined by identifying the low velocity zones detected from the solution of SA algorithm (Bernabini and Cardarelli 1997)

$$(J^T J + W)\Delta s = J^T \Delta t \qquad (3)$$

where W is the diagonal matrix of damping factors, **J** is the Jacobian solution matrix, $\mathbf{J}^T$ transpose of **J**, $\mathbf{\Delta s} = (\mathbf{s} - \mathbf{s_0})$ and $\mathbf{\Delta t} = \mathbf{t(s)} - \mathbf{t(s_0)}$.
To check the quality of results different parameters were used, in the synthetic example the Euclidean distance δ between the true and calculated velocities and the relative error with respect the mean velocity were used. In the case study the data distance σ_t between the field data and the calculated data and the relative error were used.

Synthetic Model

To evaluate our proposed sequential approach we considered a previous field study (Cardarelli, De Nardis 2001) where a column of the Pronaos of Antonino and Faustina Temple, located in Rome (Italy), was investigated. The columns of the Pronaos are formed of cipollino marble, this marble is composed of mica and calcite beds and shows characteritics of schistosity. This schistosity makes cipollino marble anisotropic from elastic point of view. In the previous study to perform seismic tomography on transversal sections, an anisotropic media, characterized by elliptical anisotropy with higher velocity along x direction and lower velocity along y direction, was assumed, LLI inversion and LTI ray tracing approximation were used (Cardarelli and Cerreto 2002), obtaining a data misfit σ_t= 0.060 ms that corresponds to 9.8 % of average of the reading traveltimes.

Synthetic model was made taking into account the field case with elliptical anisotropy, similar shape and size. The body has a radius of 0.75 m (Fig.1) and we divided the domain in 5x5 square cells with 0.3 m side grid model. We located 13 sources and 13 receivers alternatively equally distributed along the edge of the section. The velocity field was defined as follows: in the x direction low velocity area at north-west (2.0 km/s) and south-east (2.5 km/s) sides, elsewhere 3.0 km/s; in the y direction low velocity area in south-west zone (1.0 km/s), elsewhere 2.0 km/s. In figure 1 are shown respectively the two fields velocity, and the ray coverage with the double values of velocities in each cell. Noisy data set was generated by adding zero-mean Gaussian random noise with a standard deviation of 5% of average value to each traveltime.

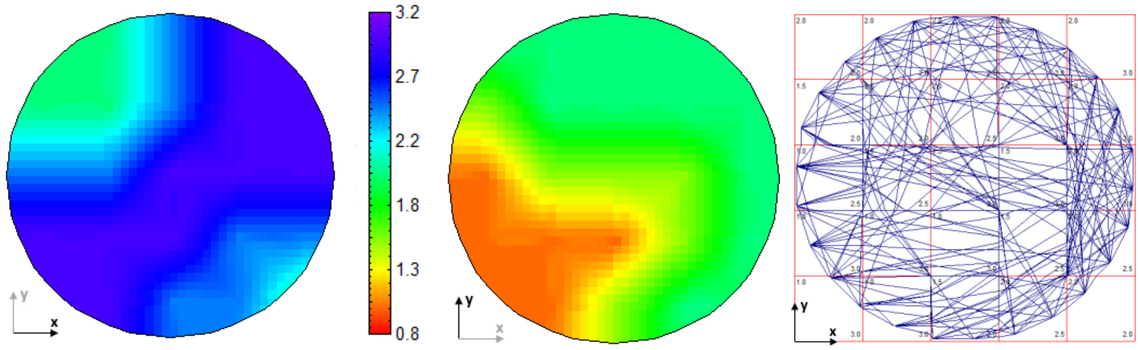

Figure 1 *Anisotropic synthetic model; Higher field velocity a), lower field velocity b), ray coverage and cell velocity values c).*

Model was solved using the following approaches:

- Simulated annealing (SA)
- Linear least square solution (LLI)
- Hybrid sequential approach (SA + LLI), assuming the solution of SA as initial guess for LLI

Simulated annealing solution

We carried out the following procedure:

- Execution of simulated annealing procedure, ten experiments, each with a different sequence of random numbers;
- Calculus of data and model misfit for traveltimes and velocities for each experiment and for an "average model", obtained assuming the velocity for each cell as the average of the ten values computed for that cell

Simulated annealing optimization was performed using the values parameters showed in table 1.

