A H$_2$ PEM Fuel Cell and High Energy Dense Battery Hybrid Energy Source for an Urban Electric Vehicle

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Abstract - **Electric vehicles are set to play a prominent role in addressing the energy and environmental impact of an increasing road transport population by offering a more energy efficient and less polluting drive-train alternative to conventional internal combustion engine (ICE) vehicles. Given the energy (and hence range) and performance limitations of electro-chemical battery storage systems, hybrid systems combining energy and power dense storage technologies have been proposed for vehicle applications. The paper will discuss the application of a hydrogen fuel cell as a range extender for an urban electric vehicle for which the primary energy source is provided by a high energy dense battery. A review of fuel cell systems and automotive drive-train application issues are discussed, together with an overview of the battery technology. The prototype fuel cell and battery component simulation models are presented and their performance as a combined energy/power source assessed for typical urban and sub-urban driving scenarios.**

I INTRODUCTION

The impetus for more environmentally friendly road vehicles and alternative road vehicle energy conversion has fostered research and development in electrically powered vehicles for road transport applications since the late 98 s This is particularly the case for medium to heavy duty vehicles where some additional propulsion system mass is not as critical as for smaller passenger vehicles Further in recent years fuel cell systems have also been proposed as a potential energy carrier and the most suitable alternative likely to displace petroleum based fuels during the first half of this century [   ] Whilst there are many technical and resource management issues associated with the displacement of petroleum fuels for transportation and the commensurate supply infrastructure requirements this paper will discuss some of the application issues associated with the implementation of hybrid energy sources for electric and fuel cell vehicles Specifically the paper will report on initial drive train design results from a research programme investigating the utility of an electric Taxi supplied via a high energy dense electro chemical battery and hydrogen fuel cell range extender for inner city operation

The aims of the research programme are to investigate and address the principal technical difficulties associated with the future commercial application of fuel cell technologies in electric vehicle traction drive trains As such a zero emission London taxi powered via two high peak power   kW) high temperature ZEBRA batteries and a kW hydrogen Proton Exchange Membrane fuel cell PEMFC) system is being developed for vehicle power train test evaluation as illustrated in Fig showing the vehicle and drive train layout schematic The prime mover for the taxi is a brushless permanent magnet pm) machine and integrated gear reduction and differential drive to the vehicle back axle   The pm machine is controlled via a three phase voltage source converter the dc supply to which is provided by the traction battery and fuel cell via a dc:dc converter The vehicle on board hybrid energy source will allow the PEMFC to operate predominantly at a steady power and at power levels associated with optimal fuel energy conversion efficiency with the battery acting to buffer peak loads recover vehicle braking energy and provide the bulk energy demand Hence the fuel cell operates primarily in a range extension function

a) London Taxi
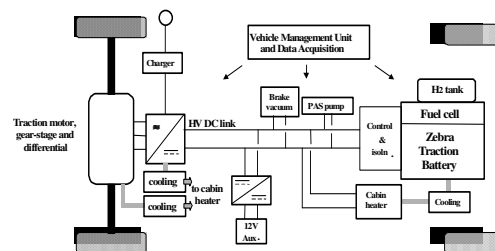
b) Drive train schematic

Fig   Hydrogen fuel cell high energy dense battery electric London Taxi and drive train layout schematic

The paper will review fuel cell systems and discuss automotive drive train application issues together with an overview of the battery technology The regulation of the traction battery and fuel cell when subject to the dynamic power loading illustrated in Fig   b)

necessitates detailed modelling to assess the functionality of the individual components once interconnected within the drive train Hence the prototype fuel cell and battery component simulation models are presented and their performance as a combined hybrid energy source assessed for typical dynamic urban and sub urban driving duty cycle scenarios It is shown that the fuel cell and battery combination are complementary for such duty loading extending the vehicle range whilst minimising the installed fuel cell power

