Constraints on Cosmological Parameters from Future Cosmic Microwave Background Experiments

To cite this article: Alessandro Melchiorri *et al* 2010 *J. Phys.: Conf. Ser.* **259** 012004

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Constraints on Cosmological Parameters from Future Cosmic Microwave Background Experiments

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Abstract. The Planck satellite experiment will soon let cosmologists to determine most of the cosmological parameters with unprecedented accuracy. In particular a strong improvement is expected in many parameters of interest, including neutrino mass, the amount of relativistic particles at recombination, the primordial Helium abundance and the injection of extra ionizing photon by dark matter self-annihilation. Here we review the constraints achievable by future experiments and discuss the implications for fundamental physics.

1. Introduction

The past two decades have seen dramatic improvements in measurements of the microwave background temperature fluctuations (see e.g. [1], [2] and[3]) and Planck's highly anticipated results (see [4]) will further significantly improved constraints on cosmological parameters. Moreover, on-going and planned ground-based and balloon-based experiments ([11], [12], [14], [13]) are exploring two important open frontiers: (a) the measurement of extreme ($\leq 5'$) small-scale temperature and polarization fluctuations [5] and (b) the search for primordial B-modes, the distinctive signature of gravitational waves from inflation, on large scales [6]. Proposals for a next CMB satellite as CMBPol [15] or B-POL [16] are under evaluation from american and european space agencies.

Here we briefly review the cosmological information that could come from these new datasets. We will mainly present the results obtained in a recent paper [7]. We consider a wide set of parameters focusing on those that mainly affect the "damping tail" of the CMB angular spectrum. We indeed consider additional parameters as the total neutrino mass $\sum m_\nu$ (that affects the growth of structure in the late universe), the number of extra relativistic neutrino particles N_ν^{eff} (that changes the matter-radiation epoch), and possible changes in the recombination process by changes in the fractional helium abundance Y_p, from dark matter self-annihilation processes and from variations in fundamental constants as the fine structure constant α and the Newton gravitational constant G.

As discussed in [7] we will consider 3 experimental configurations: the Planck satellite [4], the combined Planck plus ACT in polarization-sensitive detectors ACTPol [14] and, finally, the next CMBPol satellite [15]. We refer the reader to [7] for a detailed explanation of the analysis and forecast methods used.

Parameter	Planck	Planck+ACTPol	CMBPol
$\Delta(\Omega_b h^2)$	0.00013	0.000078	0.000034
$\Delta(\Omega_c h^2)$	0.0010	0.00064	0.00027
$\Delta(\theta_s)$	0.00026	0.00016	0.000052
$\Delta(\tau)$	0.0042	0.0034	0.0022
$\Delta(n_s)$	0.0031	0.0021	0.0014
$\Delta(log[10^{10}As])$	0.013	0.0086	0.0055
$\Delta(H_0)$	0.53	0.30	0.12

Table 1. 68% c.l. errors on cosmological parameters from future surveys. Standard case.

2. Constraints on a "minimal" $\Lambda - CDM$ scenario

In Table 1 we report the future constraints on the parameters of a "minimal" cosmological model. As we can see in the Table, the CMBPol experiment can provide a strong improvement (factor ~ 5) in the constraints on the baryon density, H_0 and θ_s, while the constraints on parameters as n_s are improved by a factor ~ 2. This Table will be useful in the following since it will be straightforward to identify the effect of the inclusion of an extra-parameter in the analysis.

3. Future Constraints on Neutrino Masses

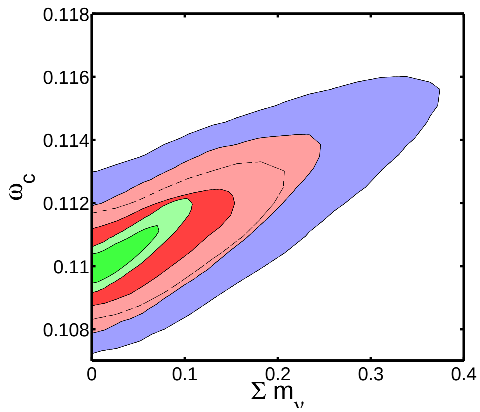

Figure 1. 68% and 95% likelihood contour plots on the $\sum m_\nu$ - ω_c plane for Planck (blu), Planck+ACTPol (red) and CMBPol (green).