DATA TYPE	STEP SIZE (KM/S)	COOLING FACTOR	INITIAL VALUE (KT)	ITERATIONS FOR EACH COOLING	TOTAL ITERATIONS
SYNTHETIC	0.01	0.97	$2X10^{-4}$	100	50000

Table 1 *In the table are shown the parameters used for simulated annealing optimization.*

In figure 2 plot of the error energies versus the iterations (50000) of the ten experiments is shown, all the experiments reach similar error energies to demonstrate the robustness of the algorithm. The interval of minimum energy ranges between 0.00110 and 0.00060.

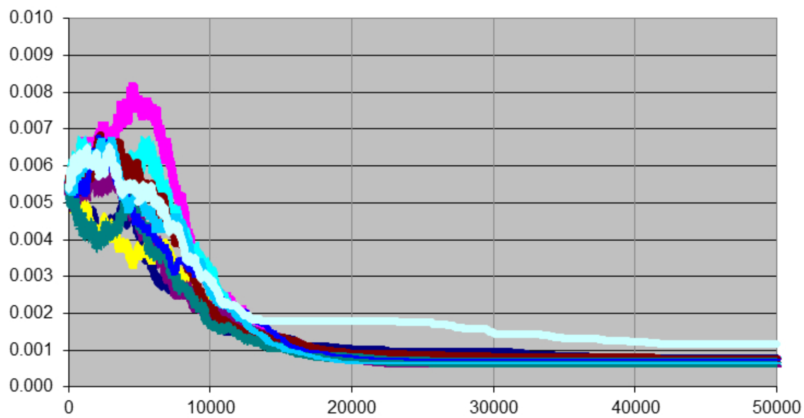

Figure 2 *Plot of the error energies versus number of iterations of the ten experiments performed in the SA approach using the synthetic data.*

As final result we considered the average that we obtained taking into account the values of each cell for each test.

Linear Least Square Inversion

Synthetic data were inverted using LLI following the formulation depicted in 3) and 4). Different damping factors λ having different values were used for both directions ranging from 0.1 to 0.3. Starting from an homogeneous model convergence was reached after 8 iterations with a σ_t=5.16%, and σ_v = 11.4%. We could not to increase the number of iterations because the data misfit diverged.

Hybrid approach

Alternatively, a LLI inversion solution was obtained starting from the "average model" obtained with SA. Biconiugate gradient algorithm was used for inversion, convergence was reached after 17 external iterations, data-misfit value was σ_t=4.96% that was less than the one that we reach with SA approach σ_t=5.6% , the σ_v = 10.2% , it was less than the ones that we obtained with SA and with LLI alone. Results are shown in figure 3.

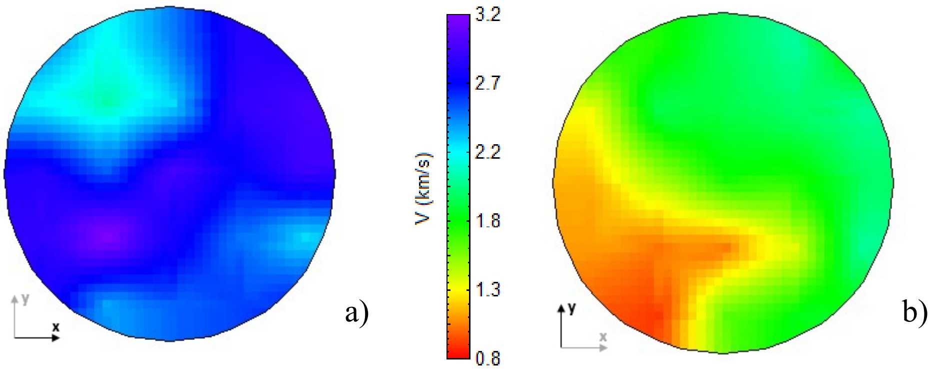

Figure 3 *Velocity fields obtained by SA + LLI approach: higher field velocity a), lower field velocity b)*

Conclusions

Results indicate that the velocity field obtained by hybrid sequential approach approximates better the synthetic model of figure 1, in fact the low velocity zones of the higher velocity component (figure 3a) are better identified with respect SA and LLI inversion, same considerations can be made for the lower velocity component (figure 3b) where, the high velocity zone is detected with higher resolution. By the other hand this is demonstrated by σ_v that is the smaller with respect the ones reached with SA and LLI. Same considerations can be made to case study, for wich hybrid approach led to $\sigma_t = 0.054$ ms that corresponds to 8.8 % of average of the reading traveltimes with respect to 9.8% obtained by using LLI alone.

References

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