II VEHICLE ENERGY AND POWER REQUIREMENTS

For road vehicle applications the on board energy and power sources must satisfy the load demand of the vehicle traction drive train The decision as to whether the energy storage medium supplies all of the vehicle load or simply the average power requirements can significantly influence the sizing of the vehicle energy power systems and hence system cost The difficulty in making this assessment is in choosing the most appropriate duty rating specification for the vehicle For example Fig   illustrates a typical 5 tonne urban electric vehicle a London Taxi which is the reference vehicle for the study The power required to propel the vehicle over the NEDC driving cycle Fig   a) that comprises of 4x enhanced European Commission R 5 4 ECE 5) urban cycles and x EC sub urban cycle [   ] is detailed in Table I showing a wide disparity in peak to average power requirements i e   7: and 4: for the urban and sub urban profiles respectively
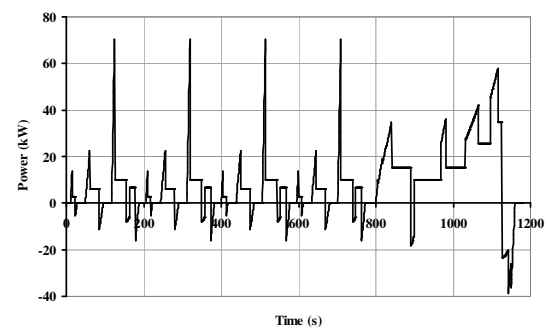
Table I Vehicle power requirement

Driving cycle	Power condition	Cycle time s)	Range km)	Power kW)
ECE 5	Max motoring	95		7 5
	Max regenerating			
	Average			4
Sub urban	Max motoring	4	9	57 84
	Max regenerating			8 8
	Average			4 57
NEDC	Average	8	47	7 7

The data in Table I is calculated via solution of the vehicle kinematics [ ] with the NEDC linear velocity driving cycle of Fig   a) The vehicle parameter data for the Taxi is given in Appendix I The vehicle dynamic power profile calculated over the NEDC driving cycle is illustrated in Fig   b) and used for subsequent vehicle performance assessment There is also a similar disparity in the vehicle peak average power for other driving or duty) cycles i e the Highway fuel economy test schedule HWFET) US 975 schedule FTP75) and Japanese   mode test schedule hence the potential to over size the vehicle energy source for single source systems as discussed in [4] Note that whilst all sources are a source of energy

reference is made here to energy and power to emphasise the functionality of the vehicle on board sources with respect to the vehicle energy management philosophy
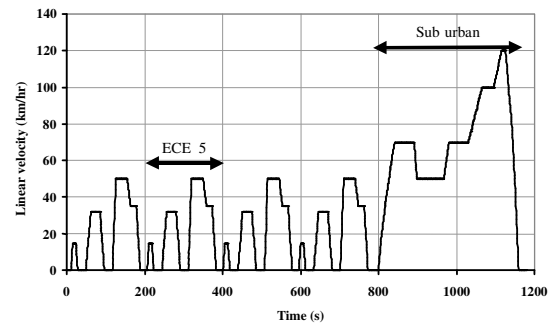
a) NEDC 4x ECE 5   sub urban) driving cycle

b) Vehicle power vs time

Fig   Vehicle linear velocity and associated dynamic power requirements for the London Taxi

III FUEL CELLS FOR TRANSPORTATION

A. Background

A major advantage of fuel cell powered vehicles is the development of cleaner more energy efficient cars trucks and buses that can initially operate on conventional fuels via local reformation i e gasoline and diesel whilst enabling the technology platform for a future move to renewable and alternative fuels i e methanol ethanol natural gas and other hydro carbons and ultimately hydrogen a particularly significant issue when considering the infrastructure and support requirements of a modern transportation network

With on board fuels other than pure hydrogen for example natural gas methanol and gasoline the fuel cell systems could use an appropriate fuel processor to convert the fuel to hydrogen Since the fuel cell relies on chemistry and not combustion local emissions from this type of a system should potentially be much smaller than emissions from the cleanest fuel combustion process emissions whilst offering the advantages of an electric transmission However in traction systems fuel cells have major operational disadvantages in turns of their voltage regulation and inability to accept vehicle kinetic energy during braking

[5] hence the consideration of a hybrid energy power source

For the taxi vehicle test platform x kW PEM fuel cells have been chosen to provide a background energy input essentially acting as a vehicle range extender The fuel cells are prototype systems developed by MES DEA Switzerland [5] and are designed to realise a very compact lightweight and simple fuel stack The stack has separate forced air flow systems for cooling and reaction air supply operate close to ambient pressure on the cathode for seal integrity have a modular layout but most significantly has no auxiliary humidification components A microprocessor manages the associated cooling and air flow fans steering electronics for membrane hydration main and purge valves Fig illustrates the kW prototype fuel cell system and control electronics the main specification details of which are given in Appendix I [5]
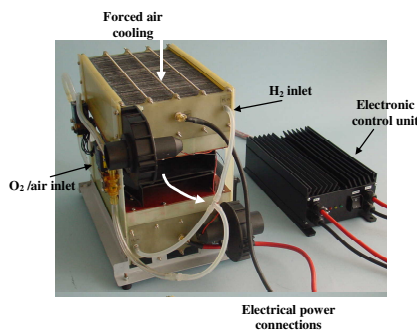