Parameter	Planck	Planck+ACTPol	CMBPol
$\Delta(\Omega_b h^2)$	0.00014	0.000081	0.000033
$\Delta(\Omega_c h^2)$	0.0017	0.0010	0.00071
$\Delta(\theta_S)$	0.00028	0.00016	0.000062
$\Delta(\tau)$	0.0042	0.0034	0.0023
$\Delta(n_S)$	0.0034	0.0022	0.0016
$\Delta(\log[10^{10}A_S])$	0.013	0.0094	0.0065
$\Delta(\sum m_\nu)$	< 0.16	< 0.08	< 0.05

Table 2. 68% c.l. errors on cosmological parameters in the case of massive neutrinos.

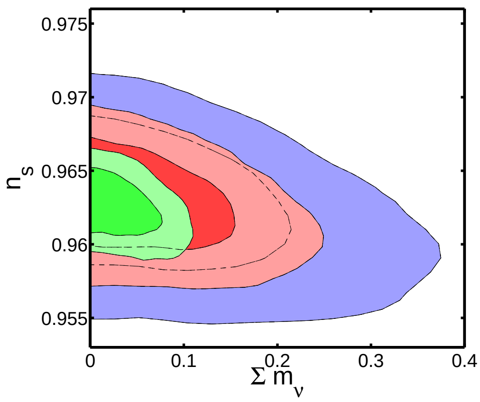

Figure 2. 68% and 95% likelihood contour plots on the $\sum m_\nu$ - ns plane for Planck (blu), Planck+ACTPol (red) and CMBPol (green).

CMB angular spectra are sensitive to a total variation in neutrino masses (see e.g. [25, 26]) defined by $\Sigma_{\nu=1,..3} m_\nu$ but can't discriminate between the mass of a single neutrino flavour (see e.g. [27])

Current oscillation experiments provide essentially two mass differences for the neutrino mass eigenstates: $\Delta m^2_{solar} \sim 8 \times 10^{-5} eV^2$ and $\Delta m^2_{atm} \sim 2.5 \times 10^{-3} eV^2$ (see e.g. [28] and references therein). An inverted hierarchy in the neutrino mass eigenstates predicts a lower limit to the total neutrino mass of about $\sum m_\nu \geq 0.10 eV$ while a direct hierarchy predicts $\sum m_\nu \geq 0.05 eV$. The goal for CMB experiments is therefore to have a sensitivity better than $\sum m_\nu \leq 0.10 eV$ for possibly ruling out the inverted hierarchy and better than $\sum m_\nu \leq 0.05 eV$ for a sure detection of neutrino mass.

As we can see from Table 2 the expected sensitivity from Planck and Planck+ACTPol fails to reach the possibility of ruling out the neutrino mass inverted hierarchy. It is however important to notice that the expected sensitivity from the KATRIN [29] beta decay experiment is of the order of $\sum m_\nu = 0.3$. Planck and Planck+ACTPol will therefore explore the same energy scale, providing a great opportunity for confirming or anticipating a mass detection from KATRIN.

Including a neutrino mass, as we can see comparing Table 2 with Table 1 as a relevant impact in the determination of the cold dark matter density ω_c that results with an uncertainty that is nearly doubled respect to the standard analysis. Moreover, also the constraints on n_s are affected. We plot in Figure 1 and Figure 2 the 2-D likelihood contour plots at 68% and 95% confidence level in the Σm_ν vs ω_c and vs n_s planes respectively. As we can see, a non negligible neutrino mass can put higher values of the cold dark matter abundance and lower values of the scalar spectral index in better agreement with observations.

4. Future Constraints on Extra Background of Relativistic Particles

An extra background of relativistic (and non-interacting) particles can be parametrized by introducing an effective number of neutrino species N_ν^{eff}. This extra-background changes the CMB anisotropies through time variations of the gravitational potential at recombination due to the presence of this non negligible relativistic component (the so-called early Integrated Sachs Wolfe effect). The main consequences is an increase in the small-scale CMB anisotropy (see e.g. [31]). The results are reported in Table 3. As we can see, comparing with the results in Table 3, the inclusion of a background of relativistic particles strongly weakens the constraints on n_s, ω_b, ω_c and θ_s. As we can see from Figures 3, 4, 5 and 6 there is indeed a strong correlation between N_ν^{eff} and these parameters. While adding ACT will improve the constraints by a factor ~ 2, CMBPol can provide constraints that could bring valuable information on the