Fig   Prototype fuel cell system and control electronics [5]

B. Fuel Cell Modelling

As with electro chemical batteries fuel cells exhibit non linear performance characteristics that can significantly influence vehicle drive system operation and component optimisation if not considered at the system design stage The three main fuel cell loss mechanisms can be summarised as:

- irreversible activation polarisation loss
- concentration polarisation loss and
- ohmic or resistance polarisation

The influence of these loss mechanisms on fuel cell performance is illustrated in Fig 4 showing measured fuel cell voltage and power output as a function of cell current density or load current) The fuel cell can be modelled by a semi empirical equation as discussed in [7] for which parameters are calculated through an identification process with experimental data viz :

$$V_{cell} = E_o - b \ln(J) - (R_{int} J) - k \ e^{(k \ J)} \qquad)$$

The equation terms are derived from the associated Nernst Tafel and Ohm s laws where V_{cell} is the fuel cell terminal voltage E_o is the steady open circuit voltage b is the Tafel s parameter for oxygen reduction J the current density R_{int} the cell ohmic resistance and k_1 and k_2 diffusion parameters Whilst the model is not universal with regard to the fuel cell fundamental

chemistry it is much simpler in form and represents the main voltage loss components Each term in) is dominant in each region of the *V-J* characteristic In region A the cell voltage decreases drastically due to the oxygen electrochemical activation reactions where the logarithm term has the main influence In region B the curve is roughly linear i e essentially ohmic resistive losses and Region C corresponds to diffusion losses i e exponential term The five parameters in) depend on cell temperature and gas pressures However for the fuel cell stack considered the stack temperature is tightly regulated via forced ventilation and the stack pressure is fixed to 4bar Fig 4 shows measured voltage regulation and power capacity with load current for a kW MES DEA fuel cell
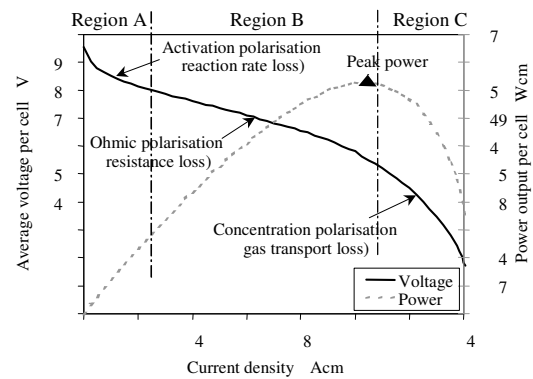
Fig 4 Measured voltage and power data for MES DEA H PEM fuel cell [ ]
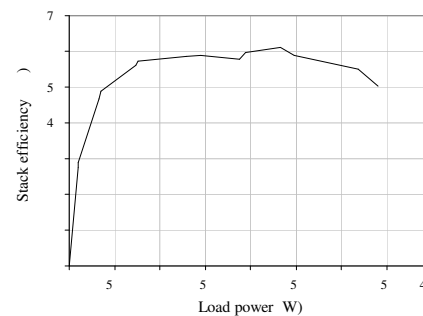
Fig 5 Measured fuel cell stack hydrogen to electrical energy efficiency with output load

For modelling purposes implementation of) can be problematic at zero current; hence a simpler quadratic fit to the measured data of Fig is used:

$$V_{cell} = k_a + k_b \ J + k_c \ J^2 + k_d \ J^3 + k_e \ J^4 + $$
$$k_f \ J^5 + k_g \ J^6 + k_h \ J^7 + k_i \ J^8 + k_j \ J^9 + k_k \ J^{10} \qquad)$$

where V_{cell} is the fuel cell terminal voltage per cell and *J* the cell current density The fuel cell terminal voltage is used in conjunction with the measured stack fuel to electrical conversion efficiency as illustrated in Fig 5 to simulate operation and predict performance Again a

curve fit to the measured data of Fig 5 is used in the fuel cell model:

$$\eta = a_a + a_b\ P_{\%}^{\ 0.5} + a_c\ P_{\%} + a_d\ P_{\%}^{\ 1.5} + a_e\ P_{\%}^{\ 2} + a_f\ P_{\%}^{\ 2.5} \qquad)$$

where η is the fuel efficiency and $P_{\%}$ the fuel cell per unit load power and the parameter values for) and) are as given in Appendix I