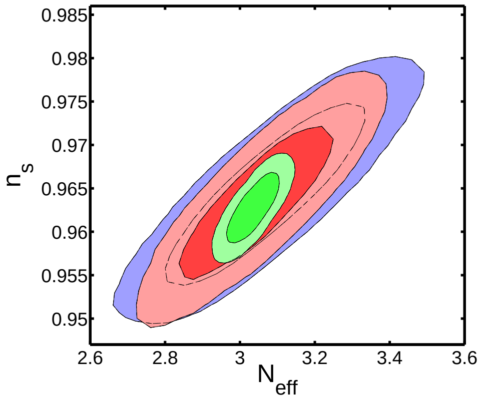

Figure 3. 68% and 95% likelihood contour plots on the N_{eff} - ns plane for Planck (blue), Planck+ACTPol (red) and CMBPol (green).

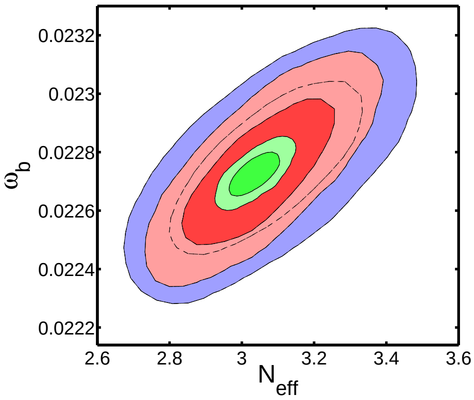

Figure 4. 68% and 95% likelihood contour plots on the N_{eff} - ω_b plane for Planck (blue), Planck+ACTPol (red) and CMBPol (green).

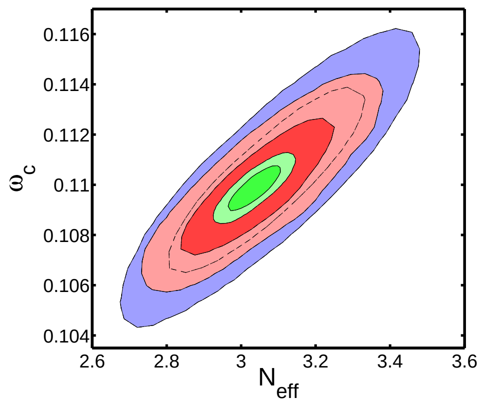

Figure 5. 68% and 95% likelihood contour plots on the N_{eff} - ω_c plane for Planck (blu), Planck+ACTPol (red) and CMBPol (green).

physics of neutrino decoupling from the photon-baryon primordial plasma. As it is well known, the standard value of neutrino parameters $N_{eff} = 3$ should be increased to $N_{eff} = 3.04$ due to an additional contribution from a partial heating of neutrinos during the electron-positron

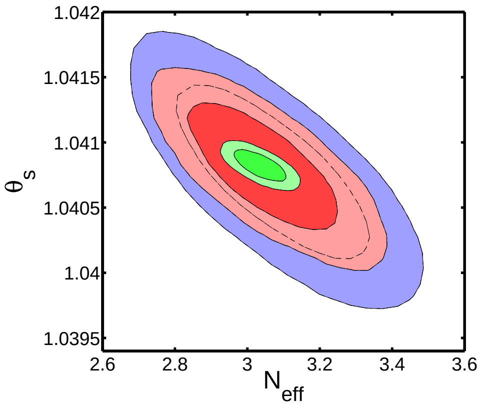

Figure 6. 68% and 95% likelihood contour plots on the N_{eff} - θ_s plane for Planck (blu), Planck+ACTPol (red) and CMBPol (green).

annihilations (see e.g. [32]). This effect, expected from standard physics, could be tested by the CMBPol experiment, albeit at just one standard deviations. However, the presence of non standard neutrino-electron interactions (NSI) may enhance the entropy transfer from electron-positron pairs into neutrinos instead of photons, up to a value of $N_{eff} = 3.12$ ([33]). This value could be discriminated by CMBPol from $N_{eff} = 3$ at ~ 3 standard deviations, shedding new light on NSI models.