C. Fuel Cell Operation

For road vehicle applications the fuel cell FC) system must satisfy or contribute to the load demand of the vehicle traction drive train. The decision as to whether the FC system supplies all of the vehicle load or simply the average energy requirements can significantly influence the sizing of the FC system and hence the FC system cost. A vehicle supplied solely via fuel cells would necessitate operation at low current densities and hence over sizing of the FC) to minimise the voltage swing on the dc supply to the vehicle traction system. Hence the taxi vehicle considered were the variation in peak average power demand is 7: and 4: respectively for urban and sub urban driving the fuel cell system would have to be rated much higher than the peak power specified. Additionally since fuel cells cannot accept vehicle kinetic energy during braking some form of transient power buffer can significantly reduce the installed fuel cell power capacity [4]. Note the time transient response of the FC fuelling also fosters FC operation in a hybrid energy source configuration

There are therefore clear benefits in terms of FC size and the recovery of vehicle regenerative braking energy for operation of FC systems in hybrid energy source configurations where the FC supplies the vehicle average energy or provides a range extension battery support function in combination with a peak power buffer such as supercapacitor or as for the taxi a higher power dense battery

IV   VEHICLE TRACTION BATTERY

A. Background

The ZEBRA technology is a serious candidate to power future electric vehicles since it not only has an energy density ~ 5x that of lead acid batteries 5 more than NiMH) but also has a relatively flat Peukert characteristic from to 8 depth of discharge DOD) has good power density for acceleration and acceptance of regenerative energy no maintenance essentially intolerant to external temperature ambient safety and fault tolerance and perhaps of greatest significance for the automotive sector the potential for low cost in volume manufacture. Originally developed by Beta R D in the UK the ZEBRA battery technology is now owned and manufactured by MES DEA Switzerland Commercialisation of the battery has made considerable progress in recent years particularly in the automation of assembly and component optimisation. Production is

batteries per year with planned staged increases in capacity to batteries and with space available on site for a further expansion to a maximum of batteries [8]. The ZEBRA battery system has been used in many applications including electric vehicles. So far the batteries had been installed in cars buses and vans from Mercedes BMW Opel VW Renault Fiat MAN Evobus IVECO Larag and Autodromo [9]

For the taxi traction system a nominal dc link of 55 V was chosen to minimise the electrical power distribution mass and fully utilise the traction inverter silicon volt ampere rating. The dc link voltage is realised via x Z5C traction batteries Fig electrically connected in series details of which are given in Appendix I [ ]

Fig   ZEBRA Z5C Traction battery

B. Zebra Battery Simulation Model

Electric vehicle traction duties are typified by high power discharge charge rates for vehicle acceleration and braking demands respectively [4] as illustrated by Fig b) Since the traction battery supplies the vehicle traction drive system the battery voltage regulation with load current is an important aspect of vehicle operation and system performance modelling. Simulation of the ZEBRA battery is facilitated via a detailed analytic model employing non linear open circuit terminal voltage and resistance characteristics derived from cell experimental test data provided by Beta R D. Since the battery dynamic current loadings are of a relatively low frequency < Hz for the NEDC cycle) the battery equivalent circuit model can be simplified to that illustrated in Fig 7 a). The open circuit terminal voltage of the battery depends on battery state of charge SoC) or discharged ampere hour and is found to be independent of load current Fig 7 b). The specified maximum discharge ampere hour Ah) capacity is taken from the manufacturer s nameplate capacity and collaborated via a series of Peukert tests. The battery internal resistance is similarly determined from test and is a function of discharged Ah and charging discharging current rate Fig 7 c)

As with the fuel cell model functions for the ZEBRA battery open circuit voltage and internal resistance are derived from curve fits to measured battery test data. These analytic expressions thus improve on simulation time when solving multiple component drive train models. The curve fit equations and battery model parameters are given in Appendix II
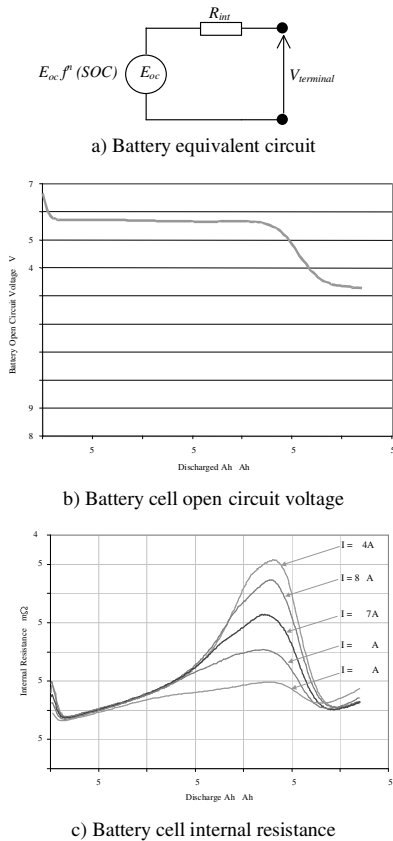
a) Battery equivalent circuit

b) Battery cell open circuit voltage

c) Battery cell internal resistance

Fig 7 ZEBRA Z5C battery cell equivalent circuit model and model parameter characteristics

C. Lead-acid Battery Simulation Model

As a comparison of two vehicle battery technologies at the initial vehicle component assessment and specification stage a similar model is implemented for a sealed lead acid battery as discussed in [4] The lead acid battery details are provided in Appendix I

V VEHICLE PERFORMANCE EVALUATION

As previously discussed the two major operational problems with fuel cell systems in electric vehicle drive trains is their poor regulation with load Fig 4 and inability to accept regenerative currents which if allowed would result in excessive chemical fatigue of the stack membranes and hence reduced life time [5] Consequently if the fuel cell system mass and volume is to be kept within limits commensurate with their utility in the drive train some form of peak power buffering is necessary and hence a hybrid energy source for the vehicle

The energy management strategy considered for the taxi vehicle is one where the fuel cell provides a range extension function supplementing the battery energy whilst allowing all dynamic power demands to be taken by the battery As an assessment of the on board energy

sources the taxi performance was evaluated via two duty loading regimes viz :

- repetitive ECE 5 cycles i e inner city driving and the more energy demanding
- repetitive NEDC cycles

The battery management strategy chosen ensures operation within nameplate voltage and current limits the minimum voltage limit dictating actual energy utilisation and hence operating time and range The fuel cell utilisation was constrained by the quoted on board hydrogen storage   However the fuel cell stack was considered as providing a light mean supply of   kW and a mean supply of 4kW hence operating over the most energy conversion efficient region of the stack Fig 5 A number of case studies were considered; Cases   4 being for pure battery mode and comparing the ZEBRA and more common sealed lead acid technologies; Cases 5 and   assess the performance of the combined ZEBRA battery and fuel cell source Part i) of Table II provides the test conditions and battery details for each case whilst parts ii) and iii) give the simulated data for the two duty cycle regimes

A. Pure battery electric mode

For both duty regimes Case   gives results for the ZEBRA battery whilst Case   compares the lead acid performance for the same battery mass as that of the ZEBRA Case   for the same battery volume and Case 4 for the same nameplate energy   These results clearly highlight the benefits of the ZEBRA battery in terms of mass and volume showing an increased range of   4x based on mass and   4x based on volume   From a vehicle system point of view minimisation of vehicle on board mass is the primary goal since this equates to drive train peak power rating   Whilst component volume is important it is not as critical for the taxi example given the available vehicle space envelope The result of Case 4 has been included for interest but such a mass impact would compromise the vehicle functionality by significantly limiting passenger payload It is worth noting that the lead acid simulation assumed a constant battery coolant ambient of   °C which would in practice necessitate a thermal management system figures for which are not included in the mass and volume audits   Fig 8 illustrates simulated battery terminal voltage variation for the repetitive NEDC duty loading for the Zebra battery a) and the lead acid battery b) corresponding to Cases   and   respectively

B. Fuel cell and battery hybrid source

Cases 5 and   compare vehicle performance with a mean fuel cell input of   and 4kW respectively The results of Table II demonstrate the benefit of the additional energy source in extending the vehicle range in both cases by a factor of   making the vehicle operating times and range attractive for fleet taxi schemes