	Planck	Planck+ACTPol	CMBPol
$\Delta(\Omega_b h^2)$	0.00018	0.00013	0.000051
$\Delta(\Omega_c h^2)$	0.0024	0.0015	0.00059
$\Delta(\theta_s)$	0.00042	0.00024	0.000075
$\Delta(\tau)$	0.0043	0.0035	0.0023
$\Delta(n_s)$	0.0065	0.0049	0.0026
$\Delta(log[10^{10}A_s])$	0.017	0.013	0.0077
$\Delta(N_{eff})$	0.17	0.11	0.046

Table 3. 68% c.l. errors on cosmological parameters in the case of extra background of relativistic particles N_{eff}.

5. Future Constraints on Dark Matter Self Annihilation

Annihilating particles affect the ionization history of the Universe in three main different ways. The interaction of the shower produced by the annihilation with the thermal gas can ionize it, induce Ly–α excitation of the hydrogen and heat the plasma. The first two modify the evolution of the free electron fraction x_e, the third affects the temperature of baryons [34].

The rate of energy release $\frac{dE}{dt}$ per unit volume by a relic self-annihilating dark matter particle is given by

$$\frac{dE}{dt}(z) = \rho_c^2 c^2 \Omega_{DM}^2 (1+z)^6 p_{ann} \quad (1)$$

$$p_{ann} = f \frac{<\sigma v>}{m_\chi} \quad (2)$$

with $n_{DM}(z)$ being the relic DM abundance at a given redshift z, $<\sigma v>$ is the effective self-annihilation rate and m_χ the mass of our dark matter particle, Ω_{DM} is the dark matter density parameter and ρ_c the critical density of the Universe today; the parameter f indicates the fraction of energy which is absorbed *overall* by the gas, under the approximation the energy absorption takes place locally. CMB is sensitive to the combined parameter p_{ann} only. Greater is p_{ann}, higher is the fraction of free electrons surviving after recombination, therefore widening the peak of the visibility function and dampening the peaks of the temperature and polarization angular power spectra.

Parameter	Planck	Planck+ACTPol	CMBPol
$\Delta(\Omega_b h^2)$	0.00013	0.000079	0.000032
$\Delta(\Omega_c h^2)$	0.0010	0.00063	0.00027
$\Delta(H_0)$	0.52	0.30	0.12
$\Delta(\tau)$	0.0042	0.0034	0.0023
$\Delta(n_S)$	0.0032	0.0021	0.0015
$\Delta(\log[10^{10}A_S])$	0.013	0.0085	0.0055
$\Delta(p_{ann})[m^3/s/Kg]$	$< 1.5 \cdot 10^{-7}$	$< 1.2 \cdot 10^{-7}$	$< 6.3 \cdot 10^{-8}$

Table 4. 68% c.l. errors on cosmological parameters in the case of dark matter annihilation. The upper limits on p_{ann} are at 95% c.l.

As we can see from Table 4 and comparing with the results in Table 1 the inclusion of dark matter self-annihilation doesn't affect much the constraints on the other parameters.

6. Future Constraints on Helium Abundance

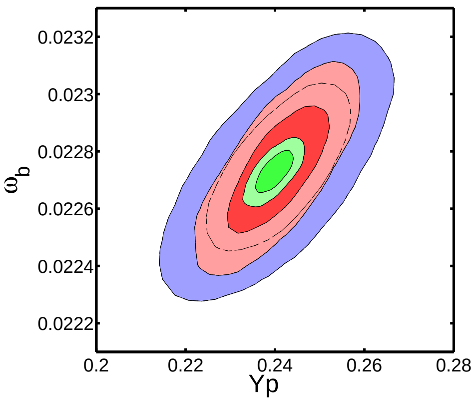

Figure 7. 68% and 95% likelihood contour plots on the Y_{He} - ω_b plane for Planck (blue), Planck+ACTPol (red) and CMBPol (green).

As recently shown by several authors ([35], [36], [37], [38]) the small scale CMB anisotropy spectrum can provide a powerful method for accurately determining the primordial Helium abundance. Current astrophysical measurements of primordial Helium converge towards a conservative aestimate of $Y_p = 0.250 \pm 0.003$ (see e.g. [39]). As we can see from Table 5 the Planck satellite mission alone will not reach such accuracy, even when combined with ACT. It is however interesting that a CMBPol-like experiment has the potential of reaching a precision comparable with current astrophysical measurements. This will open a new window of research for testing systematics in current primordial helium determinations.

Comparing the results in Table 5 with the constraints obtained in the case of a standard analysis it is easy to see that the major impact of including this parameter is on the determination

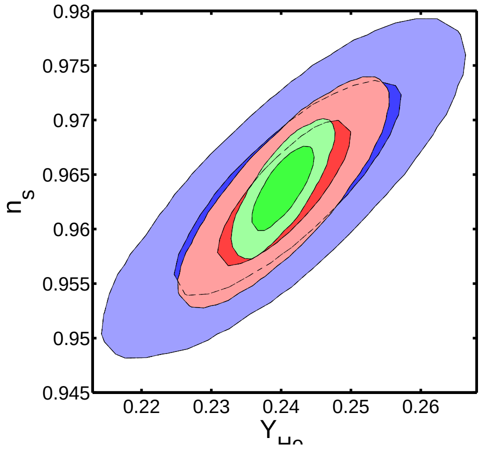

Figure 8. 68% and 95% likelihood contour plots on the Y_{He} - ns plane for Planck (blue), Planck+ACTPol (red) and CMBPol (green).

Parameter	Planck	Planck+ACTPol	CMBPol
$\Delta(\Omega_b h^2)$	0.00019	0.00013	0.000051
$\Delta(\Omega_c h^2)$	0.0010	0.00065	0.00027
$\Delta(\theta_S)$	0.00046	0.00026	0.00010
$\Delta(\tau)$	0.0043	0.0035	0.0023
$\Delta(n_S)$	0.0063	0.0043	0.0025
$\Delta(\log[10^{10}A_S])$	0.013	0.013	0.0079
$\Delta(Y_p)$	0.010	0.0061	0.0029

Table 5. 68% c.l. errors on cosmological parameters in the case of helium abundance.

of the scalar spectral index n_s and the baryon abundance, with the 1-σ c.l. increased by a factor ~ 2. In Figures 7 and 8 we plot the 2-D likelihood contours at 68% and 95% c.l. between Y_p and these parameters.

7. Future Constraints on Variations of Fundamental Constants

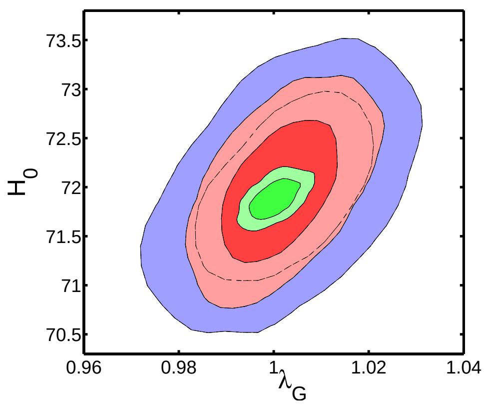

Figure 9. 68% and 95% likelihood contour plots on the λ_G - H_0 plane for Planck (blue), Planck+ACTPol (red) and CMBPol (green).

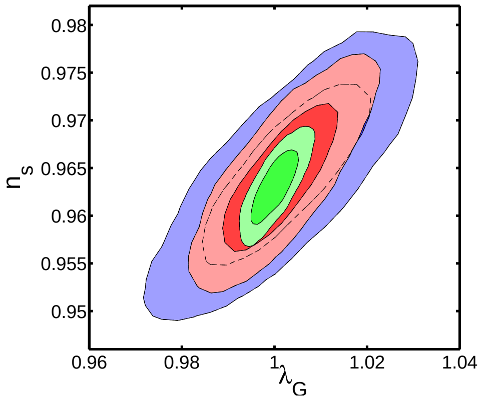

Figure 10. 68% and 95% likelihood contour plots on the λ_G - ns plane for Planck (blue), Planck+ACTPol (red) and CMBPol (green).

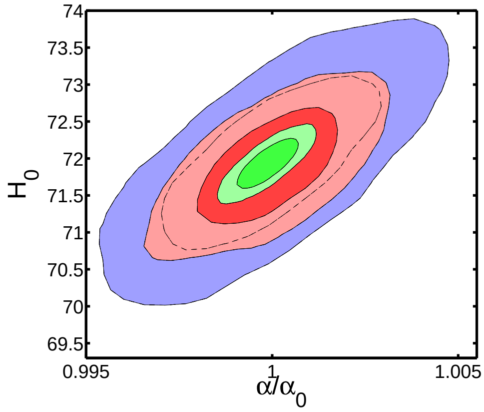

Figure 11. 68% and 95% likelihood contour plots on the α/α_0 - H_0 plane for Planck (blue), Planck+ACTPol (red) and CMBPol (green).