Table II Vehicle performance evaluation

i) Test conditions

Case	Battery	Battery rated capacity Ah)	Total battery No	Total battery	
				Volume l)	Mass kg)
	Z5C			4	9
	Pb acid			57	9
	Pb acid	5 9		4	58 4
4	Pb acid	7 5		5	9 4 8
5	Z5C FC			4	9
	Z5C FC			4	9

ii) Repetitive ECE 5 cycles

	Battery	Ave FC power kW)	Time duration hrs)	Range km)
	Z5C		5 79	7
	Pb acid		48	5 7
	Pb acid		4	87 9
4	Pb acid		5 75	9 9
5	Z5C FC		4	8
	Z5C FC	4	95	49

iii) Repetitive NEDC cycles

	Battery	Ave FC power kW)	Time duration hrs)	Distance km)
	Z5C		4	9 8
	Pb acid		58	9
	Pb acid		5	89 8
4	Pb acid		4	9 8
5	Z5C FC		4	9
	Z5C FC	4	9	4

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Per cycle values      Duration  s)     Distance  km)
ECE  5                    95                 9
NEDC                       8                 47
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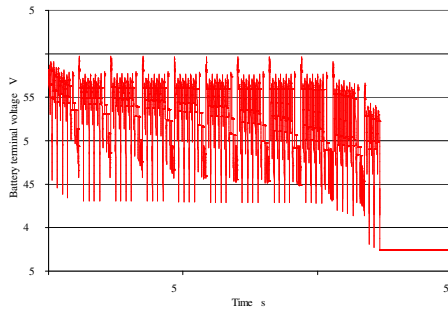

a) ZEBRA battery
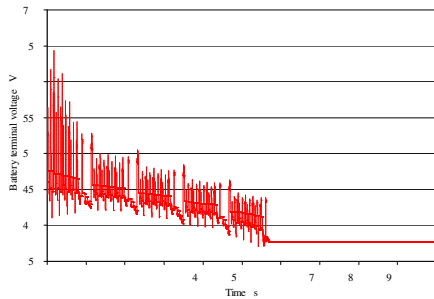
b) Lead acid battery

Fig 8 Simulated battery terminal voltage vs time for repetitive NEDC duty cycles

VI CONCLUSIONS

The paper has reviewed fuel cell systems for transportation and discussed the main application issues in automotive drive trains specifically their poor voltage regulation with load and inability to accept regenerative braking energy   A detailed model of a prototype fuel cell and ZEBRA traction battery suitable for vehicle performance simulation has been presented and vehicle performance assessed for various battery and fuel cell combinations and for typical dynamic urban and sub urban driving duty cycle regimes   It has been shown that the fuel cell and battery combination are complementary for such duty loading extending the vehicle range whilst minimising the installed fuel cell capacity   The results demonstrate the utility of combining energy and power sources for electric vehicle propulsion

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APPENDIX I: MODEL DATA AND COMPONENT SPECIFICATIONS

London Taxi Vehicle Data

Gross vehicle weight	5 kg
Drag force coefficient	
Equivalent frontal area	75m
Coefficient of rolling resistance	
Road wheel radius	74 m
Wheel inertia	4 kgm
Traction machine inertia	57x kgm

MES DEA kW prototype fuel cell system [5]

Performance data	
Unreg dc output voltage range	7 4V
H consumption at full load	9ln min kg h)
Max power output	kW
Open circuit voltage per cell	95V
Number of cells per stack	
Active cell area	cm
Operating conditions	
Stack temperature	Max ^{o}C
Hydrogen pressure	4 7 bar
Air pressure	Ambient
Fuel supply	Pure hydrogen dead end mode
Ambient temperature	to 5°C
Gas humidification	none
Working cycle	continuous
Cooling	Force air cooled
Stack volume	4 x 5 x 5 mm
Control unit volume	95 x 55 x 95 mm
Stack weight	9kg
System weight	kg
Vehicle on board Hydrogen storage	
Gas storage	Compressed H at bar
Storage medium	x carbon composite cylinders
Total storage capability	9 litres

ZEBRA Z5C battery data [9]

Type	Zebra Z5C
Capacity	Ah
Rated energy	7 8kWh
Open circuit voltage	78 V
Max regen voltage	5V
Max charging voltage	8V
Min voltage	8 V
Max discharge current	4A
Weight	95kg
Specific energy	9 Wh kg
Specific power	4W kg
Peak power	kW
Thermal Loss	< W
Cooling	Air
Battery internal temperature	7 to 5 °C
Ambient temperature	4 to 7 °C
Dimensions WxLxH)	5 x 8 x mm
Number of cells per battery	
Cell configuration	parallel strings of 8 series cells

Fuel Cell Model Parameters:

Parameters for equation):		Parameters for equation):	
k_a	95 847	a_a	5587
k_b	858	a_b	9 985 9
k_c	8 4 8 9 8	a_c	4 58 5 5
k_d	455 89	a_d	7 47 79
k_e	7757	a_e	45 45
k_f	84 4 7 87	a_f	8 84 98
k_g	89 84 8 54		
k_h	7 77 8 4		
k_i	7 757557		
k_j	5 99 7		
k_k	44		

Sealed lead acid battery data

Model	HAWKER Genesis EP Series
Cell number	cells per module
Nominal voltage	V V per cell)
Max voltage	5V open circuit 5V per cell)
Max charging voltage	V 7V per cell)
Min voltage	V recommended 7V per cell)
Max current	45 A
Rated energy	8 to 84 kWh
Rated capacity	7 Ah
Rated temperature	°C
Mass	4 kg
Volume	9 79l
Dimension	mm x 8mm x 7 mm

APPENDIX II: ZEBRA EQUIVALENT CIRCUIT MODEL

The following equations model the ZEBRA battery as a function of discharge Ampere hour q) and battery terminal supply current i) The equation for cell open circuit voltage is given by:

$$E_{oc} = \frac{v_a + v_c\, q^{-5} + v_e\, q + v_g\, q^{-5} + v_i\, q + v_k\, q^{-5}}{+ v_b\, q^{-5} + v_d\, q + v_f\, q^{-5} + v_h\, q + v_j\, q^{-5}} \qquad \text{A)}$$

From experimental test data the battery internal resistance is found to be a complex function of discharge ampere hour or SoC) and current charge discharge rate Curve fit functions are used to analytically model the internal resistance However due to highly non linear relationship it is not possible to fit one   dimension equation to the full data set without the model losing accuracy To overcome this problem the model uses a set of eleven equations determined by the current rate and discharge ampere hour For example for discharge currents higher than 4 A a constant resistance of R_{max-2} is used For all other regions i e when discharge current is between   A to 4 A the appropriate resistance equation is chosen according to the defined conditions

The choice of battery cell internal resistance is dictated by the following algorithm:

Loop start
if '$q > ah_{ref}$ then 'simulation stop   else
if '$i \leq$ then '$r_{int} = r_{chg}$ else
if '$i > 4$ then '$r_{int} = r_{max-2}$ else
if '$i \leq 5$ and $q \leq$ then '$r_{int} = r_{a0}$ else
if '$i > 5$ and $q \leq$ then '$r_{int} = r_{a1}$ else
if $< q < 8$ then '$r_{int} = r_{d0}$ else
if '$q \leq 5$ and $< i \leq 8$ then '$r_{int} = r_{b0}$ else
if '$q > 5$ and $< i \leq 5$ then '$r_{int} = r_{b1}$ else
if '$q > 5$ and $5 < i \leq$ 5   then '$r_{int} = r_{b3}$ else
if '$q > 5$ and $5 < i \leq 8$ then '$r_{int} = r_{b2}$ else
if '$q \leq 5$ and $i > 8$ then '$r_{int} = r_{c0}$ else
if '$q > 5$ and $i > 8$ then '$r_{int} = r_{c1}$
if '$r_{int} > r_{max}$ then '$r_{int} = r_{max}$
Loop end

Hence the equations used to calculate the cell internal resistance R_{int} in mΩ are:

$$R_A = \frac{a_a + a_b q + a_c q + a_d i}{+ a_e q + a_f q + a_g i} \qquad \text{A)}$$

$$R_B = \frac{b_a + b_b q + b_c q + b_d i}{+ b_e q + b_f q + b_g i} \qquad \text{A)}$$

$$R_B = \frac{b_a + b_b q + b_c q + b_d i}{+ b_e q + b_f q + b_g i} \qquad \text{A4)}$$

$$R_B = \frac{b_a + b_b q + b_c q + b_d q + b_e \ln i}{+ b_f q + b_g \ln i} \qquad \text{A5)}$$