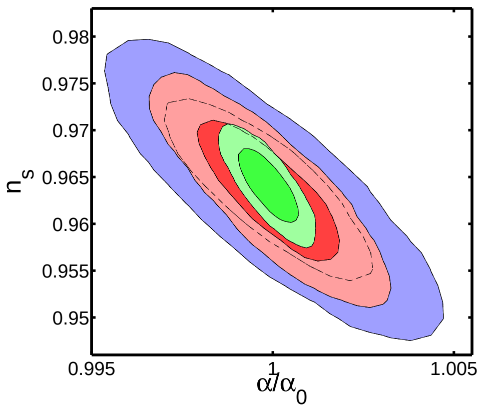

Figure 12. 68% and 95% likelihood contour plots on the α/α_0 - ns plane for Planck (blue), Planck+ACTPol (red) and CMBPol (green).

CMB anisotropies are sensitive to variations in fundamental constants as the fine structure α (see e.g. [42], [43]) or Newton's constant G ([44]) through changes in the recombination scenario. Varying α changes the ionization and excitation rates and could delay or accelerate recombination. Varying G does not affect recombination directly but "rescales" the expansion rate of the Universe, changing the epoch when recombination takes place.

The constraints are reported in Table 6 and Table 7 for variations in α and G respectively. In order to parametrize the variations with dimensionless quantities we have considered variations in the parameters $\Delta_\alpha = \alpha/\alpha_0$ e $\lambda_G = G/G_0$ where α_0 and G_0 are the current values of these fundamental constants, measured in laboratory[1] $\alpha_0 = 7.2973525376(50) \times 10^{-3}$ and $G_0 = 6.67428(67) \times 10^{-11} m^3 kg^{-1} s^{-2}$.

	Planck	Planck+ACTPol	CMBPol
$\Delta(\Omega_b h^2)$	0.00014	0.000089	0.000034
$\Delta(\Omega_c h^2)$	0.0012	0.00070	0.00031
$\Delta(\tau)$	0.0042	0.0034	0.0023
$\Delta(H_0)$	0.77	0.40	0.20
$\Delta(n_s)$	0.0064	0.0035	0.0025
$\Delta(log[10^{10} A_s])$	0.0086	0.011	0.0041
$\Delta(\alpha/\alpha_0)$	0.0019	0.00093	0.00051

Table 6. 68% c.l. errors on cosmological parameters from future surveys in case of a variable fine structure constant α.

	Planck	Planck+ACTPol	CMBPol
$\Delta(\Omega_b h^2)$	0.00019	0.00013	0.000048
$\Delta(\Omega_c h^2)$	0.0010	0.00068	0.00025
$\Delta(\tau)$	0.0042	0.0037	0.0022
$\Delta(H_0)$	0.60	0.40	0.13
$\Delta(n_s)$	0.0061	0.0046	0.0023
$\Delta(log[10^{10} A_s])$	0.018	0.013	0.0073
$\Delta(\lambda_G)$	0.012	0.0076	0.0030

Table 7. 68% c.l. errors on cosmological parameters from future surveys in case of a variable gravitational constant G.

As we can see from Tables 6 and 7 a variation in these fundamental constants has important effects in the determination of the scalar spectral index n_s and the Hubble costant H_0. This can also be seen in the 2-D likelihood contour plots in Figures 9, 10, 11, and 12.

8. Conclusions

Here we have briefly reviwed the future constraints achievable from CMB experiments on several parameters. Other than the 5 parameters of the standard Λ-CDM model we have considered new parameters mostly related to quantities that can be probed in a complementary way in laboratory and/or with astrophysical measurements. We found that CMB experiments as CMBPol could have a very important impact in the understanding of neutrino physics. CMBPol could indeed discriminate between the neutrino mass hierarchy and shed light on the physics of neutrino decoupling before BBN. Moreover, the primordial Helium abundance can be constrained with the same accuracy of current astrophysical measurements but with a much better control of systematics. Moreover, also constraints on fundamental constant can reach a level close

[1] See http://www.codata.org/

to laboratory constraints. This overlap between cosmology and other sector of physics and astronomy is definitely the most interesting aspect of future CMB research.

9. Acknowledgements

It is a great pleasure to thank our collaborators Blake Sherwin and David Spergel.

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