$$R_B = \frac{b_a + b_b q + b_c q + b_d q + b_e i}{+ b_f q + b_g q + b_h i} \qquad \text{A)}$$

$$R_C = \frac{c_a + c_b q + c_c q + c_d q + c_e \ln i}{+ c_f q + c_g q + c_h q + c_i \ln i} \qquad \text{A7)}$$

$$R_A = a_a + \frac{4 a_b e^{\left(-\frac{q-a_c}{a_d}\right)}}{\left[+ e^{\left(-\frac{q-a_c}{a_d}\right)}\right]} + \frac{4 a_e e^{\left(-\frac{i-a_f}{a_g}\right)}}{\left[+ e^{\left(-\frac{q-a_f}{a_g}\right)}\right]} + \frac{a_h e^{\left(-\frac{q-a_c}{a_d}-\frac{i-a_f}{a_g}\right)}}{\left[+ e^{\left(-\frac{q-a_c}{a_d}\right)}\right]\left[+ e^{\left(-\frac{q-a_f}{a_g}\right)}\right]} \qquad \text{A8)}$$

$$R_C = c_a + \frac{c_b}{q} + \frac{c_c}{i} + \frac{c_d}{q} + \frac{c_e}{i} + \frac{c_f}{qi} + \frac{c_g}{q} + \frac{c_h}{i} + \frac{c_i}{qi} + \frac{c_j}{q\,i} \qquad \text{A9)}$$

$$R_D = d_a + d_b \ln q + d_c \ln i + d_d [\ln q] + d_e [\ln i] + d_f [\ln q][\ln i] \qquad \text{A)}$$
$$+ d_g [\ln q] + d_h [\ln i] + d_i [\ln q][\ln i] + d_j [\ln q]\,[\ln i]$$

Zebra battery model parameter values

v_a	855	d_{0h}	5 4 7	b_{0a}	8 5 8 9 54	b_{2e}	5 8	c_{0h}	848 8 4 85
v_b	54 495	d_{0i}	8 7847	b_{0b}	4 479 9	b_{2f}	79	c_{0i}	9 4849
v_c	5 7 8 5 987	d_{0j}	4 8784 8	b_{0c}	48 9	b_{2g}	45 954 8	c_{0j}	4 5 4
v_d	478 4445	a_{0a}	8 97 9	b_{0d}	545 8 4	b_{3a}	7 44557	c_{1a}	998 94
v_e	54 58 9	a_{0b}	54 9	b_{0e}	8 78 8	b_{3b}	755	c_{1b}	8 4 9
v_f	949 54	a_{0c}	77 95 54	b_{0f}	4 79	b_{3c}	88 9	c_{1c}	9 95 7
v_g	5 47	a_{0d}	4 85 4	b_{0g}	5 49	b_{3d}	9	c_{1d}	4
v_h	44557	a_{0e}	5 7	b_{1a}	8 4 55 5	b_{3e}	5 757	c_{1e}	5 84 4
v_i	8 49	a_{0f}	7 9 945 5	b_{1b}	784 7	b_{3f}	4 4	c_{1f}	98 7889
v_j	7 5 8	a_{0g}	9	b_{1c}	595 9	b_{3g}	4 7	c_{1g}	4
v_k	7	a_{0h}	97447 5	b_{1d}	5 94978	b_{3h}	4	c_{1h}	5 9 7E 5
d_{0a}	888 7 7	a_{1a}	84 89	b_{1e}	779 877	c_{0a}	598	c_{1i}	54 5
d_{0b}	849 7774	a_{1b}	8 4859 87	b_{1f}	55 88	c_{0b}	5 879 9	Ah_{ref}	to 4
d_{0c}	5 7577 54	a_{1c}	854 45	b_{1g}	7 89 5	c_{0c}	94 5547 7	R_{max}	8
d_{0d}	8 9 579	a_{1d}	57 4	b_{2a}	48 9 7849	c_{0d}	95	R_{max-2}	75
d_{0e}	7 4 775 98	a_{1e}	449 7	b_{2b}	5 5 5 9	c_{0e}	958 45	R_{chg}	5
d_{0f}	97 4	a_{1f}	7 547 8	b_{2c}	898 889	c_{0f}	7 7 798		
d_{0g}	4 57	a_{1g}	95949	b_{2d}	9 574	c_{0g}	9 98 8		